Statics and Dynamics
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Lecture - 21

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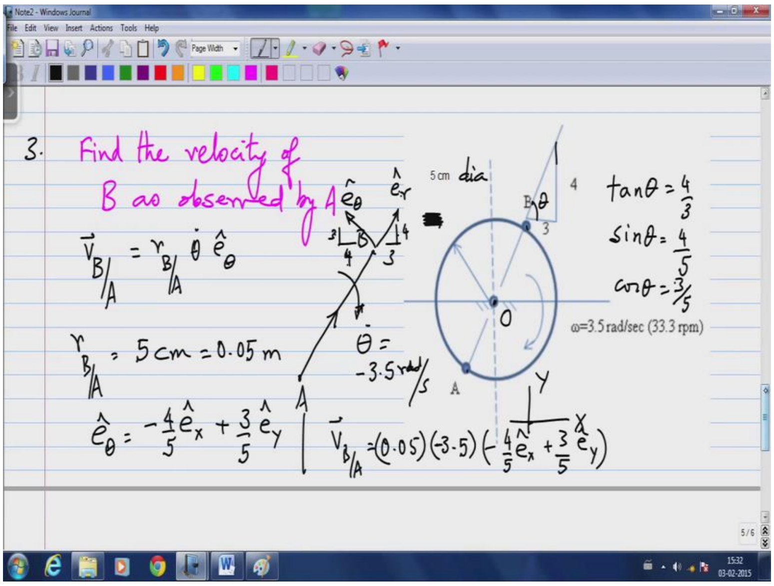

We will take on a couple of example problems to understand relative motion and absolute motion in rigid bodies. So, let us look at the first example; I have a disc, the disc is of diameter 5 centimeters, and I have two points located on that disc A B, and this A and B are located along a line which makes this familiar 3, 4, 5 triangle with respect to the horizontal. So, the meaning of this is that this angle theta is such that tan theta equals 4 over 3. So, that is the meaning of showing this kind of a triangle 4, 3, 5, triangle. So, I have this line segment A B that makes this, that this theta such that tan theta equals 4 over 3.

And we are asked to find the velocity of B as observed by A and I will just mark this for completeness that I have a fixed point on this rigid body O and that fixed point is, I mean the body is pivoted at that fixed point, I want to make this clear once more from the last lecture. You see this body is rotating clockwise with an angular velocity 3.5 radians per second or roughly 33.3 RPM that does not mean the rigid body is rotating about O.

Although, it looks like it is rotating about O only, because the point O is fixed. But, what actually is happening is that every point on the rigid body is rotating about every other point on the rigid body with exactly the same angular velocity. So, I can be any point on the rigid body, let say O every point on the rigid body is rotating about O at 3.5 radians per second you will probably not dispute that. But, if I am A every point on the rigid body is rotating about A with exactly the same angular velocity 3.5 radians per second.

And that includes the point O, the point O is rotating about A with exactly the same angular velocity 3.5 radians per second and I want to let this example bring that concept to you in a slightly more clearer fashion. So, let us start making our calculations we need to find the velocity of B as observed by A which we use this notation to denote that is r of B as observed by A theta dot e theta.

So, if I take the vector B as observed by A there is an e r and then e theta, theta dot in this case is 3.5 radians per second and in keeping with our notation that counter clockwise will always be positive. I am going to write this as minus 3.5 radians per second, r of B as observed by A is 5 centimeters I like to deal in SI units, so I am going to convert that to meters. So, this... So, if I want to notice two things now, so this e theta is unit vectors, so we will put a little cap on it, I wanted to notice two things, this disc which was pivoted at the point O is no longer relevant to our calculation.

What is relevant is this line segment A B? This disk could have been any other shape, as long as it is rotating with an angular velocity 3.5 radians per second clockwise, all I need is the line segment A B, to answer a question that we have laid out for ourselves. So, e theta let us calculate what e theta would be, so if e r is on a 4, 3, 5 triangle e theta would be on 3, 4, 5 triangle. I am in a frame of reference fixed at some point, I want to write e theta in terms of e x and e y.

Because it is easy to represent velocity components in Cartesian coordinates and although I could have left it in e theta terms, I want to show you how to convert back to Cartesian coordinates also that just involves a little bit of trigonometric. This e theta is pointing in the negative x direction and in the positive y direction. The magnitude of the unit vector in the negative x direction is 4 over 5. So, you are walking 4 units to your left and 3 units up in order to make the triangle that completes e theta.

So, the unit vector along e theta let me in terms of e x and e y for this case is minus 4 over 5 e x plus 3 over 5 e y. So, the 5 comes from the fact that sin theta is 4 over 5 and cos theta would be 3 over 5. So, now, completing the calculation V of B as observed by A is 0.05 into theta dot which is minus 3.5 times a unit vector minus 4 over 5 e x plus 3 over 5 e y. If you do this multiplication, the answer you get is the velocity of B as observed by A, rate in Cartesian coordinates fixed in Cartesian coordinates, where e x e y represent the velocities at that point.

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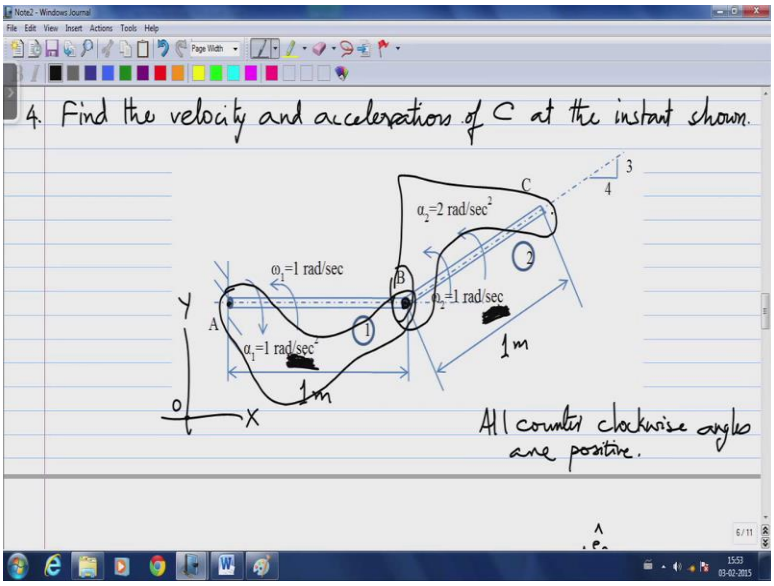

So, this is the fairly simply problem let us move on to a problem involving two rigid bodies that are coupled at some point called B. So, let us look at, let us understand what this is. I have a bar A B where one end of the bar is fixed at A and a bar B C, where the point B happens to be a pin joint that is shade by both the bars A B and B C. And we ask the velocity and acceleration of C at this instant, where the picture is shown.

The angular velocity of bar one which is A B is omega 1 given as value 1 radian per second in a counter clockwise sense, it is angular acceleration is 1 radian per second squared in a clockwise sense. Likewise, bar B C has an angular velocity of 1 radian per second counter clockwise and the angular acceleration 2 radians per second square counter clock wise. So, let us first find the velocity of C, so we have to work our way to see and to get to see we first need to understand the motion of B.

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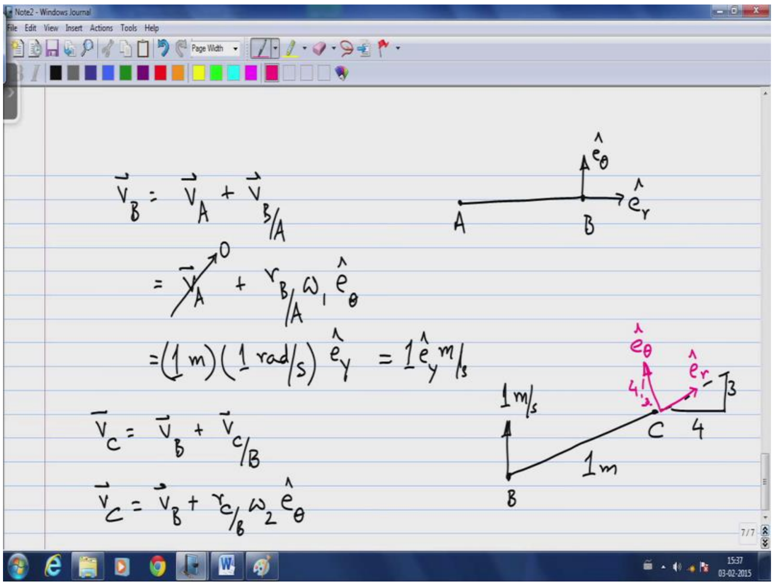

So, velocity of B is velocity of A plus velocity of B as observed by A. So, this is velocity of A which happens to be 0 plus r of B as observed by A times theta dot or omega 1 will use the notation from the figure and that is in a unit vector e theta. So, the velocity of B is simply 1 meter into 1 radian per second and if this is A and this is B, this is e r locally and this is e theta in a Cartesian coordinate system fixed at some point.

Now, remember I just need to define a unit vector set e x e y, I do not need this origin O to coincide with A. Because, I am only looking at velocities and accelerations not positions, in a form in that particular coordinate system e theta is simply e y. So, the vector velocity is 1 meter per second of magnitude and it is 1 e y meters per second, so this is the velocity of B. So, now, the velocity of C equals velocity of B plus velocity of C as observed by B, again the same D.

So, I am going to draw the trigonometry a little carefully, this is my B, this is my C, this is on a 4, 3, 5 triangle as shown here. The length is 1 meter, this point B is moving up with the velocity of 1 meter per second, we know that from the previous calculation. I want to find the velocity of this point C, the velocity of C as observed by B is simply r of

C as observed by B times omega 2 e theta. In this particular case, e r is again in this direction here e theta is perpendicular. So, if e r is on 3, 4, 5 triangle 3 being the vertical height, 4 being the base of this right angle triangle and 5 the hypotenuse, e theta is on a 4, 3, 5 triangle.

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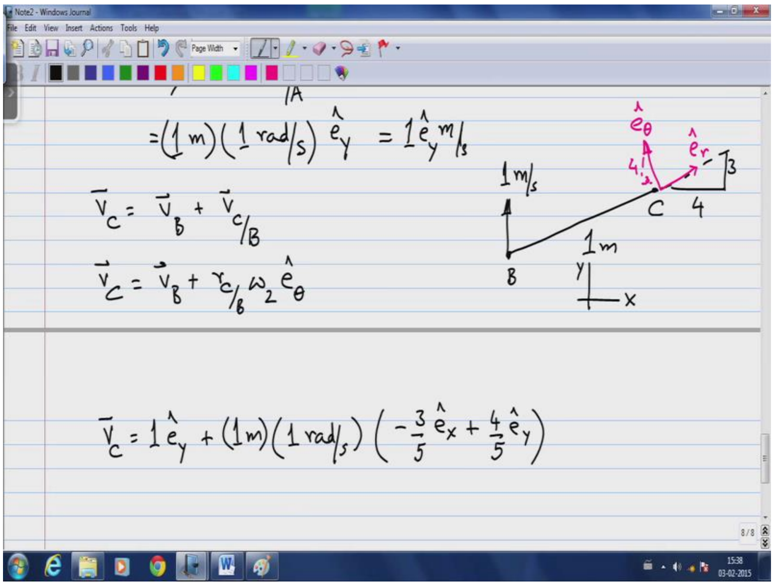

So, let us use that knowledge in our calculation V C is V B is now 1 e y plus RCB is 1 meter, omega 2 is 1 radian per second counter clockwise. So, we are going to stick with an notation that counter clockwise is positive, let me write that down up front ((Refer Time: 12:21)) and e theta is given by I have to get, to define the unit vector e theta in the x y coordinate system, I have to walk 3 units to my left which is in the negative x direction and 4 units up which is in the positive y direction. So, that happens to give me the unit vector that looks like z.

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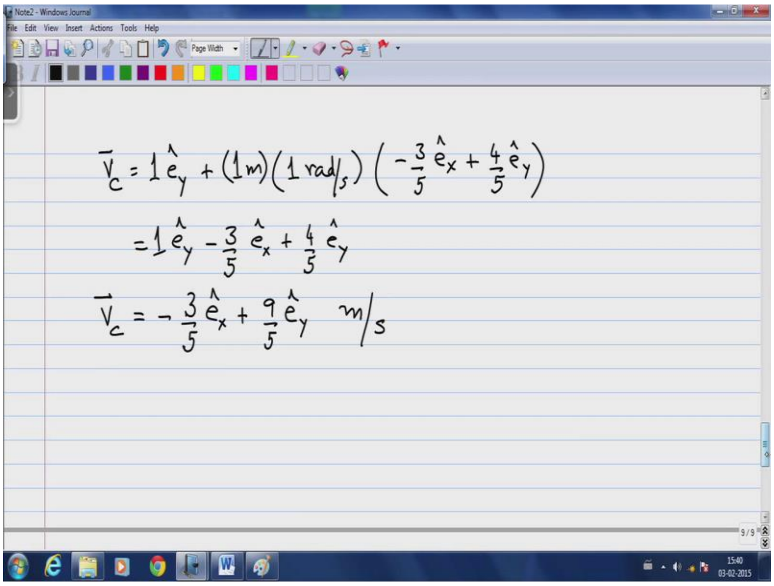

So, if I now simplify this one step further 1 e y plus 1 into this which is minus 3 by 5 e x plus 4 by 5 e y. So, V C is given by minus 3 by 5 e x plus 9 by 5 e y and this is in units of meters per second 3 by 5 units of velocity in the negative x direction and 9 by 5 units of velocity in the positive y direction. So, just to reconcile this with what we understand from physical reality, if the point B is moving up, the point B is moving up and B C is rotating all happening simultaneously.

So, there is a net translation of this whole bar upwards by the velocity given by the velocity of B super imposed on top of that is this rotation of the bar B C. So, the velocity of C especially in the upward direction should naturally be more than the velocity of B primarily, because the bar is rotating in a counter clockwise sense. So, this additional velocity component due to the rotation is of magnitude 4 by 5 e y that when added to 1 gives us 9 by 5 e y as the vertical velocity, the rotation is the only part that causes the x direction motion, so now let us get the acceleration part going.

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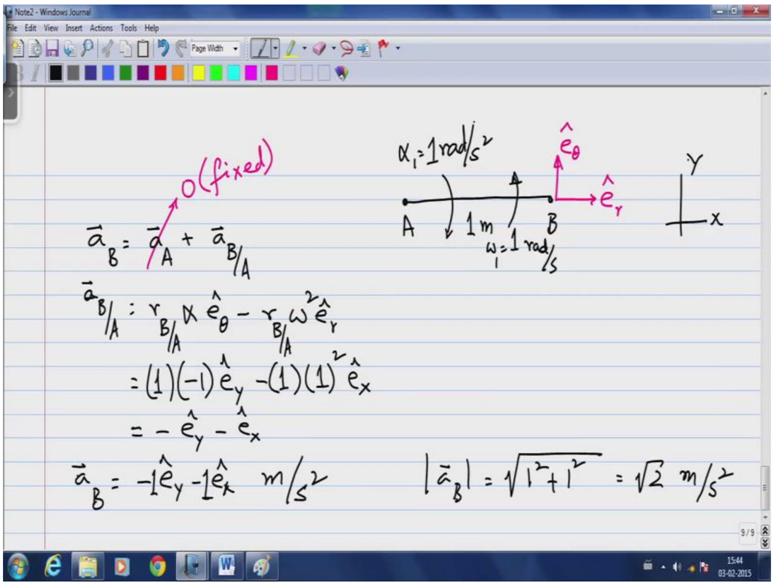

Again I am going to draw this bar A B of length 1 meter, acceleration of B is acceleration of A plus acceleration of B as observed by A. Now, this part has an omega in a counter clockwise sense of 1 radian per second, and then angular acceleration in the clockwise sense of 1 radian per second square. So, this alpha is the same as theta double dot that we had in our previous calculation.

So, the acceleration of A first of all is 0, because A happens to be completely fixed, the acceleration of B as observed by A has two parts to it, there is an r alpha e theta minus r omega square. Let me write it the way we did before, r of B as observed by A times alpha e theta minus r of B as observed by A omega square e r. Now, in this particular coordinate system e r is in that direction and e theta is that way.

So, the acceleration of B as observed by A is this 1 meter times alpha given that it is in a clockwise sense is minus 1, e theta is in the e y. So, I am going to draw the x y coordinates just for reference again minus r of B as observed by A is 1. Now, omega 1 is 1 radian, so that square e r is e x, so when I resolved this, this is minus e y minus e x. So, the acceleration of B is equal to minus e y minus e x meters per second square. So, the magnitude of this acceleration of B is square root of 1 square plus 1 square, because I have a 1 here and a 1 here. So, that is the magnitude of acceleration of B.

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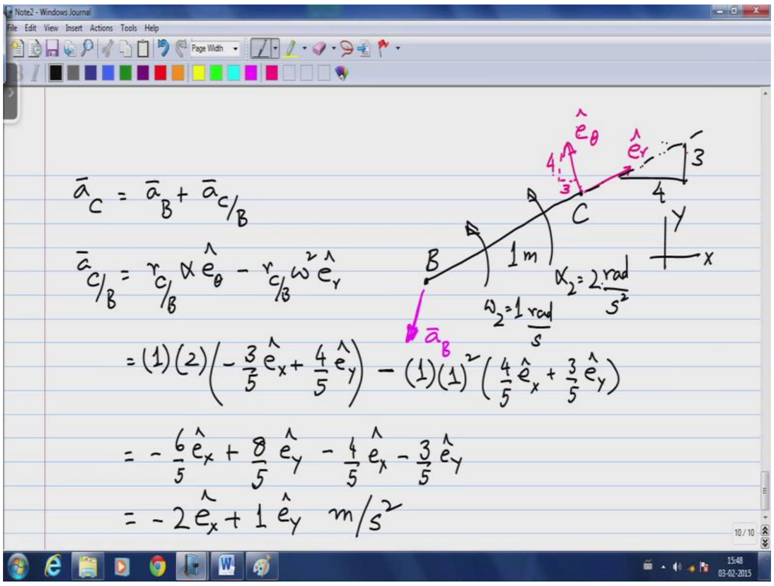

So, let us continue on to bar B C we are told the bar B C is on this 4 through 5 triangle. So, given that e r is along that bar and e theta is perpendicular to it and that as we said will be on a 3, 4, 5 triangle. So, 4 base 3 height is what e r is on 4 height, 3 base is what e theta would be on. We already found the acceleration of B and just for the sway of exactness I am going to draw the acceleration vector exactly the way it came out.

So, B is accelerating in this direction, where the magnitude square root of 2. So, the acceleration of C is now equal to the acceleration of B plus the acceleration of C as observe by B. And the acceleration of C as observed by B has the same two components r of C as observed by B which is the length of the bar B C, which in a case 1 meter times alpha times e theta minus r of C is observed by B omega square e r.

So, r of C is observe by B is 1 alpha is get those numbers from or initial problem statement omega 2 happens to be 1 and alpha 2 happens to be 2 will get those on here, omega 2 happens to be 1 and alpha 2 happens to be 2. So, alpha is plus 2 because we said counter clock wise rotations should be positive and e theta, the unit vector e theta is obtained by walking 3 units to my left which is the negative side of x axis and 4 units up which is the positive side of y axis.

So, the e theta is minus 3 by 5 e x plus 4 by 5 e y minus r of C as observed by B is still 1 omega 2 happens to be 1 square e r is a unit vector that is obtained by leaving 4 units in the positive x direction and 3 units in the positive y direction. And therefore, the unit vector is given by 4 by 5 e x plus 3 by 5 e y. So, let us complete this calculation, so if I multiply this out I have minus 6 by 5 e x plus 8 by 5 e y minus 4 by 5 e x minus 3 by 5 e y.

I simply by this further minus 6 by 5 and minus 4 by 5 added give me minus 2 e x plus 8 by 5 minus 3 by 5 give me 5 by 5. So, that is plus 1 e y and this has units of meters per second square, when not done yet this is the relative acceleration of C as observed by B.

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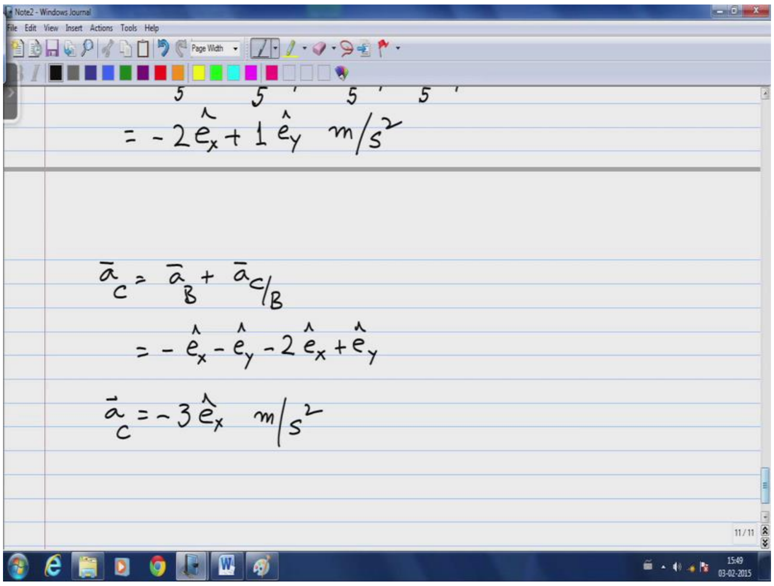

So, let us complete what we started out, the acceleration of C is the acceleration of B plus the acceleration of C as observed by B, we go back we calculated the acceleration of B and that was minus 1 e x minus 1 e y. So, this is minus e x minus e y acceleration of C is observe by B is minus 2 e x plus e y when I add these I get minus 3 e x and there is no component along e y. So, the acceleration of C is only in the negative horizontal direction and it has a magnitude 3 meters per second square.

So, this example is used to illustrate how you could use relative motion to understand kinematics. So, I first computed let us go back to the problem statement, a first computed the motion of B up and it is not motion of B about A, I computed the motion of B in a absolute sense. After that after I know the motion of B both velocity and acceleration is matter of fact that computed velocity is first and then accelerations, but you could do it anyway you choose.

So, I first computed the velocity of B, then computed the velocity of C, because I only need to add the component of velocity is of C as observe by B to the already computed velocity of B. Once, we are done with the velocity computation I move onto the acceleration computation. So, the first thing we did is compute the acceleration of B and then all I need to do is add the relative acceleration of C is observed by B, the acceleration of C is observed by B as two parts; one towards B, which is the centripetal acceleration and the second tangential to the bar B C, which is due to the angular acceleration of the bar B C. Now, I will rewrite this once more that when we show that this bar B C has an angular velocity 1 radian per second and it has an angular acceleration 2 radian per second square that is not the angular acceleration about B, it is simply the angular acceleration and angular velocity of that bar, it is not about any point it is a free vector, angular accelerations and angular velocities are free vectors they are not tight to an access about which the body rotates.

The angular acceleration in this case is 2 radians per second square and it is a counter clock wise. So, if you are writing the angular acceleration is a vector it could essentially say that it was 2 e z, e z being the unit vector pointing out of the plane of r bore. I hope this example illustrated the use of relative motion to compute accelerations. Now, these bars were shown as just little straight bars, in reality these could have been arms of a robot.

For example, the arm itself could have been a weird shape kind of like that with the second perverted arm that looks like this, it really does not matter what the actual shapes of the bars are, because we are dealing with rigid bodies, the distance between any one pair of points is fixed and we will take the pair of points of interest was and replace the real rigid body with the simple line segment joining the pair of points of interest to us.

So, any time you see a line segment, the line segment is a representation of a rigid body and that rigid body could have been of any shape, the shape itself does not matter and that forms the basis for all are future computations. So, I hope this illustrated the use of relative motion and we will start looking at sliders which adds another small degree of complexity in the next class.

Thank you.