Statics and Dynamics
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Lecture - 18

To continue our discussion of particle kinematics, by taking on an example problem.

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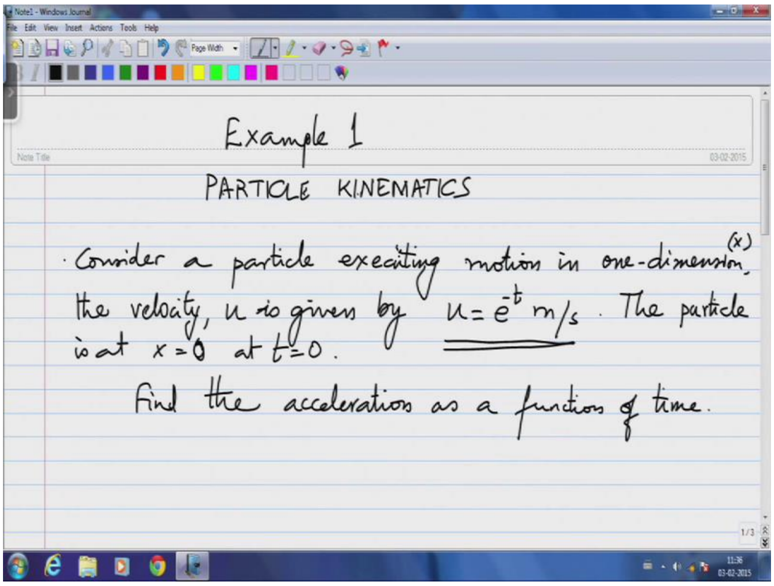

So, let us take a very simple example to start with, consider a particle executing motion in one dimension and we will denote it by a coordinate x. The velocity u is given by u equal to e power minus t meters per second, this is given. The particle we are told is at x equal to 0 at time instant t equal to 0, and we are ask to find the acceleration as a function of time.

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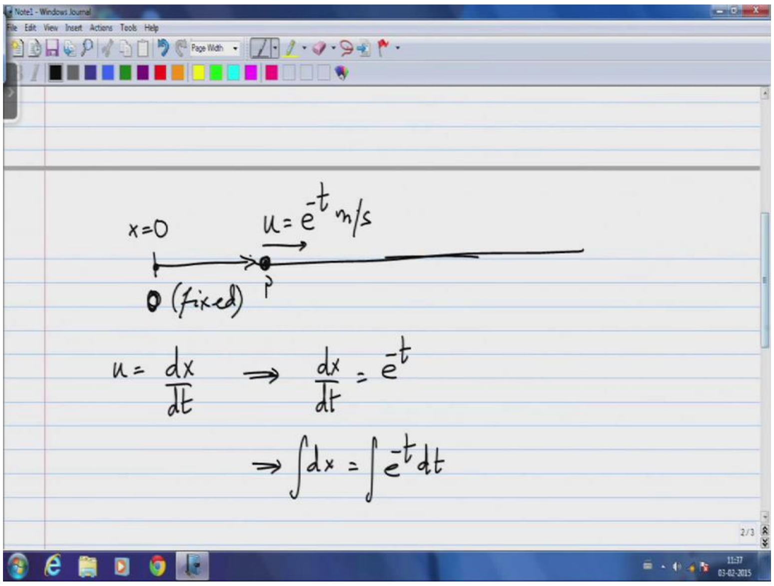

So, let us first draw a schematic show in the system or we first have to make sure, we understand who the observer is. Observer let say in this instance it is fixed at the origin let us say and we know the particle p is moving with the velocity u, instantaneously given by e power minus t meters per second. And we know the relationship between u and x from the earlier discussion, so this implies d x d t equals e power minus t.

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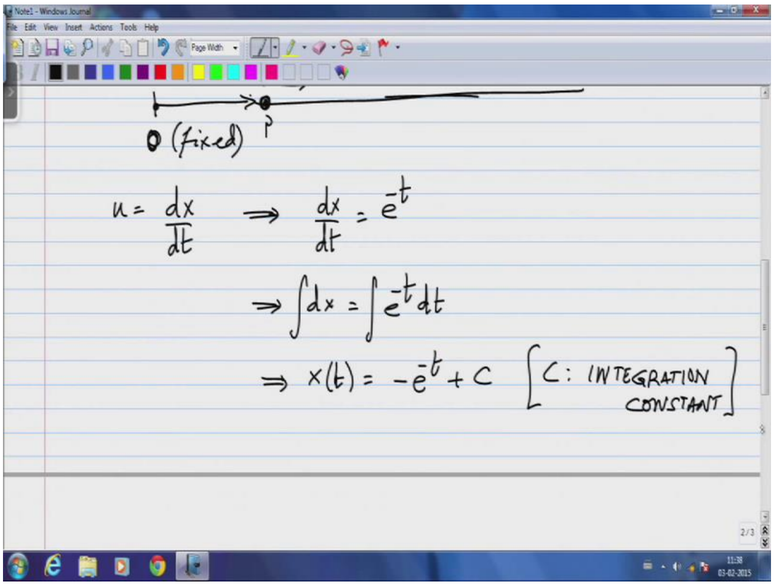

And if I integrate both sides of this equation, this is x as a function of time equals minus e power minus t plus an integration constant C. C is what we usually referred to as the integration constant.

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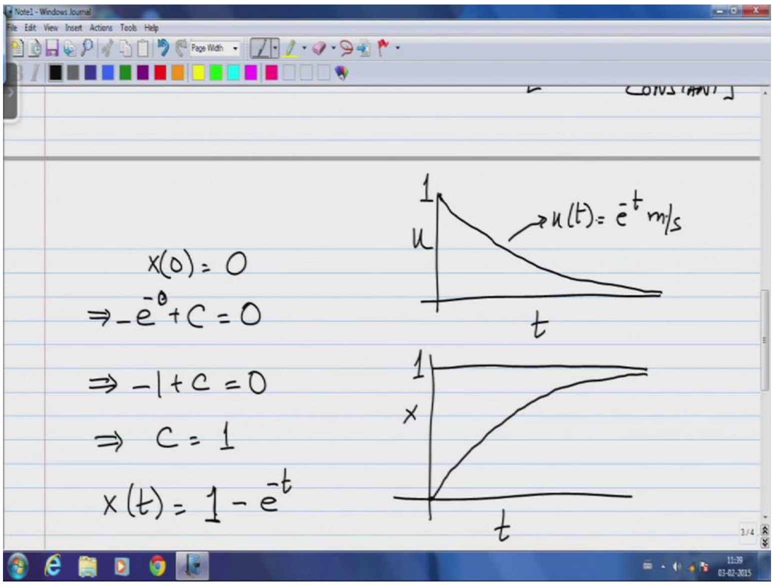

We have told that x at 0 is 0, this implies minus e power minus 0 plus the integration constant equal to 0, which again implies minus 1 plus C equal to 0, which also implies C equal to 1. So, from this, we know that x of t is 1 minus e power minus t. So, if you now look at the position, this essentially starts at 0. If I plot the position as a function of time, the positions starts at 0 and for very long time as syntactically reaches this position 1.

So, this is the graph of this and similarly, if I plot u as a function of time, u is initially 1, it slows down towards 0. So, this is the graph of u of t equals e power minus t meters per second.

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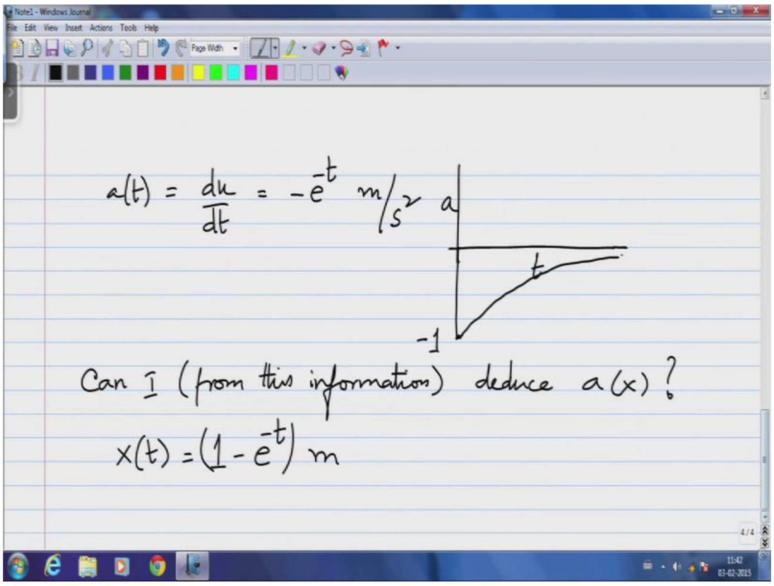

Now, this is not the end of the story, we need to find the acceleration of the function of time, which we know is given by d u, d t. The acceleration in this case is simply differentiating the known velocity by ones with respect to time and that gives me… Let us make sure we put the units and all of these quantities, x is a function of time is 1 minus e power minus t meters and the acceleration is minus e power minus t meters per second squared.

So, the acceleration at time instant 0 is really minus 1 and it goes as syntactically towards 0, but always remains negative. Acceleration being negative implies that the system or the particle is decelerating. So, you are looking at a decelerating particle; that is moving to the right of the observer given by here, by there by o. So, let us completes the problem statement as far as what was asked, let us go one step further and ask the question, can I from this information, deduce acceleration as a function of x.

Can I write the acceleration as a function of x? So, what that means, let us go back and see what; that means, I know x as a function of time, which is given by 1 minus e power minus t meters. So, if I go back to this graph of x as a function of time and if I want to make the shift of the independent coordinate from time to x.

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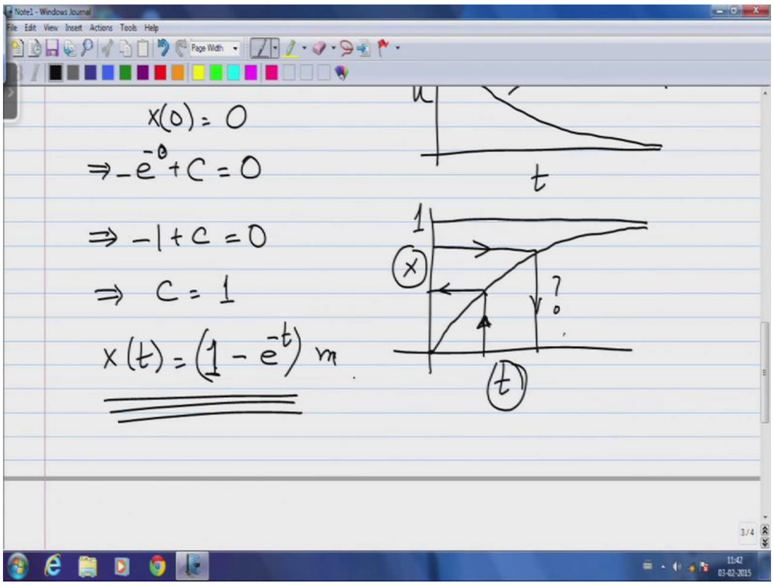

At the moment, if you give you a time, I am able to tell you, what the corresponding x location would be; that is what this equation tells us. What we want to do is, if I give you a corresponding x location, can I identify a time. If I am able to do this; that is the condition and in which I will be able to answer this question, let see that is possible with this particular example.

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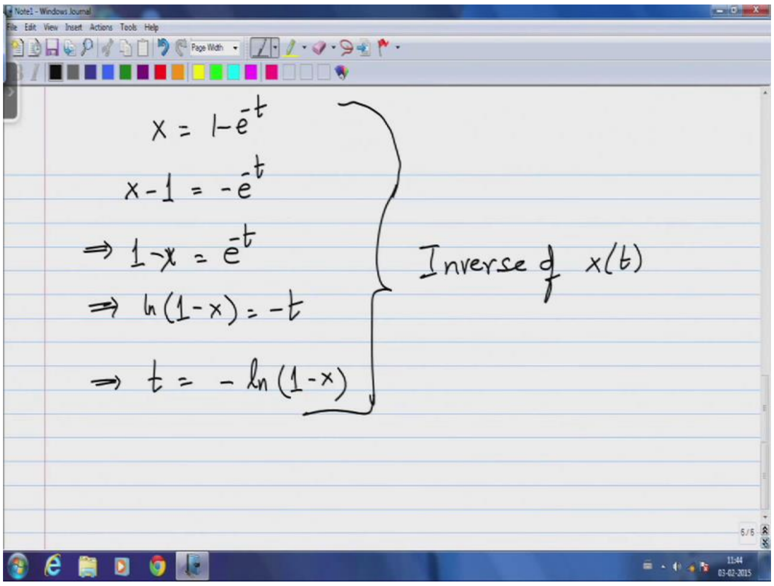

So, if I said x of t is equal to 1 minus e power minus t, I can rewrite this as x minus 1 equals minus e power minus t, which also implies 1 minus x equals e power minus t. If I take natural logarithm on both sides 1 minus x equals minus t or I can write this a simply t equals minus natural logarithm of 1 minus x. In this process, what I have done is I have calculated the inverse of x of t, this is the mathematical operation that we have performed in these four steps starting with x equals 1 minus e power minus t.

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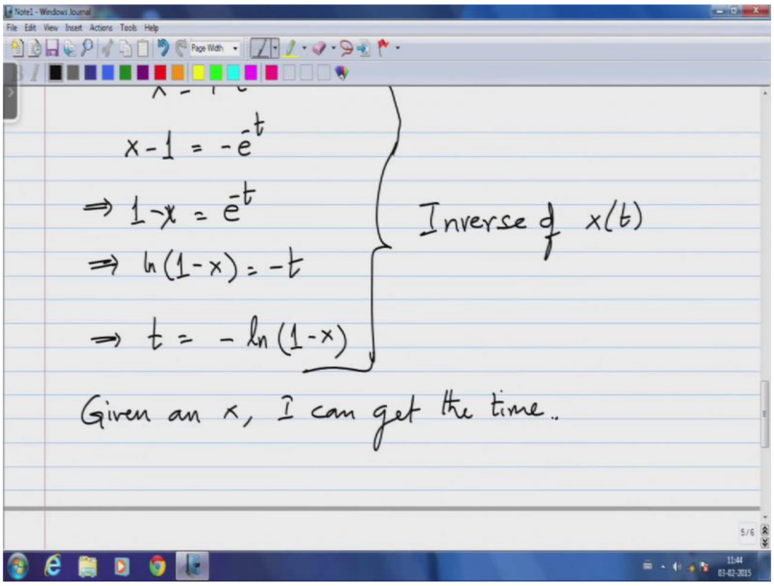

If I able to do this as I was in this particular case, then what this allows me to do is, given an x, I can tell you the time instant at which the particle is at that x location. Now, have we completed our discussion, whether I can write a is a function of x not really.

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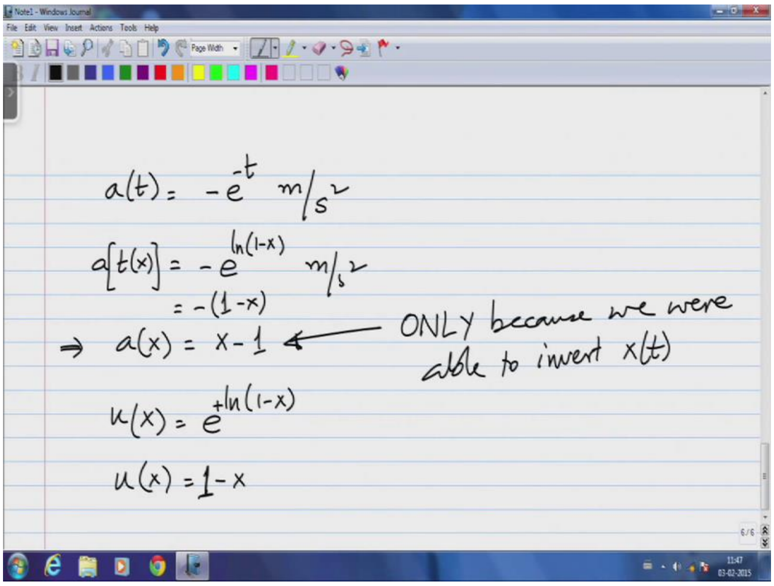

What we do now is that, a is equal to minus e power minus t meters per second squared, we do know this. So, this and if I know a, where t itself is a function of x, which is known from the previous inverse operation, inversion of the function x of t. I am able to make that substitution and say minus t equals log of 1 minus x. So, this is natural log of 1 minus x, e power log of 1 minus x is simply 1 minus x minus 1 minus x.

So, the acceleration as a function of x is equal to x minus 1. So, this is the meth simpler function to deal with, then acceleration is a function of time being e power minus t e minus e power minus t. But, as it turns out, we were able to get to this step, only because we were able to invert x of t and this inversion of a function may not always be possible. We will look at some examples, let say if simply had an even a harmonic motion with the simplest of complexity, this is kind of an inversion may not be possible.

So, this kind this acceleration is now a function of x the spatial coordinate and is given by x minus 1, I could I can also invert u as a function of x, similarly minus log 1 minus x. So, u as a function of x is simply 1 minus x. So, if I choose x, the spatial location of this particle as the independent coordinate.

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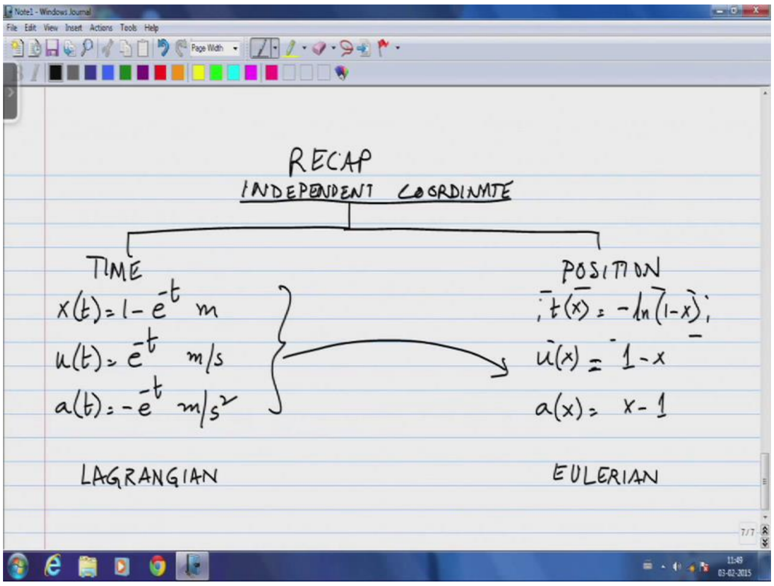

Let us quickly recap, using time as the independent coordinate, I can either use the time as the independent coordinate. In which case, I know x as a function of time is 1 minus e power minus t, u as a function of time is e power minus t. In acceleration is a function of time is minus e power minus t. If I use position is the independent coordinate, then u as a function of x is 1 minus x, acceleration is a function of x is x minus 1 and t as a function of x is given by minus natural logarithm of 1 minus x.

So, this is the enabling step to go from this to this kind of an observation, to go from time as the independent coordinate choice to position. Now, this kind of observation making where I focus on time as the independent coordinate is typically called a Lagrangian, frame of making observation. And this often referred to as the Eulerian frame of making observations.

But, if I am able to find this t as a function of x, from a given x is a function of time, then these two frames of reference, these two ways of making observations are interchangeable and they are analog us. We will continue our discussion on a kinematics in the next lecture.