Statics and Dynamics
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Lecture – 31
Impulse/Momentum – Example 3

So, we will continue or discussion of impulse momentum formulations and we will take on another interesting problem. Let us say anybody that has played billiards, essentially starts with a q stick in hand and hitting the ball at some point, that is, it could be below if I want to cause a spin in this direction or above if I want to cause an additional spin in the forward direction and is the force acts on this ball for a very short period of time. But, by nature of the impulse, the actual magnitude of that force is very high. And, we are asked to find out the net change in the momentum of this hoop as a result of that. So, let us look at this example problem.

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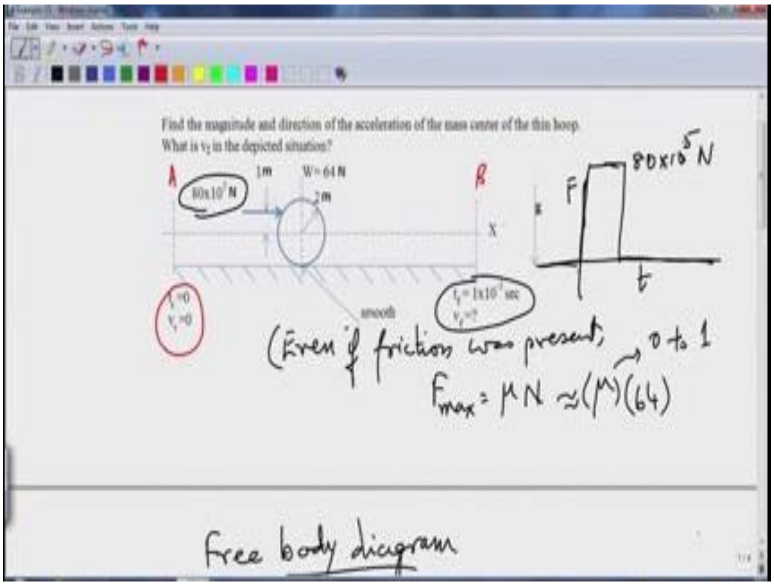

The hoop itself is sitting on a smooth plane. Now, the smooth part is not very important. In other words, even if friction was active; even if friction was present, we have to think of this way – F max would be some mu times N. This should be in the order of some mu times 64 Newtons. So, even if mu is a number between 0 to 1; F max should be on the order of 64 Newtons. But, look at this the force acting from the q stick is 10 power 5 Newtons. And, this is specifically introduced to illustrate the idea that, even though the surface may have some friction between the ball and the table, you can treat it as being smooth for the duration of the impact. This is primarily for the time over which the impact occurs; which in this case is 10 power minus 5 seconds. For such a short span of time and such a large magnitude force acting on this ball, one could treat the surface as being smooth. So, this is another example of where we model the system. We have to develop the appropriate model for the system, which would tell us whether we do need to include friction, whether it is significant or not. So, in order to analyze this, what we are doing is – for all time t less than 0, the force was not present. At some time t equal to 0, the force goes up to this higher magnitude; remains there for a very short span of time and becomes 0 again. So, this is 80 into 10 power 5 Newtons. That is the magnitude of the force acting on. We are asked to find the velocity v f under this condition.

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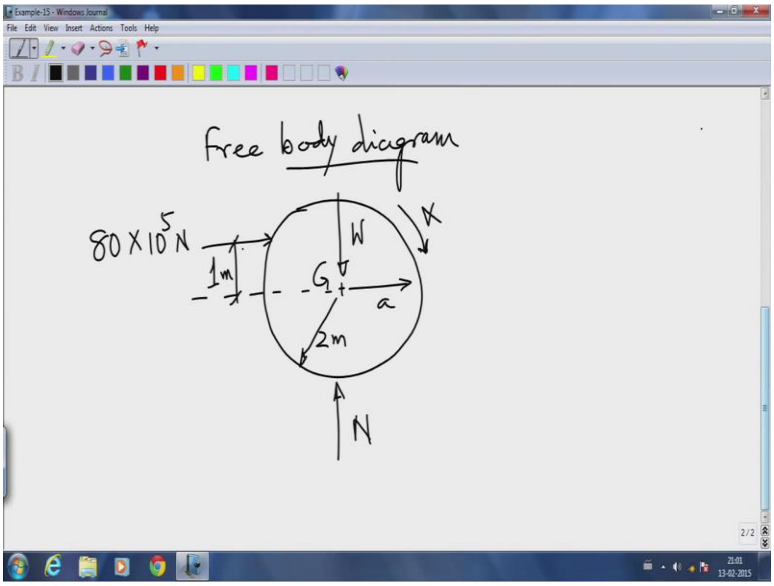

So, let us first draw a free body diagram. There is a normal reaction N acting at the table, there is a weight W. And, there is this force 80 into 10 power 5 Newton acting at a distance 1 meter away. The radius of the hoop is 2 meters. Now, remember – because the surface is smooth, there is no friction force; which also means that, the system can neither be under no slip. Remember we said – the only way to determine whether a hoop or an object slips is to first do the calculations under no slip condition and then check if that friction force is – if the maximum section force is greater than the admissible or required friction force. In this case, the maximum friction force is 0. Therefore, the kinematic motion in the angular sense, which is due to the angular acceleration alpha, and in the linear sense, which is due to the angular linear acceleration a or decoupled. So, let us find out what the independent motions – independent degrees of motion tell us.

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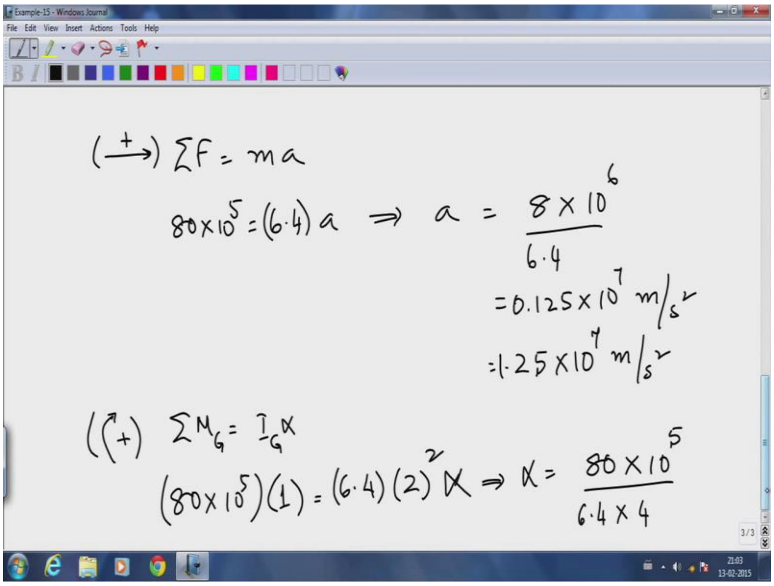

If I take all forces to my right as positive, sum of all forces is mass times acceleration. 80 into 10 power 5; the mass of the hoop is 6.4 kilograms times acceleration; which implies at the acceleration is 8 into 10 power 6 divided by 6.4. So, this is the instantaneous acceleration that is resulting from a very large force acting on this little hoop. So, let us also complete the angular calculation if the sum of all moments about the center of mass equals the moment of inertia about the center of mass times alpha. The only force causing a moment about G is higher force, that is, 80 into 10 power 5 Newton. And, that has a moment arm of 1 meter. The moment of inertia of a hoop is m r square alpha; which implies alpha equals 80 into 10 power 5 divided by 6.4 into 4.

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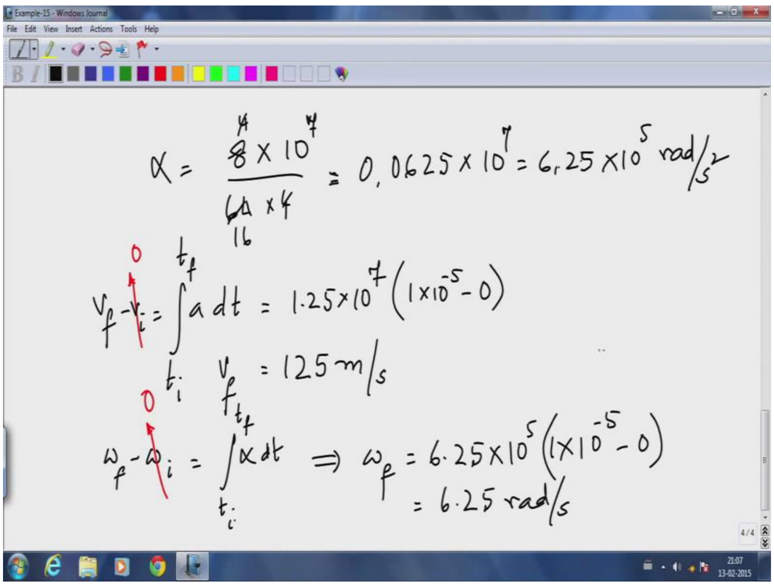

So, the angular acceleration is 6.25 into 10 power 5 radian per second square. These are instantaneous quantities, because the force itself is acting for a very short period of time. So, let us now find the final velocity v f at which is t i to t f a d t. This of course, assumes that, v i at the time instant t i is 0, which we had told is the case here. So, this hoop starts at this point A goes till this point B in a time span of 10 power minus 5 seconds. So, in this time span, the force 80 into 10 power 5 Newton is acted on it and it then went to 0. That is which means we need to find the condition – the kinematic condition of the hoop at the instant the force disappeared.

The acceleration a is 1.25 into 10 power 7 meters per seconds per t f minus t i is 10 power minus 5 minus 0. So, the velocity – the final velocity is 125 meters per second. The angular velocity at the final instant minus the angular velocity at the initial instant can also be related to the instantaneous or the angular acceleration over that same period of time. What we find is again omega i is 0. This implies omega f equals 6.25 into 10 power 5 into 1 into 10 power minus 5 minus 0; which is 6.25 radians per second. So, this force acting on the body for a very short period of time causes a radial velocity of 6.25 radians per second. But, a linear velocity of 125 meters per second; and, the net result is at the hoop is slipping at the point of contact. So, I hope this illustrated the idea of a using impulse momentum for both angular as well as linear velocity calculations. We will continue this discussion in next example.