Statics and Dynamics
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Lecture – 32

So, let us we are looking the example the in the last lecture related to where a billiards ball is impacted by a force that sets the ball on motion. So, now let us look at example problem where a ball has be certain to motion, there are no forces acting on the ball except natural forces due to friction between a table and the billiard ball. Now we are asking to analyze the motion of this subsequent to the disappearance of the questic originated force. So, let us look at this example problem, and see what we can make of this.

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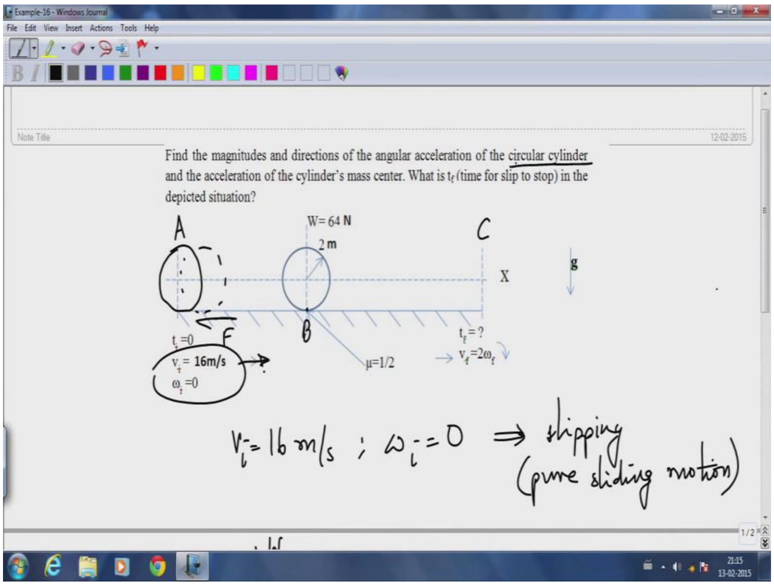

Again we have this case is circular cylinder, that is starting at this point A, that was this point A at some time instance 0. The linear velocity of this hoop of the circular cylinder was 16 meters per second, and the angular velocity is 0, and we are ask to find the time instant at which slipping would stop. So, we have to find the time instant at which the hoop would could slipping. Now, initially the hoop was here, and initially the hoop is only translating there is no rotational motion, but over some period of time, there is friction force that is acting at this point of contact, I call this B, and the friction force at this point of contact B is going to initial angular motion of this circular cylinder, where by some other distance C by the time is cylinder reaches another distance C, the system would have reached no slip rolling condition.

So, let us try to identify this condition. So, we start again just by looking at the initial condition that V i is 16 meters per second and omega is 0 implies it is slipping. So, it is in a pure sliding motion.

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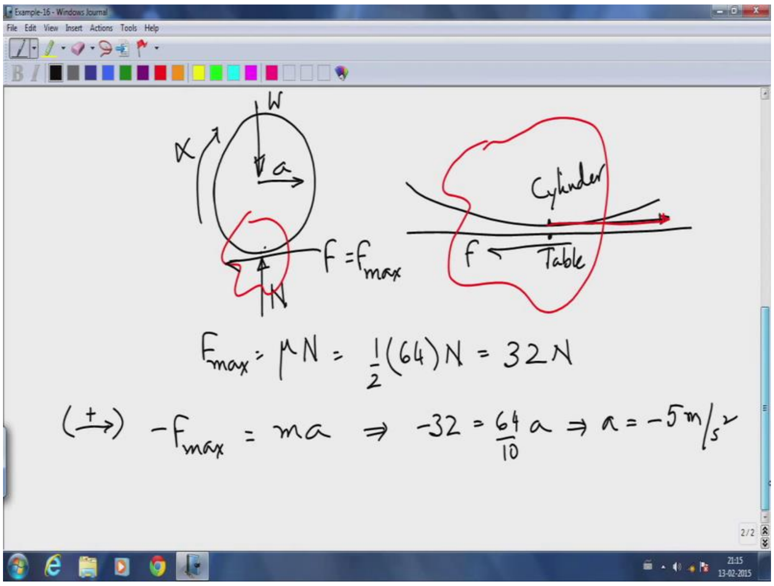

So, now let us draw a free body diagram of this circular cylinder, this the wait; that is counted by a normal reaction from the table. The only other force acting is a friction force F, and because we know because we know the velocity is to be right, at this point of contact if now I mark two points; I will try to zooming here, and draw magnified picture between the table and the cylinder; the cylinder surface is moving to the right in a table is at rest. Therefore, friction like we said earlier always acts to appears a little motion, therefore it acts from right to left. Now just that we know that this would have to equal F max, because there is relative motion; friction acts in it is fullest strength to cos elective motion to sees... And this particular problem F max equals nu times N which ((Refer Time: 04:09)) is half, and N would simply equal W. If I take on which a 64 Newton's . So, the maximum possible fiction force is 32 Newton's, and the force that acts at this point of contact will be equal to 32 Newton's, because the hoop as a whole is slipping at that time instants 0.

Now as this friction force acts, the cylinder begins to develop an angular velocity. So, let us now do will apply Euler's laws of motion, and get the angular as well as linear acceleration, I am going to replace. So, if alpha is the angular acceleration, and A is the linear acceleration at this hoop, what I do now is that minus F max equals mass times acceleration, which implies minus 32 equals 64 divided by ten times the acceleration, which implies the acceleration is minus 5 meters per second square. I choose the acceleration to point to my write, but the value came out to be minus 5; that means, the cylinder actually decelerating, remember the velocity is plus 16 to start with. So, the velocity is plus 16 to the right, and the acceleration is to the left, which means it is going to be decelerating.

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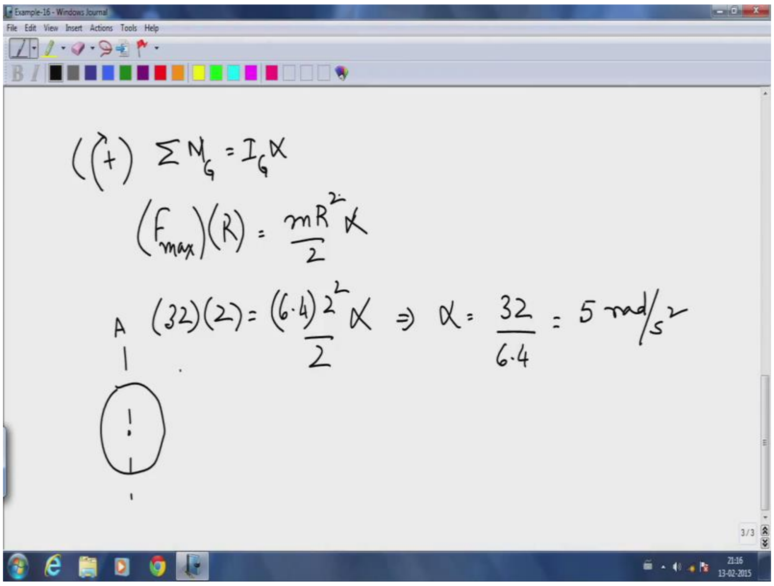

So, let us now also do the moment balance again, I choose to do this about the cylinder center of mass, the only force that causes a moment is the same F max nothing else. So, F max times the radius equals m R square over two times alpha. F max is 32 times the radius where ((Refer Time: 06:58)) is two meters, that means alpha is 5 radians per second square. So, now let us understand what is happening here at this point A, let us understand the kinematic of this cylinder.

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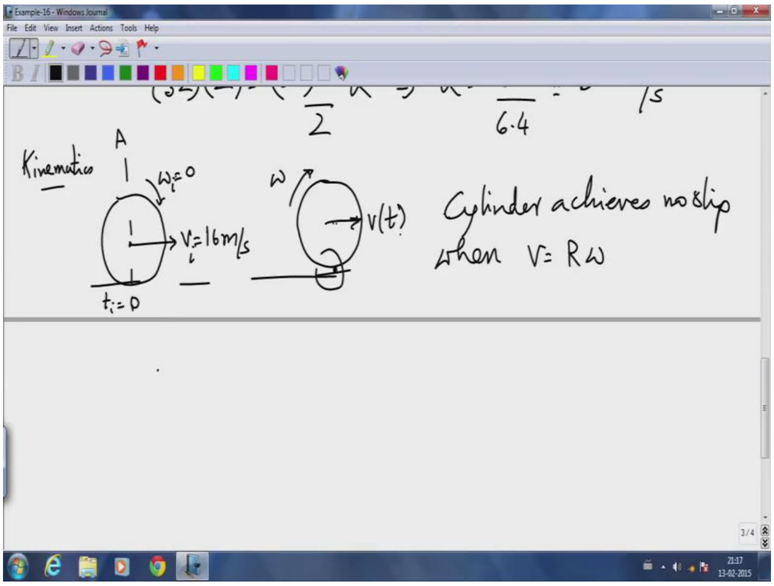

At this point A the velocity equal 16 meters per second, and the angular velocity omega is 0; these are the initial conditions at t i equals 0. At some other instant t, let say omega is the angular velocity and V is the final velocity. How do I know the cylinder is rolling without slip. So, these are the same surface, cylinder achieve no slip when V equals R omega. So, until at this point of contact you know we did that until the point of contact see is no ((Refer Time: 08:50)) motion, the friction force will continue to act.

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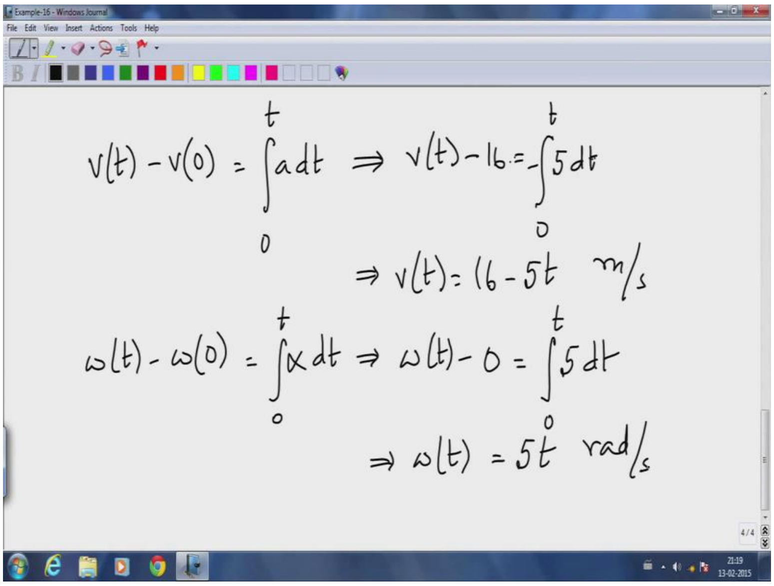

So, if I write what is V of t? v of t minus V at the instant 0 equals 0 to t a d t, we calculated the linear acceleration to be minus 5 meters per second square which implies V at any instant t minus 16 0 to t 5 with the negative sign d t, which also implies that V of t is 16 minus 5 t, in this has units of meter per second. Similarly if I be the angular velocity calculation omega of t minus omega 0 equals 0 to t alpha d t, which implies omega of t minus 0 equals plus 5, because the angular acceleration is plus 5 radians per second square d t which also implies omega of t equals 5 t radian per second.

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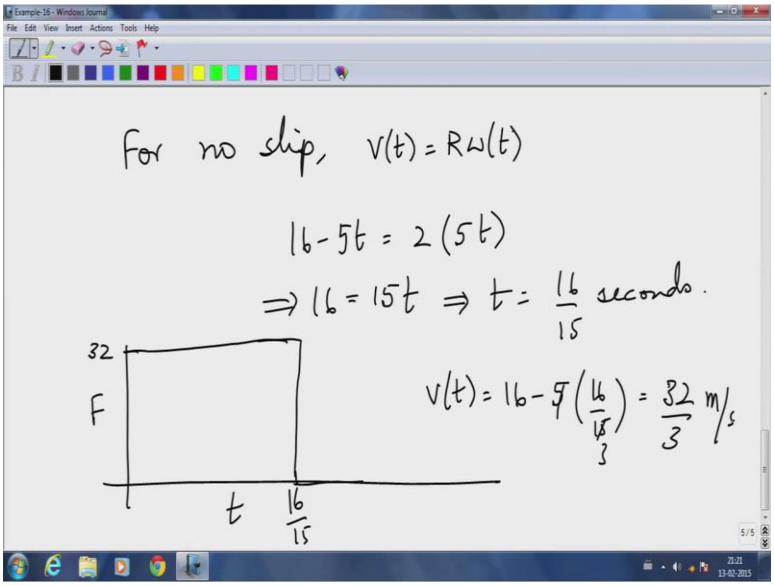

Now for no slip, we said at the instant when v of t, I call this V s of t equals R times omega are when V of t becomes all time omega of t, slip would I have completely cheesed. So, let us take 16 minus 5 t equals two times 5 t, which implies 16 equals 15 t which also implies t equals 16 over 15 seconds. So, until this time instant, the hoop would be slipping at the point of contact. So, if what would be interesting is to draw the friction force as a function of time. So, F max is 32 Newton's. So, F max will be constant till this time instant 16 over 15 seconds, and at that point in time a friction force would instantaneously become 0. This is our model of cool and friction. What is interesting is that as you approach the point, where slip is beginning go to 0, a friction is persist long as there is some sliding motion between the surfaces.

The instantaneous sliding cheeses simultaneously with the friction going to 0, and as soon as the friction disappears, this body now has a velocity, that is given by for v of t at this time instant is given by 16 minus 5 into 16 over 15, that is 32 over three meters per second and the angular velocity would be 16 over the three meters per second. So, the angular velocity and the linear velocity from this point forward or fully conserve that is they will not changing magnitude, because no other force would continue to act on the system. The only force that was acting to bring slip at the point of contact to 0 was friction, and the moment is it is job is done, the moment friction force becomes the moment relative motions cheeses, friction force also become 0, and naturally the body continues to remain in a state of uniform motion as Newton’s first law would tell us. That brings us to the end of this week lectures, we will continue our discussion of work energy relationship, and some problem and vibrations in the next week.