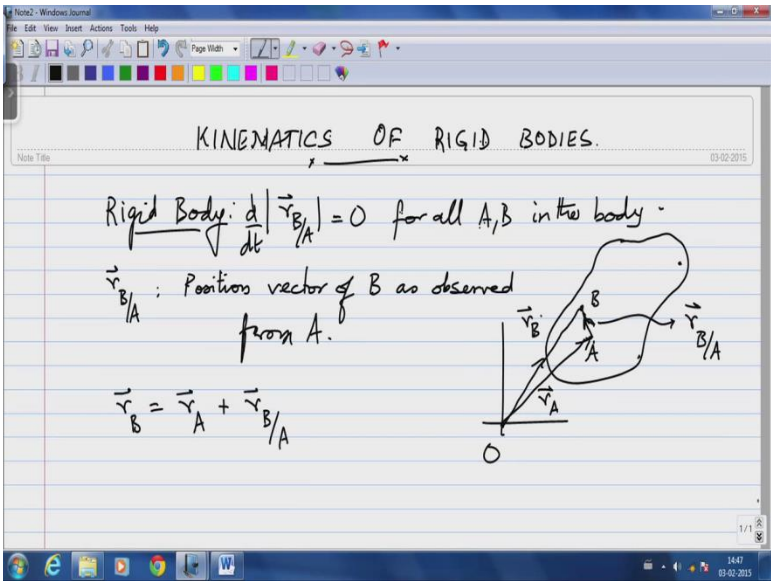
Statics and Dynamics
Dr. Mahesh V. Panchagnula
Department of Applied Mechanics
Indian Institute of Technology, Madras

Lecture - 20

We will switch one more gear; you are still in the realm of kinematics which is understanding motion without considering the forces that cause that motion. But within the realm of kinematics we are going to move on and relax our assumption of point particle. So, up until now we delt with point particles which have no designable shape or size, all of the dynamical systems, all of the bodies we see around us are not point particles, we want to understand how rigid bodies move.

(Refer Slide Time: 00:41)

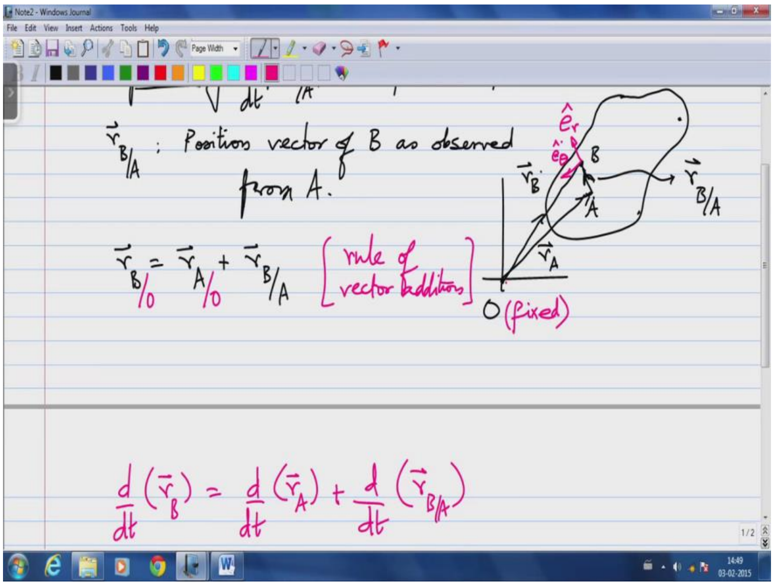


Now, rigid bodies we will define what a rigid body is, a rigid body let say I will draw a quick schematic. If I take two points and construct a vector r B as seen by A. So, we are going to use this notation to be read as position vector of B as observed from A. So, given this our definition of rigid body is the magnitude of r of B as seen by A d d t is 0 for all A comma B in the body, that is I can take any pair of points in any two corners. The position vector of one point in relation to the other point may change only in direction, but not in magnitude.

The magnitude of the vector B as observed by A is unchanged that is essentially our definition of a rigid body. So, we will construct this a little more deeply, so what I have shown here is I am going to now start by defining an origin. From the origin I can define two points I will call this A and B and this bit is r of B as observed by A. So, when I show r of B as observed by A that is the position of B as observed by A and the arrow will point towards B and the tail will point towards A.

So, these three vectors r A, r B and r of B as observed by A form a triangle and I can write them as r of A plus r of B as observed by A. So, the position vector of the point B is the same as the position vector of point A plus the position vector B as observed from A.

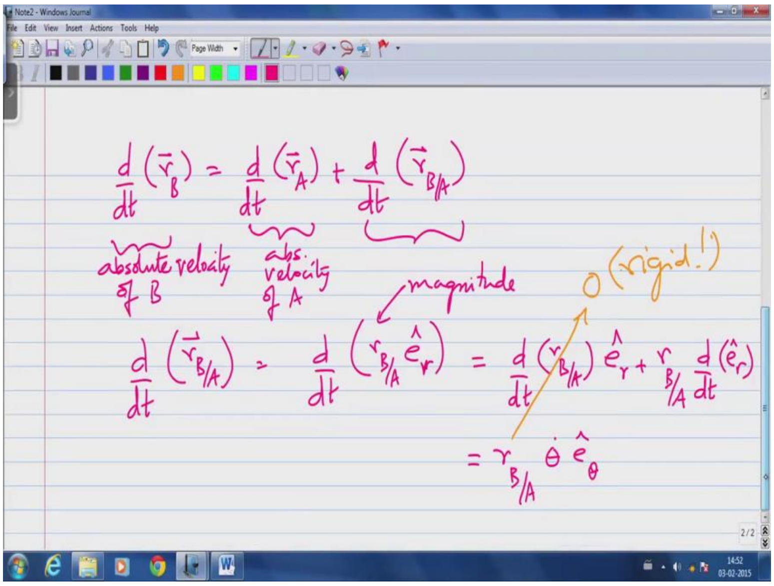
(Refer Slide Time: 03:42)



Now, what I have essentially done is I am not indicated this, but this is the position vector of B as observed from O, the position vector of A as observed from O, that O itself is a fixed point in this case. So, this forms, this is basically rule of vector addition, so when you add to the test you essentially go from the tail of the first vector to the head of the second vector that is the, let essentially what this rule represents.

So, if I now take the time derivative of this that r of B as observed by A has the unit vector e r pointing away from B, and then e theta that is pointing towards O in this case going to a right hand rule.

(Refer Slide Time: 05:13)

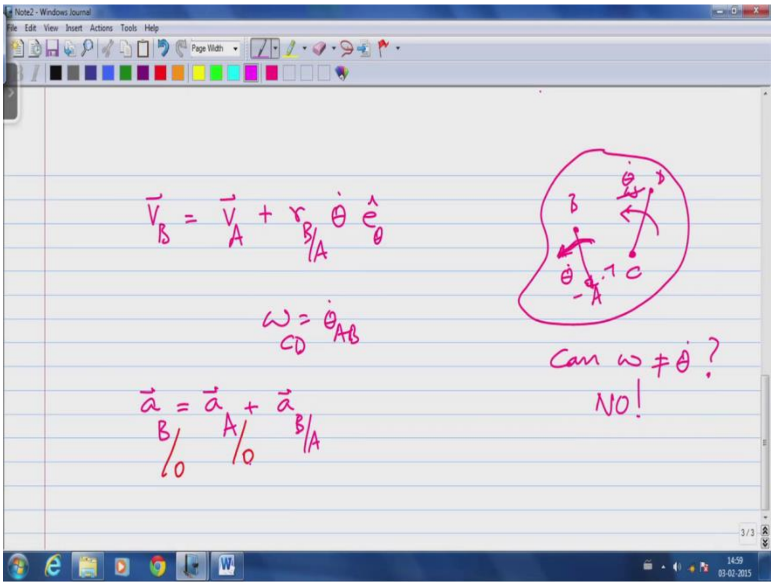


So, what essentially this means is, this is the absolute velocity of B and this is the absolute velocity of A, this third part can be… We are going to look at this third part a little more closely, I will write that out here following our previous notation. So, this is d d t of r of B as observed by A, this little r this happens to be the magnitude of the vector times e r plus r of B is observed by A d d t of e r itself.

Remember, in curvilinear coordinate systems the unit vector set are also functions of time, the unit vector set is a function of time. So, this from our basic definition of a rigid body we find that this part has to go to 0 that is because our body is non deforming, that is any two points B and A remaining at the same distance with respect to each other irrespective of what is happening to the motion of the body.

So, the only term that we have remaining is this, which is given by theta dot e theta that is, if I take two points B and A in the body given that the distance does not change, the only motion possible is that, that line A B rotate about O. In fact, there is no such thing has rotate about O, there is only such a thing has rotate. So, if the line segment A B rotates with an angular velocity theta dot.

(Refer Slide Time: 07:48)



So, what we now found is that r of B or I will write it slightly more succinctly V of B equal to V of A plus r B as observed by A theta dot e theta. So, that is the absolute linear velocity of B in the O in the fixed frame of reference is given by the summation of two vectors. The absolute linear velocity of A in the O frame of reference plus a magnitude plus a tangential component of velocity given by r of B as observed by A times theta dot, theta dot is the rate of angular rotation of the line segment A B.

So, given that if I take a rigid body, if this line segment A B is rotating with an angular velocity theta dot, we let us ask ourselves the question can another line segment C D rotate with different angular velocity omega. Can omega not equal this angular velocity theta dot? We can do a simple thought experiment and find out, if this line segment is rotating differently. So, if I had two line segments and if this line segment was not rotating with the same angular velocity as this line segment, then the position the distance between these two points in other schematic B and D would change.

So, if every pair of points remain at the same distance with respect to each other for all times, then the only way that can happen is that these two line segments A B and C D are both rotating with the same angular velocity. So, first point to take home is that the angular velocity of the rigid body is the same at all points in the rigid body. It is not restricted to a particular region of the rigid body.

Let us I think fairly simple, but important to ((Refer Time: 10:06)) understand, the second point is what is this point rotating about. So, if I now take let say the line segment A B and I, the way we drawn the schematic, if line segment A B is a rotating it essentially gives as a connotation that B is rotating about A with an angular velocity theta dot. So, if for the first part we found that can omega not equal to theta dot, the theta dot. The answer is no, omega has to equal theta dot or omega C D equals theta dot which is the angular velocity of A B.

So, if I, the way we drawn this let say it gives us the impression that B is rotating about A at an angular velocity theta dot with an angular velocity theta dot that if I now go back the line segment C D in the same schematic, it gives us the impression that D is rotating about C with an angular velocity theta dot. I am going to scratch out omega and put theta dot, because we found that omega has to equal theta dot.

So, given that some point B is rotating about A with theta dot at least in the schematic, I want to ask the question if I was the observer at B instead of being at A, if I was the observer at B what would be line segment A B look like it is doing. If I was at B, A in my frame of reference would be going in this direction at exactly the same angular velocity theta dot. So, if I was at A, B would look like it is rotating about me with an angular velocity theta dot.

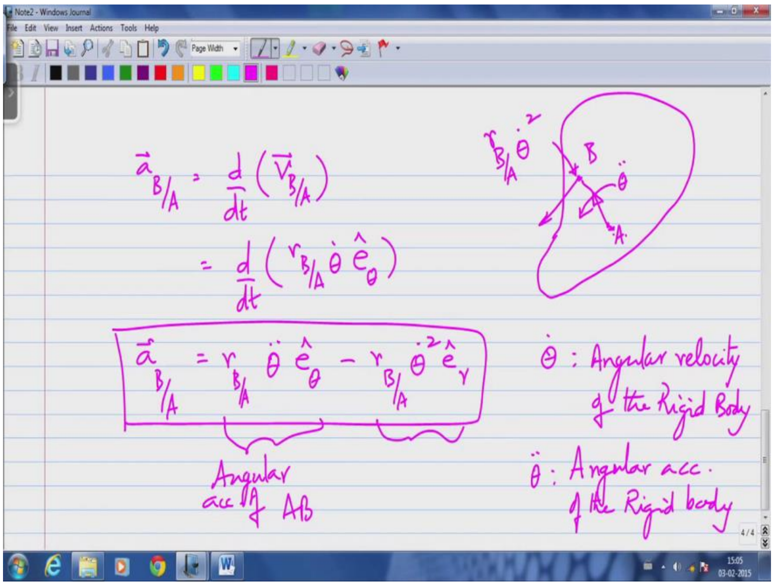
If I was at B, A would look like it is rotating about me with an angular velocity theta dot. So, this is a kind of an illusion that you get that you are at the center of the attention in a rigid body, wherever you may be in the rigid body. A simple playground example you can relate to understand this a little further is, let us say you are all sitting on a merry go round. In a merry go round is going around a little central axle. But, if I was sitting in a little chair in the merry go round and I look at the patient away from me 180 degrees on the diameter across the diametrically opposite to me that person would look like they are moving, they are rotating about me in my frame of reference.

If, for that person I would look like I am rotating at that same angler velocity. In fact, the axle which is sitting in the middle, if I was rotating about the axle, but from my frame of reference the axle would look like it is rotating about the me, even though in the eyes of the observer standing on the play ground the axle is fixed. But, in my eyes which where I am sitting in one of the chairs and going around the axle, in my frame of reference the axle would look like it is rotating about me and the angular velocity of all of these motions are all exactly the same.

So, the bottom line take home message here is, there is no such thing as a point about which the rigid body rotates. The rigid body has in angular velocity in plane kinematics where we are restricting the motion of the rigid body to a plane. The angular velocity is the vector that points normal to the plane, since the vector does not change in direction we usually denote that angular velocity by the scalar number.

But, in full three dimensional motion the angular velocity of a rigid body is a full vector and that vector is not tight to an axle or a point about which the rotation happens that is the angular velocity of that rigid body does not matter which point is your frame of reference are the observer located on. So, that is what we want to learn with velocity, now we want to go on I am look at accelerations, the same rule of vector additions still wholes. So, the acceleration of B and like I said this essentially has a as observed by O as the unset frame of reference. But, the acceleration of B in the fix frame of reference is equal to the acceleration of A in the fix frame of reference plus the acceleration of B as observed by A.

(Refer Slide Time: 15:26)



Now, this acceleration of B as observed by A has two parts to it. So, that is basically that is the acceleration of that is basically the rate of change of the velocity of B as observed by A. And these are kinematic relationship that is simply saying the derivative of velocity is acceleration and when I do this the we found that in plane kinematics, the velocity of B as observed by A ((Refer Time: 16:07)) is r B as observed by A theta dot times e theta.

So, when I go through and simplify this knowing that r B as observed by A is not a function of time that is simply the distance between the two points B A it is a number I have theta double dot e theta minus r of B is observed by A theta dot square e r. So, the acceleration of this point, so let me redraw the rigid body here once more, I have a point A another point B this acceleration of B as observe by A has two parts to it, one is a part related to the angular acceleration theta double dot of this point.

Angular acceleration of this line segment A B and the second is a ((Refer Time: 17:13)) acceleration given by r of B as observed by A theta dot square and that happens to be in the negative e r direction. So, this acceleration of B as observe by A has two components to it, one a component perpendicular to the line segment A B and second a component from B towards A and the magnitude of the component from B towards A is the length of the line segment r of B as observed by A times theta dot squared.

Theta dot is the angular velocity of the rigid body, remember it is not the angular velocity of the line segment A B. So, I am simply going to write this as angular velocity of the rigid body theta double dot following the same arguments that I gave with two line segments A B and C D for the angular velocity is also not the angular velocity angular acceleration of the line segment A B, but it is the angular acceleration of the entire rigid body.

So, every line segment in the rigid body experience is the same angular acceleration theta double dot, it does not depend on which point is my A which point is my B or which point is the point of point about which it rotates. In fact, like we said there is no such thing as a point about which the body rotates in the frame of reference we are dealing with. As for as relative motion is concerned a point B is moving relative to a point A and that motion where both A and B around in a rigid body has only two parts.

The line segment A B can rotate with an angular velocity theta dot and the line segment A B can accelerate and every line segment A B on that rigid body will accelerate in an angular sense with the same magnitude theta double dot. So, we said if you go back to the velocity triangle, the only motion possible if you look at this the only motion possible of the line segment A B is a rotation, this part has no velocity along e r the radial the unit vector along the length along the line A B.

That means, that is because the line segment itself cannot belong it, if the velocity of B with respect to A has a component along A B that automatically means the line segment A B is either increasing or decreasing in length which is not allowed. Therefore, the only velocity possible of B with respect to A is where this B is moving perpendicular to the line segment A B there can be no motion in this direction that is not the case when we come to acceleration.

If you look at this acceleration, the acceleration of B with respect to A has two terms associated with it, this first term has to do with rotational acceleration of B of the line segment A B that is more precise way of saying it or rotational acceleration of the rigid body or angular acceleration of the rigid body itself. The second part is actually a component of acceleration of B towards A.

So, remember this I have a line segment A B this is B and this is A, B can have no velocity towards A or away from A in this direction. But, B actually does have an acceleration towards A, the magnitude of that acceleration of B towards A is the length of this r B as observe by A, the length of this line segment A B times theta dot square. So, every point B as observe from a given point A exhibits a center petal acceleration towards the observer.

That does not mean that the line segment is shortening, line segment shortening comes from a velocity of towards A, there is no velocity of B towards A, B accelerating towards a does not automatically mean the line segment is shortening. I want to make sure that you get this concept of acceleration clear in our mind primarily, because we have an understanding of velocity very usually not necessarily acceleration. So, this concludes or discussion of rigid body rotation plane or kinematics of rigid bodies, we will take up an example problem, where we will study an application of relative acceleration.

Thank you.