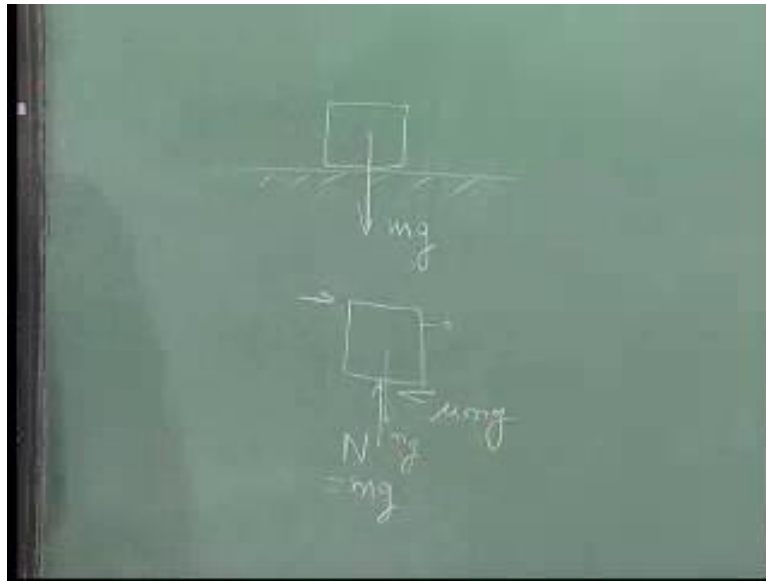


**Statics and Dynamics**  
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**Lecture - 15**  
**Statics – 5.1**

This is Mr. Venkat Rao, I am going to ask him a question, so simple question do not be nervous.

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The question is simple, I just have to know, there is a body here, just like what you see here. It has a weight on it equal to  $mg$ , I want to know the frictional force on this block, how do you find out. I want to know the frictional force on the block offered by the surface.

Student: Offered by the surface

Right,

Student: ((Refer Time: 00:51))

Can you write now the magnitude of that, you can come this side, can you write the magnitude of that?

Student: unless, it is subjected to a moment.

But, there is a frictional force

Student: We do not know, what is the frictional force, direction of the frictional force?

But, there is a frictional force.

Student: exactly,

So, what is this frictional force, if we have a weight of  $m g$ ?

Student: We get exactly opposite to the weight of the normal reaction.

That is a normal reaction, this is the normal reaction, let us call this as  $N$ ,  $N$  is equal to  $m g$ . What will be the frictional force? If  $N$  is the normal force, the frictional force is related to normal force, this is not there.

Student: Frictional will be there,

Supposing, I am applying a force like this, the frictional force is,

Student: Opposite to the direction of the applied force

Which is equal to  $m g$  is given,

Student: 0,

No, what is the frictional force; if  $\mu$  is,

Student:  $\mu u m$  is 0,

Write it down,

Student: Suppose the body is moving the direction,

No, supposing I am applying a force here, the frictional force here is related to this

Student: Coefficient of friction into

Write it down,

Student: Coefficient of friction into  $\mu m g$ .

Very good, thank you,

Student: Thank you.

Often, this is the mistake many students make, here I am not pin pointing one student, most students will immediately write the frictional force to be  $\mu$  times  $m g$ , which is I am going to argue or discuss that, this is not always the case. So, let just look at it more seriously and understand this particular problem.

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So, let me start off with a simple idea here, it says this is the body, there is a weight acting on it and if I draw the free body of this. I may audible? If I draw the free body of this, then  $mg$  is acting like this, this is the normal reaction  $N$ ; that much is correct. When, I apply a force, let say like this equal to  $F$ , the question I am asking now is, what is the frictional force? Yes, the answer is partially correct; the frictional force is opposite to the force applied.

But, saying that this is equal to  $\mu mg$  is not always correct, this is the notion that many students have a problem with. Of course, in this particular case, when I say  $\mu$ ,  $\mu_s$  is the coefficient of friction that I am taking about static friction. This is true only when the bodies touch to move or is about to move. So, if I draw the frictional force  $F$  versus the actual force applied like this.

Mind you, I am applying at the bottom, so that, it is easily understood that these two do not form a couple. What will be the force  $F$ ? If supposing I apply, let us apply 5 kilo Newton over here, let say that is less than  $\mu$  times  $mg$ . What will happen is, this is the 45 degree line, which means that, the  $f$  is equal to capital  $F$  in magnitude, but when it reaches a particular value, which is equal to  $\mu$  times  $mg$ ; that is when the whole thing changes.

The frictional force cannot be above the particular value, let us assume  $\mu_{static}$  is equal to  $\mu_{kinetic}$ . Just for now, so that the argument is simple, the discussion becomes simple. So, when it reaches this particular value, please remember the frictional force

cannot increase at all. Can the force  $F$  increase? It can increase, supposing it goes like this, what happens to the additional force that has been offered. The additional force offered goes towards accelerating the body.

Is this understood? This is very important. So, if I am looking at static situation, it is only one instant, when  $F$  is equal to  $\mu$  times  $m g$ ; that is at the threshold of starting to move. ((Refer Time: 06:19)) So, when I have this body and I am applying the force, for it to move, I need to apply a force; that is equal to  $\mu$  times  $m g$ , for it to start move. When, I starts to move  $\mu$  kinetic, coefficient of kinetic friction will play a role.

This is often a mistake; that is students make and please remember small  $f$  is equal to capital  $F$  comes purely by equilibrium. If I look at this free body and find out, let me write it down here,  $\Sigma F$  along  $x$  direction, if this is  $x$  direction and this is  $y$  direction is equal to 0 for static condition. Immediately, I have only two forces taking part, this is capital  $F$ , this is small  $f$ , means capital  $F$  minus small  $f$ , because the direction is negative to this is equal to 0, which automatically tells me that capital  $F$  is equal to small  $f$ . When is this true? It is true only, when the right hand side is 0. What is right hand side? It is nothing but, saying that it is in static condition.

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Handwritten equations on a chalkboard:

$$\Sigma F_x = m a_x$$
$$F - f = m a_x$$

In motion or at impending motion

$$f = \mu m g$$
$$F = m a_x + \mu m g$$

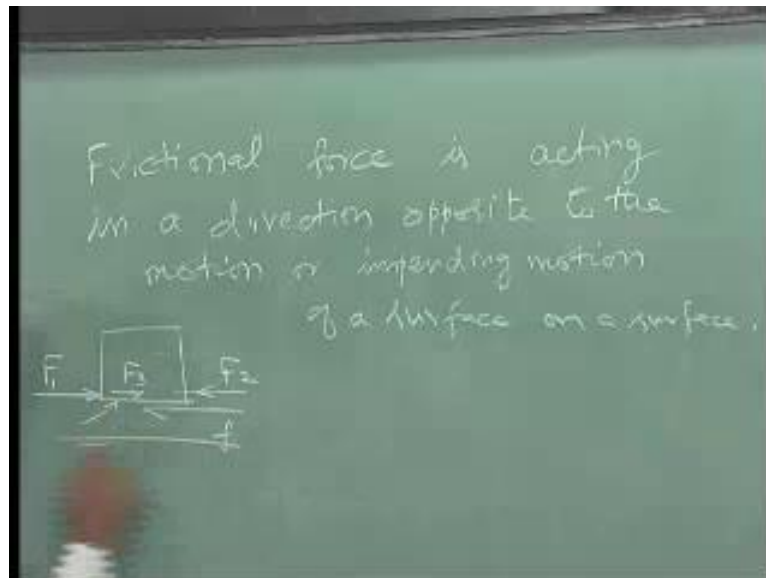
Supposing, it is in dynamic condition then  $F$  equals  $m a$ , which means that  $\Sigma F_x$  should be equal to  $m$  times  $a_x$ . So, if I insert these values, I have capital  $F$  minus small  $f$  is equal to  $m$  times  $a_x$ , when  $a_x$  is equal to 0 at this static condition, this is equal to 0, capital  $F$  equals small  $f$ . But, if  $a_x$  is non-zero, in the condition when it is non-zero,

remember there is a motion involved, which means this reaches...

So, in motion or at impending motion about to move, then  $f$  is equal to  $\mu$  times  $m g$ . What is this? This is the property of the surface, the two surfaces that interact with each other, these has nothing to do with equilibrium. Unless, I am told  $f$  equals  $\mu m g$ , it is indeterminate. Once, I know it is  $\mu$  equals  $m g$ , then I can write  $F$  is equal to  $m$  times  $a$  plus  $\mu$  times  $m g$ .

So, part of the force is taken by the frictional force and part of it goes towards accelerating it. This is a very crucial point that has to be understood. There is one more point; that is very important, when I ask to Mr. Venkat Rao, he immediately answer that, if I apply a force like this, the frictional force will act opposing the motion.

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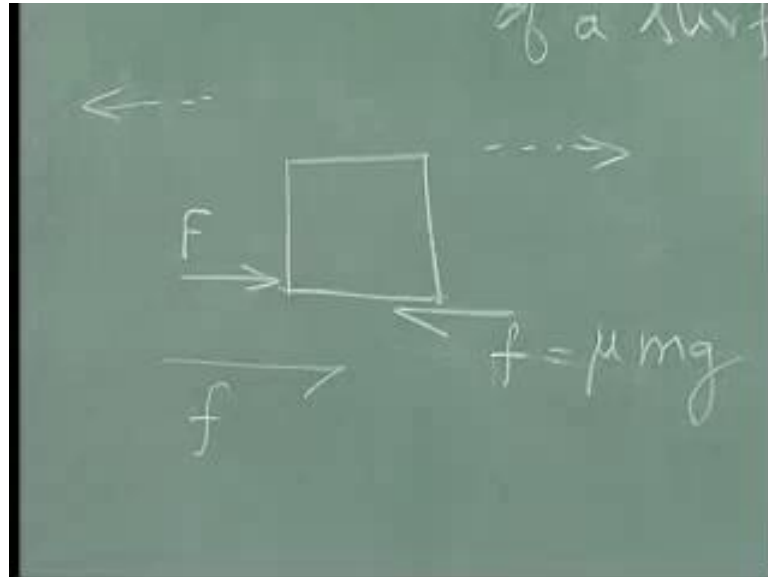


Please remember the statement I am making, let me go over there and write it down, frictional force is acting in a direction, there we come back to the direction. Supposing this is the body and let say, this is the force acting. He said that, the frictional force acts in the opposite sides. Now, I am going to twist it a little bit, I have a  $F_1$  acting like this,  $F_2$  acting like this,  $F_3$  acting like this.

Now, tell me what is the direction of  $f$ , small  $f$ ? So, a simpler way of looking at it, what he saying is, it is opposing the resultant of this. A simpler way of looking at it is, it is actually opposing not the force, but the motion. It is the resistance to the motion or in other words in a direction opposite to the motion or impending motion of, what? Of a surface, on a surface, of a surface which is this surface, on a surface which is this

surface, this is on, this is of. This should be understood clearly, which means what, if I have a body, so let me just make it very clear to you in this scenario.

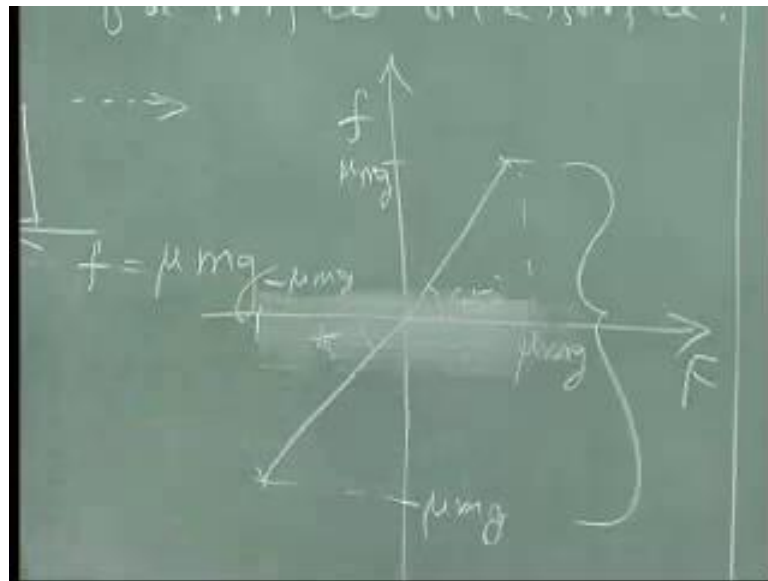
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If I have a body like this and I am applying a force  $F$  and if I say the impending motion as in this direction, I will apply a force, frictional force. Since, the impending motion is in this direction, it should be opposing it, I will put this  $f$ , since it is impending, it is reached the maximum value. I can turn around and say, what if this is negative, I am pushing it a little bit and then, I pulling it.

Remember, when I pull it, again once I reach and impending motion on the opposite direction. If the impending motion in this is in the direction, then  $f$  equals  $f$  will act in this direction and this should be understood, which means between a force that reaches  $\mu mg$  in this direction. And a force that reaches  $\mu mg$  in the direction, the body is in static equilibrium and the frictional force changes from positive value to negative value.

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So, if I plot this  $F$  versus the frictional force, when I am applying in this direction, let say it is positive, it will keep on increasing this is  $f$ , till it reaches  $\mu$  times  $m g$ . Similarly, if I move in this direction, if the impending motion is in this direction, till the motion occurs will keep on decreasing. Again, when it reaches the particular point  $\mu a m g$ , this case minus  $\mu m g$ , it will not be moving.

So, between this and this of this region, so if I have this value and this value specified, this is  $\mu$  times  $m g$ , the reason is this is set of 45 degree angle, this is minus  $\mu$  times  $m g$ . So, when the force is within this region, it is in static equilibrium and the small  $f$  is equal to capital  $F$ . Beyond this, there is motion, beyond this there is motion in two different directions.

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Third point supposing, I have a block like this, this is again mistake that is often made by the students. So, I just want to point out, such a mistake. Let say homogenous body  $m$   $g$  is acting like this; that is a force acting like this,  $F$  basically, I have something like this and I am applying a force like this, I need to now draw the reaction forces. Well, that is very simple, one made remove this, well where is normal reaction here, where is the frictional force, since it is supposing motion in this direction is equal to  $f$ .

If it is static, this is equal to this, that condition away of it is static is very important, the condition of motion these are difference story have. The answer is not correct, this is not correct, most of the students, who are used to drawing this from physics, get confused, unless this force is over here, you will see, what is this equal to, this is equal to capital  $F$ , which means, this and this form a couple, which is like this. This is the height  $h$ ,  $F$  times  $h$  is a couple that is acting.

Where is a resistance to the couple? The resistances is offered by a variation for example, if I draw normal reaction to be uniform like this. Then, it is resultant is along  $m$   $g$ , actually since I have to offset this couple in this static case, it will actually  $N$  will shift like this, so that this will form an opposite couple. This is equal to  $m$   $g$ ,  $N$  equals  $m$   $g$ , when I find the vertical equilibrium and the distance between these two a such that, it is equal to  $F$   $h$  divided by  $m$   $g$ .

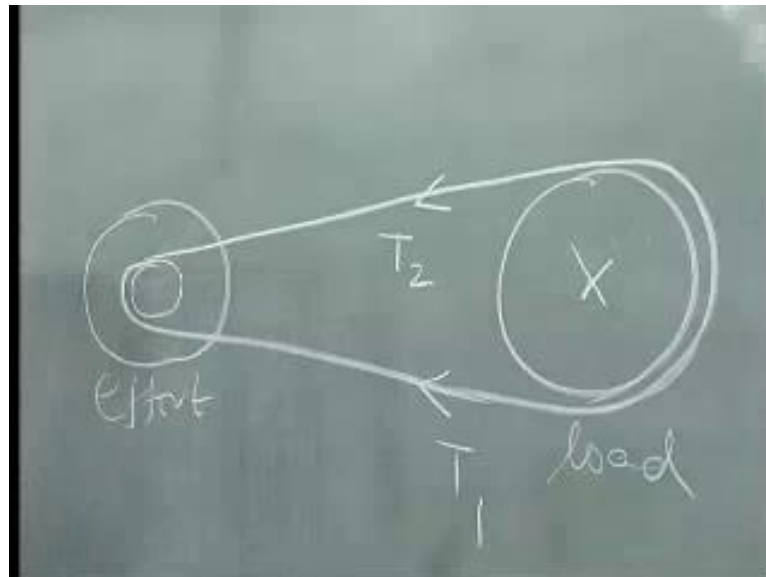
Or, in other words  $m$   $g$  times this distance will offer the opposite reaction. What is this  $N$ ? This is the net resultant reaction that comes from the pressure that is offered by the



surface. So, this is the resultant, please remember that. Sometimes, when I cut like this, one of the mistakes people do is, they will say  $N$  is acting like this, there is no  $N$  here. This is nothing, but the representation of a distribution of this sort.

The resultant what is wrong like this, so when I cut like this, I will have this reaction coming, this is the third mistake that is often made. With this as the basis, most of frictional problems can be solved. So, let us take a one problem which is often used in friction type of applications, where all to be encounter friction, where all do we use friction to or advantage, where all do we encounter a problem with friction is often already talked about. One of the cute applications in which friction is used positively is a belt drive.

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What is a belt drive, let say I have a motor over here, the motor has a roller over here rotor and thus that is rotating, I have another let say this is a wheel, I need to rotate this wheel. So, what do you usually do, I connect it with the belt, let me draw it with another color, I connected with the belt. So, what is basically happening is when this is rotating the friction between the pulley and this.

So, here when it is rotating the belt has a friction on this rotor that takes it up along the belt. The belt transfers the friction through this. So, that this rotates and this rotates that could be slipping, there need not be slipping. The two conditions are handling very similar to handling motion and statics over here. So, let us do this kind of problem, this is one of the problem that we will encounter in mechanical engineering and similar

engineering in have applications.

The aim is to find out supposing I fix this, so I fix this and I apply a force over here, tension over here, say  $T_1$ , let us say it is, it is in motion. What is the tension in this, this is not moving, I am pulling this without tension  $T_1$ , what you realize is to  $T_2$  less than  $T_1$ , the answer is yes, because there is some friction that as eaten away some of the tension.  $T_2$  will be less than  $T_1$ , can I find out  $T_2$ ; that is a question, let us answer that question.

Why is it important by answering this question, I will be able to tell you depending on  $T_1$  and  $T_2$ , whether slipping as occurred in this or in this. If no slipping as occurred, then the entire thing is transferred directly to this, this is usually call the effort and this is the load. You do not want the power to be wasted in friction, because friction is going to converted to heat and there is a loss of energy unnecessarily, in addition to the other losses that we want to encounter.