Statics and Dynamics
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Lecture - 22

Let us continue our discussion of kinematics; we had looked at some example problems in the past. We are going to continue that process, we are going to look at a couple more example problems.

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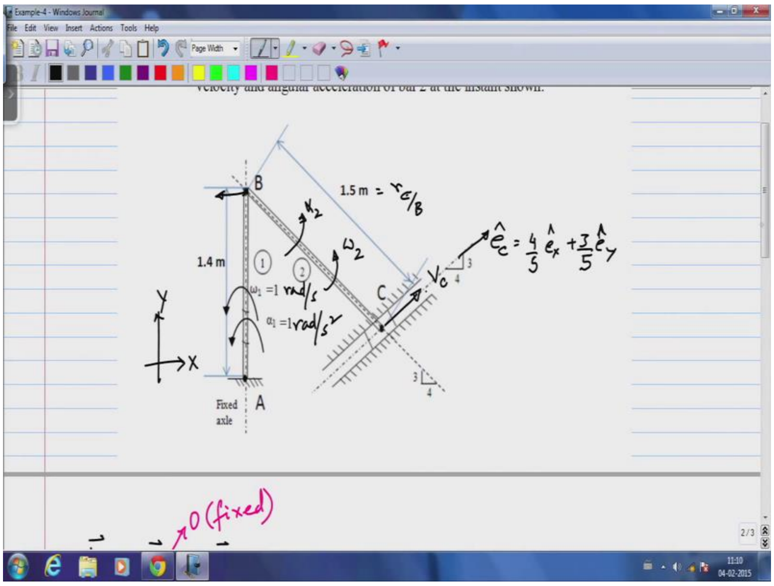

So, here is our first one for today, this is what we will call crank slider mechanism. So, essentially this is a bar A B that is free to rotate about A and there is a pin jointed at D at which the two bars A B and B C meet. Now, C is a slider that is constrained to move on a grove that is on this 4, 3, 5 triangle and we have some lengths. The bar A B, A has a 1 radian per second angular velocity and 1 radian per seconds squared angular acceleration.

We are asked to find the angular velocity and angular acceleration of the bar B C, as well as the linear velocity and the acceleration of the slider. So, we are going to use our relative velocity and relative acceleration calculations that we start it to that we used in the previous two examples as well.

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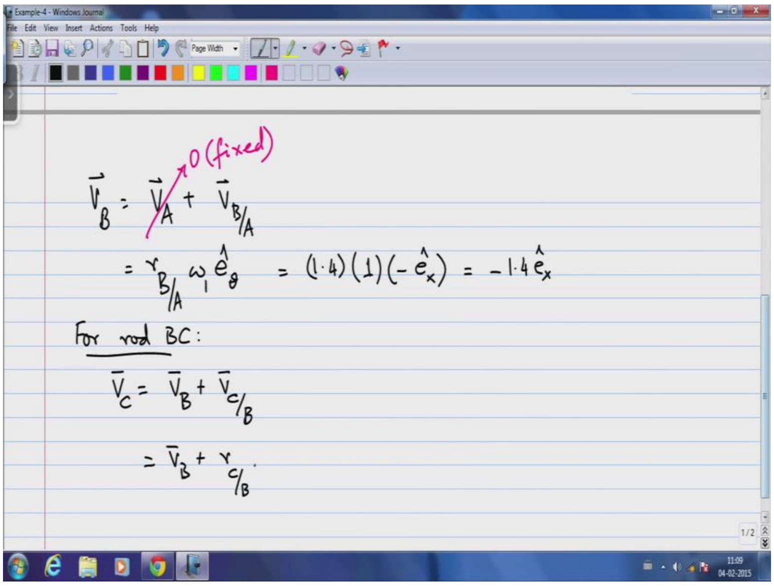

So, we will start with the bar A B and velocity of B equals velocity of A plus the velocity of B is observed by A. This is a law of relative velocity, velocity of A happens to be 0, because it is fixed. The velocity of B as observed by A is r of B as observed by A times omega times e theta omega 1. So, in this particular instants the r of B as observed by A is 1.4 meters in length, omega 1 is 1 radian per second e theta ((Refer Time: 02:03)) in a Cartesian coordinate system e theta happens to be minus e x.

So, that is this point is moving in this direction, so therefore, the velocity of B is minus 1.4 e x. Let us get the velocity of… Now, do the same analysis for the rod B C, the velocity of C equals velocity of B plus the velocity of C as observed by B. The velocity of B is what we know plus the velocity of C as observed by B is this r of C as observe by B ((Refer Time: 03:02)) the magnitude which happens to be the length of this rod 1.5 meters.

And we do not know the angular velocity omega 2 and too we do not know the angular acceleration, these are two quantities we have to find. And in addition, we do not know this B C the speed of this slider along the grove, so let see what that tells us times omega 2 and e theta of C as observed by B. So, let us see what we do know about the velocity of C the vector is that it violates speed is V C. The unit vector along this direction e C I call this e c is 4 by 5 e x plus 3 by 5 e y, that is the unit vector along which the bar can move, so let us draw that picture to the side here.

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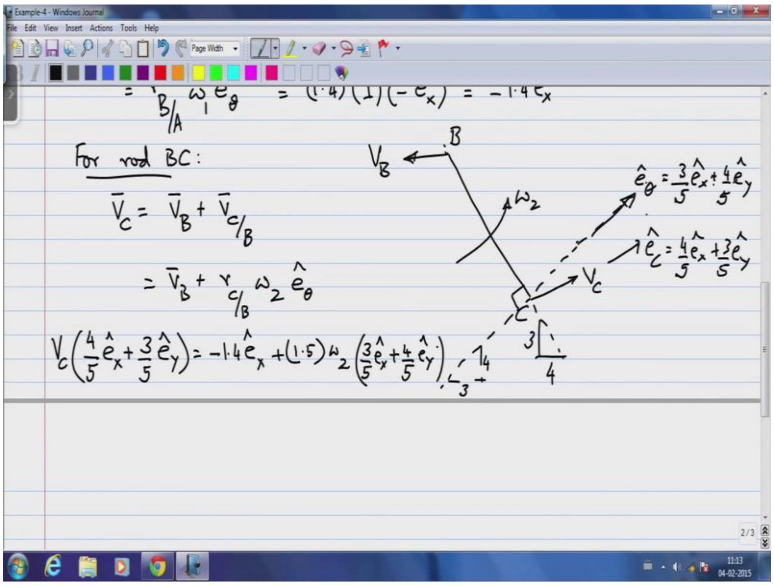

So, this is my bar B C, this is V C it has to be on the unit vector e C equals 4 by 5 e x plus 3 by 5 e y, V B is what we know which is in this direction. Now, if I know that this triangle is on a 4, 3, 5 in this fashion; that means, if I now draw a line perpendicular to that through here, a unit vector along this is my e theta. Because, I know omega 2 is in this direction, so this e theta if this is on a 4, 3, 5 triangle, this would be on a 3, 4, 5 triangle.

This is basically a fact that these two lines are perpendicular and for the 3, 4, 5 triangle you just interchange the sides and that is essentially amongst to the fact that the tan theta 1 times tan theta 2 equals minus 1. So, now on this triangle e theta is 3 by 5 e x plus 4 by 5 e y, so this e theta is the direction along which V of C as observed by B can fall. We talked about this the other way that C cannot have a component along the rod C B, it can only have a component perpendicular to the rod C B.

So, let us complete this, whereas could all the information we know in this equation, we know that the speed of this is V C and that the direction is along this V C that equals minus 1.4 e x, this is my V B plus r of C as observed by B is 1.5 meters times omega 2 is an unknown times the e theta unit vector is 3 by 5 e x plus 4 by 5 e y.

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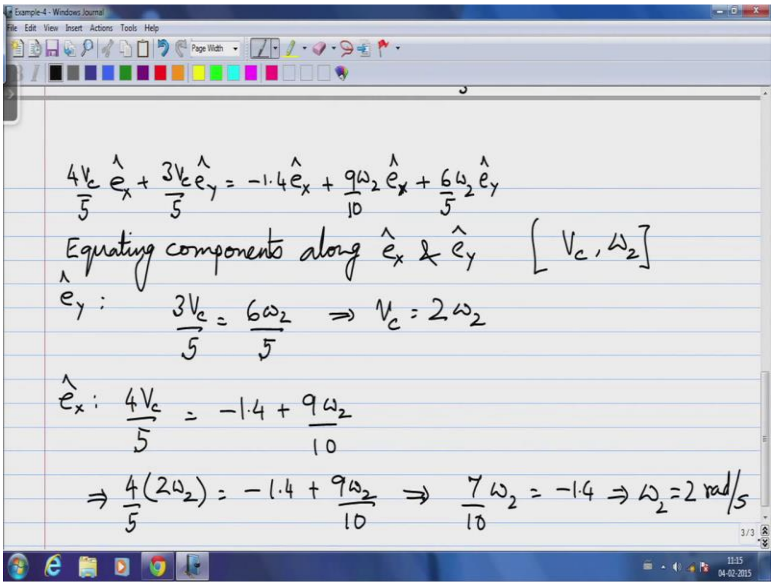

So, if I complete this I have a 4 V C over 5 e x plus 3 V C over 5 e y equals minus 1.4 e x plus 9 by 10 omega 2 e x plus 6 by 5 omega 2 e y. So, the first, so I can equate components along e x and e y, I have two unknowns V C and omega 2 that I can solve for from these two equations. So, I begin with the e y equation which tells me 3 V C over 5 equals 6 omega 2 over 5 which implies V C equals 2 omega 2 and the e x component says 4 V C over 5 equals minus 1.4 plus 9 omega 2 over 10.

So, if I substitute V C equals 2 omega 2, 4 V C meaning 2 omega 2 equals minus 1.4 plus 9 omega 2 over 10. So, I have 16 minus 9 7 by 10 omega 2 equals 1.4 which implies omega 2 equals 2 radian per second.

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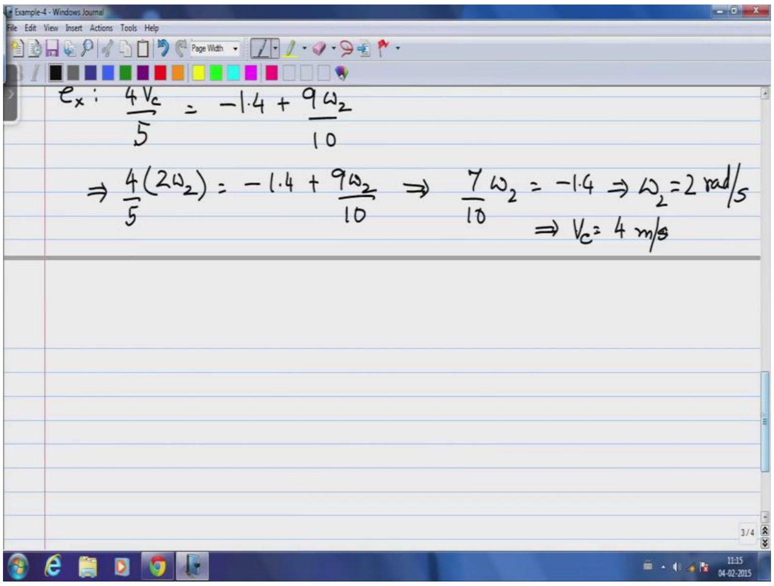

And which also implies V C equals 4 meters per second, so this rod this slider is moving with the speed of 4 meters per second it is of course, constrained to move inside the grove. The rod itself has an angular velocity of 2 radians per second, I will be… I say this once more that the rod has an angular velocity of 2 radian per second, it is not about any point, every point has the same angular velocity about every other point.

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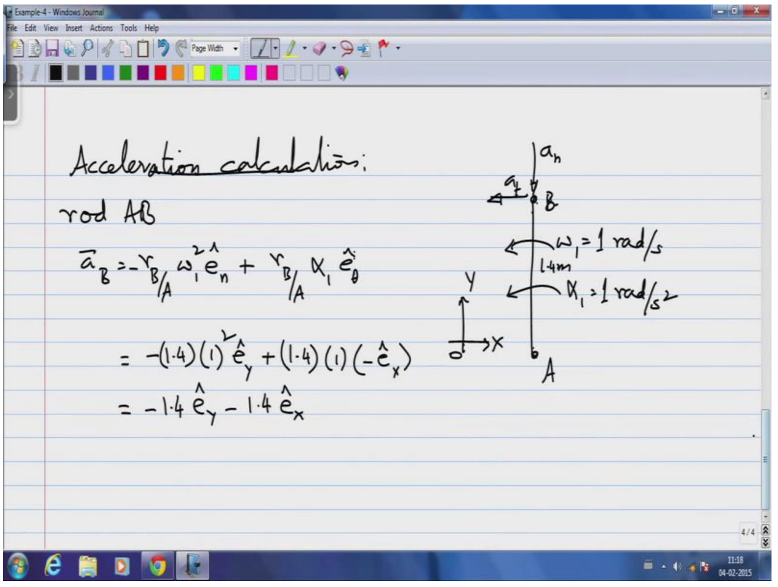

So, now, let us get to the acceleration part, so we will start with the rod A B which translate the information we had in the figure, omega 1 happens to be 1 radian per second and alpha 1 is 1 radian per second squared and this length is 1.4 meters. So, the acceleration of B has two components which is this r of B as observed by A omega 1 square minus e n plus r of B as observed by A alpha 1 e t or e theta. So, just to relay the coordinate system down to get our reference frame.

So, the acceleration B itself has two components, there is a normal component and then there is a tangential component given by each of these terms. The normal component is 1.4 omega 1 square e x, let us write it in a, if we substituted omega 1 and that gives me minus 1.4 times 1 square e n e x plus saying 1.4 into 1 times minus, this happens to be e y. So, this unit vector e n is along minus e y and a t is along minus e x. So, this is minus 1.4 e y minus 1.4 e x, so that is the acceleration of B.

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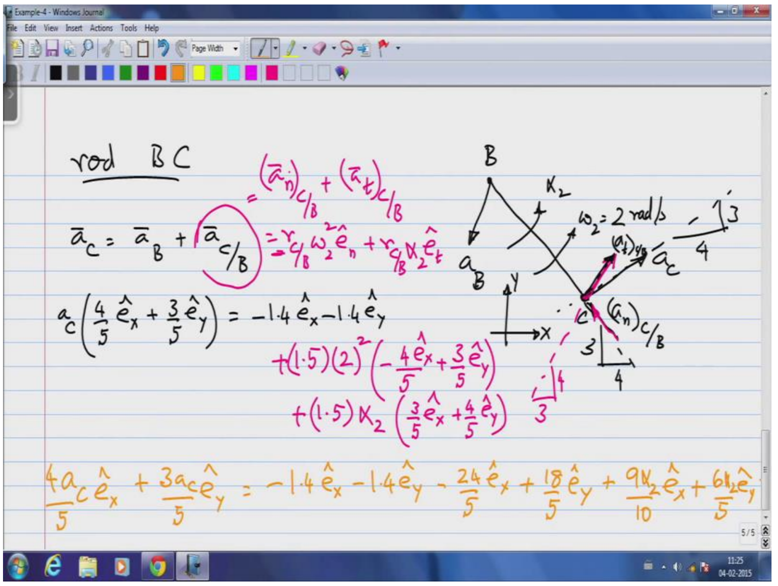

So, now, let us get the acceleration, let us do the analysis for rod B C, again will draw this, the acceleration of B is in some direction C happens to be in that slider. So, the rod B C has, is on the 4, 3, 5 triangle the slider C is constrained to move on a grove that is on a 4, 3, 5 triangle plus 4 plus 3, 5 triangle as shown in this figure. If the slider point C is constrained to remain in the grove it cannot have, because it is only moving on a straight line.

Because the grove itself has more local curvature, the only possibility is that the acceleration of C is also co linear with the grove. So, cannot have a component of acceleration perpendicular to the grove, if the grove itself has a curvature that cos that then there exist a possibility that the component of acceleration perpendicular to the grove may be non zero that since the grove is just a straight dove, the slider C is constrain to move on a straight grove which also means that the acceleration is only along that straight grove.

So, these are all information we know the solve for omega 2 and that has an magnitude of 2 radian per second and we do know what alpha 2 s. So, alpha 2 and a C or a two new unknown variables. So, will use the same aloof relative accelerations, so the acceleration of C is acceleration of B plus the acceleration of C as observe by B, again a C is the magnitude of that acceleration, the unit vector along which that acceleration occurs, y the acceleration of A B is minus 1.4 e x minus 1.4 e y, the acceleration of A of C as observe by B has two parts to it.

So, those are normal component of C is observed by B and then there is a tangential component of the C as observe by B, I will use the different color to denote these two. So, these are the two components of this acceleration I will write each of this separately in the same color plus this part here is r of C is observed by B omega 2 square e n with the negative sign primarily showing it is coming from C to B plus r of C is observed by B alpha 2 e t.

So, the magnitude of the centripetal acceleration is 1.5 omega 2 square, the e n unit normal vector is of the form is going in the negative x direction, but positive y direction. So, it is unit vector is minus 4 over 5 e x plus 3 over 5 e y, so that is the normal vector going from C towards B which is the way centripetal acceleration works A t is along another unit vector that is going perpendicular to this 4, 3, 5 triangle it is magnitude firstly, is this 1.5 times alpha 2 we do know what alpha 2 is as yet.

But, the unit vector is on a 3, 4, 5 triangle, so this would be 3, 4, 5 and so it going both in a positive x direction and the positive y direction. So, let us write one simplifying step here and then... So, 4 A C over 5 e x plus 3 A C over 5 e y equals minus 1.4 e x minus 1.4 e y plus 1.5 times 4 is 6. So, this becomes minus 24 over 5 e x plus 18 over 5 e y, so write 1.5 as 3 half's, so this becomes 9 alpha 2 over 10 e x plus 6 alpha 2 over 5 e y. So, we left with the same problem as we had with the velocities I have a vector equation involving e x and e y.

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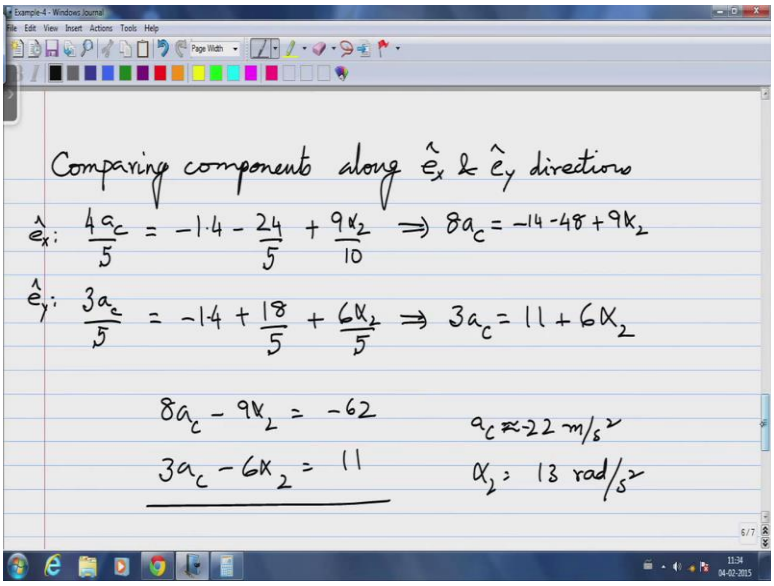

I can compare it essentially means that I have two scalar equations, one for the e x direction, one for the e y direction and both this equations involves only two unknowns A sub C and alpha two comparing components, the first one is 4 a c over 5 equals minus 1.4 minus 24 over 5 plus 9 alpha 2 over 10. So, I simplify this slightly sum multiplied everywhere by 10 I have 8 a c equals minus 14 minus 48 plus 9 alpha 2. The second equation I have is this is from the e x comparison this is from e y comparison threes a sub c over 5 equals minus 1.4 again plus 18 over 5 plus 6 alpha 2 over 5.

So, if I multiply throughout by 5, so this becomes 18 minus 7 11 plus 6 alpha 2. So, I will write those equations 8 a c minus 9 alpha 2 equals minus 62 3 a c minus 6 alpha 2 equals 11. So, if you solve these two equations you will find that a c is approximately equal to some 22 meters per second square with the negative sign and alpha 2 is 13 radians per second square. So, this gives as essence for what how you can use the laws of relative motion to solve problems involving multi body systems.

So, this is an example where we now have 3 bodies A B is a rod, it is a rigid rod, B C is another rigid rod in the slider C being in a grove is a constrain that is a post at C. So, it is too rigid rods with a slider C, potentially the slider is also a third rigid body, but it is only constrain to linear accelerations and linear motion. So, you could think of this as a 3 digit body system.