Statics and Dynamics
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Lecture – 30

Let us take on the next example problem; this is also pertaining to impulse momentum, but in an angular sense to start with.

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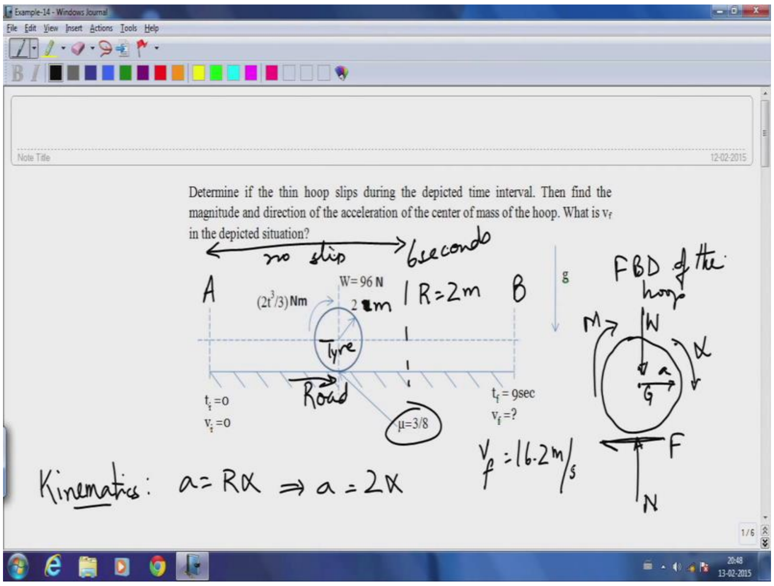

So, we want to see, if we know we have a thin hoop and if it slips during the depicter interval. So, the thin hoop starts at this end A and an initial moment, it is case has the time t 2, t cube over 3 is the initial moment acting on the hoop and this moment causes the hoop to roll from left to right. We are asked that we have told that it reaches a point B some distance away at a time t F equals 9 seconds. We ask to find the final velocity and we are asked to determine, if the hoop would have slipped in this entering time.

The only way to find, if a hoop slips is to calculate the friction force necessary to keep the hoop from slipping, and then compare that necessary friction force to the maximum possible friction force that is available. So, let us do that calculation. So, first of all I draw a free body diagram of the hoop, there is a normal reaction, there is a moment M that acts on it, they told there is a friction force F, this is center of mass of the hoop. There is a weight W that acts through the center of mass, this completes our free body diagram.

So, now, for the kinetics of this problem, kinematics, if the angular acceleration is alpha and if the linear acceleration is a, then a equals R alpha, this is what the kinematic relationship would tell us. So, this implies a equals 2 alpha, R equals 2 meters in this particular problem.

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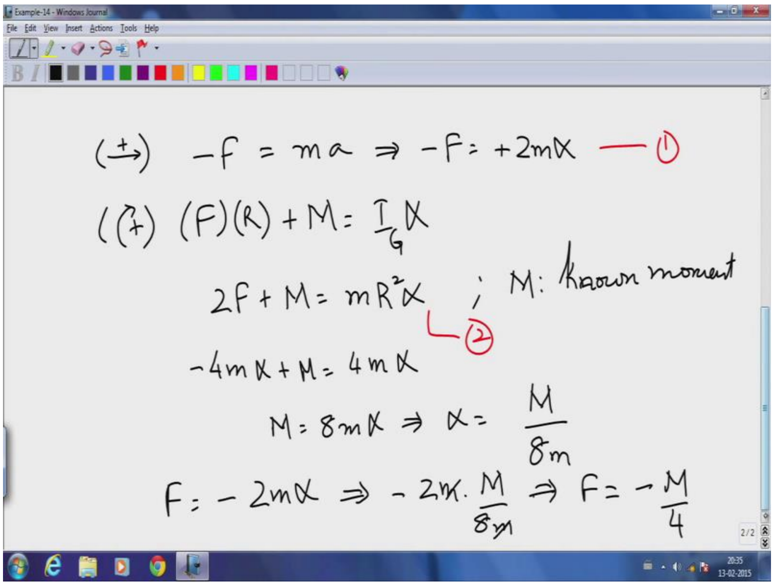

So, let us take a free body diagram and apply Euler's first law, if I take all forces to the right being positive; that says minus F equals m times a, let us implies minus F equals my is equals to 2 m alpha, I call this our first equation. Now, taking clockwise moments positive, the only set of forces that cause a moment the only force in fact, that causes a moment is F and that causes a moment F times R plus M equals I G times alpha.

Like always, this is written in the frame of reference with the center of mass being the reference point. So, this says 2 F plus M equals m R squared alpha, M is a known moment or a couple; that is acting on this body. So, I will substitute equation 1 into equation 2, let says F equals minus 2 m alpha. So, this is minus 4 m alpha plus M equals 4 m alpha R being 2.

So, M equals 8 m alpha, which implies alpha equals M over 8 m, M is the mass of the hoop itself. So, now, F is minus 2 m alpha minus 2 m times alpha is capital M over 8 m, F is minus M over 4.

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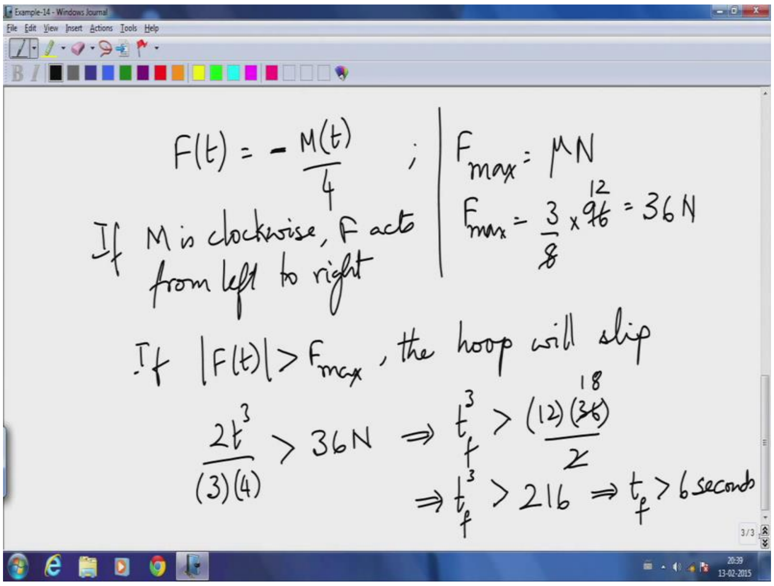

So, I want to draw our first observation from here, for any M in fact, I will generalize this and say that F of t can be a time varying friction force. Because, M of t is a time varying moment acting on the hoop and this F of t is simply proportional to M of t. Now, what are you do know is that F max equals mu times N, it cannot F of t cannot exceed mu times N. So, which also implies F max equals being this case we have told it is 3 over 8. And every N would simply the equal to W; N is 96 Newton's, so this is 3 over 8 into 96.

So, F max is 36 Newton. So, the maximum possible friction force is 36 Newton's. So, let see what the implications of these are. So, the first thing we see is that if M is clockwise, F acts from left to right, not right to left as we have assumed, go back to our free body diagram ((Refer Time: 06:56)), I assume F is acting to my left. But in reality, we found that if M is clockwise, F is actually negative, which means it is acting from right to left from left to right, which means that this is similar to a front wheel in the car.

If the front wheel, if the engine produces a torque of magnitude M, then the friction between the road and the tyre, so if this was a road and this is a tyre, the friction force would actually provide attraction to move the car in a forward sense. But, as far as F max is concerned, this sign negative sign does not matter. What I do know is that if the magnitude exceeds F max, this F of t is the magnitude of the friction force required for no slip.

If this magnitude exceeded F max, then we know the hoop will slip. ((Refer Time:

08:20)) The magnitude acting is 2 t cubed over 3, so let us plug that in there. So, 2 t cubed over 3 times 4 is greater than 36 Newton's. Whenever, this condition is achieved, the hoop is slightly to slip. So, if I take care of this calculation.

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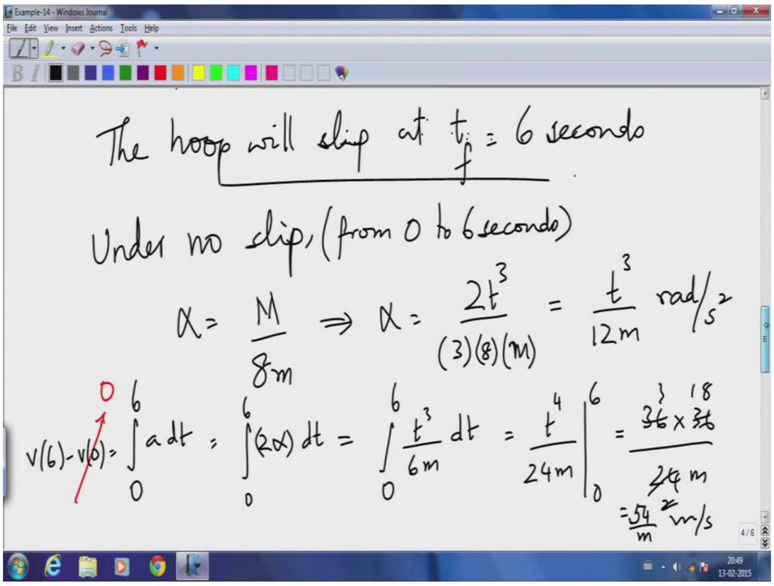

So, whenever t of cubed is greater than, which implies if t f is greater than 6 seconds the hoop will slip, so let us go back to this problem ((Refer Time: 09:41)). So, we found that whenever that at this instance 6 seconds, the magnitude of the friction force required for no slip will exceed the maximum possible friction force which is mu times N.

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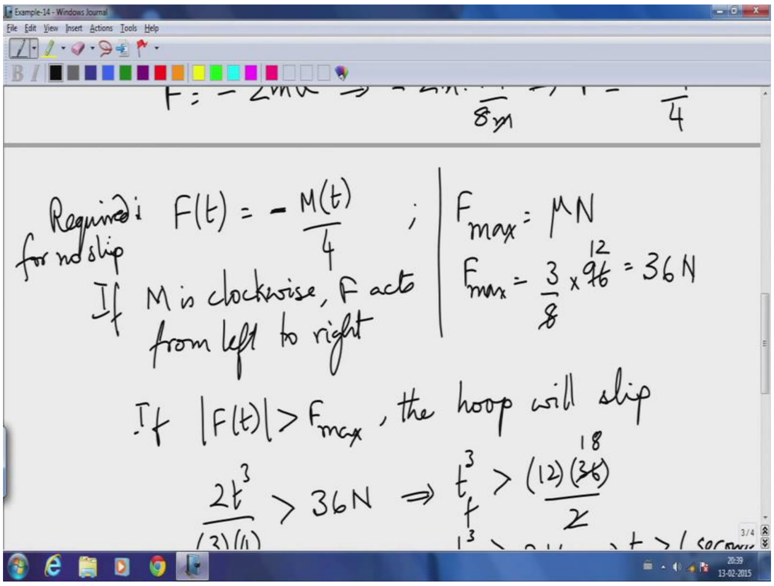

So, let us see, what that looks like ((Refer Time: 10:05)). So, we have told that the total span of time over which the motion is being analyzed is 9 seconds. So, up until 6 seconds in this span of time, there is no slip. Beyond that, for the 3 seconds from 6 seconds to 9 seconds, the hoop will slip. Preserve, we have two motion configurations happening in this span of 9 seconds and we will lead to analyze the two separately.

So, let us first do the calculation for the no slip part. So, under no slip, what we do know is that ((Refer Time: 11:08)) F is that alpha equals M over 8 m and for this hoop ((Refer Time: 11:34)) is 9.6 kg t cube over 12 m radian per seconds square. So, if I integrate this from 0 to 6 seconds and knowing that the linear acceleration is 2 alpha, so this will be at 6 seconds would be equal to this. More specifically v at 6 seconds minus v at 0 seconds equal to this, but v of 0 seconds is known to be 0. This is 0 to 6, twice alpha is t cubed over 6 m d t, so this is 36, 36 by 24 m.

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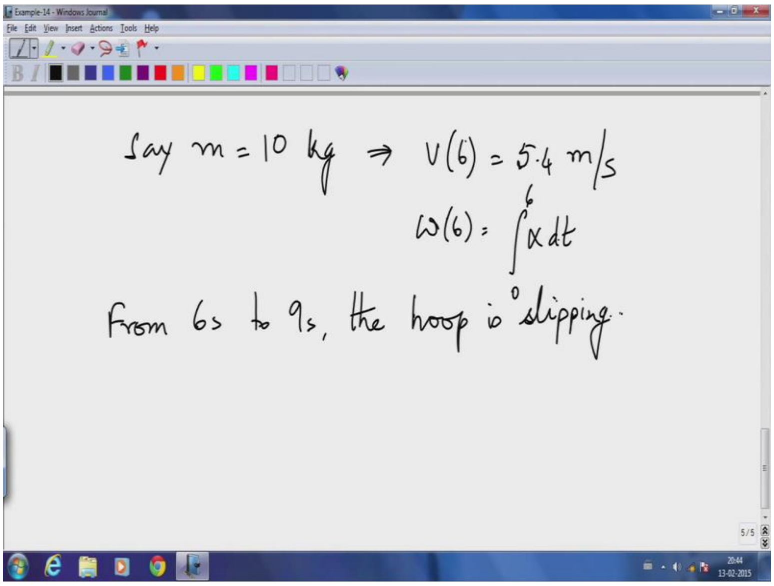

So, I am just for the sake of simplicity, we want to assume m is 9.6 kilograms or to make it simple I am going to assume m is 10 kilogram. Because, it really not going to make much of a difference, which implies velocity at 6 seconds is 5.4 meters per second. This is the linear velocity. The angular velocity is equal to alpha at 6 seconds; alpha is essentially, I have to do the same integration from 0 to 6 seconds, alpha d t.

If I need to find the angular velocity that is a way I would divide. Now, from 6 seconds to 9 seconds, the hoop is slipping, we do know that. So, the previous free body diagram we drew would no longer be valid.

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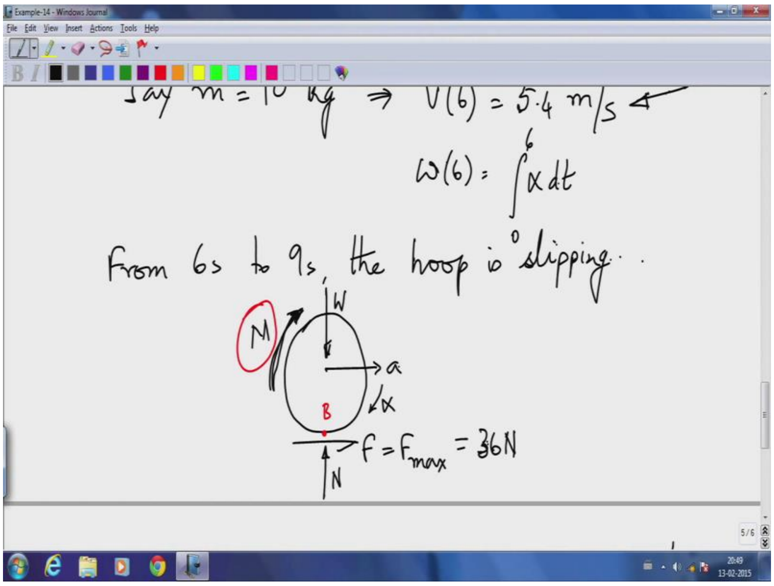

For this case, what we need to do is that, the same moment M is acting. The rest of the free body diagram is an alter, except now the friction force acting to move this forward is equal to F max. So, in other words, this counter acting moment M has exceeded the value; that is below which it needs to stay for the friction force to cos know slip at the point of contact B.

So, for all other points, the traction force between the tyre and the road, takes on the maximum value of the friction possible, which we calculated to be 36 Newton's. So, now if I take the free body diagram, there is no kinematic relationship between a and alpha. So, the linear acceleration and the angular acceleration for this instance at no longer couple, the angular acceleration is whatever is cos by the moment turning the v or turning the shaft and the linear acceleration is whatever the traction force F can sustain.

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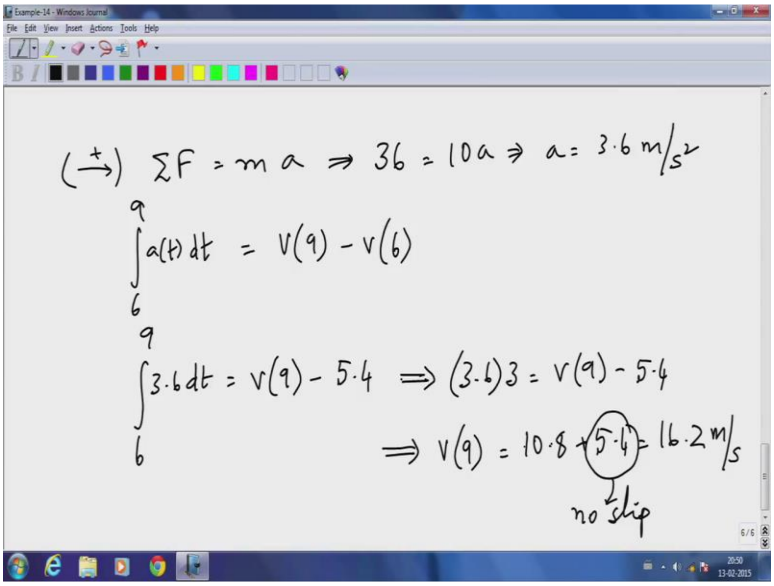

So, let us apply our loss of motion some of forces equals mass time acceleration, I am going to assume m a time kg. So, 36 Newton is a force acting. So, a is 3.6 meters per seconds squared, this is the linear acceleration for the duration that the hoop is slipping at the point B. Now, the we do know that, if I integrate this acceleration, although this is not time varying and going to generalize this equation and say, if I perform an integration have this acceleration from time duration 6 seconds to 9 seconds; that gives me the total change in the velocity between the final instant 9 seconds and the instant 6 seconds.

We already know the velocity at 6 seconds. So, if I this v of 9 minus 5.4, it is also implies that the velocity at 9 seconds is 10.8 plus 5.4, which is 16.2 meters per second. So, ((Refer Time: 18:38)) the final velocity at 9 seconds is 16.2 meters per second. So, let us go back and quickly recap here. We encountered two separate situations for the duration ((Refer Time: 18:58)) from 0 to 6 seconds; the hoop was rolling under no slip.

So, this like you are at a traffic light, you are a engine exist a torque on the shaft and that torque itself, let us say time varying with M of t being in 2 t q over 3. Initially, since the torque itself, it is quite small, there is no slip, but as the torque increases as the moment increases. At a time instant of 6 seconds the attraction force between the tyre or the hoop on the row reach the point, where it could move long the sustain motion and the no slip and that point essentially the hoop started to slip.

So, the first instants is analyzing motion under no slip, and then from 6 seconds 9 seconds ((Refer Time: 19:51)), the hoop is now slipping, which means the attraction force is the maximum possible, which is 36 Newton’s. And under that condition, I get an acceleration of 3.6 meters per second squared. If end result is motion under no slip, so these 5.4 meters per second of velocity was acquired under no slip conditions and an additional 10.8 meters per second was a square under slipping conditions and the actual velocity of the hoop at 9 seconds is 16.2 meters per second.

So, this tells us, how to analyze motion, when you have two separate situations happening, a part of time, when slip was not happening, and then a sub sequence part of time, when slip was happening. And in order to determine, whether slip happens, the only way to get there is to first solve the no slip problem. Understand the magnitude of the friction force, needed to sustain motion under no slip and check, if you have that much friction.

Once you have checked, whether you have friction, you can then moving to the face, where you analyze the problem into two half’s, one where the hoop was rolling under no slip and the second, where the hoop was rolling under slip conditions. So, we will now continue this discussion with are next example problem.