Statics and Dynamics

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Lecture – 13
Statics - 2.9

I have a pair of scissors and a small sheet, just to illustrate to you, how I can find out bending moment and shear force in a particular situation. For example, this is a sheet of paper and I am so used to doing this operation from a childhood, cutting it a sheet of paper using a pair of scissors. Can we analyze what kind of force is going on to this paper? So, that I cut this particular sheet of paper.

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Let us do and see, let say this is a sheet of paper, I mean just write nicely. What is happening here, there is one leg of scissors pair of scissors, this one leg I have drawn here, so that is cutting it. It has to be this way, because it is cutting along this. There is the other one which is just near this, like it, like this. So, that, when I cut, I am applying force equal and opposite force at these two points; that is not very difficult to understand.

If I apply a load over here and the load over here, the two loads that appear on the paper or exactly one on the same. So, essentially I am applying a force P here and a force P here and I wish to use this set of forces in order to cut the sheet of paper. What is the distance between these two? Very small some delta, let just below this of and see, what kind of force is appeared.

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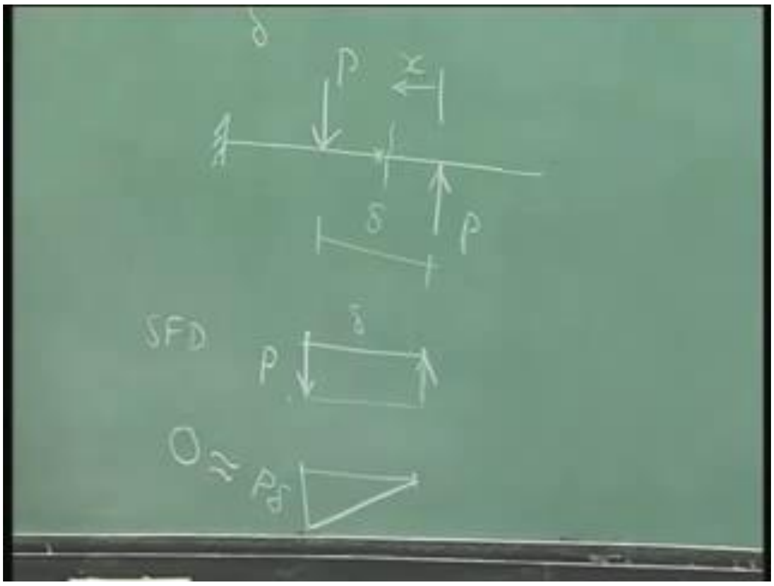

So, I have a P over here and a P over here, they were small delta that rotates it. I am not showing that particular side of it. By holding it tight and applying, there is a slight moment that takes place and that desistance is taken by my hand. If I draw the shear force for this, pretty simple there is no force in between, there is a shear force is equal to P here, constant over this delta region and goes to 0 at this region.

How about the bending moment diagram? I may make it small, so that I can draw the bending moment diagram. Bending moment diagram, if I take any particular point over here, I ask the question is it in equilibrium? The answer is no, I should have applied for example; I should had a cantilever or something like that, which is nothing but, the hand that is holding the sheet of paper.

There is a moment that appears over here, but then, if I look at only between these two and look at this particular side, I cut it off like this, the moment is P times. If I take the distance from this as x, it is P times x. So, it is some kind of moment like this and at this particular point, it is P times delta. What do I know about delta? Delta is very small, very small compare to the total in the paper and all this.

So, and therefore, P delta will be very small and therefore, I can say this is roughly equal to 0. So, in this particular case of applying force using a pair of scissors, all I am doing is applying a shear force equal to the force that I have applied through my hand.

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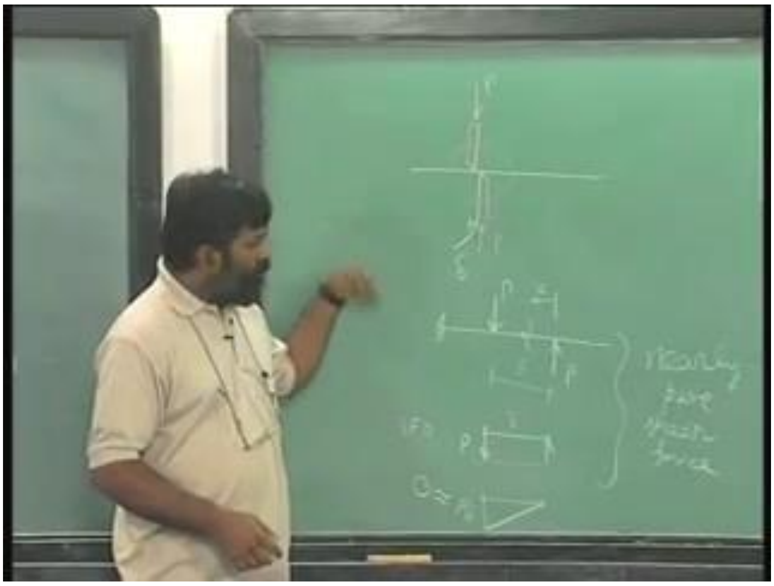

And therefore, I have a situation of nearly pure shear force. So, I am applying a pure shear force on this and that is enough to cut the paper and it is delta becomes smaller and smaller, the efficiency of this scissors will become better and better.

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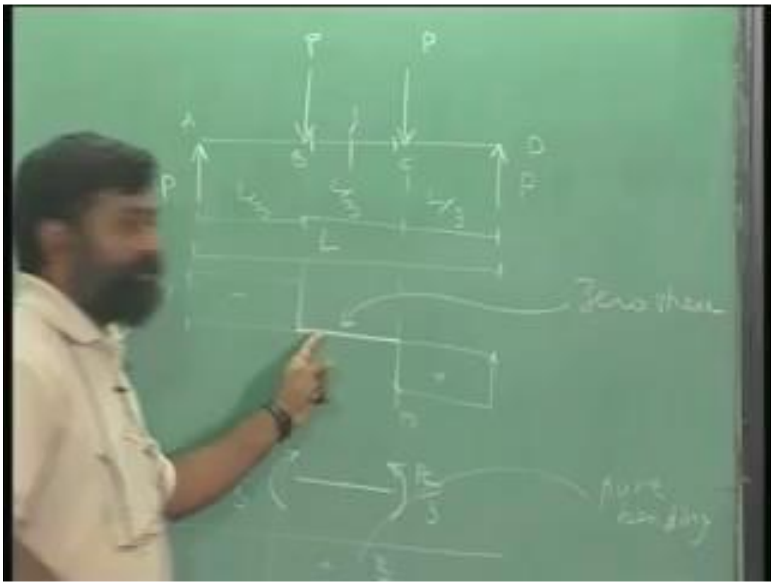

Now, there could also be a situation, where you can have pure bending moment for whatever reason. I will give you one example of that, this is usually called either two point bending or four point bending, so situation like this. This is a force P, this is a force P acting at L by 3 away, I shown here. What will be the reaction? Reaction is P and P over here. If I take the bending moment variation in this region, let me call this as A, B, C, D.

Let us first draw shear force diagram, I am going to focus only on this zone B C. If you notice, there is a P over here like this, like this and then, this is P and I have bring it down by P. This is negative, it is going to be 0 over here and I have a positive P shear and it ends like this. So, I have negative shear and a positive shear, but if you notice in between you have a 0 shear situation.

However, bending on bending moment diagram, you will find that the bending moment diagram can be written like, I just copied over here. If I take a section of in between, this P and this P both form a couple equal to P times L by 3. So, I can just replace this by a couple equal to P times L by 3. Similarly, these two form a couple, which is 3 times L by 3. There is no other force acting and therefore in this particular zone, it is a pure shear bending moment acting.

Just to make it clear here, you have like this I am cutting close to C, close to B and C and it reveals just pure bending moments equal to P L by 3 and P L by 3 and therefore, any other section here will be equal to P L by 3. And therefore, we have a constant moment within this zone B C, which is a positive equal to P L by 3, thus we can generate a pure bending situation, this is called the pure bending situation.

Why pure bending? There is no shear, there is no axial force, the only force resultant that is present is pure bending. Now, question, if I move this particular set anywhere around, you think, I will generate constant bending moment over the B C.

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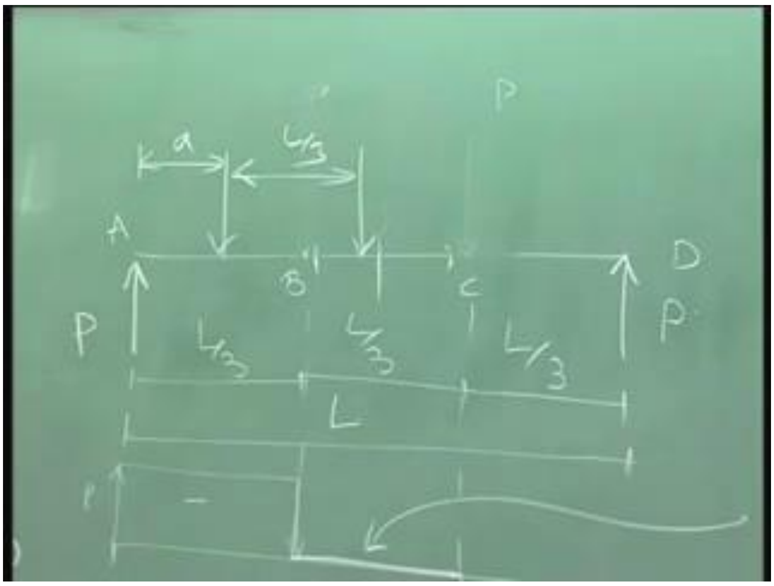

For example, instead of this being L by 3, L by 3, L by 3, supposing I moved to something that is at distant a from the left hand side, retaining this as L by 3. Do you think I will have a constant bending moment over here? Please, find out by yourself.

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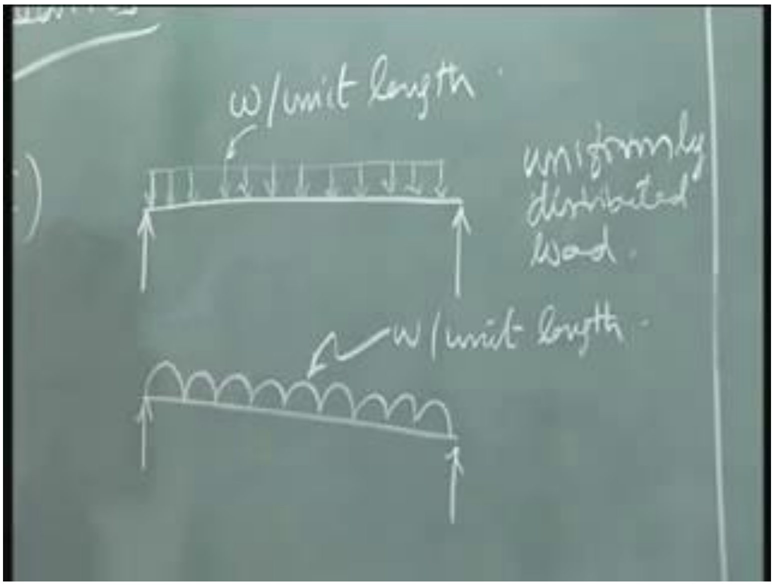

Now, let us look at different scenarios of loading that can appear on a beam. Supposing I ask this question, let say this is the simply supported beam, immediately I can draw like this. Let says it has, it is own weight to be taken. So, it is own weight, let say it is weight per unit length given, this is nothing but, mass times the area of cross section per unit length.

Now, how do I represent it? People represent in different ways. One of the simplest ways in which you can represent is draw something like this and say, this is equal to w per unit length. They also represent it using a diagram like this, we call this is uniform, because it is, let say if it is a uniform cross section, it is a uniformly load, distributed load.

What is the direction of these loads transfers to the access reveal? Remind you, it will always be transfers the access of the beam, if I am taking this particular structural member system to be a beam. And therefore, the reaction, the support reaction will also be perpendicular to the access of the beam.

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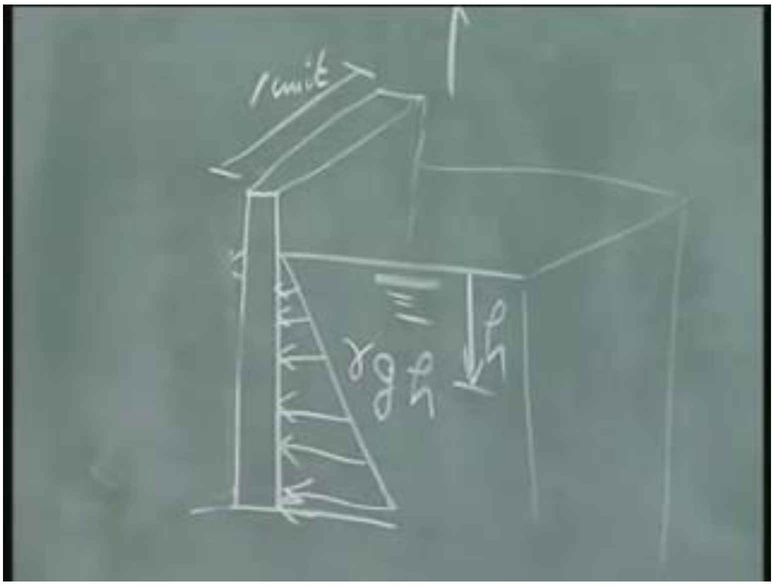

Another situation is for example, I have a beam like this, let say it could be a wall and it is basically supporting a column of water. And if I have to find out the distribution of force on this particular, let say beam, it could be a wall, basically it can be idolized to be a beam. What will be the force on it? Supposing, I take this to be a unit length, then at the top, I have no pressure, as I go below, it will be gamma times g h, where gamma is the specific weight of, specific mass of this particular.

So, in this wall, since the pressure is acting this side, the loads to be distributed like this equal to gamma g h, gamma g is the specific weight times the height that will give you the pressure at any particular point and that is acting. Remember, this h is varying with from this and therefore, it is a linear variation over here. You can also imagine if you have a cross section that is varying, that will introduce variations in the load ((Refer Time: 15:21)).

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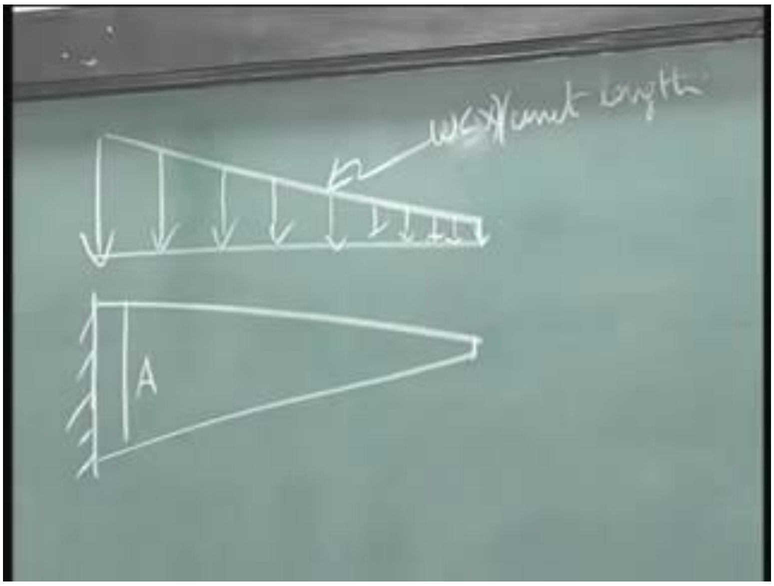

Simplest example could be, supposing I have something like this and a beam with the cross section this arc, I know the unit weight per unit length is varying linearly from one end to other. And this will introduce, for example, if the cross section with area A is varying linearly from here to here, then the load also be varying linearly. This w per unit length is now a varying load. So, I should write this as w of x per unit length and then, define what is w of x.

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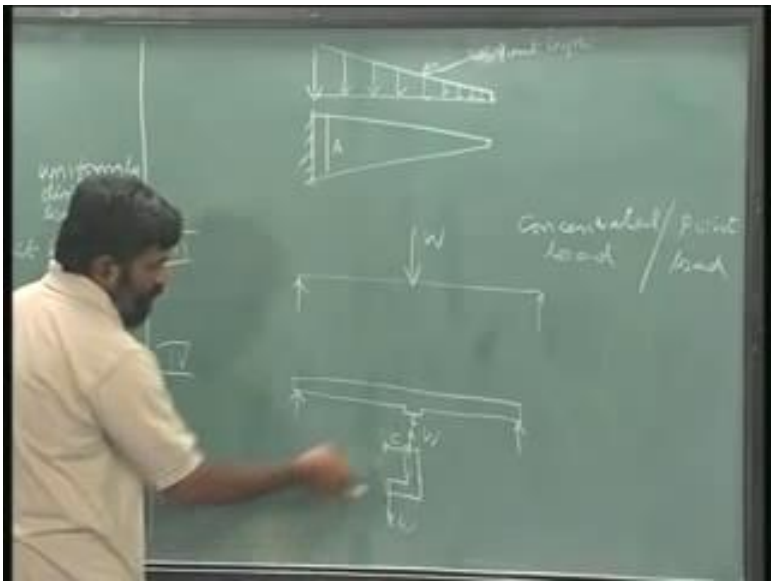

The third type is the type that you already know. For example, if this is a platform and a man is standing on top, the weight of the man can be replaced by a load equal to is weight and this is usually called a concentrated load. The width where which this load is applied is very small compared to the length of the beam, that you can take this to be a load, which is a pointed load or a concentrated load. So, this is also called point load, for this very fact that, it is acting at a point on the beam.

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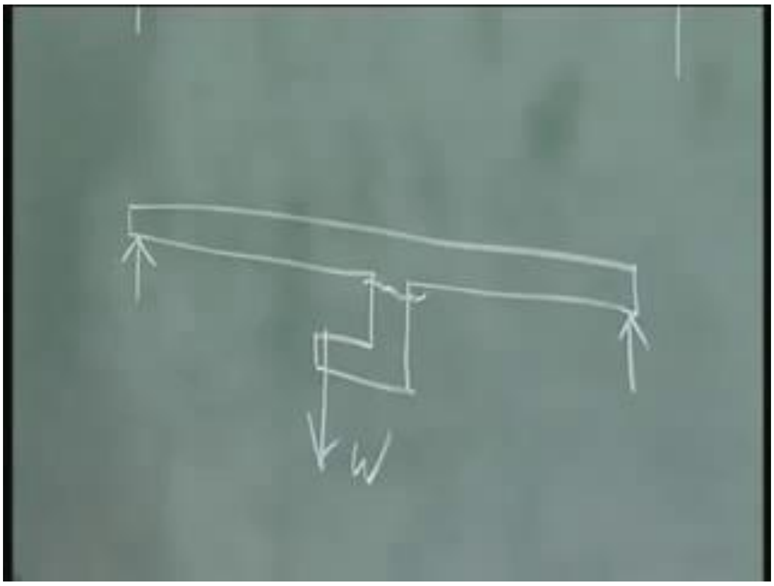

In other case could be think of something like this, I have something like a hook here and a weight is hanging over here or like say the here is a load, let us take this particular example. If I have to treat this particular length as just a beam, what I can do is, I can cut off here and treat this as a separate body.

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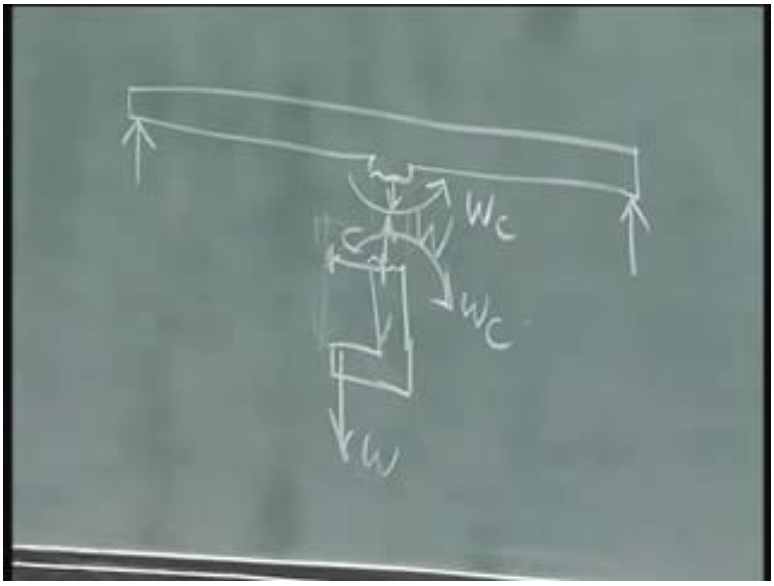

So, if I have to cut it off, I am revealing the internal forces over here, there is a force over here and that will create equivalent opposite force. This is very simple to solve, it this will be equal to w and if I take moment of this force about this particular point, let say this distances I have. So, ((Refer Time: 18:46)) say c, in which case, there is a bending moment acting in this direction. Therefore, I will have something like this, which is equal to w time c.

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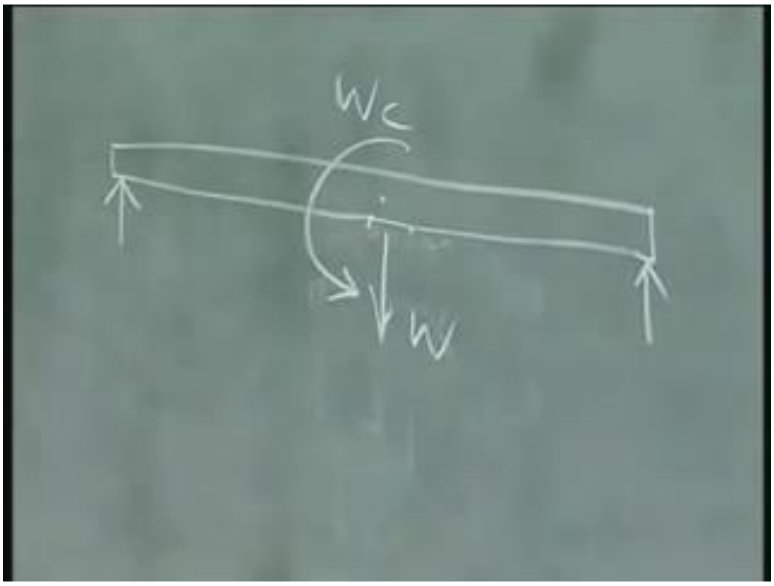

So, I can cut this half and replace with these two a weight and a bending moment equal to w c, w times c.

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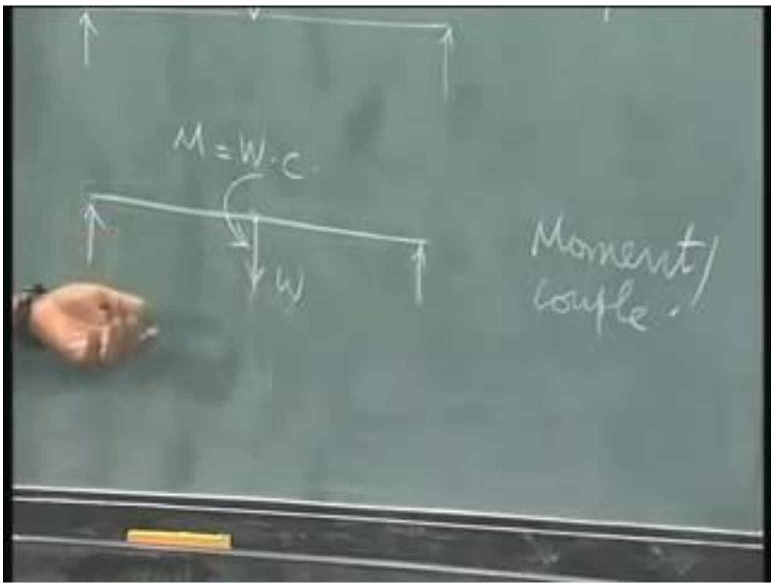

Now, if you notice, since this is a one dimensional member, I can jolly will write draw a line over here, represent the bending moment, a moment applied at this particular point and the force applied at this particular point as w and w c. In addition to a point load here, you also have a point moment acting of this. The other situation is, this is your example that I showed you, supposing you have a scissors, a pair of scissors that is blend the center of the forces will be search that will be a distance and that will apply a moment.

And if you the first sufficiently having a gap, then you are basically applying a moment; that is also can be replace like this. These are the some of the loads that may appear in a beam.

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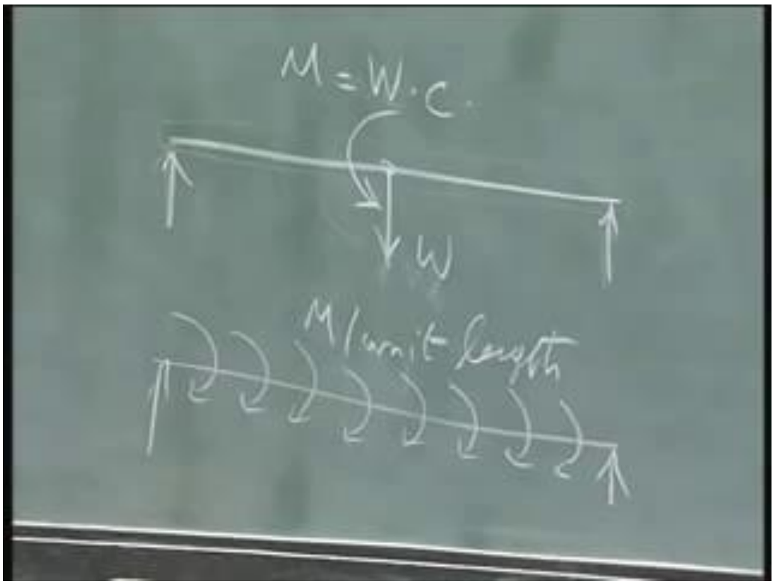

One last scenario, which may not expect in most of situations is something like this. I uniformly distributed moment, let say c per unit length, let me call this is m, small m per unit length, per unit length. The acting over the entire length of the beam, given this there will be combinations of these forces that will come into picture, while analyzing for various structures that use beams.

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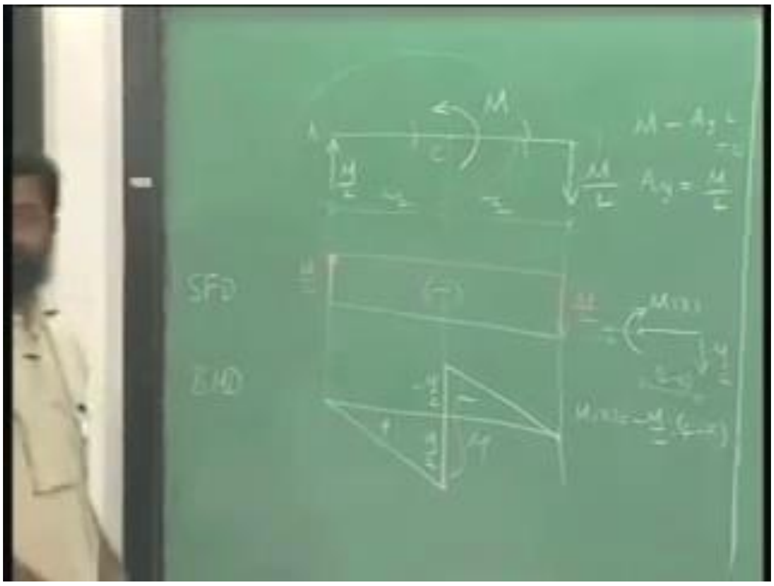

One of the tricky examples is an example of the sort; I have a simply supported beam, let say at center, I have a moment acting M. So, let say this length is L and therefore, this is L by 2, L by 2, I need to be find out the shear force and bending moments. What would be the reactions; that is not very difficult to find out. If I have to find out the left hand side reaction, I have to take moment about this, I have anticlockwise moment here.

M due to this and this is a clockwise, this force applies a clockwise about this and let say this reaction is A y, this is A and this is let say B y and this is C, A y times the length L. So, this minus A y times L equal 0, this implies A y equals M by L. And therefore, I can replace this by M by L. How about this side, there is no other force acting on this free body, if this is M by L, this will be minus M by L, on another words, I can change is direction and write it as M by L.

And this is important to note, I have an equal and opposite forces here. In fact, they form of a couple equal to M; that is one a way of understanding. How do I draw this shear diagram, when I shear force diagram I do not have to care about the moment that is acting on this. Let us pretty simple, I have M by L acting here and therefore, there will be a shear, there is no transfers force acting on this, also on this. And therefore, it will continue to we like this and at this particular point it will dip down by M by L and this is the shear force diagram.

What senses this like this, negative sense, remember, upward is negative and downward is positive is what we used to discuss in the final lecture. How about bending moment diagram, run that is, I question, the most of us have problems answer, but if you think rationally, it becomes very simple. Like before, if I cut anywhere this is the one that is going to take part, this M by L is going to take part in the bending moment.

And it is going to have a positive sense and therefore, automatically I will have like this, positive, there is no transfers force on it and therefore, the shear forces constant. Is the shear force is constant, bending moment has to be linear, starting from a 0 over here to a value equal to M by L times L by 2. So, M by L times L by 2 is M by 2, at this particular point.

What is the moment, what is the sign of this, it is equal to positive. You can use the trip that I told you earlier and have to find out for the right hand side, instead of taking and section that is this, if you take the section that is to the right. I have a force acting downward. So, if I have a force acting downward like this and I wish to find out the bending moment, what is the positive sense of this bending moment is it this a the answer is yes. If clockwise is positive, because negative direction is this is minus x direction.

If I need to find out this M at x, it is equal to M by L times L minus x and therefore and substitute M of x will turn out to be, is it positive or negative, M of x is in the clockwise direction, M by L times L minus x is also in the clockwise direction. And therefore, I will have a minus M by L times L minus x and let may make it clear here M of x is equal to minus M by L times L minus x.

Sometimes I may go from Goa to Chennai, what is a value of this at L x equal to L, naturally equal to 0, is it going to linearly vary, the answer is yes, is L minus x always positive, the answer is yes. There is a negative sign here, which means the bending moment will all negative, it is linearly varying at this particular point this equal to 0. And what about at x equal to L by 2, it is L minus L by 2, which is L by 2, M by L times L by 2 is M by 2 minus M by 2.

And therefore, I will have in this, this is M by 2, this is the bending moment diagram. Most often than not you will have a confusion over here, because you are have minus here plus here does not make sense, why should always have a continuous curve for M and that creates a problem here. What we have done here is correct, please always remember this particular notion, if I am drawing the shear force and there is a concentrated load. For example, in this case, I have a concentrate load that push is the value of shear force by that value abruptly at that particular point.

I go on at this particular point, I have a force and therefore, it pushes it down. Similarly, if I have a bending moment diagram, bending moment diagram will be continuous until I see a moment, concentrated moment like this. That will create a jump, sudden jump here and there is a jump that you see here, equal to total M, M by 2 and M by 2 total together, this will be equal to this M that your operator.

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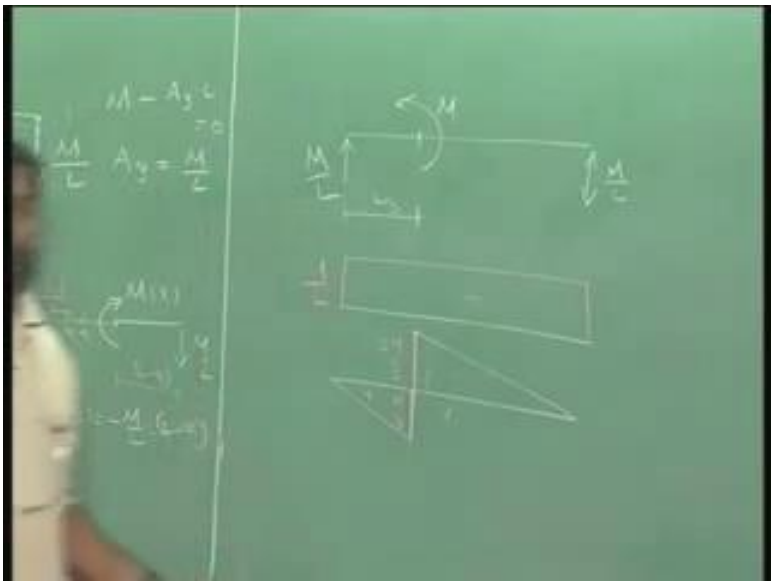

Now, the question, supposing I move this M from away from this center to some other point over here, let may draw it and ask you that question. Let say I have moved it to L by 3 away, what happens to the reactions, most of in the not, if I ask you no answer quickly, you will say that, this force will change will increase compare to M by L. But, if you do the calculation it will determine M by L here and minus M by L, here is a often confusion that you will have.

I can draw the shear force diagram; shear force diagram will not change it all, irrespective of my moving the moment over here. So, shear force diagram will be essentially the same. How about the bending moment? I will give this understanding that this concentrated bending moment will give a sense of sudden shift. I know that is no transfers force acting here, which means, it is derivative as I have already pointed out here, ((Refer Time: 31:14)) d square M by d x square is equal to minus w by x, w and this particular zone is 0.

Means this is 0 ((Refer Time: 31:30)), d square M by d x square is equal to 0, means d M by d x is constant. If d m by d x is constant M is linear x and therefore, this M by L will create let me draw till this, this a positive sense like this what is this it is M by L times L by 3, which is M by 3. This is positive moment, you can do it yourself and see, I can use the same principle that I used over here for this concentrated moment, what is this concentrated moment going to do, it is going to up, because it is counter clockwise, it is going to up by a value equal to M.

I already have M by 3 here, which means this is 2 M by 3, I know it is linear between this and this, because there is no transfers force acting and I just connected at this end, it has to be equal to 0. I finish it and no time; please practice a lot of the these, if you have to immediately solve some problems. By practice and understanding how these inter cases are useful it would not be able to solve the problems faster and correctly.

Thank you.

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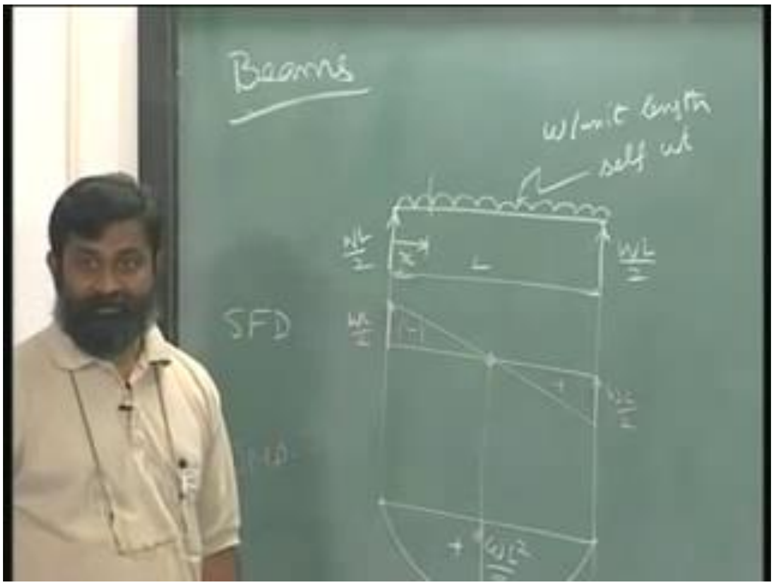

Let us take another simple example, this time use in a distributed load. So, this is also a simply supported beam with self-weight acting on this. Let us assume that the cross section is uniform all through. So, we have I uniform fully distributed load, we can draw something like this, how do I draw shear force in bending own diagram for this. Now, if you look at it, we have force everywhere. So, unlike the other case, where we had two concept that will load and we were able to separate them.

Here, there is no need to separate that is no distinct change that is occurring from one point a one zone to other zone. So, let us now start to draw shear force diagram. Let us adapt the method, there it was given are here I take an x from the left hand side; I need to take a section.

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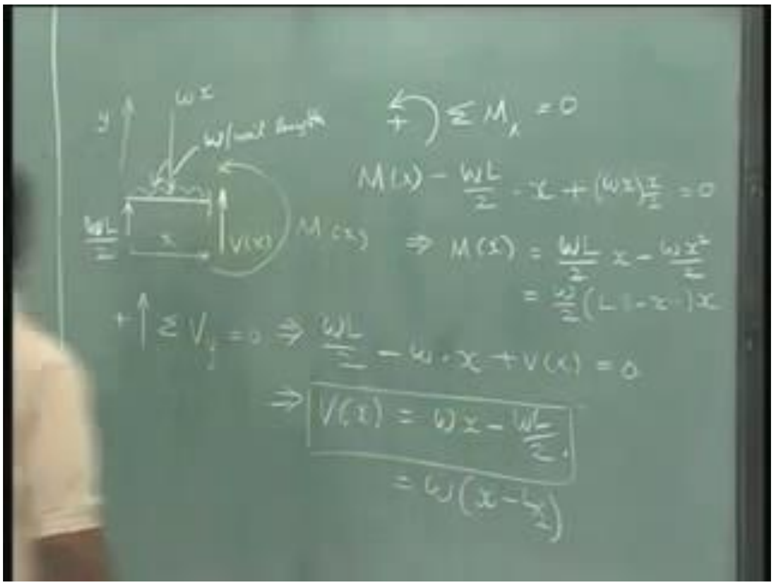

So, what I have basically is the self-weight acting on it, cut at a distance x, there is a reaction over here, let say this self-weight is w per unit length. Let say this total length of the beam as L and therefore, the reactions will be w L by 2, w L by 2. So, w L by 2 and what is, it w L by 2, the answer is yes, because w is per unit length and this is act over the entire length L's of the total weight is equal to w times L.

And since, it is symmetric immediately I can distribute half and half here and therefore, it is w L by 2 here and this is w per unit length. Upon section, it I should be revealing the shear force again I draw in the positive sense of these the shear force and the bending moment at x. What shear force on this, it is if I take this can be positive, sigma V long the y direction, if this is y is equal to 0 implies w L by 2 minus w times this length is x and therefore, w times x, then we use a same symbols so that, this is not confusion plus V of x is equal to 0.

And this immediately gives us V of x is equal to w x minus w L by 2, this can also be written as w into x minus L by 2. Notice here, unless x goes to over L by 2, this value is negative. So, we have a negative shear ((Refer Time: 37:25)) till half the distance and the rest of the distance will have positive. Now, doing it to write separately for this half not necessary, this will completely give the value given x varying from 0 to L.

So, how do I draw this, we look at this, this is a linear variation in x, at x equal to 0, it is w L by 2 and that is naturally true ((Refer Time: 37:56)), because I have a w L by 2 over here. So, this is w L by 2 and then, we have a uniformly decreasing value to this at center x equal to L by 2 V of x is equal to 0 from this. And therefore, it passes through this and it is linear, we have something like this, started with w L by 2 and this is a negative, it becomes positive and finally, we close this with w L by 2.

Bending moment diagram, how do I draw the bending moment diagram, again the same principle, let me use the same free body diagram over here. In order find out M of x as it choose this point over which I find the equilibrium of bending moment.

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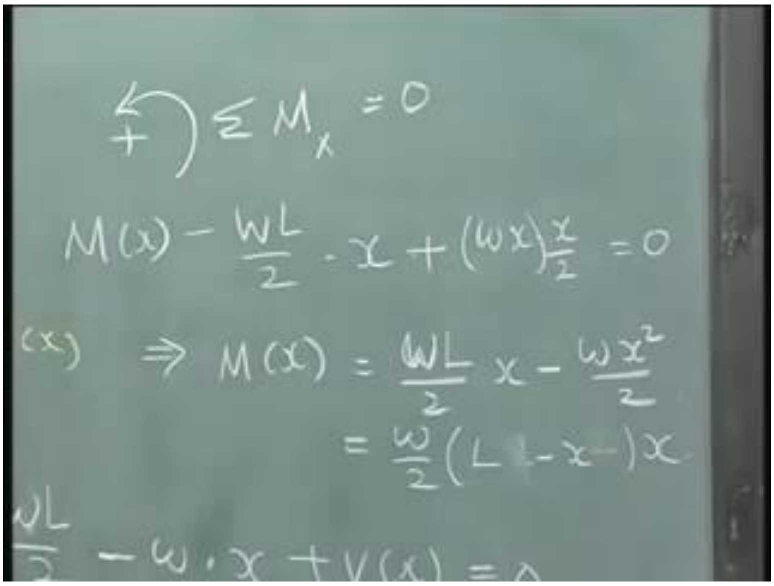

So, m at x equal to 0, this will give me V of x will not take part w L by 2 will take part in the clockwise sense and therefore, I have minus w L by 2 times x. Many times people may forget this x and how about this, this is uniformly distributed self-weight over this entire length. Now, there are two ways of doing, you can integrate from one into the other. The other way of doing it is, the total force can be replaced by this equal to w times x, because that is a total force and that is acting at the center of this x, which is at I distance x by 2 from this x.

So, w x times x by 2 will be the moment created by that and that is in the anticlockwise sense. So, we have plus w x times x by 2 and this is the positive moment equal to 0. So, this will give us M of x is equal to w L, w L by 2 x minus w x square by 2. You can also write this is as w by 2 into L x minus x square, we can take the x also outside and write it as M minus x times x, look at this symmetric L minus x times x.

If I go from the left hand side it is x ((Refer Time: 41:32)), if I go from the right hand side, it is L minus x. So, it is L minus x into x, if I put x is equal to 0, this an entire expression is equal to 0, because equal 0 here. If I put x equal to L, this will go to 0 and therefore, the entire expression 0, which is true ((Refer Time: 41:58)), because these two points are simply supported points.

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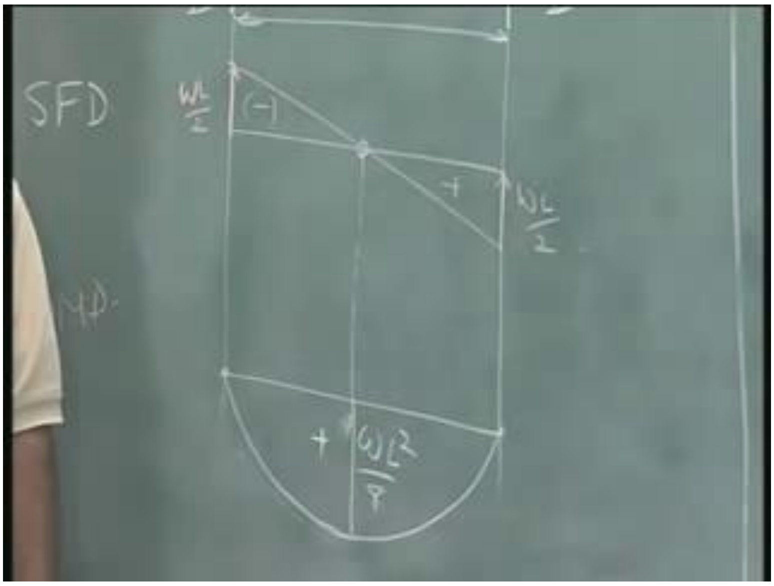

Now, anywhere in between, what is this sense of the particular value x is a positive value, L minus x is also a positive value as long as I am between 0 and l and therefore, this entire thing is a positive value. Positive we denote by down below, what kind of variation is this, it is a quadratic variation. Therefore, primarily I will get something like this. How about the center, what is the value of the center, the value of center is ((Refer Time: 42:55)) x L minus x is L minus L by 2, which is L by 2, times L by 2, L by 2 times L by 2 is L square by four times w by 2, you get w L square by 8.

So, this is w L square by 8 parabolic variations, 0 at this point and this point and this is the positive value. This one thing that you should note over here at the middle the shear force is equal to 0, whereas bending moment is the maximum. Shear force is a linearly varying, whereas the bending moment is quadratic. There is the previous examples that we solves, we had cost and shear forces and the variation of bending moment was linear, for a constant we had a linear, for a linear shear force diagram, we have a quadratic. There seems to be a relationship between these two diagrams and that is what we going to explode next.

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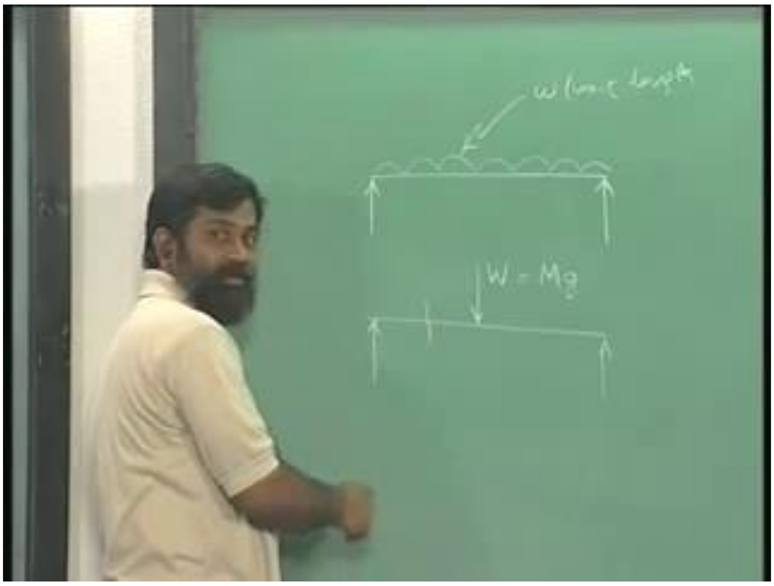

Before doing that I have one simple question and that is this, I have something like this as shown over there. This is uniformly distributed load due to itself-weight w per unit length. Can I find out the total result and force on this, the answer is yes and if I replace this uniformly distributed load with that, it is nothing but, the total weight acting at this particular point, which is equal to m times g, where M is the mass of the entire body.

So, remain use the capital M, the question is can I use this diagram in act to draw bending moment and shear force, the answer is natural, the answer is no. Because, if I cut anywhere here, what I have if a constant shear force and linear bending moment, which is not the story over here, this is a common mistake done by many. So, I just want to points is out.