Statics and Dynamics
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Lecture - 26

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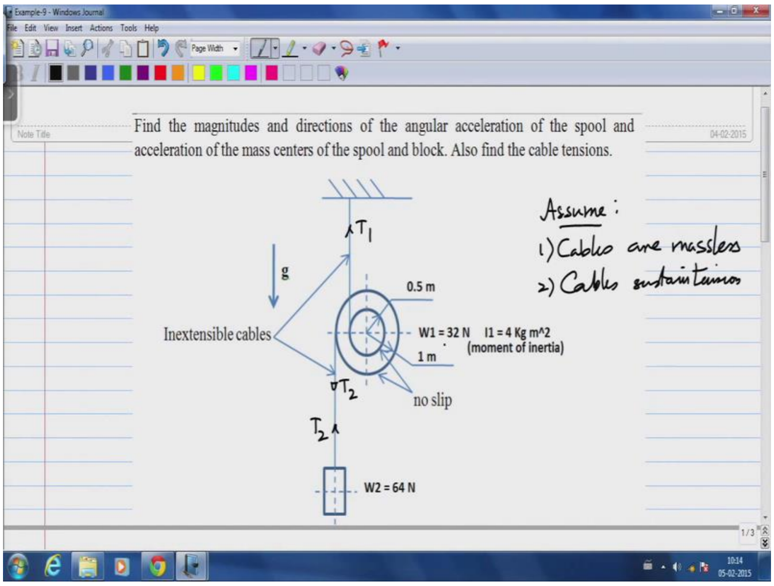

Let us look at another example problem involving two bodies. So, we are going to look at a system, where now two bodies that couple through a set of cables and we still learn the process of a client, the laws of dynamics to this system. So, let us look at the example problem. The example problem here involves a spool, this spool has two pulleys on it, there is an inner pulley on which a cable is warm and that is debate to the top. The inner pulley has a radius of 0.5 meters and the outer pulley has a radius of 1 meter. And what we ask to find is the dynamical condition, the accelerations of the spool as well as a dead weight that is tight to the outer spool.

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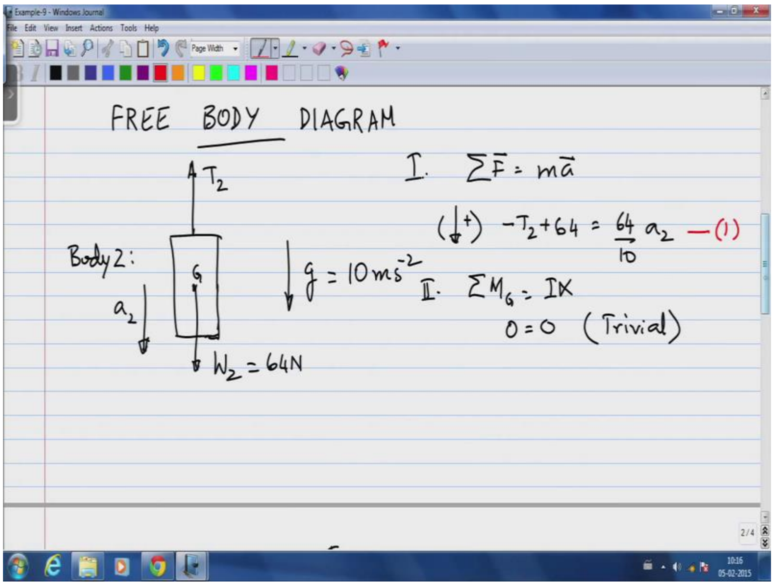

So, let us start drawing our first step is to draw free body diagrams like we said, if there is one skill we walk away from this entire 8 week class, it should be to draw quantitatively accurate free body diagrams. So, just a set of simple assumptions, we assume the cables are mass less. So, they do not have an inertia of their own and secondly, they only sustain intension.

The second assumption is really not mean it, but information you need to be aware off. So, let us look at the dead weights first; that is the easier one to draw the free body diagram of. So, I have a cable, in the whole idea of a free body diagram is to free the body, body 2 in this case of all external influences and replace the influence with simple forces. So, I am going to call the tension in the cable 2 as T 2 inside the only action that the cable has on this body 2 is to exit a force T 2 on the body 2.

And the weight is known that is 64 Newton's and for all these problems I am going to assume g is 10 meters per second square as we said earlier on, you not going to complicate the divisions. So, we are going to assume this body has an acceleration a 2. Now, some of the several text books will show you, what is called a kinematic diagram; that shows the relationships between the two in a kinematic sense and we will see how to do that in a short while.

But for now, I know this body has an acceleration a 2 and I am going to without laws of generality assume it is downwards. Now, the first law, the first Euler's law says sum of all forces is mass times acceleration. And I am going to take downwards force is positive as my convention for this particular for applying this law here, minus T 2 plus 64, which is my W 2 equals the mass, which is 64 divided by 10 times acceleration.

Now, in this case, the acceleration is a scalar, because we are only looking at one dimension of motion of the scalar. In this particular instance, the two forces under question W 2 and T 2; both pass through the center of mass of this body. Therefore, the second law of motion is somewhat trivial; it just becomes 0 equals 0. But, I do not want to write this time, because in a general sense, you have to remember that, there are two laws of motion when it comes to rigid bodies.

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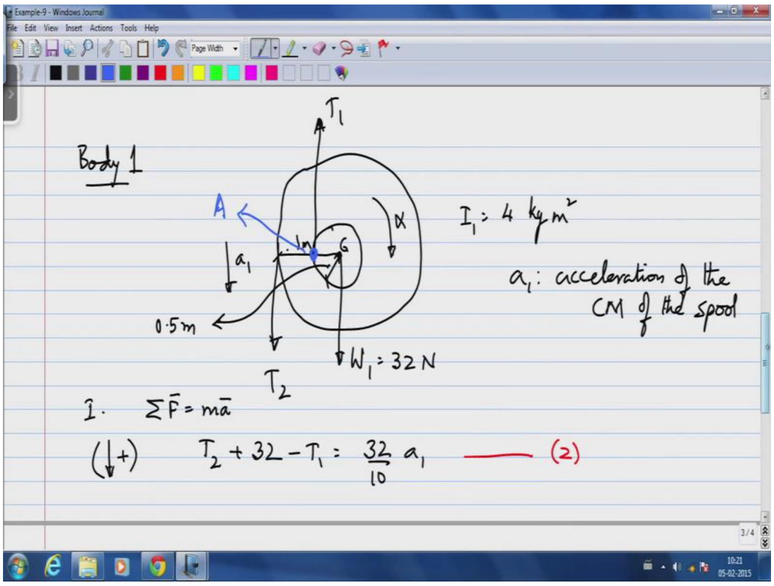

So, now, for body 1, which is our spool slightly more complicated, let us draw this spool, this spool has an inner cable, this has a tension T 1 and the tension in the cable is already in the outer cable is already been designated as T 2. So, there is a force T 1 that acts here; this is a force T 2 that acts here. T 2 in general, ((Refer Time: 05:03)) let just to make a point clear, T 2 is not equal to W 2, because the acceleration of this body is non-zero and the spool itself has a mass associated with it ((Refer Time: 05:15)), the weight of the body is 32 Newton's.

So, we will use that and I 1 the moment of inertia of this body is 4 kilogram meter square. Now, we did not learn how to compute moments of inertia of various bodies, I am assuming, you would have learnt that in earlier course in Physics. A spool such as this is not a regular geometric body that you would have learnt, how to compute the moment of inertia and it is possibly not, it is not even possible to compute.

In his particular instance, you have told what the moment of inertia is, because this number really depends on the masses of the two discs separately. So, it is like two bodies that I have been attached to make a spool. So, one could not in a very general way calculate that from without being given that number. So, we know we want to identify the center of mass of this spool and let us say, the acceleration of the spool is a 1 in a downward sense.

So, this is a 1 is the acceleration of the center of mass of the spool, let us be very clear. So, when we went from Newton's laws to Euler's laws, the first point we learnt is that, Newton's second law is Euler's first law, when apply to the motion of the center of the mass. So, if I take downward forces positive again says T 2, so let me write the law first and then this is Euler's first law, T 2 plus 32; the weight of the body minus T 1 equals the mass of the body, which is 32 divided by 10 times a 1.

So, I am going to designate these equation numbers, because we are going to come back and use them later. So, our first equation of motion came from Euler's first law apply to body 2. The second one is coming from Euler's first law apply to body 1 and now, we look at the rotation of this body. So, let say the spool has a clockwise angular acceleration alpha. The spool is one rigid body and the point to understand here is that a 1 and alpha and as a matter of fact a 2 is well are related. So, the way would be this, is… So, let us first write this and then we will draw the relationship.

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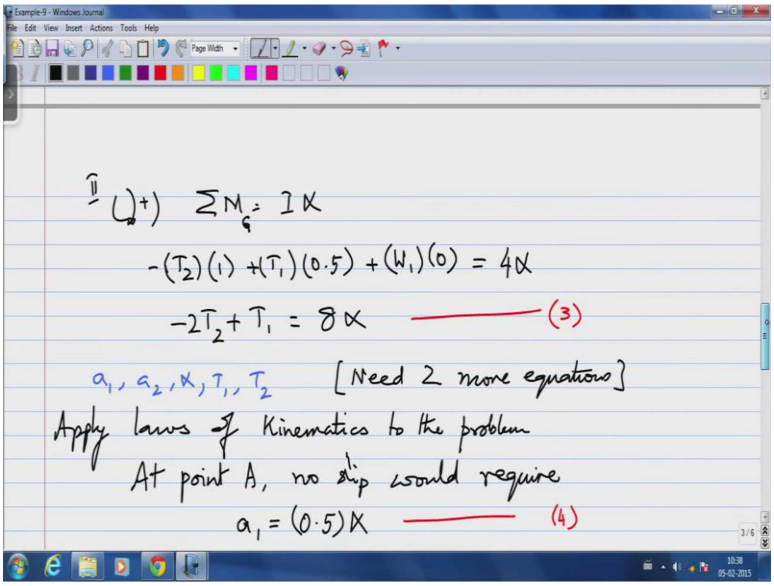

So, if I take clockwise positive as my sign convention for applying the Euler's second law and like we said we will always choose the center mass as our point about which we take the moment and the moment of inertia. The simple reason is, even if the center of mass is accelerating, sum of moments equals I times alpha and this is my simple form of the equation to deal with. You do not have to remember the additional terms that arise, when you are dealing with a point of reference that is moving in general.

So, we are going to take moments about the center of mass. So, in this particular instance T 2 times the radius associated with this is 1 meter ((Refer Time: 09:21)) and the inner radius here is half a meter, we are given those numbers. So, the radius times 1 meter produces a counter clockwise moment as you can see and so by our sign convention that would be negative. T 1 on the other hand produces a clockwise moment, so that would be T 1 times 0.5 and the weight passes through the center of mass, so it does not create a moment.

But, I will still write that just for completeness, this equals I which we are given is 4 kilogram meter square times alpha. So, if I simplify this minus T 2 minus 2 T 2 plus T 1 equals 8 alpha and that is my equation 3. So, let us quickly list out our unknowns, I have a 1, a 2 and alpha, T 1 and T 2 as a five unknowns and we have three equations arising out of the applying the laws of dynamics to body 2 and to body 1.

We still need two more equations. So, let see, what the two equations would look like, the first equation would come from studying this point of contact. So, if I now look at what is happening at that point of contact and I will designate that as A. If this spool is unwinding an angle theta, then the center of mass will move down a distance R 1 times t theta, where R 1 is this number 0.5. It is akin to a cylinder rolling on an inclined plane under the way slip conditions.

So, at this point A, there is no movement there is no slip between the cable and the spool. So, the first equation, we need to more equations, let us and these two equations come from applying the laws of kinematics. So, at the point A, no slip, we required a 1 equals 0.5 times alpha. So, a 1 ((Refer Time: 13:17)) the center of mass of this body 1 moving downwards would have to exactly equal the rate at which the cable is unwinding.

Now, if we look at the sense of the two quantity, sense meaning the directions I have chosen, alpha is clockwise. So, if alpha is positive that would automatically resulting positive a 1. So, we have been consistent, but if these you are choosing to not be consistent. Say for example, if I have chosen alpha to be counter clockwise in this particular problem, then I would have to introduce a negative sign here, to a count for the fact that the sign of alpha is not the same as a sign on a 1. So, this becomes are 4th equation, a 5th equation comes from relating the motion of the 2 and 1 put together.

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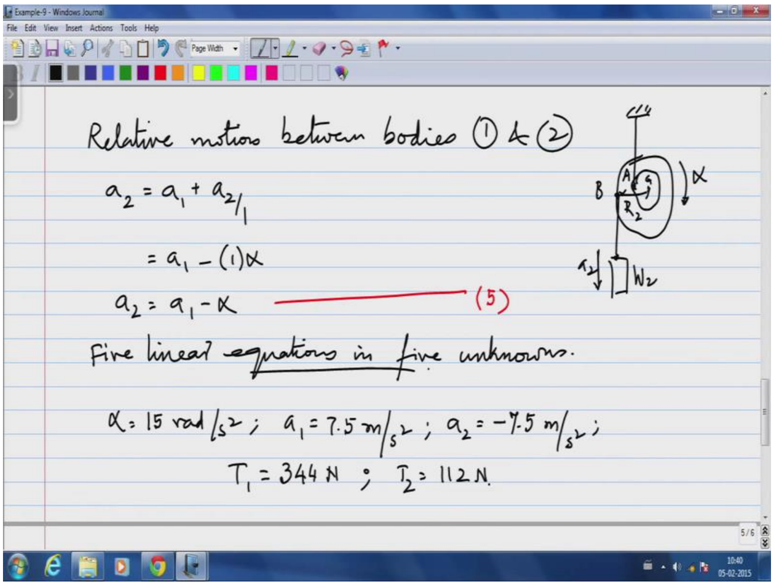

So, we use the laws of relative motion between bodies 1 and 2. So, I will just diagram of this to see, what this spools do together, the body is do together. So, if the angular acceleration of this body is alpha, then at this point A that we said. If the spool rotates an angle theta, then the center of mass comes down a distance 0.5 times theta, which also means a 1 is 0.5 alpha, we have already done that.

So, now, the acceleration of 2, which is the point here, so which is our point body 2 down here, which is moving down with an acceleration a 2 equals a 1 plus the acceleration of 2 as observe by 1. And acceleration of 2 as observe by 1 can be found from understanding how the spool works with the two cables. So, if this spool turns an angle theta that also means that on the point B, the cable vines on to the cable, the cable vines on to the spool systems R 2 times theta.

So, this it pulls up the cable a distance R 2 times theta that is the acceleration of 2 as observe by 1 is R 2, which is 1 times alpha. So, a 1 is R 1 times alpha and acceleration of 2 as observe by 1 is R 2 times alpha. So, I can write this as a 2 equals a 1 minus alpha, which is our 5th equation. So, if you go back we will quickly recap, we have equation 1 ((Refer Time: 16:43)) that came from applying Euler's first law to body 2. Equation 2; which came from applying Euler's first law to the spool are body 1, equation 3 came from applying Euler's second law to body 2 to body 1, which is our spool.

And the last equations came from relating the accelerations; the angular acceleration of the spool to the linear acceleration of a 1 of body is 1 and 2. So, the net result is that we have five equations in five unknowns, the five unknowns being a 1, a 2, a 3, T 1 and T 2. So, when you solve those five equations is the five linear equations in five unknowns and the result, I am going to write the result, you can check this on your own time.

Now, what you need to observe here is that, our initial assumption of alpha being clockwise is correct, because alpha comes out to be 15 radians per second square, a 1 is positive that is a spool is moment down. But, a 2 is negative 7.5 meters per seconds square, which means that a 2, the a 2 is negative means, the body is actually moving up. Another and this is the way to kind of intuitively check the calculations that you have performed, T 1 is a large number in comparison to T 2, meaning T 1 is greater than T 2.

That should also be obvious, because the cable one is barring the weight of the spool plus the that dead weight 2 in a static sense. So, when nothing is moving the cable one is barring both of those weights. So, it should you should be able to you should kind of expect this. I do not want to close with one point that notice that going we applied the Euler's second law to the spool, we choose to take moments about g, which is the center of mass of the spool.

If you did that, if you do not do that, they choose to apply some of moments equals I times alpha about any other point, you get an incorrect answer. Primarily, because you are not guaranteed that those are the points of reference or non accelerating. So, a simplest choice, I will really to write again for applying Euler's second law is to ensure that the point about which you take moments and calculate the inertia, moment of inertia is the center of mass. We will continue this in another example problem next time.