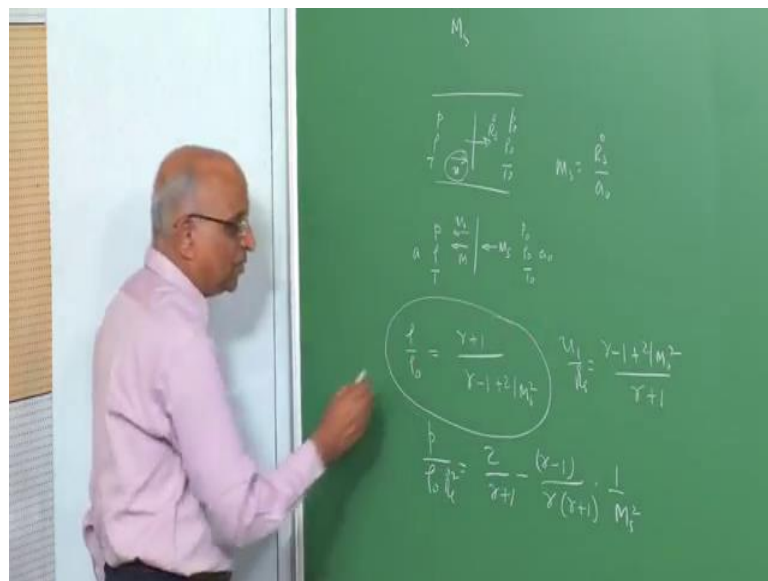


**Introduction to Explosions and Explosion Safety**  
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**Lecture - 06**

- 1) **Blast Waves: Concentration of Mass at the Front**
- 2) **Snow Plow Approximation**
- 3) **Energy Conservation in a Blast Wave**

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Well, good morning in today's class we will try to extend what little we learnt for a constant Mach number shock which we did in the last class to be able to predict the motion of a strong blast. Let me rephrase what we have done in the last class, we set shock moves in a medium whose pressure is  $p_0$  density is  $\rho_0$  temperature is  $t_0$  behind the wave. Well, the particle moves with a velocity  $u$  the shock velocity is  $R_s$  dot and what we did we predicted the value of  $p$   $\rho$  and  $t$  and also the velocity behind the shock wave mind you the shock is moving at  $R_s$  dot.

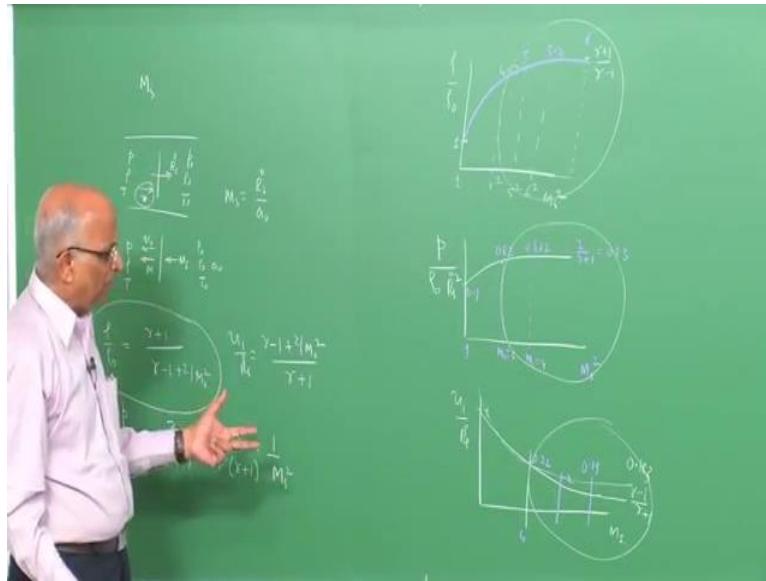
The particle is moving with velocity  $u$  over here and in the frame of reference of the shock stationary what did we get we got the medium moving towards it with a velocity  $R_s$  dot or equivalently with a Mach number  $M_s$ . The upstream property where  $p_0$   $\rho_0$   $t_0$  and the sound speed in the undisturbed medium was  $a_0$ . The Mach number behind in the shock in the frame of reference of the shock was  $M$  the properties were  $p$   $\rho$  and  $t$  this is what we did.

That means we were able to get the properties of pressure density temperature and the velocity in the frame of reference of the shock that is shock stationary how particles are moving back we were able to get it in. We were able to get this in terms of  $p_0$ ,  $\rho_0$  and  $t_0$  this is what we did, let us write one or two of these expressions down we had  $\rho_1$  by  $\rho_0$ . The density behind the shock to the upstream density was equal to  $\frac{\gamma + 1}{\gamma - 1} \frac{2}{M^2}$  by the mark number of the shock by the Mach number of the shock. That means we are looking at mark number  $M^2$  is equal to  $\frac{R s \dot{}}{a}$  over here, therefore we were able to get the density.

Similarly, we got the ratio of  $p_1$  divided by  $\rho_1$  into  $\frac{R s \dot{}}{a^2}$  is equal to  $\frac{2}{\gamma + 1}$ . Then, let me put the number over here minus  $\gamma - 1$  divided by  $\gamma + 1$  into  $\frac{1}{M^2}$  if the value of  $\rho_1$  by  $\rho_0$  is  $k$ . Now, we were also able to get the velocity that means for the velocity for that velocity  $M$  into the value of  $a$  which is equal to  $u_1$ , we were able to get the value of  $u_1$  divided by  $R s \dot{}$ , which is just the inverse of this the mass balance.

We got it as equal to  $\frac{\gamma - 1 + 2}{M^2}$  divided by  $\gamma + 1$  these are the relations which we want to use these relations to be able to predict some motion of the blast wave and how do we do that. Well, a blast wave is something which travels with varying velocities that means the Mach number is continuously changing in a blast wave and that creates some problems, but to be able to do that, let us examine these three expressions in somewhat greater detail. When I when we look at this particular expression namely the density ratio, what did we get?

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We got the value of rho by rho naught as a function of M s square we said when the when the wave has a Mach number one when the density ratio was 1 when the when the Mach number was very high, let us say infinity. The density ratio from this particular expression was gamma 1 divided by gamma minus 1 because this is very high, it comes out to be gamma plus 1 divided by gamma minus 1 at a very large Mach number. Let us put down the value of 2 or 3 intermediate value, let us put down the value, let us say Mach number let us say 4 at Mach number 5, maybe at Mach number 6.

That means 4 square, 5 square and 6 square that is Mach number of 4, 5 and 6, when I put it down, when I put it down, what do I get? I get the value of gamma plus 1 divided by gamma minus 1 by 2 by M square at 4 square, it is 2 by 16, it is 0.185 gamma of 1.4. We assume it to be here gamma is 1.4 plus 0.12, 5.62, 5.2, 0.4 divide by the value of point 4 plus 2 by 16. Over here, it gives the value over here and this value work out to be I calculated this value comes out to be 5.47.

The value for Mach number 5 let me get a color chalk over here comes out to be around 5, the value at 6 work out to be 5.27 and this value as we saw yesterday was 6. That is 2.4 divided by 0.4 which is 6 and therefore, if were to plot the variation, what is it I get, the variation is something like this.

That means beyond a value around 4, 4.5 or so the value does not really change very significantly the value of rho by rho naught is nearly a constant value. After this

particular value, let us take the value of let us say  $p$  by  $\rho$  naught  $R$  s dot square. Similarly, let us try to plot it again let us say I have the pressure behind the shock divided by the three stream density divided by the shock velocity square over here.

Now, if I look at the expression in in the in for the condition when  $M$  s that is the Mach number is very large that means  $M$  s is infinity this term drops out the value is 2 over gamma plus 1. Well, ultimately the value I get is 2 over gamma plus 1, this is the value very large value of  $M$  s over here or let us say I am plotting with respect to  $M$  s square. Then, if the  $M$  s is slightly smaller, then what happens the value is going to get subtracted from this value.

Lets us see what is going to be the value, let me plot when the  $M$  s is 1 that is 1  $M$  s is 1, therefore,  $M$  s square is 1, the value is 0.7. We found that  $M$  s is 1, it is 2 over gamma plus 1 minus gamma minus 1 divided by gamma into gamma plus 1, this gave you the value 2 gamma. It became gamma plus 1 divided by gamma plus 1 into gamma, that is equal to 1 over gamma, the value came out to be 0.7, we did this yesterday when the value is around 3. The value of  $M$  s is equal to 3, the value was 0.82 for a gamma of 1.42 over gamma plus 1 is equal to 0.83, when the value is of  $M$  s is equal to 4, the value is 0.822.

Therefore, we also find, well even I do not need to work anything, it is all 0.82 to 0.83, this is 0.7, therefore the value goes like this. In other words, even for anything exceeding around 3.5, the value of  $p$  naught divided by  $\rho$  naught  $R$  square is a constant. Similarly, if I look at the value of particle velocity behind the shock in the frame of reference of the shock divided by the shock speed in  $R$  s, I get well  $u$  1 divided by  $R$  s dot. Now, I plot let us say  $M$  s over here I just should get the inverse of this because it is going to be the opposite over here, that means this is just the inverse of this.

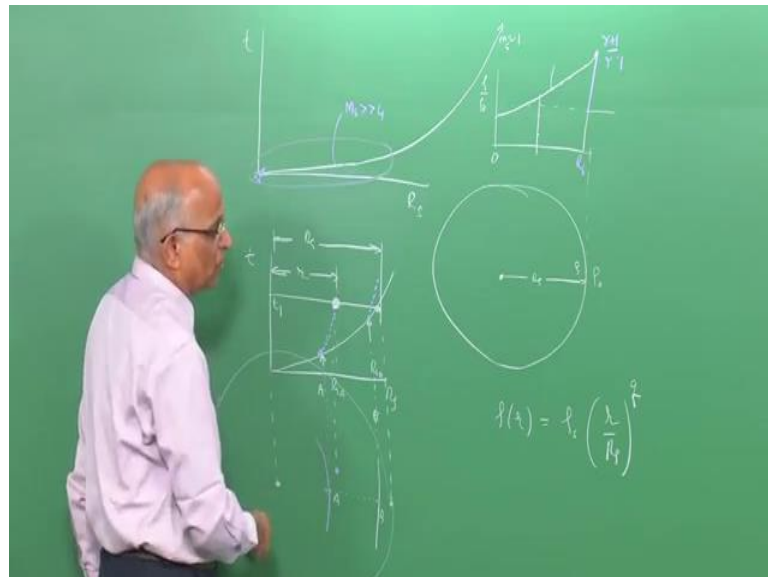
Therefore, if 1 in over here, the curve should be like this and let us now put down the value, the value at 4 is around 0.22 at Mach number of 4 at Mach number of 5, it is 0.2 at 6, it is 0.19 and for the condition  $M$  s is infinity. It is equal to if I look at this expression, this becomes 0, it is gamma plus 1 divided by gamma minus 1, which is equal to 0.183.

That means you know beyond this something like it is dropping, it comes down and it is almost a constant over here. That means beyond a certain Mach number, the value tends to a constant tends value around gamma minus 1 divided by gamma plus 1, the value  $p$

by  $\rho_0 R^2$  tends towards this. Beyond a certain Mach number, the value of  $\rho_0 R^2$  tends to a constant around  $\frac{\gamma + 1}{\gamma - 1}$ .

Therefore, if we are really interested only for Mach numbers greater than some limits somewhere in this region of Mach numbers. We can very well say that the Mach number of the shock will not very significantly influence or affect the value of  $\frac{u}{c_0}$  by  $\rho_0 R^2$  and  $\rho_0 R^2$ . Let us use this fact to try to model a blast wave, that means we will try to predict the blast characteristics the Mach number of the shock is a little high.

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Therefore, now we go back to the shock and see how the shock strength in a blast wave varies now we again go to this diagram  $t$  by  $R$  versus the distance what did we find. Well, the shock or the blast wave is started at the velocity and it keeps on moving and it ultimately becomes an acoustic wave and here Mach number towards the end almost near one  $M_s$  is 1.

Here, the Mach number is quite large, therefore in the portion like what we have may be in this portion, wherein the blast wave, the Mach number is around greater than around 4 or something. Then, I can use these conditions and we will try to see what happens when the initial or in the near field of the explosion wherein a blast wave get started for which let us say the Mach number is greater than around 4. How will the blast wave get affected? When I talk of these things well the Mach number also influences the pressure

density and temperature behind the shock wave and therefore it might be a little more difficult.

Therefore, let us do this problem to be able to do this problem I take this part again and again draw it again, let us take a look at the blast wave I just take these strong part of it I have  $t$ , I have  $R_s$  over here. Well, I say the shock started here and it goes like this I am interested in a strong blast region. Let us therefore, say well maybe I have the explosion coming taking place over here the shock keeps moving the blast wave keeps moving. I am talking in terms of a high Mach number shock, which is formed and it continuously decaying this we found it to decay.

Let us say a point  $a$  over here when the shock is at point  $a$  that means this is point at the radius is  $R_s a$  what happens at this radius may be the point  $a$  enters the blast wave over here. What happens when it enters the pressure behind it increases very rapidly and what happens to the pressure  $p$  by  $\rho$  naught  $R_s$  dot square. Since the Mach number is quite high is given by the value  $2$  over  $\gamma$  plus  $1$ . That means this is the pressure behind the blast wave and then once the wave propagates out or once this particle has entered it is highly compressed.

Then, it expands out and it follows a trajectory like this that means the shock particle of gas once blast wave goes away like for instance I have the blast wave coming over here the shock particle is at a high pressure. The high pressure particle expands out and this we said is the particle as it enters over here. If we assume that this expansion is isentropic may be we are talking of particle isentropic or something type of an expansion that is particle isentropic, but the motion need not be isentropic as we will see a little later. Similarly, this wave continues to travel let us say to point  $b$  the wave is here blast wave is here it has decayed slightly less, but it has decayed around  $4$  or  $5$  and then what is going to happen a particle, which enters at  $b$ .

Now, the distance of the shock from the origin from  $R_s d$ , it enters over here this particle enters over here and this particle enters when the shock is slightly lower, but it is still strong. Therefore, it also expands once it enters because it is highly compressed it expands thereafter, therefore what we find is may be different particles as they enter the shock wave as they expand out. If I am interested in let us say in this particular particle

the properties at for the properties, may be in this particular region means when the shock, let us say slightly qualify this.

Suppose, let us say I am interested at this particular time is equal to  $t_0$  at time  $t$  is equal to  $t_0$  the shock is here in other words the shock is over here, let us for the present assume that the shock is spherical. Therefore, may be let us say this is the origin let us say this is the shock like this over here and now what is going to happen the shock is over here. I am interested in property over here, therefore the property over here is something which corresponds to this gas which is particulate gas, which enters at a as it expands this is also being processed by the lead over here. I am interested in this particular one.

Therefore, the question is can I express the property at this particular point, which let us say is at a distance  $R$  from the origin from the center when the shock is at a distance or from the origin from the center when the shock is at a distance of  $R_s$  away. I think this requires some element of further deliberation because what I am trying to say is well you have an explosion caused over here a spherical blast wave or a lead blast wave is something over here. The lead blast wave at a given time  $t$  or  $t_0$  is at a distance  $R_s$  what is happening the lead shock wave compresses the gas from  $\rho_0$  to  $\rho$  and that means it compresses the gas over here.

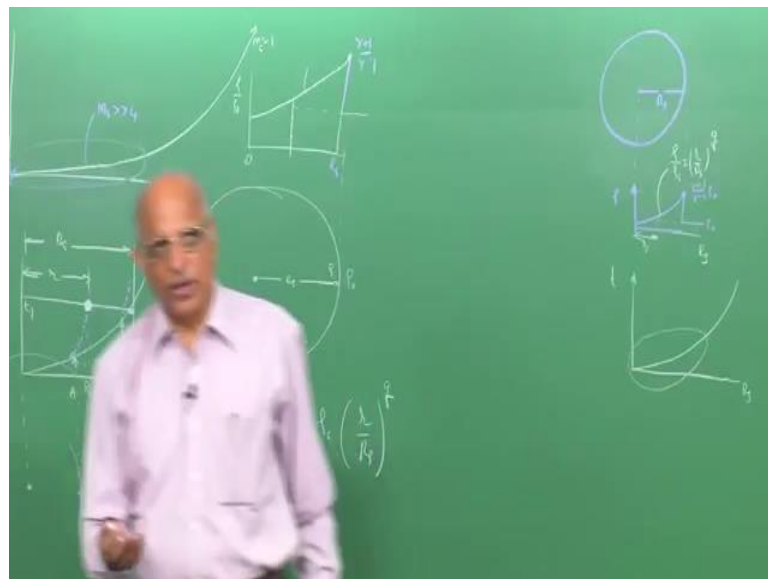
Let us put it down in terms of a figure, let us say  $\rho$  by  $\rho_0$  over here you have the lead shock, you have the ambient density is  $\rho_0$ . That means I have the value 1 over here this is the ambient density ahead of the shock. When the shock reaches this point, it compresses the shock to a higher value and this value since Mach number is greater than around 4 can be approximated as  $\gamma + 1$  divided by  $\gamma - 1$ . Thereafter, may be at any distance this is equal to  $R_s$  any distance  $r$ , well the gases are expanding and therefore, it is to be expected that may be the value of the density will be a little lower.

That means we can sort of assume that I can have something like a profile like this which is the value of  $\rho$  by  $\rho_0$ . This is from 0 at any particular radius or rather I can say density at any particular radius I can express as a function of density at behind the lead shock divided by  $R + R_s$ . This could be anything could be linear could be exponentially dropping could be a large value. I can say let it be denoted by  $q$  and such

type of profile sort of profile for density has been used earlier in some analysis and it leads to good results it is very illustrative that people who did this, where he did it?

He did this while working at the law Salamose science lab at Caltech, this is also followed by several other peoples work was followed by work by professor Bark at Megaline University followed by John Lee. Again, a number of people worked on this namely with the power law assumption and they were able to sort of model, the blast characteristics of a string wave. We will do something similar, but we will do something which is quite preliminary not advanced level, which level of work which may be he was followed by on this Zach around this let us try to do this problem.

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What I am trying to say is let us consider only a spherical lead shock. Let us assume that the lead shock originates from an explosion over here at the center and let us assume that the lead shock originates form an explosion over here at the centre.

Let us assume that the wave is spherical and the shock distance is  $R_s$  over here and when it is at  $R_s$  we are telling that well Mach number of the shock is still high. Therefore, if this is my value if I were to plot the value over here of the density the initial density ahead of the shock is  $\rho_0$ . Here, there is a jump in density and this jump in density because their shock strength is quite high is equal to  $\frac{\gamma + 1}{\gamma - 1}$  divided by  $\rho_0$  over here. This is the value and then because of the expansion

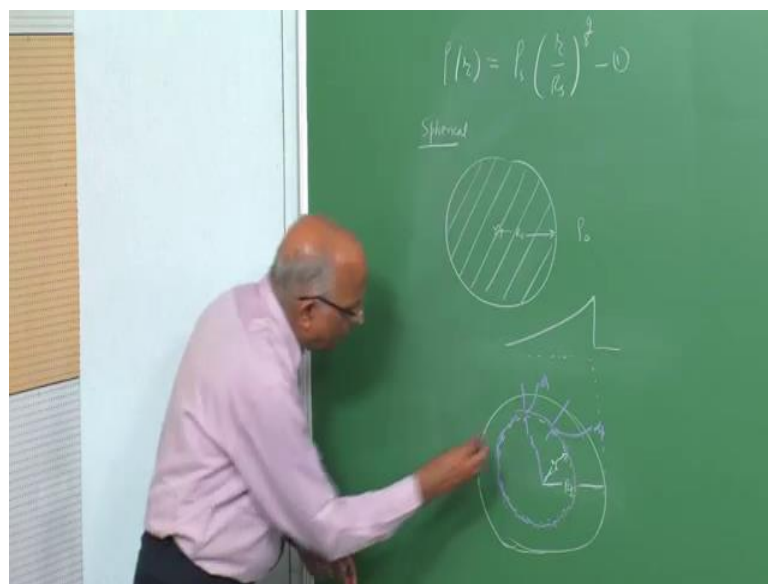


behind the shock, this keeps falling and this particular profile that means this is equal to  $R^{-q}$ .

This is equal to value we say  $\rho$  at the surface  $\rho$  is equal to  $R^{-q}$  by  $R^{-q}$ , this is at any radius, let us say any radius  $R$  To the power  $q$  is what we are assuming with this assumption. Let us try to predict something like the motion of a blast wave, let us try to solve the mass conservation equation and see if we are able to deduce anything further about blast, which is propagating.

Let us be very clear on this  $R^{-q}$  diagram when we have  $t$  versus  $R^{-q}$  and the blast wave is propagating out we wish to find out what are the characteristics of this initial region which we say is the strong blast region. This is what I will do in the next five or ten minutes to be able to do that let us put down the problem a little more clearly, well we tell ourselves.

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Well, the density at any at any distance  $r$ , when the shock front is at  $R_s$  is given by the density at the shock front divided into we said  $R$  by  $R_s$  to the power  $q$ , this is the equation 1. We assume that this is a power law, now we want to look at the mass conservation equation what is the mass conservation equation. Well, we had the explosion over here, the lead shock is at a distance let us say  $R_s$  from the source over here what is the total mass which is contained by the lead shock wave. Well, the lead

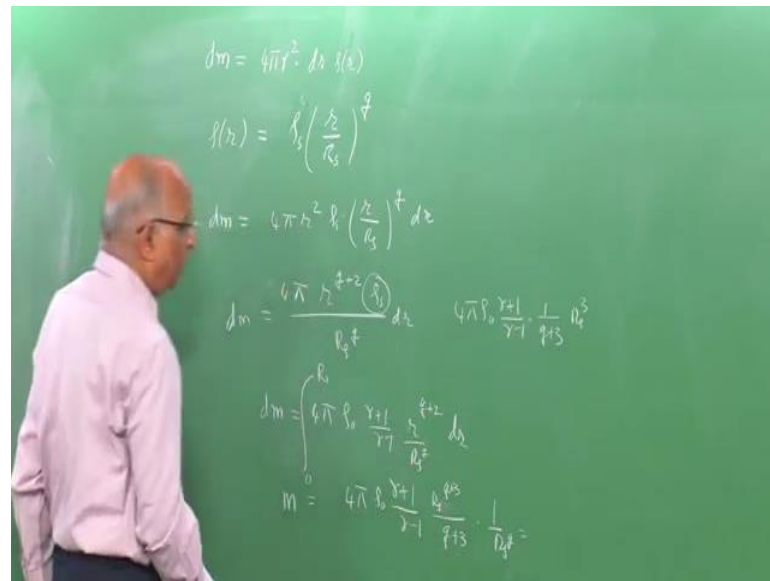
shock wave starts from the explosion site of the explosion keeps moving forward it has moved a distance of  $R$  s we assume it to be spherical.

This is only for the spherical which could be done for cylindrical could be done for the plainer, what is the total mass contained. Let us assume that the mass of the explosive is quite small and we will not consider it, we assume that when the total  $R$  is the total mass here when the ambient density is  $\rho_0$  the total mass is after all the mass which was initially available. For a spherical diameter of a radius  $R$  is it is equal to the mass is equal to  $\frac{4}{3} \pi R^3 \rho_0$ , this is the mass which is contained.

Now, this is mass is the total mass and in practice what did we tell ourselves this is the lead shock, now in practice what is happening the energy dispersion this explosion point overall changes the density pattern. What happens is at the front you have a large value of density, let us put it down over here and the value of  $R$  is the density jumps from  $\rho_0$  to  $\rho_1$  and then it keeps falling. Let us consider may a particular radius  $R$  small  $R$  That means this radius is  $R$  is let me consider small value of radius  $r$ . Around this radius  $r$ , let us consider a small element of thickness  $dr$ , in other words I am interested in what is happening in this element.

I will try to find out mass which is contained within this element, which is between the radius small  $R$  plus  $dr$ . Let us write is out this is  $R$  the inner radius the outer radius is  $R$  plus  $dr$ , therefore the element of mass which is contained within this small annular spherical passage between these two can be written as let write the expression for that.

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Here,  $dm$  that is the elemental mass in the particular shell is equal to the surface area  $4\pi R^2$  into the volume which is  $dR$  in the particular shell the density is  $\rho$  at a distance  $r$ . This  $R$  is within  $R_0$  so here this is equal to  $dm$ , which is contained over here, you will also recall we had the density at the distance  $R$  is equal the value was given by the ratio of the density at the surface. We said it is equal to  $R_0$  by  $R_0$  to the power  $\gamma$  at the surface at the lead shock radius divided by  $q$ , therefore we substitute this expression into this value here.

Then, we get the value of the elemental mass in this small spherical shell of width  $dR$  between  $R$  to  $R + dR$  for whose for which case within this elemental shell the density is  $\rho(r)$ . We get the value is equal to  $4\pi R^2$  into  $dR$  into  $\rho(r)$ . I substitute the value of  $\rho(r)$  I get  $\rho_0$  into  $R_0$  to the power  $\gamma$  into  $dR$  over here. Let us simplify this, therefore, I get this is equal to  $4\pi$  into the value of  $R_0$  to the power  $2 + \gamma$  into  $dR$ .

This is the value of the elemental mass  $dm$  over here, now if we look at the value of the density at the surface of the lead shock. We told ourselves well  $\rho_0$  by  $\rho_0$  is equal to  $\frac{\gamma + 1}{\gamma - 1}$  in the limit of strong shock.

What happens is you had that term  $\frac{2}{M^2}$  which for  $M$  is large, it knocks out over here, therefore, this becomes equal to  $\frac{\gamma + 1}{\gamma - 1}$ . If I were to substitute this value upstream over here I get the value of  $dm$  is equal to  $4\pi$

into I get  $\rho_0$  into  $\gamma + 1$  divided by  $\gamma - 1$ . That was the value of  $\rho_0$  over here into  $R$  To power  $q + 2$  into  $dR$  and of course I also have the value of  $R$  to the power  $q$  over here.

I want to solve this and therefore what is the total mass which is enclosed by the sphere that is by the lead shock by the radius  $R$ . Well, I have to integrate it out between  $R$  is equal to 0 to  $R$  is equal to  $R$  and if I integrate it out, what is it I get, let me write the value here, which is equal to  $4\pi$  into  $\rho_0$  into  $\gamma + 1$  divided by  $\gamma - 1$ . Now, I get  $R$  To the power  $q$  by  $q + 2$ , I integrate it out, it becomes  $R$  To the power  $q$  by 3, that means  $R$  To the power  $q + 3$  divided by  $q + 3$  into  $1$  over  $R$  to the power  $q$  is the integral.

For 0, it becomes 0 well for this  $R$  will become  $R$  and therefore, the value of the mass which is enclosed by the lead shock wave is equal to the  $4\pi \rho_0 R^{\gamma + 1} \gamma - 1$   $R$  to the power  $q + 3$  divided by  $q + 3$  divided by  $R$  to the power  $q$ , which I can also write as equal to the value if I were to write, I get  $4\pi$ . Let me write it over here,  $4\pi$  into  $\rho_0$  into  $\gamma + 1$  divided by  $\gamma - 1$  into here, I have  $1$  over  $q + 3$  into  $R$  to the power  $q + 3$  over here. We had  $R^{q + 3}$  over here, we have  $R^3$ , therefore, I have  $R^3$  over here therefore, I have  $4\pi \rho_0 \gamma + 1$  divided by  $\gamma - 1$  into  $R^3$  to the power cube divided by 1 by 3.

What is the total mass which is enclosed by the lead shock wave after all the lead shock wave has processed the gas, which was initially at the density  $\rho_0$  to a distance of  $R$ . Therefore, this will be equal to the value of  $4\pi$  into  $R^3$  divided by 3, this is the volume into  $\rho_0$  and this will be identically equal to this, which is equal to  $4\pi$  divided by I take this  $q + 3$  over here.

I get  $4\pi$  divided by  $q + 3$   $\rho_0$  into  $R^3$ , that means  $4\pi$  divided by  $q + 3$  into  $R^3$  into I get  $\rho_0$  into the only thing which I am left with  $\gamma + 1$  divided by  $\gamma - 1$ . Let us simplify this further, I find that four and four gets cancelled on the two sides  $\pi$  and  $\pi$  gets cancelled over here  $R^3$  also gets cancelled. The lead shock radius also gets cancelled, well the  $\rho_0$  also get cancelled over here and what is it I get, I get  $q + 3$  divided by 3 is equal to I get the value as  $\gamma + 1$  divided by  $\gamma - 1$ .

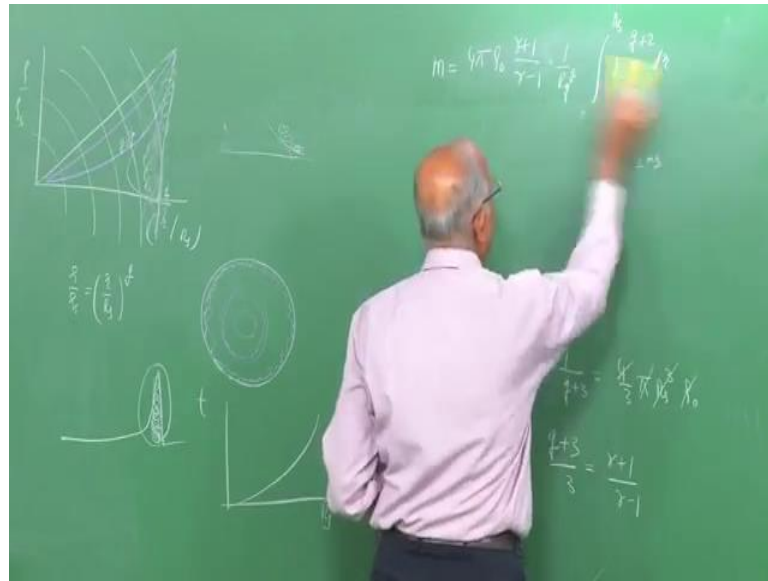
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The image shows a green chalkboard with handwritten mathematical equations. The first equation is  $\frac{q}{3} = \frac{\gamma+1}{\gamma-1} - 1$ . The second equation is  $q = 3 \left[ \frac{\gamma+1}{\gamma-1} - 1 \right]$ . The third equation is  $\gamma = 1.4$ . The final equation is  $q = 15$ .

Now, I can simplify this further and therefore, what is it I get  $q$  by 3 plus 1 is equal to gamma plus 1 divided by gamma minus 1 or rather the value of  $q$  is equal to 3 into gamma plus 1 divided by gamma minus 1 minus 1. For a particular value of gamma, may be for 1 we are considering gamma is equal to 1.4, which is equal to 2.4 by 0.4, which is 6 minus 1, 5, 5 into 3 is 15, I get the value of  $q$  as equal to 15.

Therefore, whenever we are considering, let us get back to the real problem, we are considering the mass enclosed by the lead blast wave and we found out, we assume some density distribution starting from the lead shock. It keeps varying and what do, we find if I assume the density distribution to be given by the value of density at the shock front and the density to be distribution to be given by something like a power law or by  $R^s$  to the power  $q$ . I find that this value of  $q$  is around 15, it is quite a large number, let us just discuss what this really means and it has some ramifications, let us see what the results indicate.

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You know if I were to consider the value of rho by rho s and I am looking at the value of R by R s, if I am looking at rho by rho s is equal to R by R s to the power q if q is equal to 1, well this is the type of distribution what I get. That means when R by R s is at the lead shock wave which is 1 well it is distributed like this when I have R by R s is a number is greater than 1.

Well, it goes like this that means what is happening is since it is greater than 1 R by R s is near to this value the magnitude is higher the R by R s is small because of the larger value, this being greater than one the magnitude changes as the value of q increases.

Let us say when q is 10, it could be like this when q is 15, all what it means is well what is going to happen is the amount of mass which is being contained in the front. That means let us say when q is equal to 15, what is going to happen, the mass is essentially contained at the shock front. There is nothing really over here or what we mean is the shock is moving most of the mass is contained, there is nothing really in depth over here because most of the mass is contained over here since q is large.

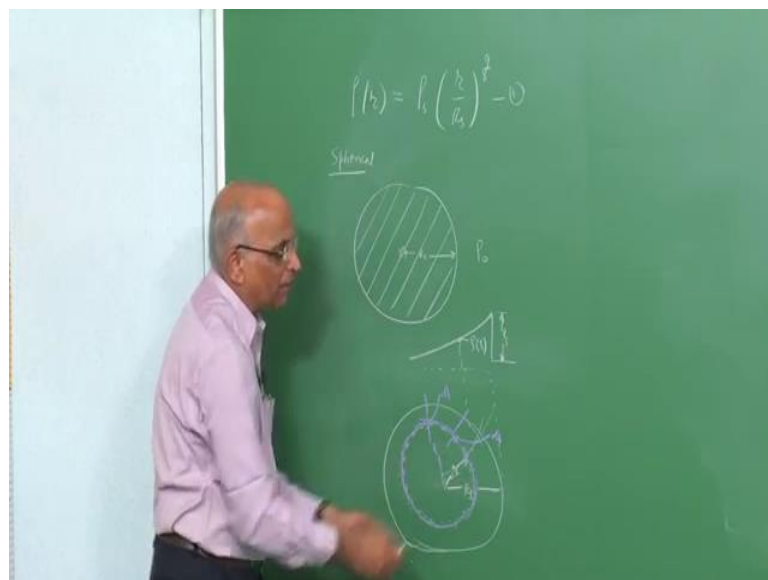
Therefore, this type of picture what we develop using for a blast wave, which has a high Mach number essentially tells us that most of the mass which is contained within the blast wave is concentrated at the shock front what does this mean really? You know we have been telling that a wave does not really transport the mass from the center and bring

it over here, but what we observe is whenever we are talking in terms of a blast wave, the blast wave processes the gases.

When the blast wave reaches the value of  $R_s$  that is  $R_s$ , you know it is able to take the mass and sort of distribute it at the front itself. If I were to say well my shock front is over here, the shock front as it is travelling it picks up the mass and puts it over here may be take the mass and distribute it over the front. If the blast wave is still strong, well all the mass is sort of localized gets localized at the surface.

It is as if the blast wave let us again plot this on the  $t$   $s$  diagram  $t$  versus  $R_s$ , well the blast wave gets started. Once the blast wave gets started, it is something like the blast wave with a broom or something it is picking up the masses locating it at the shock front. Therefore, all the mass which it collects in the medium gets concentrated at the shock front it is something as if it is like a broom which sweeps the air and keeps all the air at the shock front itself or something like you have snow which is there on the road. The snowplow comes and picks up the snow and all the snow is accumulating at the snow plow this becomes empty over here.

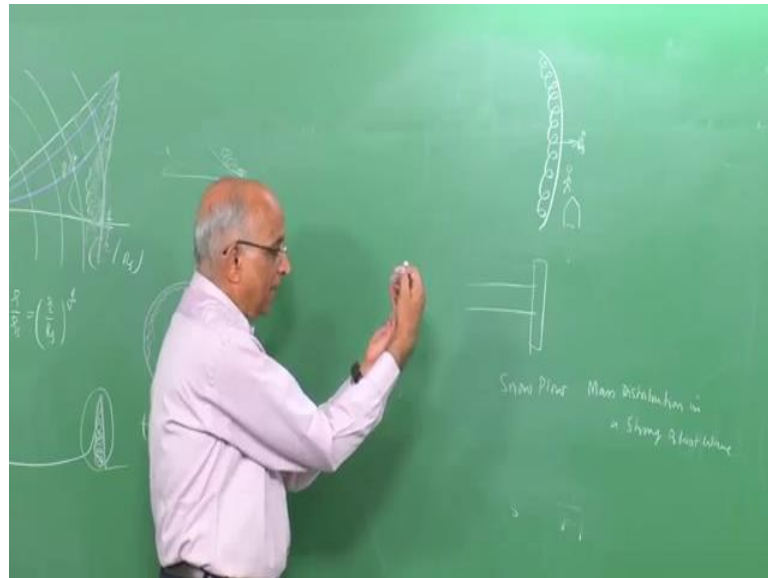
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Therefore, the type of picture we get is when a when a blast wave gets started, the blast wave picks up the mass concentrates the mass at the front and in the depth that is in the zone far from the blast wave, there is hardly any mass over here. Therefore, this is the

type of picture we are able to get from the power law profile and then let us pursue on that little more and see what it really implies.

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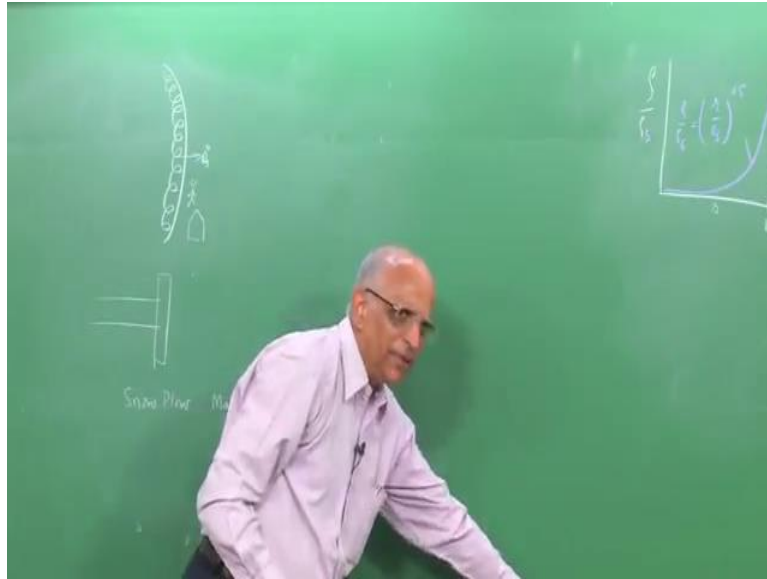
It implies that may be the front of the shock wave or lead blast wave provided that the lead blast wave is still strong enough contains most of the mass, which it is picking up and accumulating over here. It is something like a hammer all the mass is contained over here and it moves with a speed  $R \dot{s}$  which is still quite high we are talking in terms of the strong blast wave. It is as if this hammer, it comes and hits a person or some building over here well it just compresses and demolishes this and this is how a strong blast wave acts.

Therefore, a strong blast wave in the context of the power law assumption which we made is something like a snow plow or something like a broom which sweeps the place and this type of an assumption we get may be for the mass distribution in a blast wave. Therefore, we are able to picture something about the type of damages which a blast wave or a strong blast wave can do namely it picks up the mass.

As it moves, it concentrates at the lead shock and this lead shock when it picks some building or so can cause the damage whatever happens, therefore may be from this mass balance, we are able to get some physics.



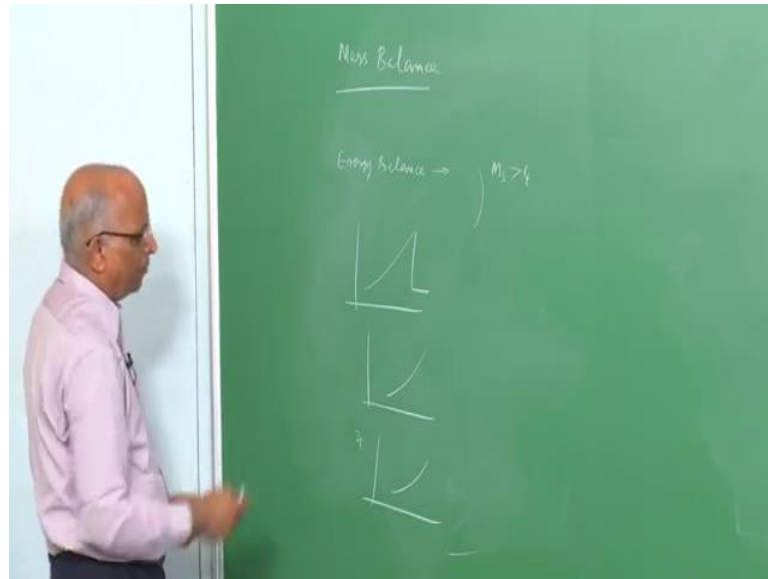
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The physics is well the density distribution behind the lead shock is such that when I plot the value of  $\rho$  by  $\rho_0$  as a function may be the lead shock  $R$  s I am interested at some station any station  $r$ . Well, at the lead shock I have a density jump and then it keeps coming that type of picture I get is not something which is gradual like this. Most of the density is concentrated over here and in depth I have really nothing over here. That means I get a value of  $q$  that is I get  $q$  by  $R$  by  $R$  s to the power around 15 for air.

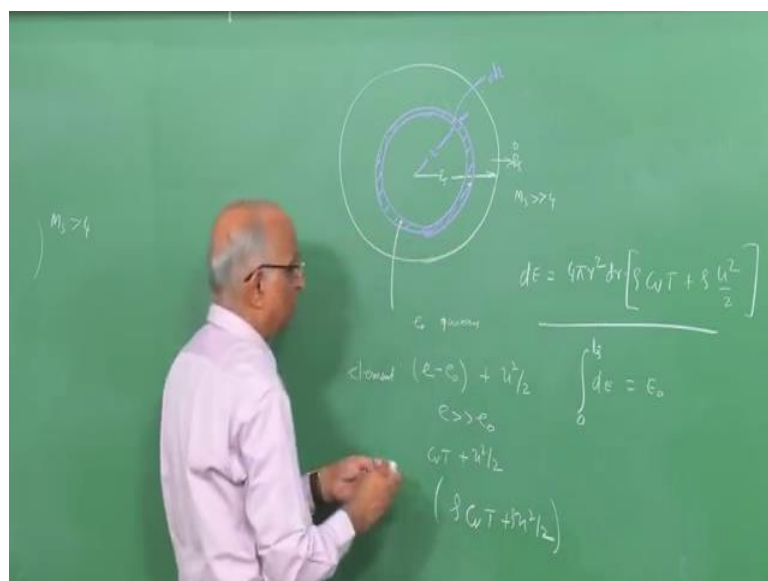
May be depending on their value of  $\gamma$ , I could get different values this is the value of  $\rho$  by  $\rho_0$  over here or it should have been at the shock front this is  $\rho$  s is equal to  $\rho$  by  $\rho_0$  s over here. Well, this is the picture of this we find, therefore it can be the lead blast wave can be assumed to act something like a hammer with which all the mass is concentrated at the front. Therefore, we tell ourselves the strong blast wave is something like a shock wave in which the mass which is processed by the blast wave is more or less contained at the blast wave front itself.

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Let us try to look at energy balance that I can draw any other information again, let us be very clear let us do the energy balance only for the condition when the lead blast wave is strong that is when the Mach number of the shock is greater than around 4. Let us see whether we can draw some physics out of it again, we assume a power law profile, we assume that well the density can be given by at the lead shock the density jumps the density decreases. So, also the value of pressure decrease, so the value of  $u$  decreases or let us try to put these things in some form and make may be make an energy balance.

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What is the energy balance, now I can go straight to the case of the lead shock being at  $R$   $s$  may be the value we are doing from the element a spherical shock the lead shock is at  $R$  now. We are considering a particular  $R$  over here we are considering a spherical segment of radius  $R$  The width is  $dr$ . Let us assume that the lead shock wave travels with a velocity  $R \dot{s}$  such that the Mach number is still high greater than around 4 such that we can still use those values which are constant with which we got started.

Now, I want to calculate the energy over here the change in energy, now initially I had this particular segment before the lead shock processed it this particular segment has an initial energy  $E_0$  it is that means there was no motion. Once the blast wave processes, this particular small segment it has an it has now an energy, let us say  $e$  which is over and above  $E_0$  above this value, it also has some kinetic energy may be  $u^2$  divided by 2  $u$  is the particle velocity.

Once the blast wave progresses, well we have presumed that there is some change the particle velocity here could be  $u_1, u_2, u_3$  here it is  $u$  and the particle velocity keeps changing. We are looking at the particle velocity within the small element over here, this is per unit mass energy per unit mass kinetic energy.

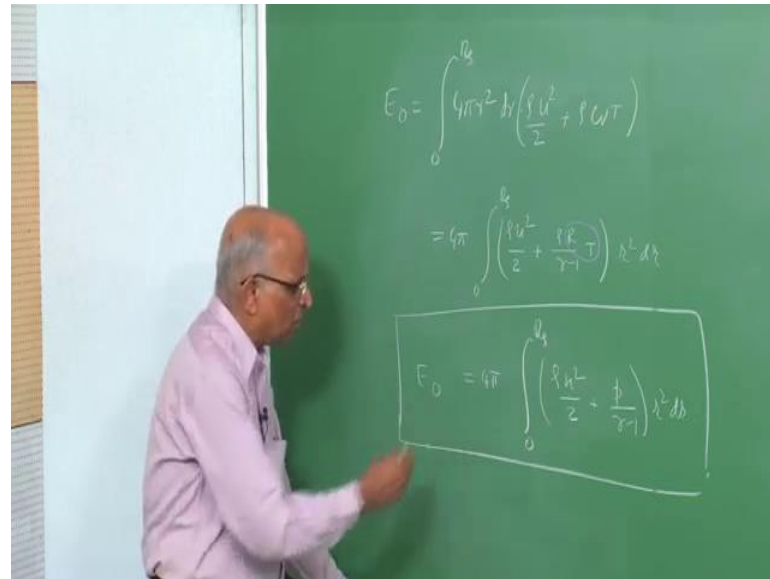
Therefore, per unit volume I say is equal to  $\rho e$  minus  $e_{naught}$ , let us presume that the value of energy after the blast wave processes, it is very much greater than  $e_{naught}$  because we are talking in terms of a strong blast wave. Let us presume that  $e$  is greater than  $e_{naught}$  this is an assumption we make, therefore we tell ourselves the energy in the element is equal to per unit mass is equal to  $e^2$  by 2. Well the internal energy is or is equal to molecular energy which is equal to  $c_v$  into  $t$  and this is per unit mass per unit volume is equal to  $\rho$  into  $c_v t$  plus  $u^2$  divided by 2 and per unit volume.

This is the value the volume of this small spherical segment again is equal to  $4\pi R^2$  square which is the surface area multiplied by  $dr$  over which the properties are constant. Therefore, the energy within this element  $d$  is equal to  $4\pi R^2 dr$  which is the volume into the value of  $\rho$  into  $c_v t$ . This is the internal energy of this particular volume this is per unit mass, they should have also been  $\rho$  plus  $\rho$  into  $u^2$  divided by 2 is the energy within this small volume.

Just like we balanced the mass in this volume and integrated out over here, we will integrate out and find out the total energy, what is the total energy what is now available

is the energy from the explosion because initially it was only  $E_0$ . We are looking across over here and therefore, the energy  $dE$  if I were to integrate out between 0 to  $R_s$  should be the energy, which is released by the explosive or during the explosion. What happens the energy released by the explosions is dispersed by the lead shock wave over this particular volume and therefore, I get the value of  $E_0$  is equal to this.

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Let us put it down over here  $4\pi R^2$  into  $dr$ , this is the volume into  $\rho$  into  $u^2$  plus  $\rho$  into  $c v t$  over here and this is equal to the energy which is released which is  $E_0$ . Now, let us try to simplify this becomes equal to  $4\pi R^2$   $4\pi$  into well  $R^2$  depends on  $R_s$ , I cannot take it out, I get  $4\pi$  into integral 0 to  $R_s$ . now, I get the value of  $\rho u^2$ , well  $u$  depends on the shock front density I still keep it as  $u^2$  divided by two plus I get  $c v$ .

Can I simplify  $c v$  you know if I look at  $c_p$  minus  $c_v$  is equal to this specific gas constant  $c_v$  is equal to  $c_p$  by  $\gamma$  minus 1 is equal to  $R$  or rather  $c_v$  is equal to  $R$  over  $\gamma$  minus 1. Therefore, I put this instead of this I get  $\rho$  into  $R$  divided by  $\gamma$  minus 1 into  $t$  over here into  $R^2$  and  $dr$  and now I make note that I have  $P V$  is equal to  $R T$  for a gas  $P V$  is equal to  $R T$  or  $t$  by  $\rho$  is equal to  $R T$ . If I were to substitute the value of  $p$  by  $\rho$  instead of  $R$  and  $t$  product of  $R$  and  $t$ , what it is.

I will get  $t$  by  $\rho$  over here and this row and this row gets cancelled and what is it I get, this is equal to  $4\pi$  into 0 to  $R_s$  of  $\rho u^2$  divided by 2. I have  $p$  by  $\rho$  gets

cancelled  $p$  divided by  $\gamma - 1$  into  $R^2$  into  $dr$ , which is equal to the energy released in the explosion.

This is the total value of energy released and what this energy release is it is dispersing a through the medium and it is changing the value of density particle velocity and pressure in the particular medium. I want to be able to solve this and to be able to solve this I need to make an series of assumption like I still consider my wave to be strong. So, the conditions the wave front is known, in other words, see we have already seen the following. We told ourselves well the value if I were to consider lead shock over here the density is  $\rho_s$ , it has jumped from the value  $\rho_0$  to  $\rho_s$  and it keeps coming down over here.

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The image shows three handwritten equations on a green background:

$$\frac{\rho}{\rho_0} = \frac{\rho}{\rho_0} \frac{\rho_s}{\rho_s} = \psi\left(\frac{R}{R_s}\right)$$

$$\frac{p}{\rho_0 a_0^2} = f\left(\frac{R}{R_s}\right)$$

$$\frac{u}{a_0} = \phi\left(\frac{R}{R_s}\right)$$

Therefore, the density at any particular point  $\rho$  by  $\rho_0$  can be written equal to the value of  $\rho_s$  into the value of  $\rho_0$ , which I can again write in some quantity like  $R/R_s$ . In other words, this is a particular value that means  $\rho_s$  by  $\rho_0$  such that I get  $\rho_s$  by  $\rho_0$  can be written as some function of  $R/R_s$ .

Similarly I can write  $t$  by  $\rho_0 a_0^2$  at any particular point that is the shock front I am interested in a radius, let us say  $R$  over here I can write it as equal to may be some function of  $R/R_s$ . Similarly, the last term which I want to write I can also write  $u$  by  $a_0$  can also be written as some function of  $R/R_s$ .

These could be expressed as power law, but now we say well it is the function of the initial condition, it is a function of  $\rho$  into  $R \dot{s}$  square it is a function of  $R \dot{s}$  over here. Therefore, what I will do in the next class is maybe I will use these particular values substitute it through the energy equation and try to solve how the strong blast wave changes its characteristics. We can also see how the blast waves behave when it propagates at high Mach number.

Well, then thank you that is about it.