Statics and Dynamics
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Lecture - 27

We will look at another example problem here, this one relates to the introduction of friction.

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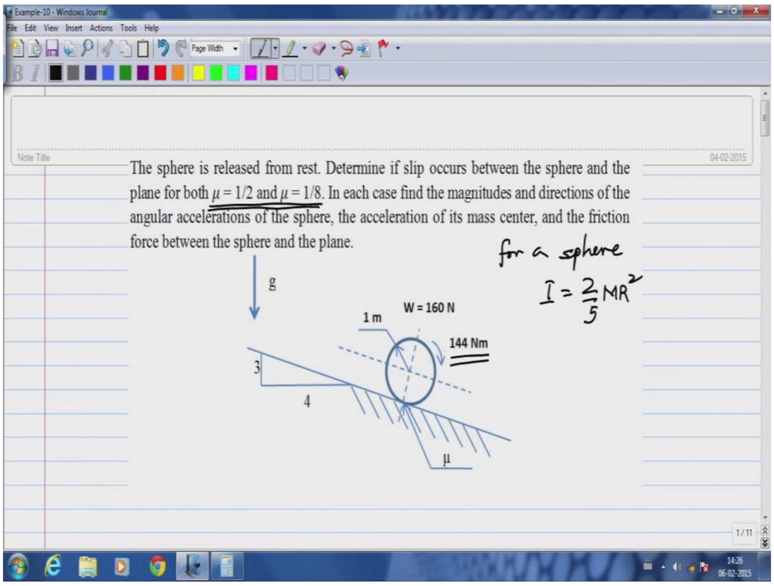

So, let us look at the problem, there is a sphere; that is rolling down in an inclined plane. The inclined plane is 4 on the horizontal and 3 on the vertical, the mass of this sphere is 16 kg or 160 Newton's in force, the radius is 1 meter. There is an external moment that is acting on this sphere in the magnitude of 144 Newton meter. So, I want to first solve this problem without the external moment to see, what we understand and then we will solve the problem with the external moment.

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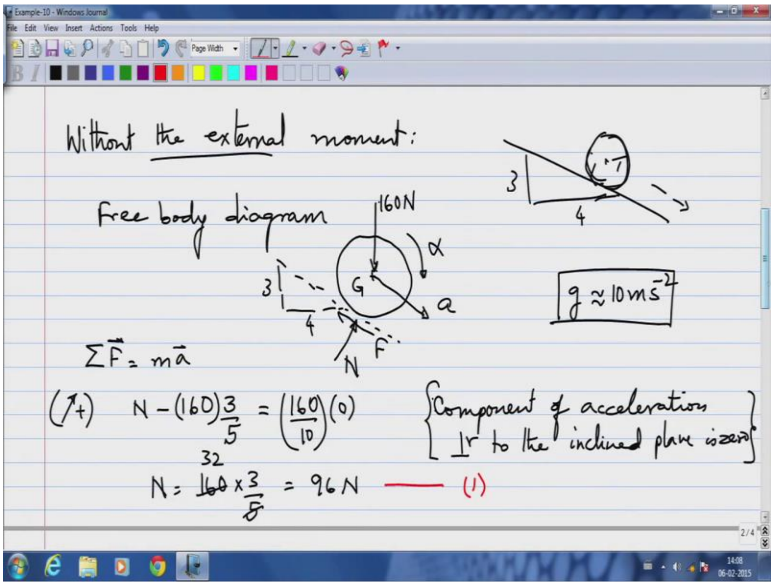

So first, let us look at the case without the external moment, which should be the case of a sphere of sin 1 meter rolling down in an inclined plane. So, let us first draw a free body diagram, I have a sphere, it is the center of mass, this is weight 160 Newton's acting at this through the center of mass. And then at the point of contact, this is normal reaction force, this is the normal reaction force of some unknown magnitude N and a friction force F.

So, this point is the center of mass is moving with an angular with a linear acceleration a and an angular acceleration alpha. So, this diagram captures all the necessary elements, all the forces, the friction force F, normal reaction N, the way to the body and the kinematic property, the acceleration a and the angular acceleration alpha. So, before we can proceed much further, I have four unknowns capital N, capital F, a and alpha.

So, let us start by writing the laws of conservation of linear momentum. So, sum of all forces is mass times acceleration. So, I am going to do two things, I am going to take all forces perpendicular to the inclined plane, I am going to write this Force balance as two separate component level balances. The first one being the component of the force perpendicular to the inclined plane and the component of the force along the inclined plane.

So, let us first do the perpendicular to the inclined plane. Then, if I take all upward forces perpendicular to the inclined plane positive, the only such force is the normal reaction.

And the downward component due to the object, due to the weight; that is pushing down on an inclined plane is this 160 Newton's times the vertical part, which is 3 fifth's and that is equal to 160 divide by 10, which is the mass of the body times 0. So, the component of acceleration perpendicular to the inclined plane is 0, so this tells me that N is equal to 160 into 3 over 5, which is 96 Newton's.

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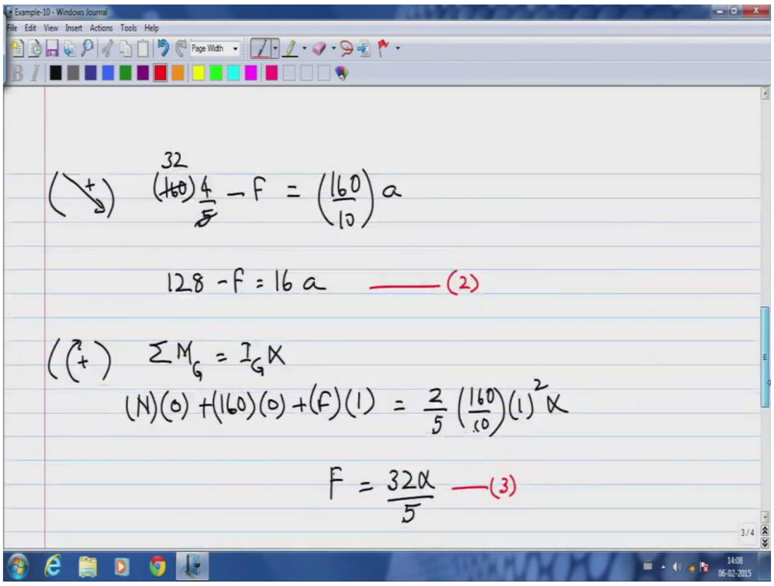

Now, taking all the forces down the inclined plane positive, so this is force balance along the inclined plane. The force is acting down the inclined plane or along the inclined plane are 160 into 4 divided by 5, which is the component of the weight that along the inclined plane minus F. So, the friction force is acting, we assumed it is acting opposite to the, it is acting up the inclined plane. So, this is equal to the mass again, like I said earlier on, I am going to assume g is 10 meter per second squared times the acceleration.

So, let us simplify this, so this is my first equation that gives me the normal reaction, this is my second equation. Now, I can still write the moment balance, so in this case, I assume alpha to be clockwise, since it is rolling down the inclined plane. So, I will take the same clockwise moments positive, sum of all moments about G equals I, G times alpha. The normal reaction has no momentum plus the 160, the weight of the body has no momentum, the only force with a momentum is the friction force and that has a momentum of 1 meter. I G for a sphere is 2 fifth times M, which is 160 by 10 times R squared alpha.

So, from here we find that F is equal to 36 alpha over 5, I call this my equation number 3. So, I have three equations, but in total of four unknowns.

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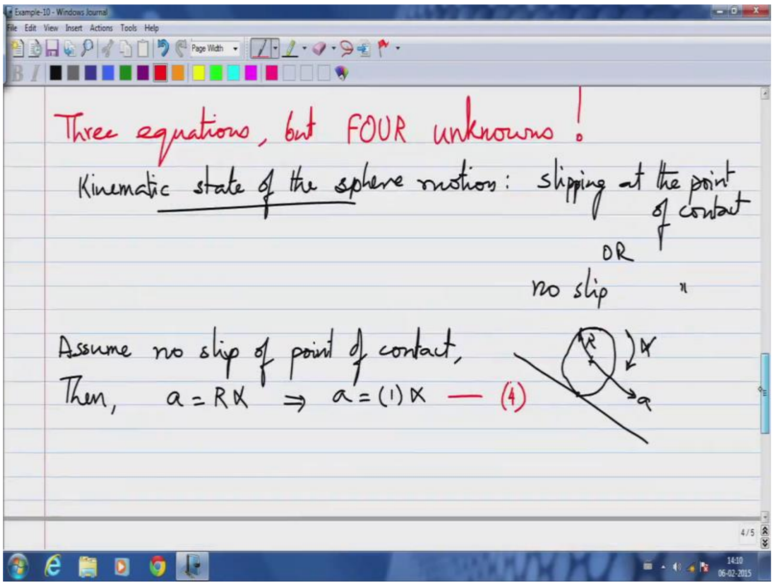

So, this is where we need a little more information on the kinematic state of the sphere motion. So, we going to ask, what is the kinematic state of this sphere motion that is, is it slipping at the point of contact or no slip at the point of contact. So, depending on which of these two is the kinematic state of the sphere motion, our approach from here are on, the 4th equation which we need to complete the system of equations would be different.

So, I first write, I will start by assume no slip at point of contact and we will solve this problem. So, if I have no slip, what that automatically means is that, I have this sphere of some radius R, if this center of mass moves with an acceleration a and an angular acceleration alpha. These two quantities cannot be unrelated, because at this point of contact there is no motion. So, when there is no slip, then a equals R alpha, which for this particular case, says a equals 1 times alpha, one being the radius of this sphere. So, this gives us our where much needed 4th equation.

So, let us write them up, write them down, I am going to ignore the first equation ((Refer Time: 09:56)), because that is just illumining the normal reaction is 96 Newton's. We are going to write down equations 2, 3, 4 and solve them.

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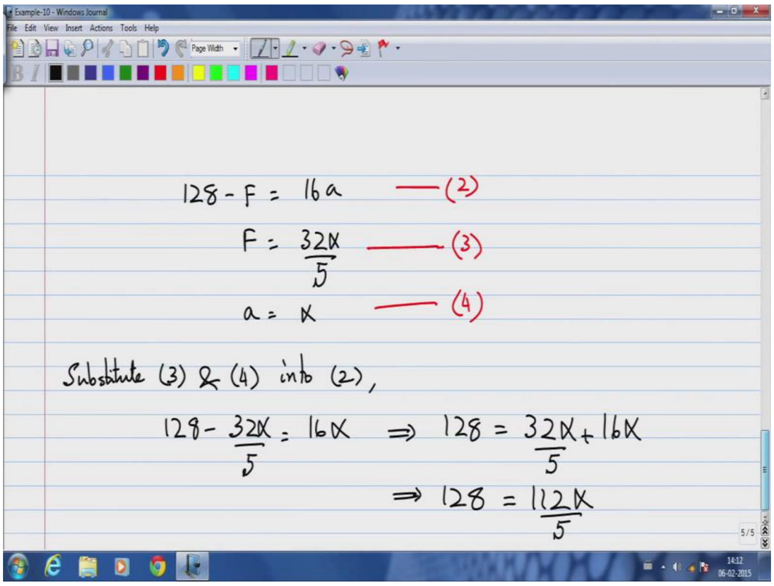

Two is 128 minus 5 is equal to 16 a minus F equal to 16 a, this is our equation 2, 3, F equals 32 alpha over 5 and a equals alpha, this is our equation number 3, this is our 4. So, we have three linear equations in three variables, so we are going to be able to solve them. So, I will substitute 3 and 4 into 2, let see what we get, so 128 minus 32 alpha over 5 is equal to 16 alpha, which implies that 128 equals 32 alpha over 5 plus 16 alpha or alpha equals 128 into 5 divided by 12.

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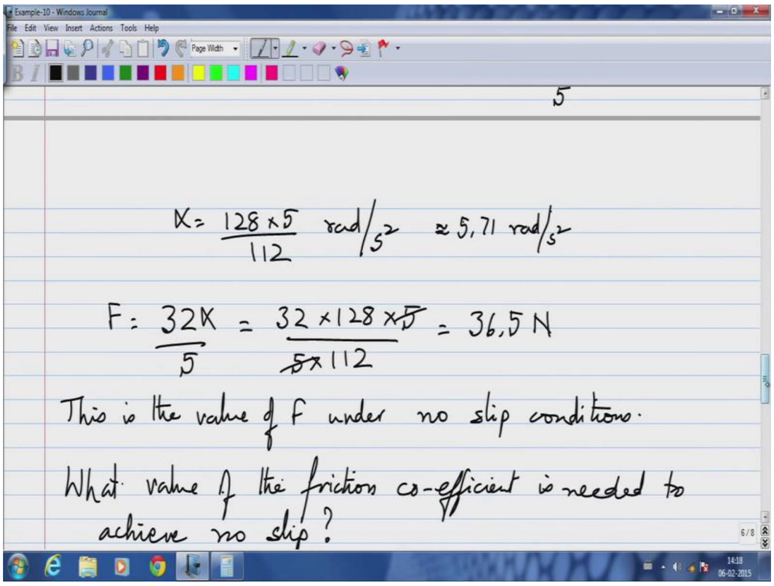

So, from here I can calculate what F should be, F equals 32 alpha over 5. So, this is roughly 36.5 Newton's, so this is the amount of friction needed, this is the friction force acting at the point of contact in order to achieve no slip. So, I leave force inside, this is the minimum value or let me actually say, the way I send it away I complete what I have going to say, this is the value of F under no slip conditions.

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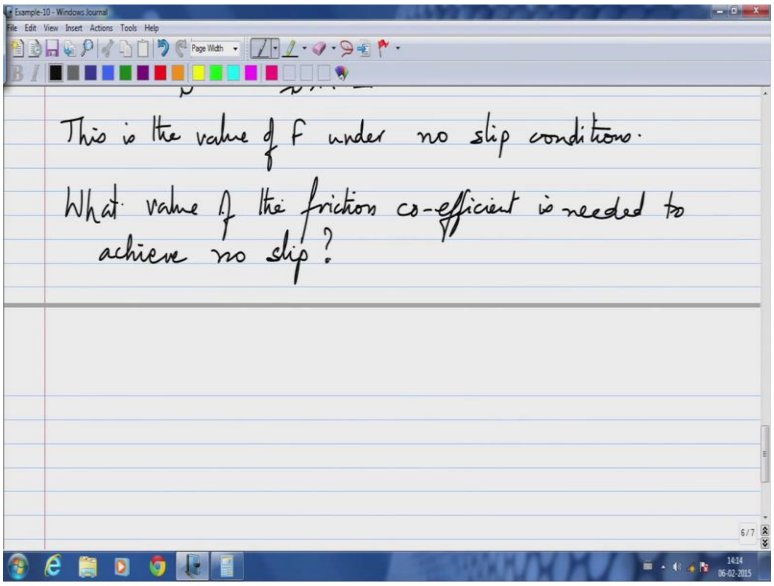

But, typically between a pair of surfaces, let say in this case this sphere in an inclined plane; I have a value of mu. So, if I replace and ask the question, what value of the friction coefficient is needed to achieve no slip?

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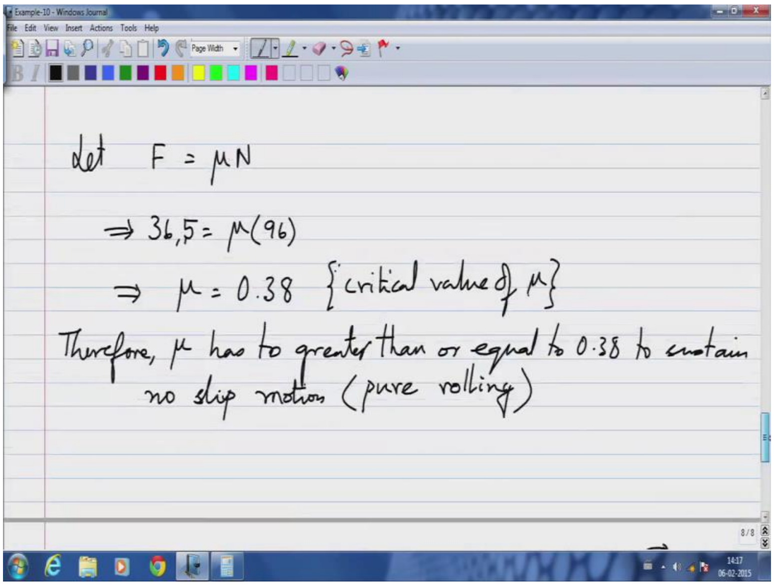

So, if I ask this question, then what I need to do is, relate this F to mu times the normal reaction. So, I shall let F equals mu times the normal reaction. So, 36.5 equals mu times 96, we already calculated the normal reaction N to be mu times 96. So, from here, I can calculate mu to be approximately 0.38, remember mu has no units. So, therefore, we can say mu has to be greater than or equal to 0.38 to sustain no slip motion, which is also referred to us pure rolling.

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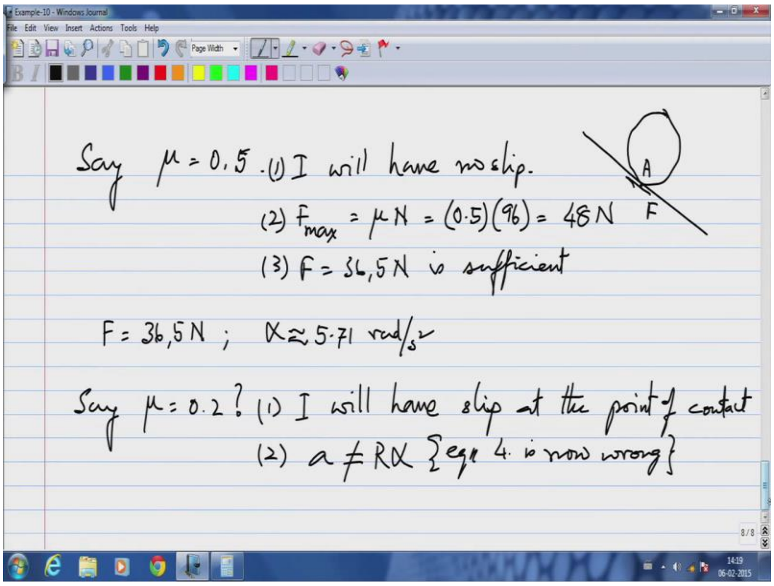

So, let us just do some thought experiments, say mu equals 0.5. So, if mu equals 0.5; that means mu is at least 0.38, mu is greater than the critical value. So, this is like a critical value of mu needed, mu is greater than the critical value. Under these conditions, what it says is that I will have no slip; that is the first observation. The second observation, the maximum value of F as we know from elementary Coulomb friction modeling is mu times N.

So, in this case the maximum value is 0.5 times 96, which is 48 Newton's. So, actually the surface at this point of contact A, this Friction force F can go to be as high as 48 Newton's. But, in reality, I do not need 48 Newton's of force to sustain no slip, only F equals 36.5 Newton's is sufficient to sustain no slip, which means that even though mu is 0.5, the value of f would be 36.5 Newton's.

And the angular acceleration alpha would be, ((Refer Time: 16:56)) whatever we get from this number, let me just do this calculation, which is roughly 5.71. So, if I now take for this case mu to be 0.5, I will actually only need 36.5 Newton's to sustain no slip. And the friction force at the point of contact, even though it can be as high as 48 Newton's we will only take on a value of 36.5 Newton's and alpha would be 5.71 radians per second squared.

Now, say mu is 0.2, then what happens. The first conclusion you can say is that, mu is left hand the critical value of a Coulomb friction coefficient; that is needed to sustain no slip, which means I will have slip at the point of contact that means the sphere will slip at the point of contact. So, which now means, I cannot say that a is not equal to R alpha, this is the first thing that you get to know. That means what I wrote down for my equation 4 is now wrong and I have to replace it with something else.

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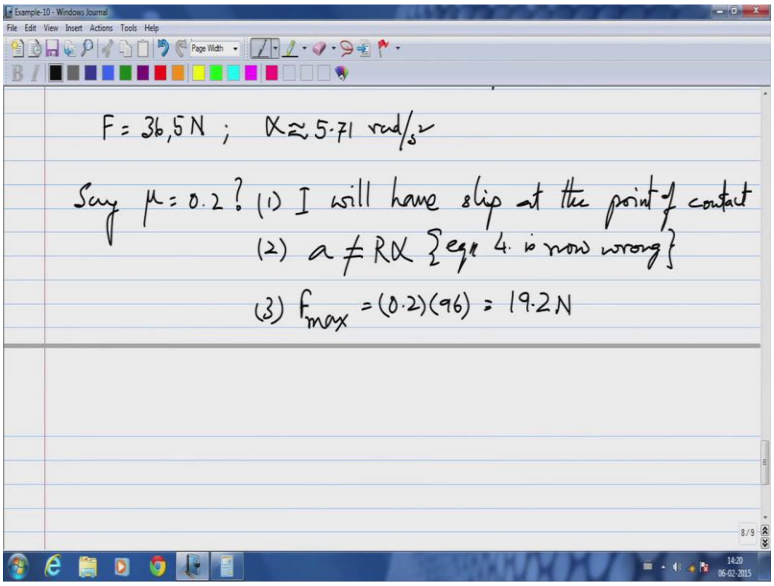

But, what I do now is that f max is now 0.2 times 96, which is only 19.2 Newton's. F max is now 19.2 Newton's, but I need 36.5 Newton's of force at the point of contact to sustain no slip, clearly we do not have. So, essentially these the two surfaces, the sphere and the inclined plane do not have sufficient friction between them, which means we will have slip, which also means that F in the problem, in the free body diagram will take on the maximum value possible.

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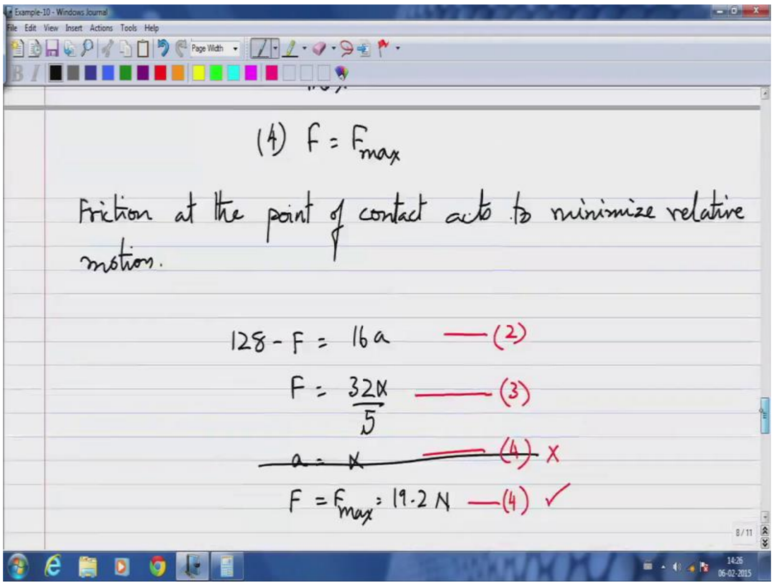

That means the friction force tries to be it is best to minimize relative motion at the point of contact. So, this is the purpose of friction at the point of contact. So, friction at the point of contact acts to minimize relative motion. So, it is going exact it is maximum force, which in this case is 19.2 Newton's and it that force is unable to bring the slip, which is essentially the relative motion at the point of contact 0, it continuous to exert the maximum possible force.

So, now, let us go back, in rewrite our three equations that we had I will copy this from here and pasted it here. What will found is that, this is new longer correct for our equation 4, we will replace it with. We still need an equation to complete the system of equations. So, we say F equal to f max which in this particular instance is 19.2 Newton's So, now, what we find is that, I have three unknowns, F, a and alpha and three equations. So, this is my, so this is what use to be my, this is now my 4th equation, correct 4th equation.

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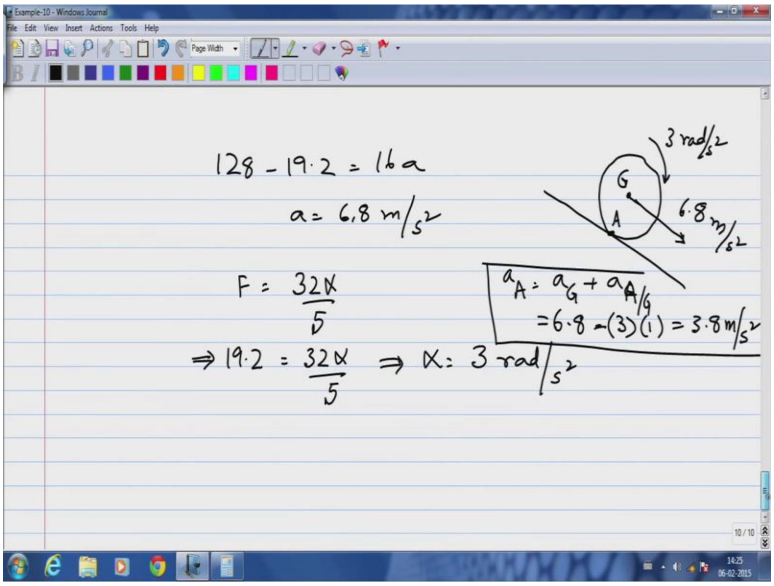

Now, I can solve these three equations, now 128 minus 19.2 equals 16 times a. So, which means a equals 6.8 meters per seconds squared, equation number 3 still valid. So, F equals 32 alpha over 5, which implies 19.2 equals 32 alpha over 5, which implies alpha equals 3 radians per seconds square. So, just to get the feel for how this is going to behave this point here is moving at 6.8 meters per seconds squared and this is rolling at 3 radians per seconds squared.

So, at this point of contact A, this sphere this sphere is moving back, if you imagine the point G, the point G is moving downwards at 6.8 meters per second squared and in relation to G, acceleration of A as observed by G is minus 3 times 1. That means, the absolute observation of the point A, using the loss of relative motion is that at the point A, which is the point of contact on the sphere side is moving down at 3.8 meters per seconds squared.

The ball is trying to move down it is not perfectly sliding, but it is essentially rolling left than it is needs to roll, while it is sliding. So, the net motion is that, the point of contact, where the sphere touches the inclined plane is actually scratching the inclined plane in a downward sense. Friction as you can see is trying to bring that point to rest, it is acting upwards, but it is unable to. So, this is the case of sliding, the sphere as a net sliding motion at the point of contact.

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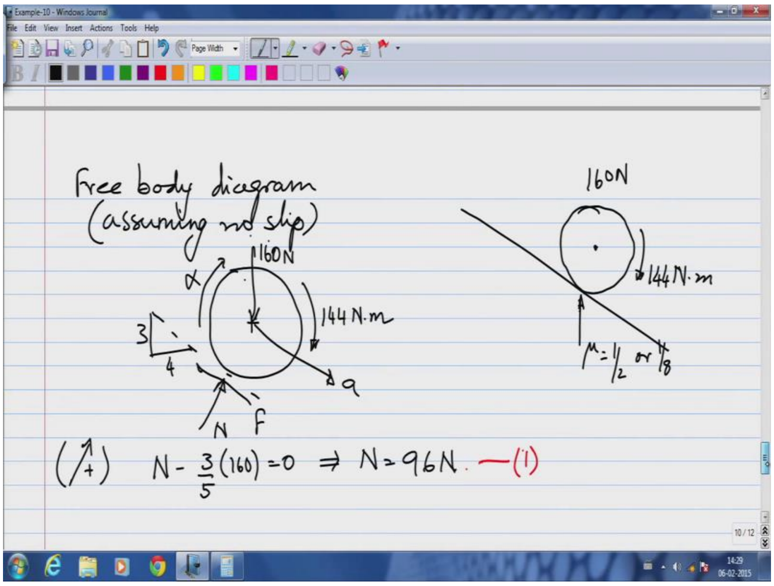

So, now, let us move on to our problem and look at the case with the moment acting on it. Go back to the top of the problem; there is an additional moment of 144 Newton meters; that is acting on it. Now, if you notices here, we are told that mu is half or 1 over 8, let see what this looks like. So, this is a net moment, there is an external moment, so if you think of this as the front wheel of a car, the front wheel of the car is the driven wheel.

So, there is an action that is trying to drive the tyre and that that engine torque creates a net moments of 144 Newton meters, you could you could think of it in that way. There is a weight which is 160 Newton's and we are ask to find, I am what we are told is that, this point of contact as two possibilities, let us look at this first and the second. So, the bottom line here is that, if I am told what the friction force, friction coefficient is at the point of contact; that it is a half or a 1 8 th; that information write at the outside is useless.

I need to know, if the cylinder or the sphere or the object is slipping or not slipping that is the kinematic condition that is the question related to the kinetics that I have to first answer. So, the way to solve a problem of rolling motion or sliding motion is to first assume no slip. Calculate the critical value of friction needed to sustain no slip, and then compare the critical value of friction to the actual value of friction coefficient that you are given in the problem.

So, let us do that. So, the first step is to draw a free body diagram and we going to solve the problem assuming no slip that is our first step. So, this is our free body diagram, this is the linear acceleration a and an angular acceleration alpha. So, we go through the same process, the fact I am going to do a vertical force balance, vertical perpendicular to the inclined plane that shows N minus 3 force of 160 equal to 0, which also implies N equals 96 Newton's that did not change.

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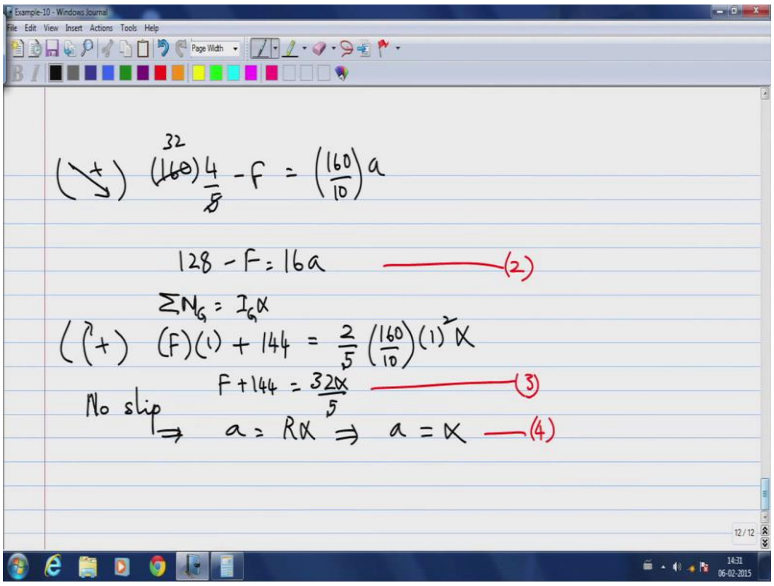

You are going to assume downward forces positive into a force balance in the downward direction. So, 160 into 4 over 5 minus F equals 160 over 10 times 8. So, this also has not change. So, I am going to call this, this is my 1 this is my modified 2. The third part is going to different, because there is I am going to assume clockwise moments positive. The only force that causes a moment is the friction force that has a moment term of 1 plus there is an additional external moment whose value itself is 144 Newton meters.

This equals, so sum of moment equals I G times alpha. So, the first moment is the due to the friction force, the second moment is the external movement, this equals 2 5th's M R squared, M is 160 divided by 10 R squared alpha. This is my modified equation 3 and the modified equation 4 is coming from no slip which implies a equals R alpha, which also implies a equals alpha. So, this is my modified equation 4. Let me simply 3, F plus 144 equals 32 alpha.

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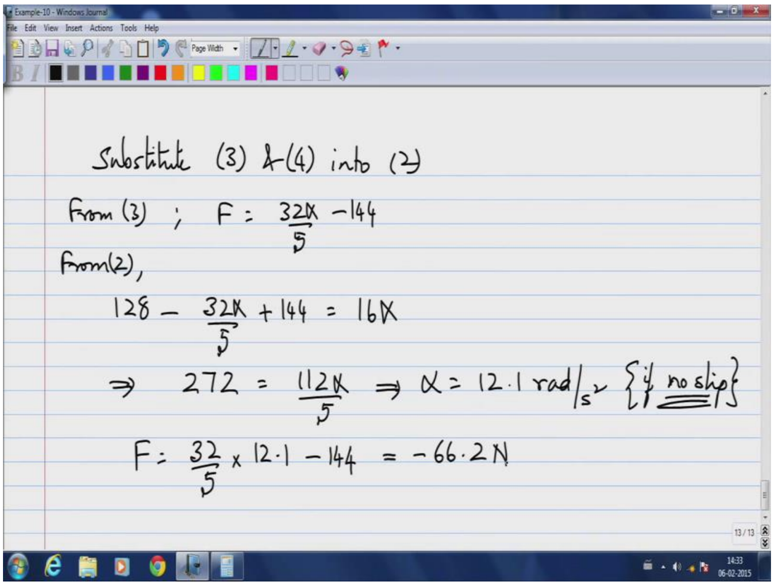

So, let us now solve for this solve these three equations, if I substitute 3 and 4 into 2, I if from 3, F equals 32 alpha over 5 minus 144, 128. So, from 2 minus 32 alpha over 5 plus 144 equals 16 alpha. So, this is 272 equals 112 alpha over 5, which implies alpha equals, alpha equals 12.1 radian per seconds squared. So, this is, if no slip, let us be clear about that, but the value of F is what I am really interested in which is 32 over 5 into 12.1 minus 144. So, this the required friction is approximately minus 66.2 Newton's.

So, under these conditions, we assume the friction force acts upwards, look at the free body diagram ((Refer Time: 32:53)). What the value of F has now come out to be negative, which means, it is not really acting up the inclined plane, it is actually acting down the inclined plane and it is magnitude is 66.2 Newton's.

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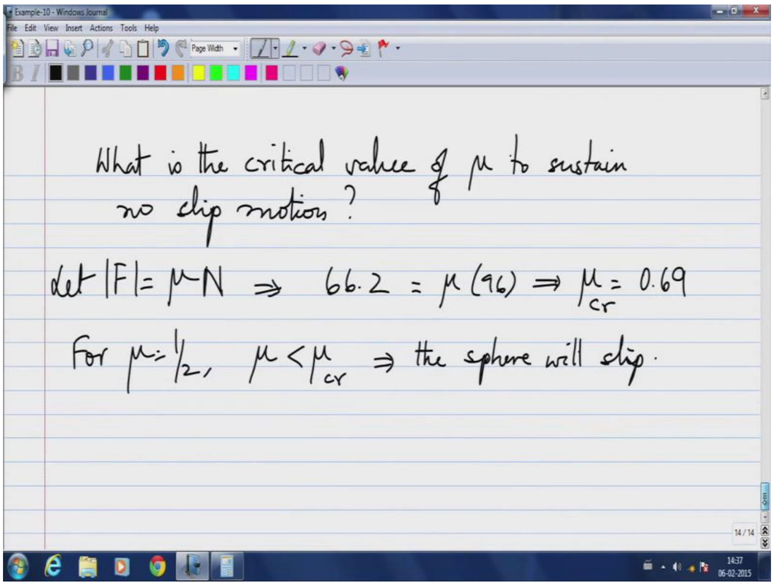

So, I ask the next question, what is the critical value of mu to sustain no slip motion, the answer to that is gotten the equating F equals mu times N; that means, this 666.2 Newton's. I can ignore the sine, because in any case, which are the direction the friction force at the mu calculation would independent of that. So, you can just take the modules of the friction forces equals mu times N, which implies mu would have to be of the value 66.2 divided by 96.

So, if we have to do at the value 0.69, mu would have to be of a value 0.69 before we see see no slip, this is the critical value of mu needed to sustain no slip motion. Any less than this, will have to do what we did earlier for the sphere without the external moment; that is I cannot assume a equal to R alpha, I will have to assume a is not equal to r alpha, but F is equal to the maximum value. So, if I know mu is half, the maximum value of friction force possible is half times 96, which is only 48 Newton's, but I really need 66 Newton's in order to sustain no slip notion.

Therefore, the surface would exert the maximum possible force on the sphere to be relative motion to rest, they would still failed, which means F would be equal to 48

Newton's not 66.2 Newton's. So, replace the 4th equation, where we said a equal to R alpha with the correct version of the 4th equation, which is F equal to 48 Newton's and solve for the motion. So, let us complete that part. So, for mu equal to half, a that is be clear, mu is less than the mu critical, which implies the sphere is likely to slip.

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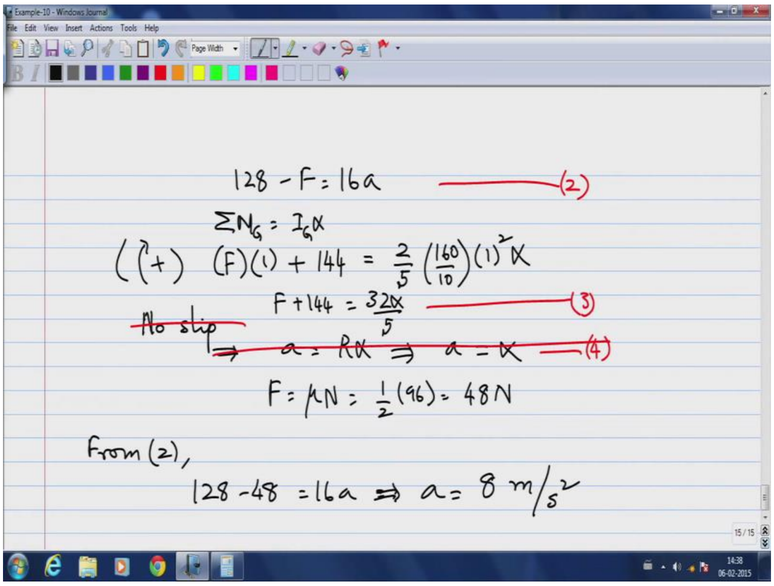

So, what we now have is again I will write down the four equations once more, except this 4th equation is incorrect. Instead what we have is F equals half mu times M, there is equal to half times 96, which is 48 Newton's. So, if F equals 48 Newton's, then from equation 2 128 minus 48 equals 16 times a, which implies a equals 8 meters per seconds squared.

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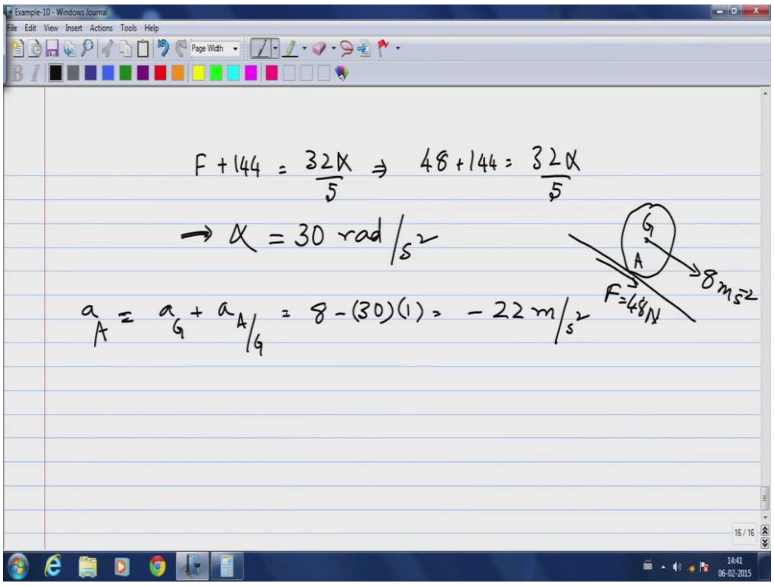

We also know that F plus 144 equals 32 alpha over 5, which implies 48 plus 144 equals 32 alpha over 5, which implies alpha equals 30 radians per seconds squared. So, notice how, if a equal to R alpha and if a equal to 8 meters per second squared, it is going to be moving within angular acceleration also that is equal to 8 radius per seconds squared. So, you can see that for this particular case alpha is 30 radius per seconds squared, which means, it is slipping and so the cylinder is spinning faster than the no slip condition would allowed.

So, let us just see what the kinematic condition look like at the point of contact a G plus a of acceleration at the point a is acceleration of G plus the acceleration of a is observe by G. This is 8 minus 30 into 1, which is minus 22 meters per seconds squared. So, that is at the point of contact, the sphere is meaning up the inclined plane, because it is spinning faster than the no slip condition would allowed to spin and that is the reason friction is now acting downwards.

So, this 48 Newton friction force mu times M, which is a maximum possible friction force is acting down in the inclined plane, it is trying to minimize this motion as it is trying to bring the relative motion to rest are to minimize it as for as possible. And the friction force essentially acts in full magnitude of 48 Newton's to make that happen, but it is still unable to bring the relative motion to 0.

The bottom line, I want you to appreciate the action of friction, friction at any point is not apposing motion as a whole it is not opposing the motion of the center of mass. So, in fact, in this particular problem the friction is acting down in the magnitude of 48 Newton's and a cylinder is also moving down at an acceleration of 8 meters per second squared. So, the friction force is in fact providing the traction to provide the forward acceleration, of course, in this case along with the weight of the sphere itself.

But, even in the options of way to this sphere, this sphere with move downward would move down at the plane just because you have the which is creating an attraction force. So, friction is in fact, the cause of the forward motion in this case, one of the causes of the forward motion in this case. So, friction in general does not oppose the motion of the center of mass, it only opposes relative motion at the point of contact and tries to bring the relative motion at the point of contact to near 0 values.

So, I hope this illustrated the value and the physical use of friction, the physical gave you a physical feel for how friction works. Now, the problem statement had the same calculation proposed for another value of mu of 1.1 over 8, 1 8. As you will see 1 8 is also less than the critical value and I would like you to take that as homework and complete it.

Thank you.