Statics and Dynamics
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Lecture - 23

We are going to start looking at the action of forces on rigid bodies, so up until now we have studied kinematics which is a study of motion without really considering forces that cause that motion. We are going to start looking at kinetics, we still going to restrict ourselves to plane kinetics, which is essentially kinetics or dynamics as it is sometime called kinetics of rigid bodies on a plane. And we will first write down the laws of motion that govern plane kinetics of rigid bodies or in a slightly more general way kinetics of rigid bodies. But, before we get there we start by recapping our famous Newton's second law, which is the, that force causes acceleration and the proportionality constant there is mass.

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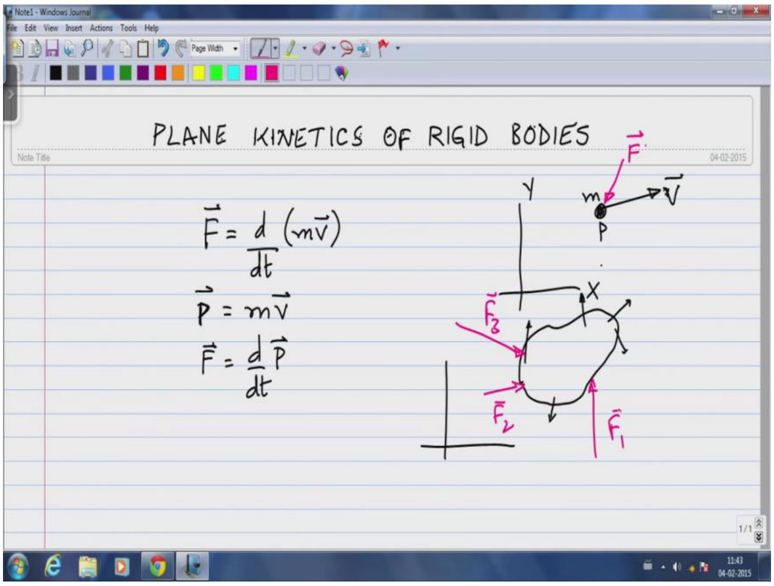

So, let us start by writing that in a vector form, except I will write it slightly differently. In fact, the way we wrote it when we first wrote down the laws of motion, if I have a particle of mass m that has a velocity v, the force acting on this particle causes a rate of change of momentum. So, if I now define G has m times v, I think we use the subscript P to denote linear momentum. So, if P is the linear momentum of this particle which is m times v, the force acting on this particle causes…

So, let say there is a force in a particular direction acting on this particle, that force causes a rate of change of momentum in the direction of that force, this is what we had for a point particle. Now, if I have to extend the same to a rigid body, so there is a rigid body, the difference between a point particle and a rigid body is that, a rigid body has additional degrees of freedom, this is what we talked about in the context of statics.

And so if I now take a rigid body, each point on this rigid body can be moving with slightly different velocities, because we said rigid body can rotate. So, if I have different rigid bodies moving with different velocities and let us say there are some forces denote these as vectors. So, if I have forces acting on this rigid body, what would this law Newton's second law look like for the rigid body? Now, our basic context in which you can make this generalization of the laws of notion valid for a point particle to a rigid body is by defining what is known as the center of mass.

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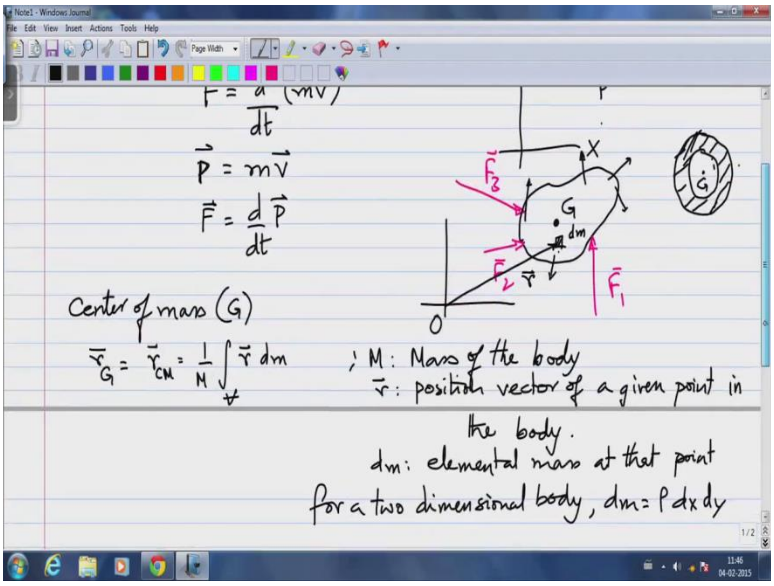

So, the center of mass, we will use the symbol G to denote the center of mass is defined in such a fashion that the integral of r. So, if I take an element d m and r as the position vector of that element d m, so the integral over the whole volume r d m divided by m. This gives me the position vector of the center of mass r the position vector of G. So, here M is the mass of the body, r is the position vector of a given point in the body, d m is the elemental mass at that point.

So, if in a, for a two dimensional body this may simply be the density times d x d y again the elemental area times the area density. So, the center of mass now has… Is a point inside the body? Typically, it may not be inside the body, if the body is not simply connected let us take for example, a simple hoop. So, if I have a hoop of a circular hoop like a bangle, the center of mass of this hoop is not in the mass of the body itself. So, as long as the body is simply connected, the center of mass lies inside the body. So, the center of...

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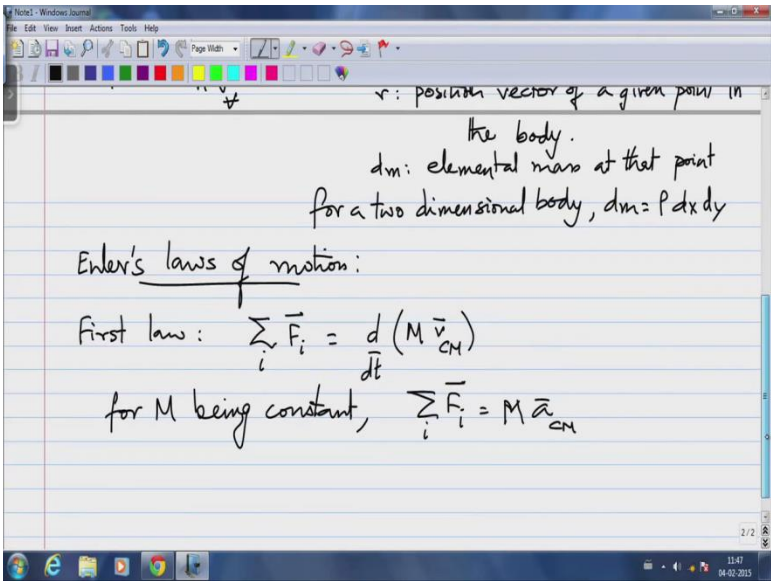

So, the extension we can make of the Newton second law is what we will call Euler's laws of motion. In the first law, the first Euler's law of motion states that the sum of all forces acting on a rigid body is equal to the rate of change of momentum of the center of mass. So, the net effect of having a large body with a distributed set of forces acting on that body is that the velocity of the center of mass or more precisely, the moment of the center of mass is altered. So, in a simply... So, if mass is constant, so for m being constant.

So, we use the same capital M to denote the total mass of the body. Summation over all the forces vector sum is the mass times acceleration of the center of mass. So, this is direct extension of Newton's laws to rigid bodies. Now, from our earlier discussion of kinematics, we know that rigid bodies are also able to rotate not just translate, meaning this.

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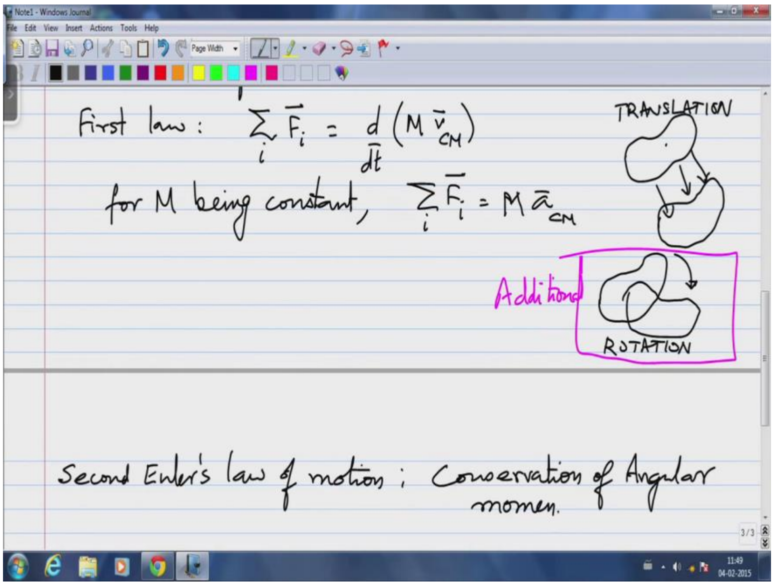

If I have a rigid body, it can move in a translational sense which would be simply that kind of motions or it could rotate. So, the whole body could rotate in this fashion, so this is translation and this is rotation, so this rotation is an additional degree of freedom that is enabled for a rigid body. So, in plane kinetics, meaning if I am constraint to move on a plane, let us say the plane of this board, the first Euler's law which was derived from applying the first this Newton's second law to a rigid body is sufficient.

That if I have to also study this additional degree of freedom, then there is a second law which pertains to conservation of angular momentum. So, how do I define angular momentum?

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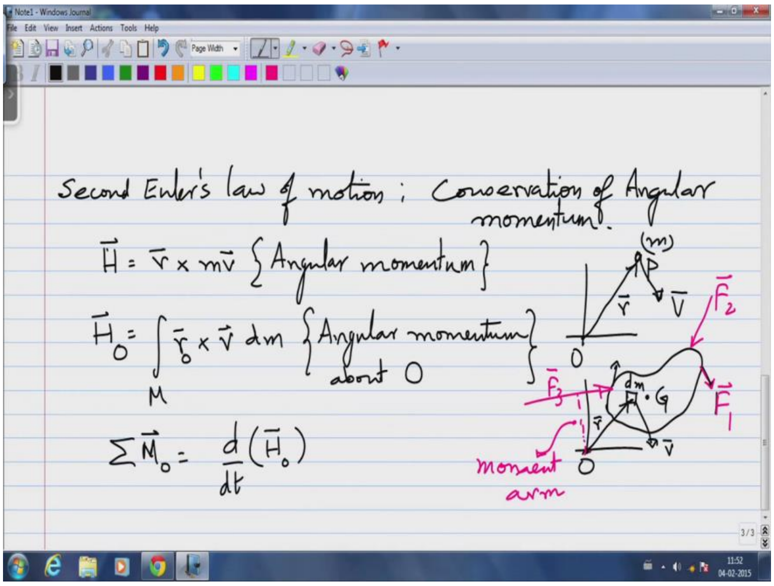

The angular, let say if I have a point particle P again at a point O, this point particle has some linear momentum, linear velocity v and a mass m and if this r is a current position vector, the angular moment is given by r crossed into m v. So, this is what is defined as the angular momentum of the particle, for a rigid body I can define the angular momentum. So, if I have a rigid body and like we said this is center of mass, but different points are moving at the different velocities.

So, I can compute the angular momentum about any point and that is obtained by performing this integration of r cross v d m over the whole volume. So, this I leave a subscript O, meaning this is the r as observed from O. So, if I take an elemental mass d m, I know the position vector r from the position O to d m r times v d m and if that point locally has some velocity v, the r times v d m integrated over the whole volume of the body gives me our… Really, this is integration over the whole mass of the body.

Since, we are looking at d m, this gives us the angular momentum about O. So, what the subscript here it is O means, it denotes angular momentum about O, so we need an additional law of conservation for this angular momentum. So, just as four force causes change of linear momentum in moment, so if I have a force F some other force F 2, each of these forces causes a moment. So, I can take a perpendicular to this, that is what is usually referred to as moment arm.

So, the sum of all moments acting on a rigid body about O is equal to the rate of change of angular momentum. So, this looks very analogous to Newton's second law.

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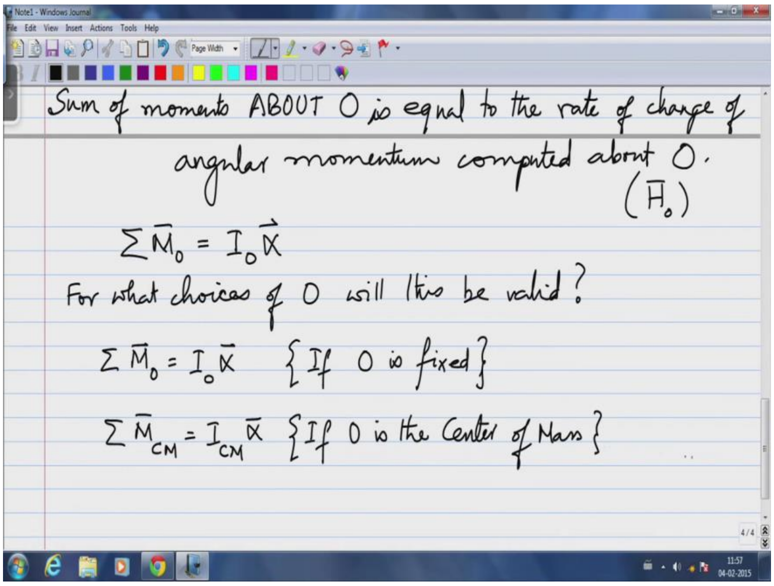

So, we will write it in English, sum of moments about O, so is equal to the rate of change of angular momentum computed about O, an axis through O. So, this second part is what we are referring to as H o and the moment is computed about m. So, unlike angular velocity, moment is a vector, but it is constraint to an axis. So, a moment is calculated about an axis, the axis in this particular instance of plane kinetics is a line passing through O perpendicular to the plane of the paper.

So, we are computing the action of all the forces in an angular sense about a line passing through the point O perpendicular to the plane of the paper. Likewise, the angular momentum is also computed about a line passing through O perpendicular to the plane of the papers. So, this H sub O is the angular momentum about an axis passing through O perpendicular to the plane of the paper. So, I supposed to angular velocities and angular accelerations, angular momentum and moments are computed about fixed axis about axis.

So, this looks nice, because this looks analogous to our Newton's second law, the sum of forces equals rate of change of linear momentum that is what we had here ((Refer Time: 14:47)). So, this is our first law or simply the sum of all forces on a rigid body equals mass times the acceleration of the center of mass. So, this looks nice and analogous, so this sum of all moments about O equals I the moment of inertia about O times alpha of that rigid body. Notice, how I did not put center of mass here, alpha is a free vector.

Now, for plane kinetics I moment of inertia happens to be just a number, it is a scalar, whereas in three dimensional kinetic, moment of inertia is not just a scalar. Now, we have to be a little carful, for what choices of O will this be valid. So, we have to ask this question, so if O is moving would this still be valued? So, there will be several applications, where I would like to choose O my origin as a point inside the body to be moving with the body.

So, with this calculation of sum of moments equal to I sub O times alpha also apply to that condition. Now, I am not going to derive this that I am going to write this down that there are three specific instances, where this simplification will apply. So, the first one is if O is fixed, it does not matter where O is it could be in the body or outside the body, if O is a fixed point about which your computing the moments and the moment of inertia, then this equation will work.

The second condition is, if I compute the moments about the centre of mass and I compute the moment of inertia about the center of mass that will also be a case, where this simple nice and elegant form that we get from Newton's second law is still valid.

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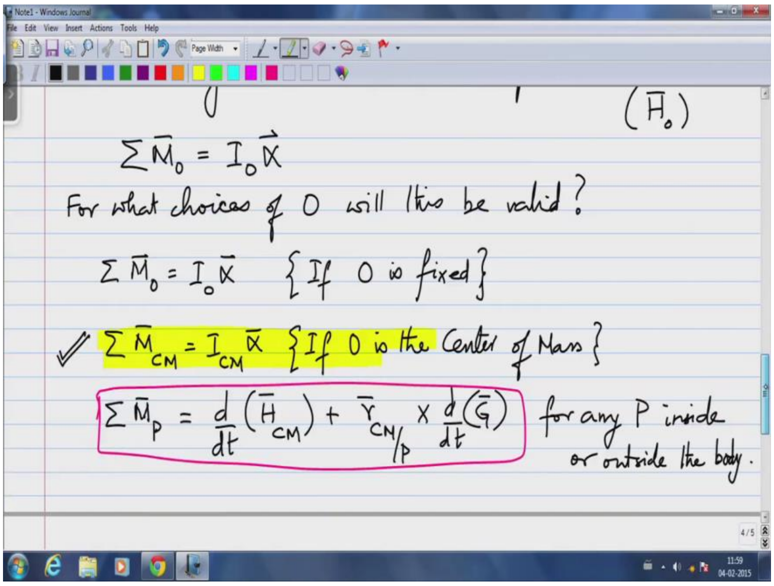

For any other point p, if I have to do this, if I compute the moments about some other point p, then I have to take and additional term involving that distance of this center of mass as observe by p. So, this r bar C M as observe by p is a vector that has a head ending at the center of mass and tail at p and you have to account for an additional cross product, where the rate of change of linear momentum computed of the center of mass is included.

So, as far as possible we will avoid this possibility, so this is the form for any p inside or outside the body. We will use this from very often, where the center of mass will be choosing as our point about which we will compute the moments. If may or may not be a fixed point, it may be an accelerating point there is no reason to believe that the point has to be fixed. So, wherever be the nature of the motion of the center of mass, if I compute moments about the center of mass, the sum of the all the moment equals I center of mass times alpha.

So, this shall be our point about which we will compute the law of conservation of angular momentum, in all the future example problems. And I strongly recommend sticking to this mode of solving problems, in the sense that if you choose the center of mass as the point about which you compute this about the moments, the law of conservation of angler momentum is very is exactly analog is to the laws conservation of linear momentum.

Therefore, it is a very elegant choice, the mathematics in seven senses we involve one or two steps more. But, the procedure would be very straight forward in you be able to follow along a lot easier, if you chose the center of mass as the point about which you compute your moments in angular momentum. We will solve some example problems in the next set of videos.