Statics and Dynamics
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Lecture – 29

We are now used to discussing the application of Euler's laws of motion to kinetics problems. We start with our Newton's laws, which says F equals m a. In many instances, F is not a constant force, F itself is a function of time. And in many instances, I am not interested in the instantaneous acceleration; I am interested in the effect of a particular time variant force acting over a certain period of time. I want to understand what change this force the action of this force would have on the body of interest. So, that gives rise to a different approach, still based on Newton's laws of motion, but a different approach called impulse momentum.

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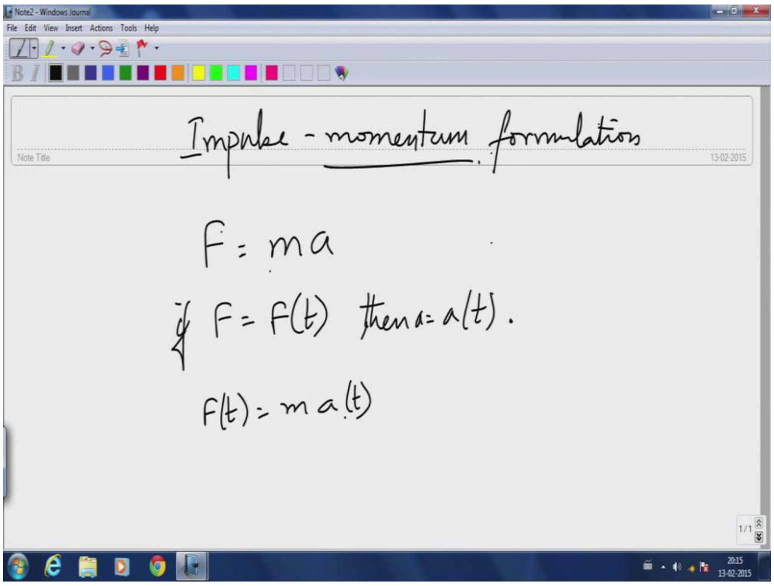

So, let see, we can first understand the whole idea of impulse momentum formulation. So, we will start with a whole Newton's law, I am going to write it as a simple scalar formulation, because just to illustrate the idea. Now, if F over the function of time, whatever be the function of time, then all are would say to start with is that, a of t is also a function of time, that acceleration is also a function of time.

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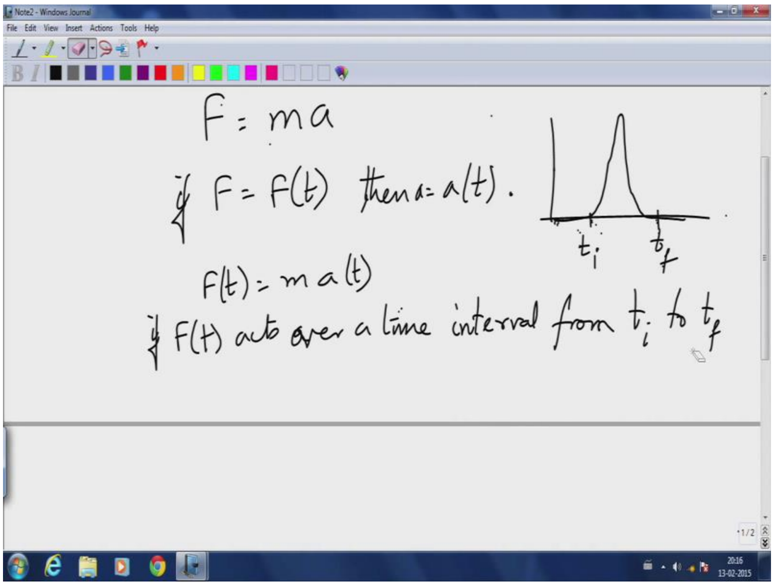

So, F of t equals m times a of t, but like we said I am not interested in the instantaneous accelerations. If F acts over a time integral from an initial time t i to t f a final time, then essentially let say I have a force that kind of was 0 all the way to some initial time t i and became non-zero at some time t f, I want to understand the action of this force.

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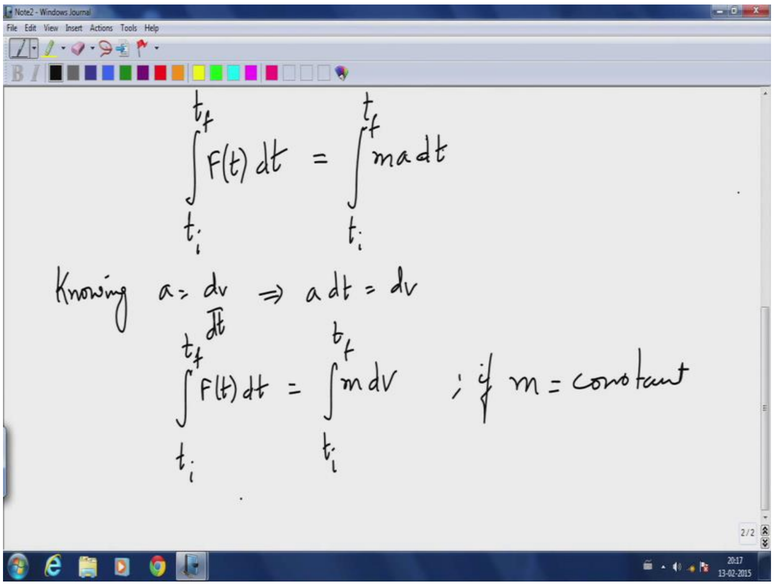

Then, the simplest way would be to write this in an integral formulation, now knowing that a equals d v d t, this is a basic definition of acceleration implies a d t can be replaced with that change in velocity.

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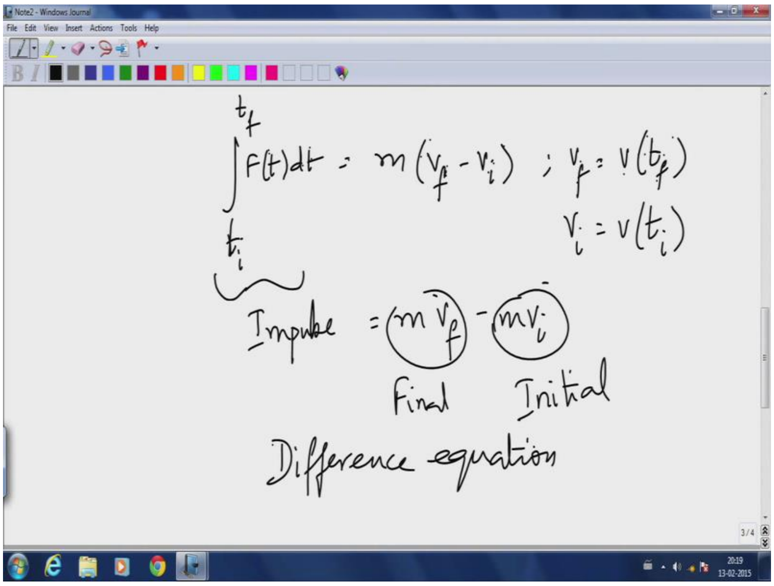

So, this means integral t i to t f, F of t d t equals integral t i to t f, m d v. If m is constant, this means integral t i to t f, F of t d t equals m times v of f minus v i, v of f is simply the velocity at some time t f and v i is velocity at some time t i. This right hand side here or left hand side here has a name; this is what is called impulse. The integral of a force which is time varying over up certain span of time is called impulse. So, impulse causes a change in momentum.

So, the final momentum is m times v f, the initial momentum is m times v i. So, in many applications, I will just be interested in this kind of a difference formulation. So, we remember, this is a difference equation that tells me that for a given impulse the rate of change of momentum can be calculated and let see, if we can use some applications of this.

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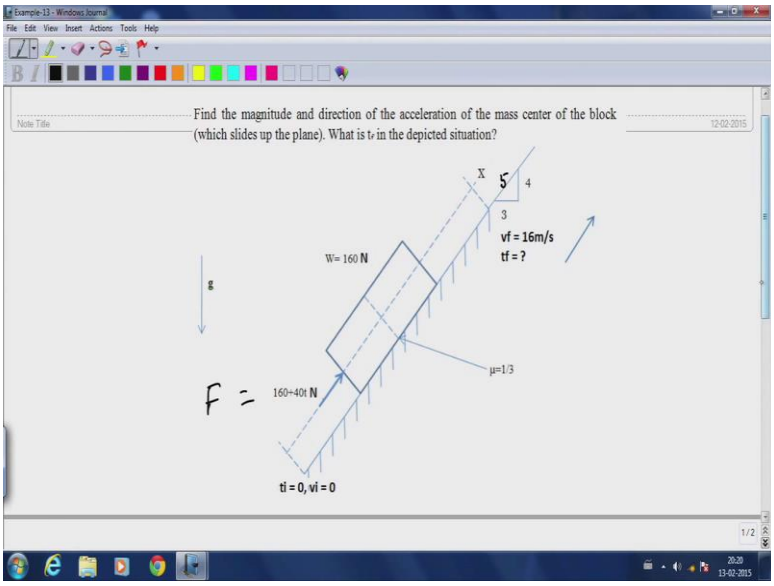

So, let us we take this example here, a block of weights 160 Newton's is on a inclined plane that has, that is on this 3, 4 5 triangle. There is a coefficient of a friction, which is got a coefficient of sliding friction, kinetic friction; that has a magnitude one-third for the coefficient acting on the bottom surface of the block and the block is sliding up. So, there is a force here, the magnitude of the force is 160 plus 40 t is acting to push this block up.

As t increases, you can see that the magnitude of this force F is increasing with time and we are asked to find, the magnitude direction of the acceleration of the mass center as a function of time. And we are asked to find, what is the time instant F at which the velocity is 16 meters per second.

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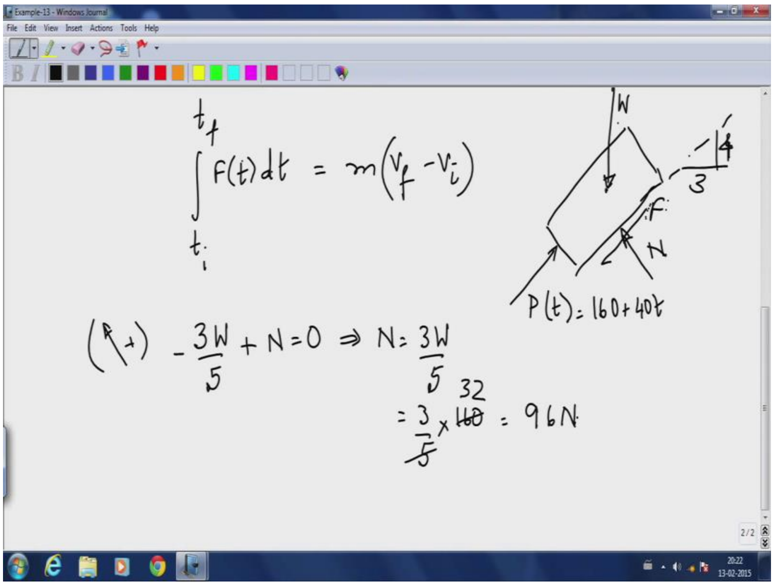

So, this is a prime candidate for our impulse momentum formulation, because the force is time varying, but I have to be a little careful here. This F which is 160 plus 40 t is not the only force acting on this block. So, in order to make sure, I have a picture of all the forces acting on the block, I have to draw a free body diagram. There is a normal force, there is this force F, there is a force W and this whole thing is on a 3, 4, 5 triangle.

So, firstly, taking all forces normal to the plane 0, by the way I forgot, there is a friction force, just for a sake of clarity, I call this as some P. Taking all perpendicular forces 0, what we find is that, 3 W over 5 minus plus N equal to 0, which says N equals 3 W over 5, W itself is 160 Newton's. So, N has a magnitude of 96 Newton's.

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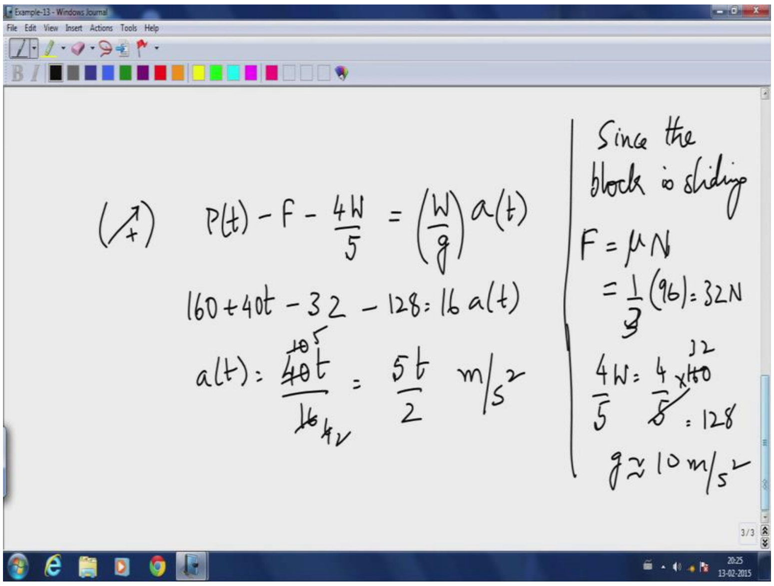

Now, taking all forces up the inclined plane is positive, P of t minus F the friction force minus m g sign alpha, which would be 4 W over 5 equals mass, which is W over g times acceleration. And in this case if P is a function of time, the acceleration is also a function of time, let place the numbers here 160 plus 40 t minus, since we know the block is sliding, F equals mu times N. So, that is one-third of 96; that is 32 Newton's minus 4 W over 5.

I am going to assume like, always g is 10 meters per second squared, which puts the mass at 16 kilograms times a of t. So, I can quickly do this 160 minus 32 minus 128 is simply, so this says a of t equals 40 t divided by 16, which is 5 t over 2 and this is units of meters per second squared.

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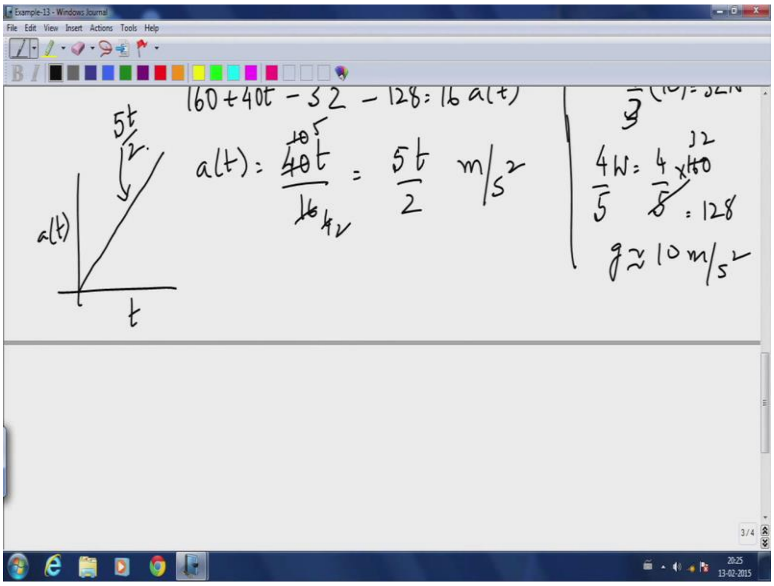

At some initial time t equal to 0, so for prop the acceleration was as time at some initial time t equals 0, the acceleration is 0 and then it increases linearly. Equation of this line is 5 t over 2; that is not what we are ask to find, you are ask to find the time at which the block would acquired a velocity of 16 meters per second given that at some initial time 0 seconds the velocity is also 0.

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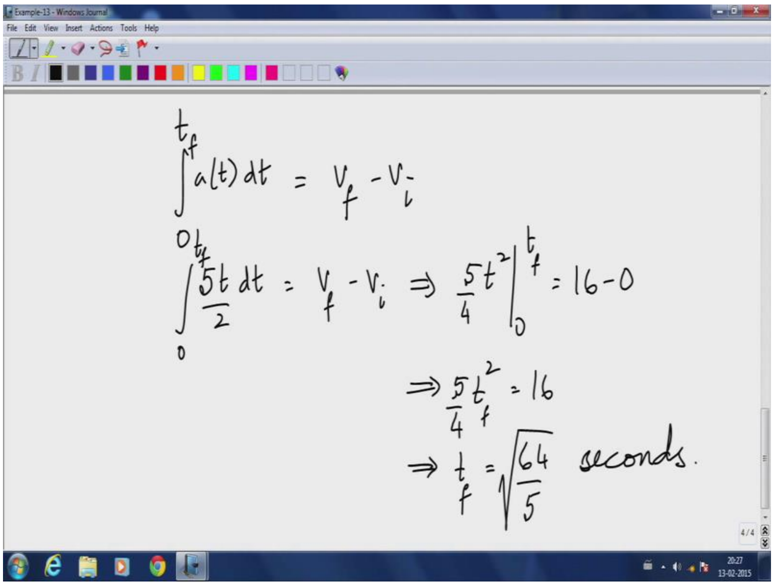

So, this amongst to integrating a d t d t from 0 to some final time t f equals v f minus v i, we are given both v f and v i, v f is 16 meters per second and v i is 0. So, let us do that integration 5 t over 2 d t 0 to some final time t f equals v f minus v i which implies 5 over 4 t square 0 to t f equal 16 minus 0, let us implies 5 over 4 t f squared equals 16, t f equals 164 over 5 seconds. So, this force acting on the block for about three seconds would get the block up the inclined plane to a velocity of 16 meters a second.

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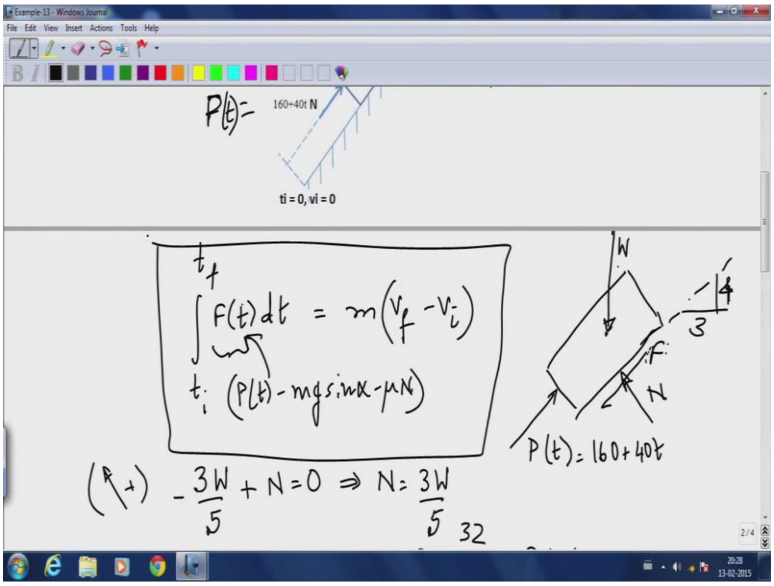

So, essentially what we are done is calculated the force F of t, which in this case is the P of t minus m g sign alpha minus mu m. So, that whole thing is use to replace this F of t and when we do that, we are able to calculate the final changing the momentum m times v f minus v i. So, that gives you the final answer, we will take the next example problem in the next lecture.