Statics and Dynamics
Dr. Mahesh V. Panchagnula
Department of Applied Mechanics
Indian Institute of Technology, Madras

Lecture – 19

We are going to take up another example this one related to curvilinear motion. So, the simplest case of curvilinear motion is rotation about a point. So, will start will take an example and work a way through there.

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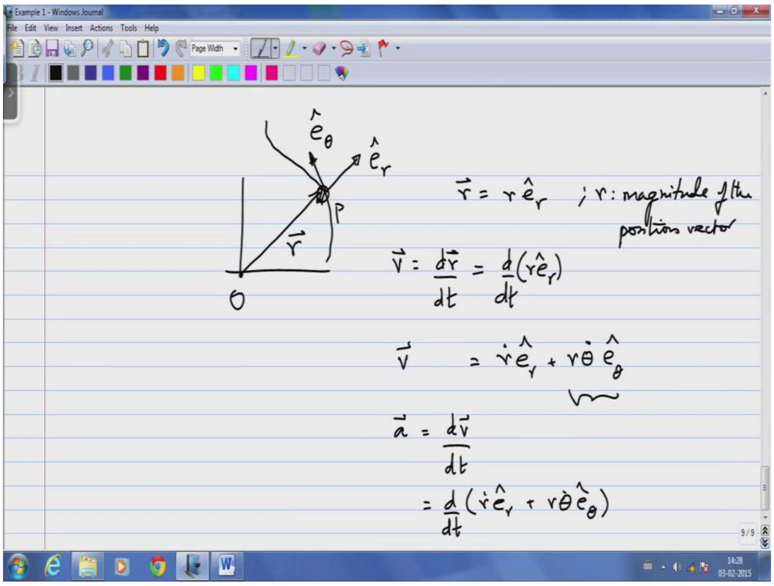

But before that I want to lay out the general rules for curvilinear motion. So, if I let say have an origin and a point p, that is tracing motion on a path - it is kind of generally curvilinear, we looked at cartesian definition of the motion of this object. Another way these to define position vector define denoted by this r and define a coordinate system in with respect to the local r. So, I can define a unit vector call e r and then e theta; e theta being locally tangential to the path being trace whether the particle p, and er being along the particle p.

So, let us write down it is some very simple definitions; r is r times er, where now r is the magnitude of the position vector. So, in the same coordinate system just like you did with the Cartesian, I can define a velocity of this particle a simply dr dt. Now when I do this you essentially get d dt of r times er. The difference between this coordinate system and the cartesian coordinate system is that the unit vector er is now subject to change with respect to time, whereas in the cartesian coordinate system ex, ey, and ez remain unchanged in both are magnitude and direction with respect to time.

So, because of the one added one complexity d or dt is can be written as r dot e or plus r theta dot e theta. So, this additional term arises, because there is a rotation of the er vector, and the magnitude of the rotation is given by this theta dot e theta. So, in a simplest... So, we going to take the simplest case of a particle p, tracing a circular rotation about a point o, and this point is located to need to on this radius; the particle p is rotated 2 meters on a circle, that is radius 2 meters with respect to a center o. And the acceleration in the angular sense is given by 4 cos 2 t radians per seconds square. And we ask to find the maximum velocity of the particle, And the amplitude of the periodic motion.

So, as you will see the angular acceleration as a cosine of 2 t kind of dependents which implies that it takes on a maximum and a minimum value and remains inside those 2 bounds for all times. Therefore, this is an example of periodic motion. So, let us start with that. So, this gives us… So, we do not try to simplify this definition of the velocity for that case, but before we do that we are having to also define and acceleration. So, the acceleration is given by the same definition that we had in the coordinate system. So, if I now do this differentiation.

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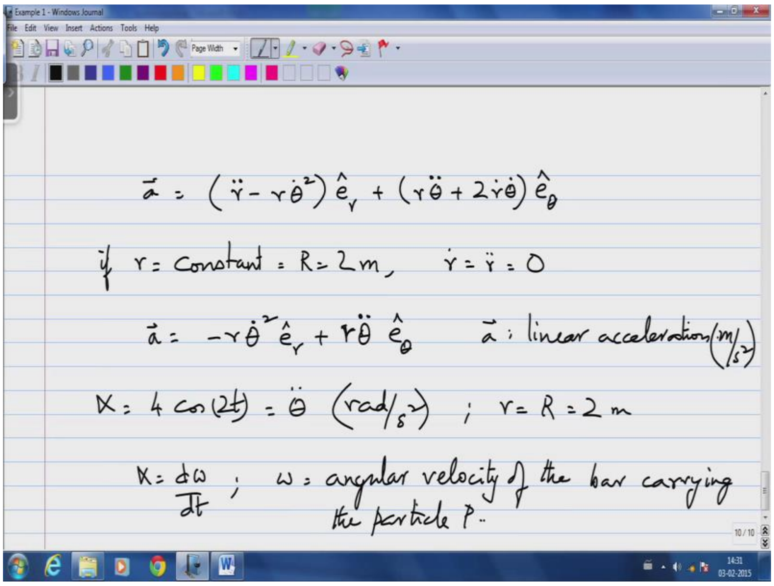

You end up with additional terms, and I am simply going to write it, because the mathematical derivation is fairly straight forward and occurs in almost all textbooks related to dynamics. So, given a particle moving in an r theta coordinates being whose motion is being define any r theta coordinate system, this part this defines the total acceleration of this particle. So, if r is unchanged in time as is the case with our example; if r is constant equal to capital r equal to 2 meters, then r dot equal to r double dot equal to 0. So, simplifying that acceleration is minus a er plus r theta double dot e theta.

This is the total linear acceleration. So, a is the linear acceleration which has units of meters per second square remember that; alpha which is what we are given is like our theta double dot that has units of radians per second square. And in other case r equal to r equal to 2 meters and constant. So, let us go through and solve this example and for that case we find first of all that alpha is d omega dt, where omega is the angular velocity of the particle with about of the bar as a matter of fact carrying the particle p.

Now a common error that is found in most in some text books is they refer to the angular velocity of a particle. A particle is of a point mass, and a point size, it has no designable angular rotation. So, you are essentially in this problem interested in the angular motion of this bar op, which carries the particle p at the end of it. And the bar has an angular acceleration alpha and an angular velocity omega.

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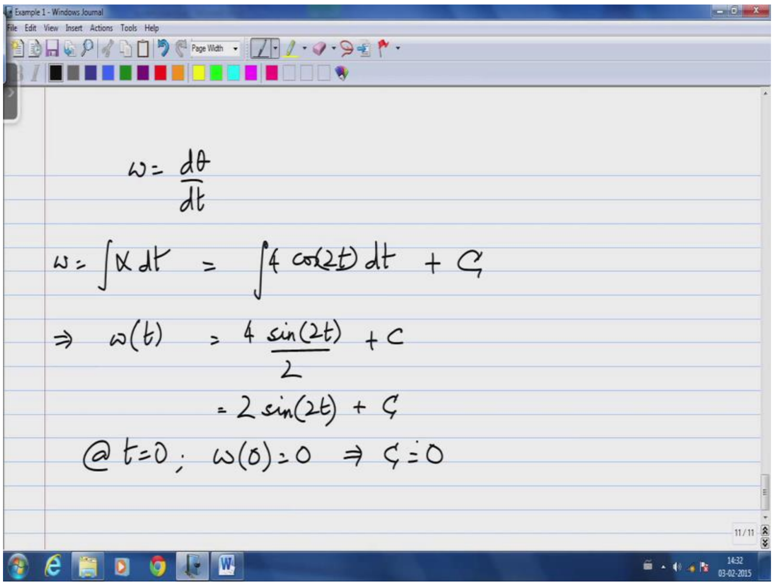

And similarly just like in the case of a linear one dimensional motion omega is also given by d theta dt. So, let us start given alpha I can find omega by doing a time integration 4 cos 2 t dt plus an integration constant that arises out of the fact. So, which implies omega, which is a function of time. So, at t equal to 0, omega of 0 equal to 0 which implies c in this particular integration is 0.

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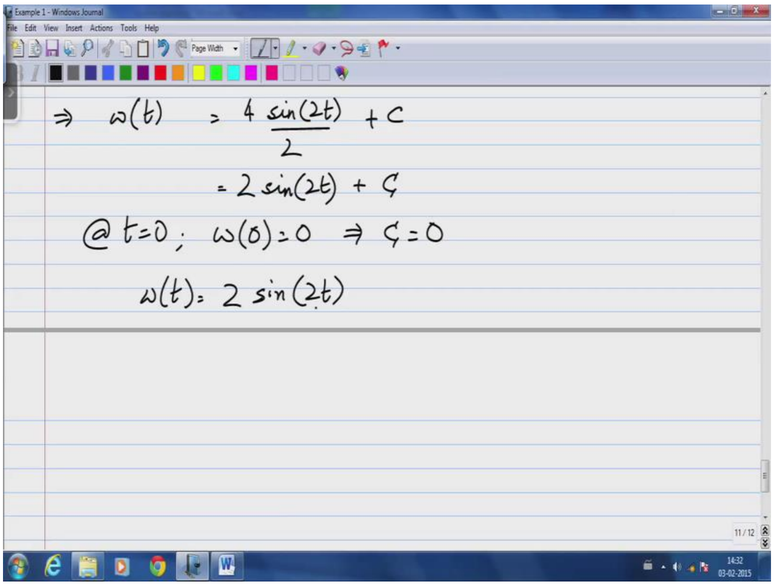

So, we determine the angular velocity with respect to the time as 2 sin 2 t.

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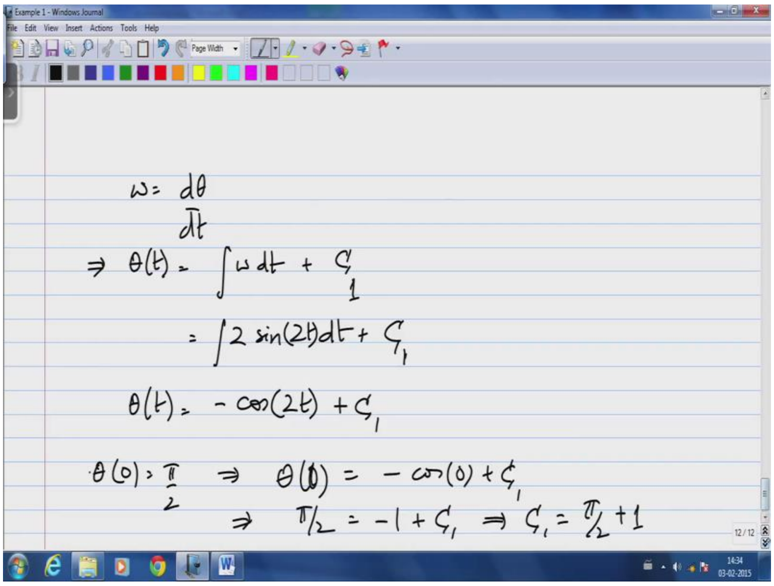

Going one step follow omega itself is defined as the theta dt, which implies theta is a function of time is integral omega dt plus another integration constant, I call this c 1. So, when I do this integration to sin 2 t dt plus c 1, and we are told that at theta of 0 at time 0 is pi over 2 which implies theta of 0 is minus cosine 0 plus c 1.

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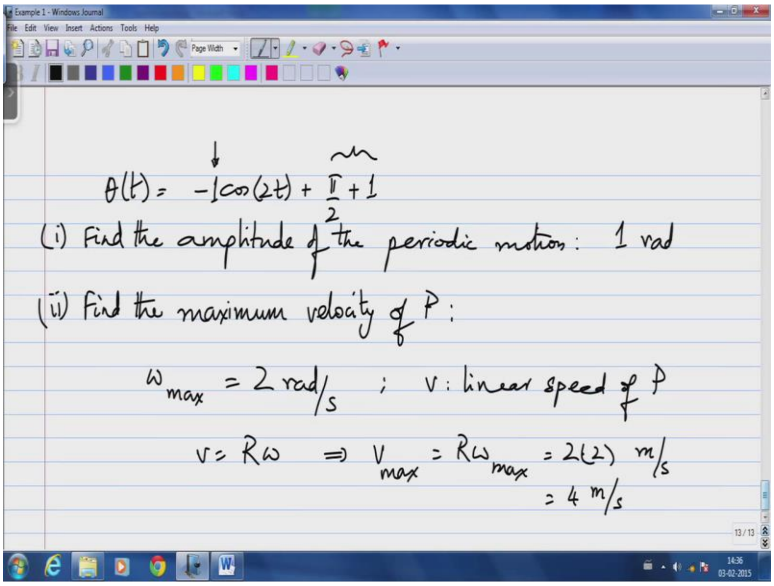

So, we ask... So, theta is a function of time is minus cosine 2 t plus our integration constant which happens to be rate. So, we asked the 2 questions. First find the amplitude of the periodic motion, this amplitude has nothing to do with this number pi over 2 plus one that is just phase shift in time, the amplitude is this number which is kind of invisible it is actually 1. So, the amplitude is 1 and it has units which is radians. The second question that is asked is find the maximum velocity of this of p.

Let us go back in order to find that, let us go back to the definition of omega; omega happens to be of the form 2 sin 2 t omega takes on its maximum value, when sin 2 t takes on value of 1, which happens t pi over 4, etcetera. And the maximum value of omega is 2 radians per second. So, first of all omega max is 2 radians per second, and v the linear velocity of p are more precisely linear speed of p is given by R times omega which implies v max is r times omega max which is equal to 2 in 2 meters per second, which is 4 meters per second.

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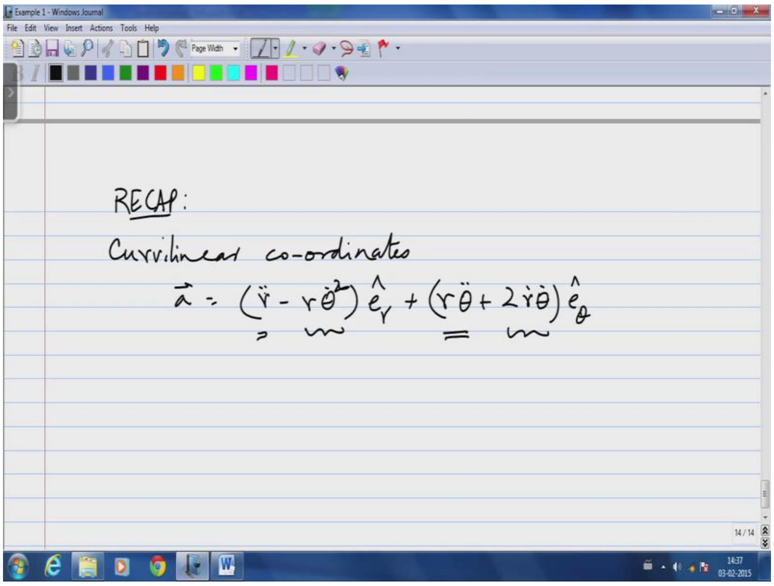

So, quick recap, what we learn is that in curvilinear coordinate systems, acceleration takes on a form, such as that now just again to recap from the previous video, why are we looking at acceleration, because later on when we deal with forces. Forces are proportional to or acceleration cost is proportional to force. Therefore, we need to understand the motion of a particle in the context of acceleration, it is not enough to understand only it is velocity.

So, given this, there is a 4 times here r double dot is simply the linear acceleration along er, likewise r theta double dot is the linear acceleration along e theta, this part r theta dot square is often refer to as a centripetal force, all the remember a centripetal acceleration and this is often refer to as a ((Refer Time: 12:26)) acceleration that they are technically incorrect. We will refer to that this point when we start talking of kinetics in curvilinear coordinates in about week or so.

Thank you.