Introduction to Explosions and Explosion Safety
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Lecture - 05
Properties behind a Constant Velocity Shock

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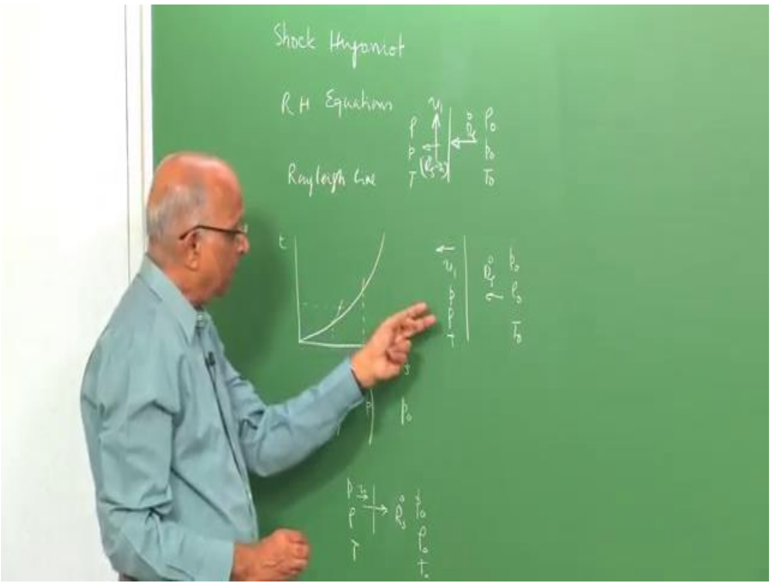

Well, good morning you know in the last class you will recall we talked about the shock Hugoniot and then we talked about the rank in Hugoniot equations and also about the ray length line, what did they represent? Let us quickly review it. We were interested when a blast wave gets generated, maybe in the streak diagram, we have t versus r f, the blast wave gets started at a high velocity and keeps decreasing, until it becomes an acoustic wave in it keeps decreasing in strength.

We are interested in finding out as the blast wave progresses. Maybe when it is at this radius, the wave is here, it gets started over here, when it is at this radius well it is over here, we would like to know what pressure behind it is, when the ambient pressure is P 0.

We would like to know what the pressure what is the jump in pressure and also the wind effect what are the velocities behind it, which give the impulse. Therefore, to be able to calculate the pressure at the different points as the blast waves progresses. We wanted to know what is the value of pressure at the different points as the blast wave progresses.

We want to know what the value of pressure is, we wanted to know what the value of the velocity is, we wanted to know what the temperature is etcetera as the wave progresses.

To be able to do that, we found that the problem is a little complicated because the wave is decaying. Well, a particle which enters here gets progressed like this it is also affected by the subsequent motion of the wave, maybe at this particular time, the wave is here, the particle is here. Therefore, the problem is terribly transient it is unsteady and therefore, it becomes difficult to do this actual problem which we will anyway do. But to get a feel for the problem, we wanted to first do the problem of a constant velocity.

Let us say d r is by d t is some fixed value, the shock is moving at the value let us say R s dot when the shock is moving at R s dot into a medium whose pressure is P r 0, whose density is rho 0, let us say whose temperature is t 0. Let us say this is the medium that means it has no velocity, I want to find out what is the pressure density and temperature behind the wave and also the velocity behind the wave. We wrote the equations for this particular one may be the mass conservation the momentum conservation and also the energy conservation relations and solve these equations in the frame of reference of the shock stationary.

Let us put it together over here, I have the shock here, instead of the shock moving at the velocity R s dot I made the medium move towards it a medium at rho 0 P 0 and t 0. Now, moves towards this shock at a value R s dot because the shock is now stationary. And I was interested in finding out well the velocity behind the shock is now u over here. I have rho P and temperature which are the parameters behind the shock wave.

Therefore, now I have R s dot over here before the velocity is equal to R s dot over here therefore, the velocity is R s dot minus u over here and this R s dot minus u I called it as u 1. Therefore, we solve the problem of may be u one may be P rho t behind it i have the value R s dot of the medium moving towards it is moving over here I have P 0 rho 0 and t 0. What did the shock Hugoniot do it related the pressure behind with the initial value of the pressure, it also related the density o the initial value of density.

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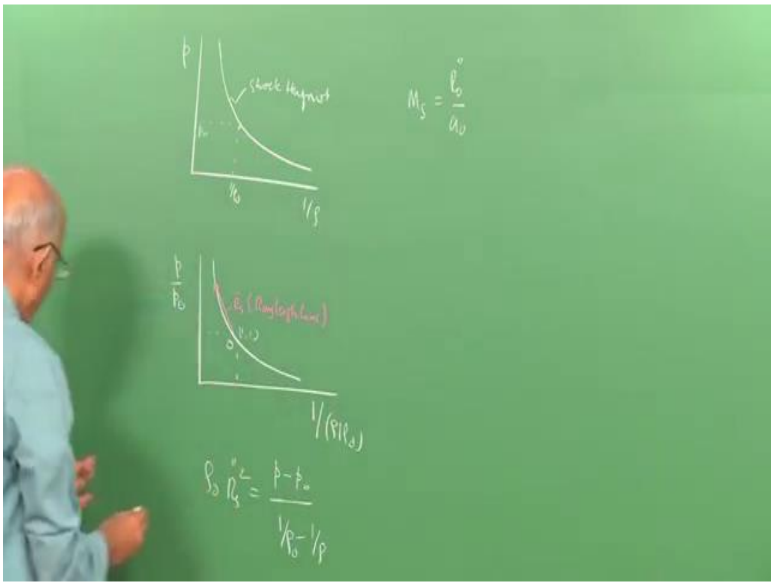

We got the curve namely we had pressure versus 1 over rho, for the initial condition let us say initial condition is P 0, the initial condition is 1 over rho 0, this is my initial point. I have a family of points for different velocities of R s dot and this is what we call as the shock Hugoniot. We went one step further, we were not interested in this alone, and we wanted P by P 0 divided by 1 divided by rho by rho 0 for which we said I could modify my shock Hugoniot equation to give me what we call as rank in Hugoniot equation. We got P by P 0 as a function of 1 divided by rho by rho 0 in which case my initial point becomes let us say o 1 dot 1.

I have the rank in Hugoniot equation, which predicted this particular curve, now if this curve was for a series of R s dots because the radius the shock velocity did not come in this picture and therefore, to be able to predict the shock velocity what did I do? I have to draw the ray length line which was momentum equation, what was the momentum equation momentum, equation of rho naught R s dot square is equal to P minus P 0 divided by 1 by rho 0 minus 1 over rho. This gives me explicit value of one minus R s dot and therefore, for a given value of R s I have a ray length line and this gives me the pressure and the density behind the particular wave.

Well, this was done pictorially using this rank in Hugoniot equations and the ray length line or the shock Hugoniot or the ray length line which gave me the velocity of R s dot over here. Let us put it down it is ray length line over here, but you know I found that

that you know even though graphically I can do the problem and get an analytical expression for P by P 0 as a function of R s dot and as the function of the initial conditions.

Let us say P 0 and rho 0 and what we wanted to do to be able to do that, we said well P by P 0 is a non dimensional number, rho by rho 0 is a non dimensional number, why not express R s dot also as a non dimensional number. We expressed, therefore we talked in terms of shock Mach number is equal to R s dot divided by initial sound speed.

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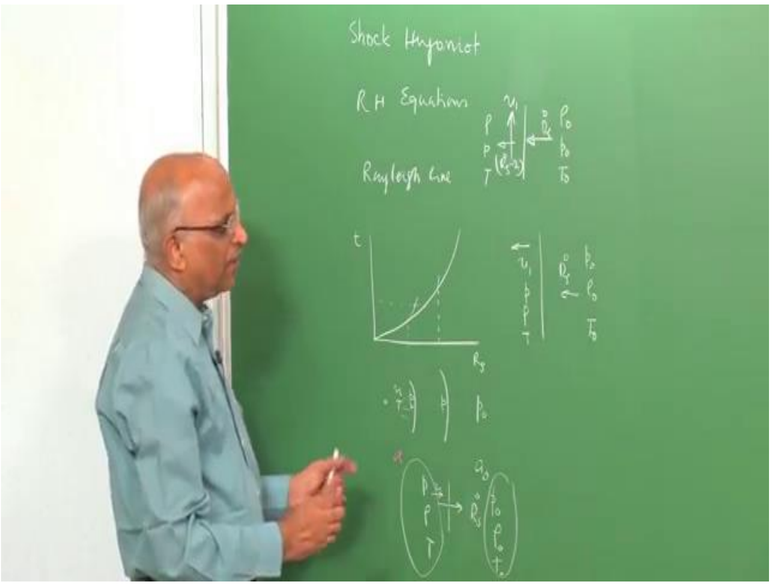

That means you have the conditions over here corresponding to sound speed in the in the in the upstream side that is in which the shock is propagating behind the shock, I have the value of sound speed which is a over here. Therefore, we did this and what did we get we got the momentum equation in the particular form, we got P minus P 0 or rather the ratio because we are not considering explicitly the value of pressure we got P by P 0.

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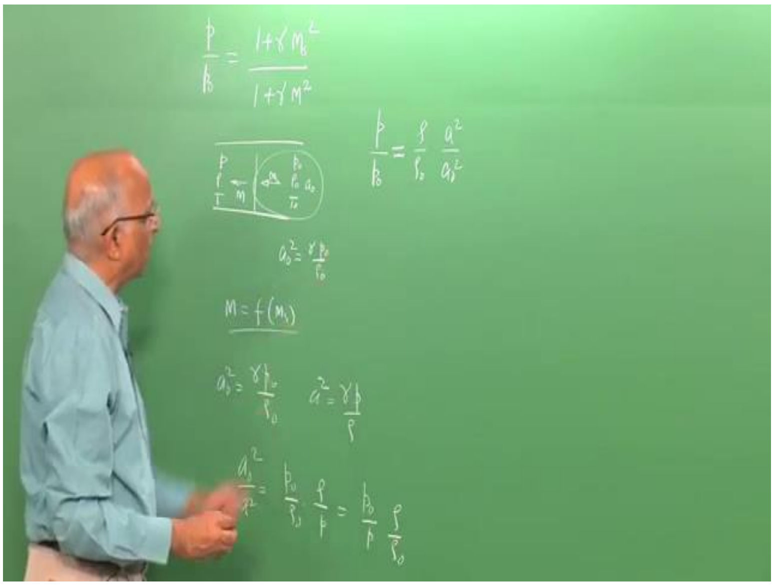

Let me put it down 1 plus gamma into the shock Mach number square divided by 1 plus gamma into m square behind the shock. Let us again qualify this equation because we are going to use this to be able to predict the P by P 0 in terms of M s square. Therefore, now we have the problem the shock stationary we have the gases upstream at condition P 0 rho 0 t 0 and the value of a 0. Well a 0 is the shock is the sound speed ahead of the medium in the undisturbed medium.

This we have derived in the first class as equal to gamma P 0 by rho 0 and this moves towards the shock Mach number is M s over here with a shock stationary this medium moves with a velocity M s over here. It moves away from the shock at condition m over here the pressure is P the condition is rho the condition is t over here. Therefore, we found P by P 0 is given by gamma, Mach number of the gases which is moving in the frame of reference of the shock towards the shock at a Mach number M s and leaving the shock at value of m well gamma is considered to be the specific heat ratio.

Being a perfect gas, we assume that gamma here and the gamma here are the same. Well today we want to solve the equation explicitly in terms of P by P 0 and having noted that already we have P by P 0 in terms of m and M s.

If I can get a relation in between m as a function of m s, if I can solve for this well, I can substitute the value of m in terms of M s over here. That means I have M s over here instead of m over I replace it by m s, I get P by P 0 in terms of M s and that is what we want to do. Therefore, to do that let us go back to this particular equation and simplify it in terms, let us say I want to get rid of the m square or rather I want an equation in terms of m and M s. Therefore, we again know well the sound speed stream over here is given by a square, I again write the expression which I just wrote is equal to gamma P 0 by rho 0.

I also know that the sound speed behind the shock wave is given by the property gamma P by rho. Therefore, if I take the ratio of these two numbers, I get a 0 square divided by a square is equal, now I get t 0 by rho 0 into I get this comes into denominator, I get rho by P or rather this gives me P 0 by P into rho by rho 0. Now, I can simplify, I can write the expression P by P 0 from this expression, let me write it here P by P 0 as equal to P by P 0 I bring it here.

I write it as equal to P by P 0 is equal to rho by rho 0 into a square by a naught square all what we have done is I take P by P 0 a by a naught square into a square by a 0 square into rho by rho 0. Now, I already know the value of P by P 0 if I can somehow get the value of rho by rho 0. Therefore, let me put it in terms of explicitly in terms of rho by rho 0, which I do over here.

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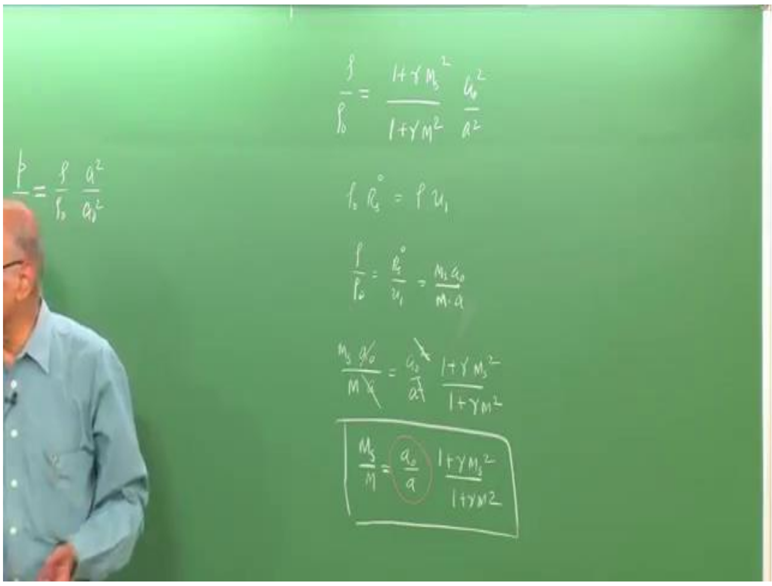

I get the value of rho by rho 0 as equal to P by P 0, which is equal to 1 plus gamma M s square divided by 1 plus gamma into m square. Well, into I get this is equal to a 0 square by a square all what we have done is a rho by rho 0, I get a 0 square here and I substitute the value P by P 0 over here, now see we already know the value of rho by rho 0 from the continuity equation. Let us go back to the basic mass conservation of the shock, let us go back over here what is it we find?

Well, you had you have the shock coming towards the you have the free stream gases coming towards stationary shock in the frame of reference of the shock with a mass flux rho 0 R s dot leaving without density rho with a velocity u 1. Therefore, you will note that this is the mass conservation we had namely rho 0 into R s dot is equal to rho into u 1, now let us simplify it a little bit.

Let us put it in terms of Mach numbers over here such that I can replace, therefore, I say rho by rho naught is equal to R s dot divided by u 1. Now, I want to express in terms Mach number R s dot is equal to M s into a zero sound speed into the Mach number of the free stream gases which is moving towards shock is equal to R s dot this is equal to m into a 0. Therefore, substituting this value over here what is I get now the M s into a 0 divided by m.

This should not have been a 0, it should have been a because the sound speed behind the gases is a m into a over here is equal to a 0 square divided by a square from this equation into 1 plus gamma into M s square divided by 1 plus gamma m square over here. Well, in this equation, you simplify again at a 0 square becomes a and therefore, the final equation. Now, I get is M s plus m is equal to a 0 by a into 1 plus gamma M s square divided by 1 plus gamma into m square.

This is the equation I get, you know in other words I have been able to relate m with M s m with M s. The only thing which is still pending is I have the value of a 0 by a, can I again get a 0 by a in terms of either M s or m both, that is the question which we have to ask ourselves. Therefore, to be able to do that we realize that the sound speed we talk of it something like the internal energy of the gases.

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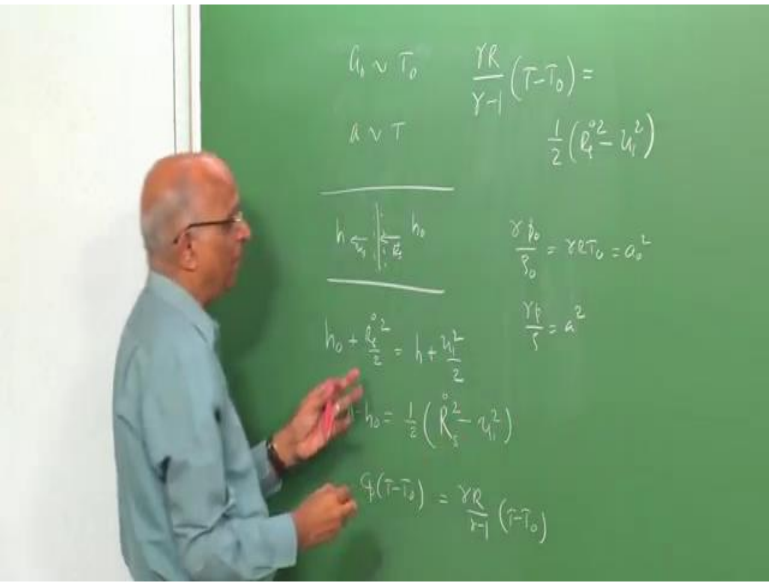

Therefore, we say well a 0 must be related to t 0, a must be related to the temperature behind the shock wave t. Therefore, can I somehow use the energy equation for the across the shock to be able to do that and therefore, we say well I have the shock, which is travelling. We have the enthalpy here, which was h 0 and I have the velocity R s dot with which the gases are moving in the frame of reference of the shock wave shock wave being stationary.

I have u 1 over here, we talked in terms of the control volume and what did we say that the equation across the control volume becomes h 0 plus R s dot divided by 2 kinetic energy plus enthalpy is equal to h plus u 1 square divided by 2. From this particular equation, I get if we were to put the terms here, we say h minus h 0, h minus h 0 is equal to half into I get R s dot square minus the value of u 1 square h minus h 0 is equal to half into R s dot minus u 1 square and what is h minus h 0.

Let us remember that we are doing this problem only for a perfect gas because we said for the perfect gas d h by d t is equal to c P and this becomes c P into t minus t 0 is change in enthalpy, which is equal to the change in kinetic energy. What is c p, you will remember we said c P minus c v is the specific gas constant r. Therefore c P we got it as equal to gamma r divided by gamma minus 1, please recall this gamma r divided by gamma minus 1 into t minus t 0.

Therefore, this particular equation which balances the enthalpy and kinetic energy ahead with enthalpy and kinetic energy after the shock gave the equation in the form. Let us put it down before I solve it in equal to gamma r divided by gamma minus 1 into t minus t 0 is equal to the value of half into R s dot square minus u 1 square. Now, what is the aim I would like to get the expression for a by a 0 because I get temperature over here, I get temperature over here. This temperature is the reflection of the initial velocity of the gases this temperature is the reflection of that is the sound speed behind the gases at the temperature t.

Over here, I know well gamma r t gamma P 0 by rho 0 is equal to gamma r t 0 rather we have gamma P 0 by rho 0 P 0 by rho 0 is equal to gamma r t 0. This is equal to the value of the sound speed in the free stream medium have gamma P by rho is equal to a square gamma r t which is equal to a square behind that. Therefore, this particular expression now can be rewritten in the following form, let me rub it out we are not doing shock Hugoniot, now we are trying to explicit between P by P 0.

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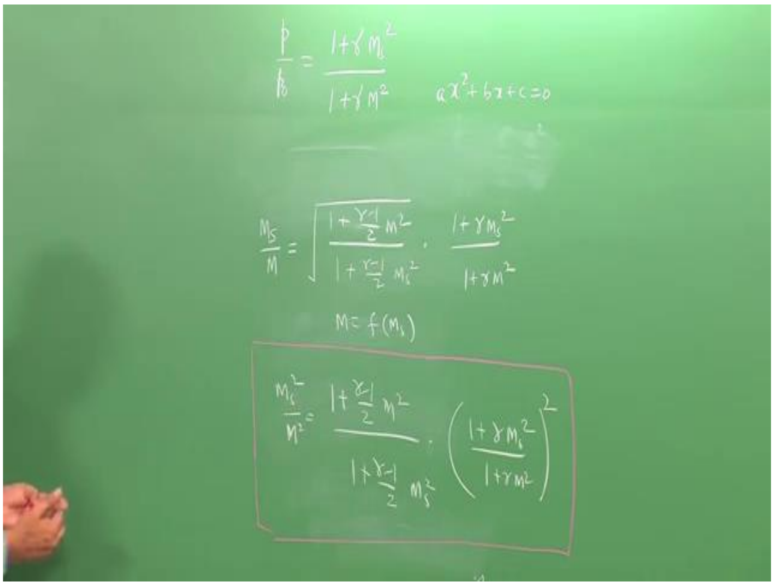

Therefore, may be using these two relations, I get over here gamma r t is equal to a square minus a 0 square is equal. I get over here into gamma minus 1 divided by 2 gamma r t minus gamma r t 0 is equal to a 0 minus a square minus a 0 square gamma minus 1 comes over here gamma minus 1, 2 into R s dot square minus u 1 square over here. Now, if I were to bring a 0 and R s dot together, I get it 1, the other side I get a 0 square plus gamma minus 1 divided by 2 into R s dot square is equal to i. Now, I am left with a zero over here plus I have gamma minus 1 divided by 2 into a 1 square over here.

Now, if I were to take a 0 outside over here, I get a 0 into 1 plus gamma minus 1 by 2 into R s dot by a 0 whole square R s dot by a 0 is equal to m s, which is the shock speed or the equivalently in the frame of reference of the shock stationary. The Mach number at which the gases are approaching the shock wave similarly, I get one plus gamma minus one into u one by a square which is equal to the Mach number behind the shock. Therefore, this was a square mind you and therefore I get a 0 square divided by a square is equal to 1 plus gamma minus 1 into M s square divided by 1 plus gamma minus 1 by 2 into M s square.

Now, I am able to get the expression for a 0 by a, which in terms of M s and m therefore, I put this expression over here and what is it I get? Let us put the final expression on the board, I get the value M s by m is equal to a 0 by a is equal to under root of 1 plus gamma minus 1 by 2 into m square divided by 1 plus gamma minus 1 by 2 into M s square. Now, this was not M s m this was equal to the value of a 0 by a, therefore M s by m, I substitute this is equal to a 0 by a into the value of 1 plus gamma M s square divided by 1 plus gamma into M s square over here.

Now, what is it you find in this expression you have M s and m, rather I am able to get the dependence of m as a function of m s, which is what we wanted to do, but then you know is equation is a little clumsy with all this. Therefore, I take square on both the side I take M s square divided by m square is equal to the square root vanishes 1 plus gamma minus 1 by 2 into m square divided by 1 plus gamma minus by 2 into M s square. I get this was square over here, similarly this becomes this expression becomes square 1 plus gamma M s square divided by 1 plus gamma m square whole square.

In other words, I square this term, I square this term, I get 1 plus gamma minus 2 m square divided by 1 plus gamma minus 1 by 2 M s square, I square this. This is my final expression, which I get, now this equation must be useful for me to be able to predict the value of m as a function of M s mind you M s in know because of the shock velocity. For a particular shock velocity, I want to get the value of Mach number behind it, therefore I can find out m, but how do I solve this equation?

We first note the following, we note this particular equation has m square term well it will have m square term therefore, m 4 term, it is quadratic in square. If I were to say square is equal to x, this equation deduces to the form something like a x square plus b x plus c equal to 0, where x in equal to m square and we know anyway M s square. Therefore, I can solve this equation for x and similarly, now the value of m specifically as a function of M s little bit of algebra is involved.

I will not solve this equation, but I will give you the result and it is quite simple you just put m square is equal to x. Therefore, m 4 becomes x square m square becomes x, it gives this form. You will say, well I have to solve this x is equal to minus this middle number plus minus under root of middle number squared minus 4 into the first into the last number divided by 2 into the first number is the value of x. When I solve for the value of m s, I get the result as the following, let us get this value let me put this value down the value I get is let me write the expression.

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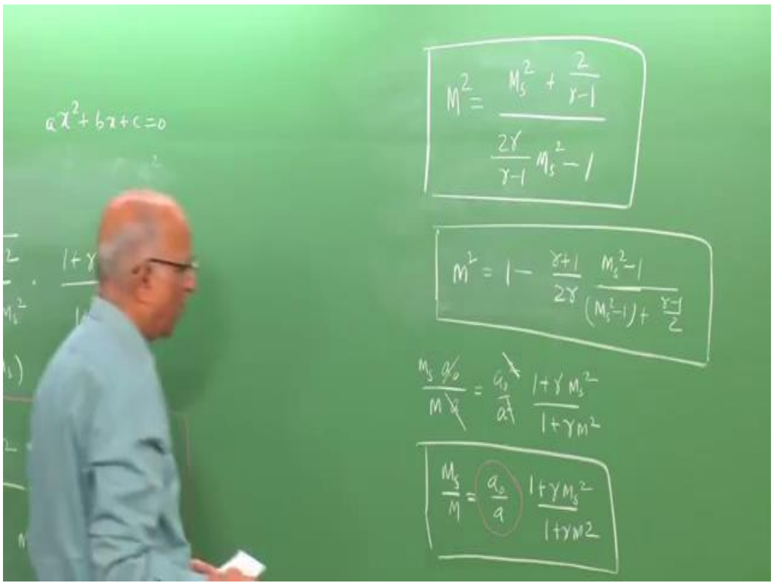

I get the value of m square as equal to M s square plus 2 over gamma minus 1 divided by 2 gamma divided by gamma minus 1 into M s square minus 1. Therefore, we have succeeded in getting from looking at the non dimensional form of M s square m m square a relation between the Mach number behind the shock and the Mach number ahead of the shock in the frame of reference of the shock itself. This particular value of m square can again be simplified, after all you have this, you bring gamma minus 1 by 2 over here.

The expression works out to be 1 minus gamma plus 1 divided by 2 gamma into M s square minus 1 divided by M s square minus 1 plus the value of gamma minus 1 divided by 2, this is the value of m square which I get. You know this form is to expect I have M s square minus one gamma minus one divided by two by simplifying it this is the final form what I get. Therefore, we are able to get the Mach number behind the shock as a function of Mach stream upstream of the shock or the particular case of the shock stationary. Let us discuss this equation before I explicitly get the value of pressure that is P by P 0 rho by rho 0 other parameters which I wanted to do in this particular analysis.

Therefore, coming back to the discussion of this equation what does this equation tell us let us quickly try to see what are the limits and what is that equation going to give. Well, if the wave is something like an acoustic wave or if the Mach number is one like M s is equal to 1 if M s is equal to 1 I find well M s is equal to 1, the numerator is 0, the value this becomes ms square is 1, 1 minus one is 0.

Therefore, M s square is equal to 1 and if m square is equal to 1, I get m is also equal to 1. On the other hand, if I get m s, I take the other limit extremely shock strong shock wave travelling at infinite speed. If M s is equal to infinity, what is going to happen, well I cannot get infinity over here. Therefore, I slightly modify this equation I want to put in terms of 1 over M s square, therefore I can put 0 over here and what is it I do well let.

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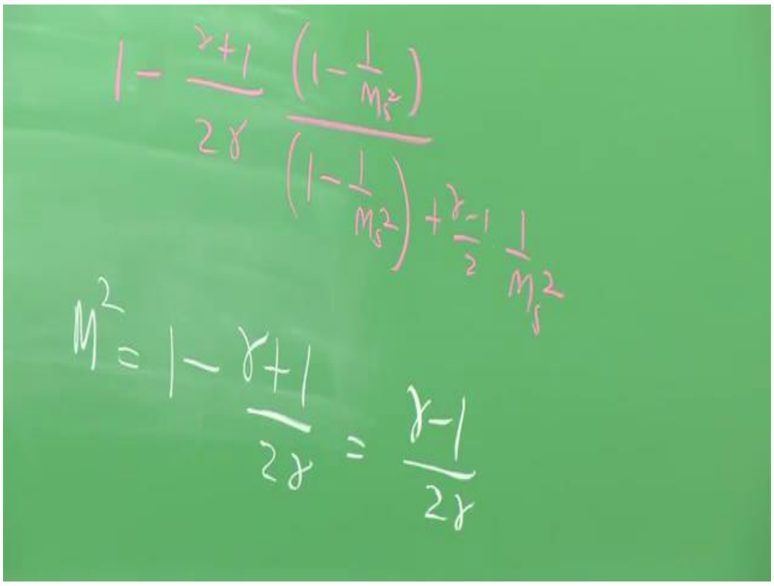

Let us put it down, I get gamma plus 1 divided by 2 gamma, let us divide the numerator and denominator by M s square. I get 1 minus 1 over M s square divided by I get 1 minus one over M s square because I am dividing plus gamma minus 1 by 2 into 1 over M s square. Now, in this equation if i put M s is equal to infinity well this becomes 0, this becomes 0 again and this becomes 0 and what is it I get the value of M s square is equal to 1 plus I get, let us put it down on this is minus over here.

The value of m square is equal to 1 minus gamma plus 1 divided by 2 gamma because this becomes 0 this becomes 0, this becomes 0 1 by 1 1 minus gamma. This becomes 2 gamma minus gamma minus 1, this is equal to I have two gamma minus gamma that is gamma minus 1 divided by 2 gamma, this is the value of M s square.

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Therefore, in the limit of very strong shock wave extremely strong I get m is equal to gamma minus 1 divided by 2 gamma. Therefore, what are the limits let us try to put it down in terms of a figure, well I have m which is the dependent which I am interested in. I am looking at the value, which is given to me for a particular value of m s, I want to calculate this value if M s is 1, well M s is also one as the value of m of M s increases. Well, m decreases in the limit of very strong shock wave that is when M s is equal to infinity what is the value, well we take gamma for air is equal to 1.4.

Let us say we are doing for atmospheric air, let us say we get therefore, 0.4 divided by 2 into 1.4 and this is under root and this was m square over here, therefore this is equal to under root over here and this works out to be 0.38. In other words, in the limit of very strong shock wave the value of m is 0.38 and the downstream Mach number keeps decreasing till reaches the value of 0.38. This is how the shock strength behind to the free stream value of the shock Mach number this is how it changes, but let us go back to this particular equation and tell ourselves.

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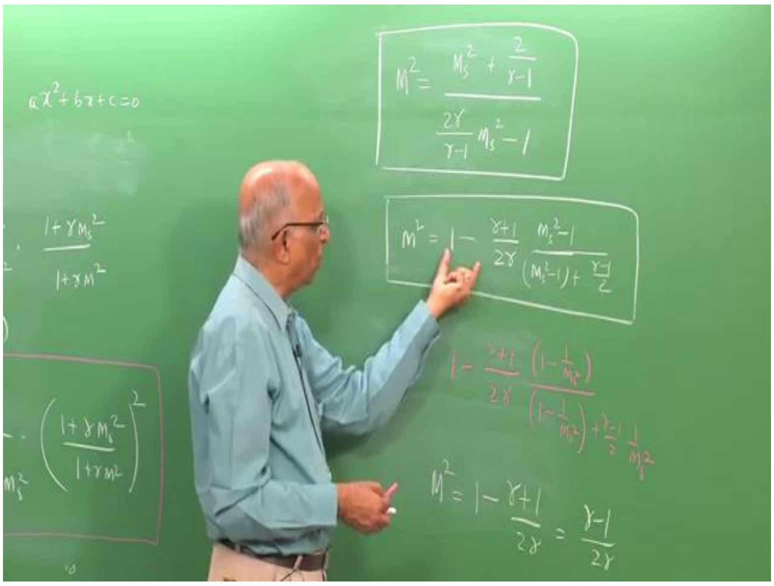

Well, we are considering the upstream Mach number that is shock to travel at speeds greater than sound speed M s is greater than 1, whenever M s is equal to one I get m is equal to one when M s is very strong I get a smaller value as the speed. The supersonic speed keeps increasing well m keeps decreasing till the value when it is infinite, when it becomes 0.38. If I look at this particular equation and I look at the solution of the equation even if M s is less than 1, in other words I am looking at M s being less than 1.

Then, if I solve for M s less than 1, I get m is equal to m to be greater than 1, but you know if I look at the other parameters in this equation what happens is the entropy decreases during the particular phenomenon, which is not possible. Therefore, we rule out the conditions for M s less than 1 leading to m greater than 1, I forgot to also tell you that when you solve this quadratic equation, we get two roots. We get one root, which is complex, which we ignore and the root what we got here is the real root, which we consider. Well, this is about it therefore, we have been successful in getting the value of the downstream Mach number as a function of M s and now I can use this to be able to predict my value of pressure issue how do I do it?

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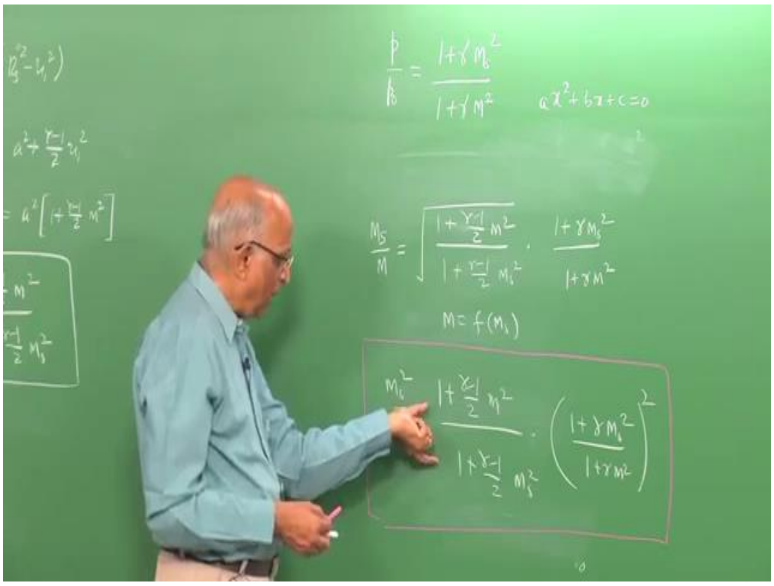

Again, I tell myself well I have the expression for m by m s, I now know square over here I can substitute the value and get the vale explicitly in terms of M s. Let me take you through the slides because the algebra or the terms are involved, therefore let me come back to the slides and do this.

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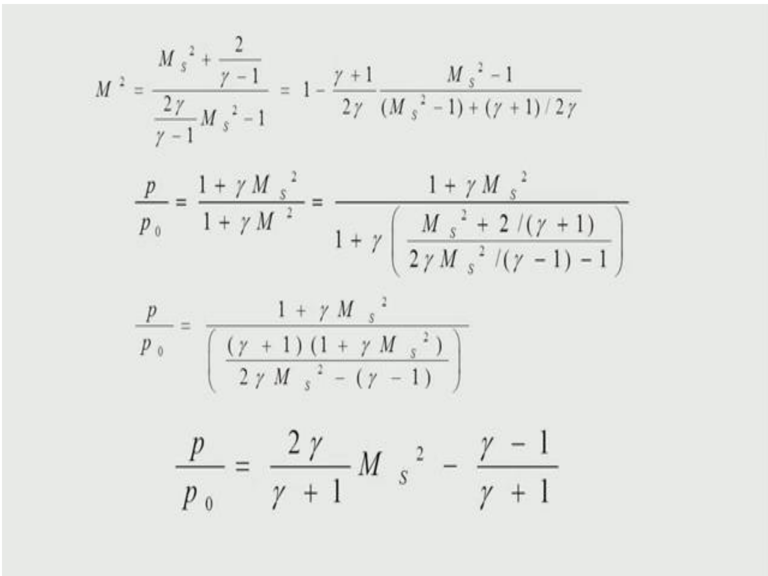

What is it, I say well I got M s m square is equal to M s square plus 2 over gamma minus 1 into this particular expression, which I also said could be written as 1 minus gamma by M s square minus 1 plus gamma minus 1 divided by 2 gamma. Over here, we also have the expression for P by P 0, which is on the board, we write it as 1 plus gamma M s square divided by 1 plus gamma m square. I write the expression for m square as a function of whatever we have over here namely m square is given in terms of M s square by this expression.

Therefore, I have 1 plus gamma M s square in the numerator being the same 1 plus gamma for m square, I substitute this particular value. I get M s square plus 2 divided by gamma plus 1 divided by 2 gamma M s square into gamma minus 1 divided by 2 over here. Therefore, now what is there I also take, similarly the value of gamma over here 2 over gamma minus 1 and therefore, I get 2 gamma M s square 2 divided by gamma minus 1 M s square minus 1 over here. Therefore, now if I simplify this particular equation which is straight forward I get P by P 0 is equal to 1 plus gamma M s square divided by in the denominator.

Then, gamma plus 1 into 1 plus gamma M s square divided by 2 gamma M s square divided by minus gamma minus 1. If I simplify further, I get P by P 0 is equal to 2 gamma by gamma plus 1 M s square minus gamma minus 1 divided by gamma plus 1. We are able to get the value of P by P 0, let us discuss this particular result. Therefore what is it I have got let us put it down, I already know the how the Mach number is behaving, let us take the role of pressure over here.

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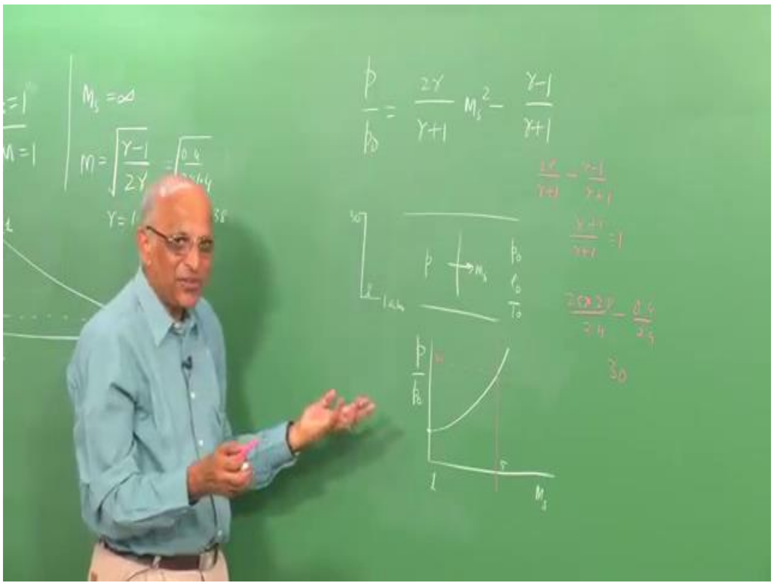

I get the value of P by P 0 is equal to what is it I say 2 gamma divided by gamma plus 1 into I get the value of M s square minus. I have gamma minus 1 divided by gamma plus 1, this is the value of the pressure ratio I get. Therefore I tell myself whenever I have the shock, which is moving at a value of M s into the free stream medium of P 0 rho 0 t 0 over here the value of t behind the shock P by P 0. The ratio is given by this and what does that tell me as the Mach number of the shock increases the pressure increases.

In other words, if I were to now write P by P 0 as a function of M s over here for the value of 1, well I get 2 gamma divided by gamma plus 1 divided by gamma minus 1 divided by gamma plus 1. Let us see what result we get, therefore, I get 2 gamma divided by gamma plus 1 minus gamma minus 1 divided by gamma plus 1.

What does it tell me, 2 gamma minus gamma the denominator is still gamma plus 1, 2 gamma minus gamma, therefore I get gamma and plus 1 over here, which is equal to 1. Therefore, when the sound speed is 1 here, well the pressure issue is just 1 as the Mach number increases, well it keeps on increasing when I have a very strong when the M s tending to infinity, well the pressure ratio is very large.

In fact, if I am interested in a particular pressure ratio, let us say a value of M s is equal to let us say 4 or 5, well I take the value of 5 as an example M s is equal to 5 that the shock Mach number is 5. Well, this becomes 25, let us put the value down 25 into 2 into 1.4 for air 2.8 divided by 2.4 minus I get 0.4 divided by gamma minus 1, that is 1.4, I get 2.4. If I look at this, it is something like 10 times into 28, something like 29 or 30 is the value, that means at a value of 5, I get a pressure ratio of 30.

In other words, we find that when the Mach number increases the crushing pressure or the pressure rise across the shock wave, what happens? Let us again plot it, I have the initial pressure, let us say one atmosphere if the shock is travelling at the Mach number 5, the pressure behind it is of the order of 30. That means it has so much of crushing power that if somebody if hit by the shock wave, he gets compressed if the shock wave is of Mach 5. He gets compressed by a factor of 30 times the atmospheric pressure and as the shock wave increases, well this pressure keeps increasing, therefore we are able to write P by P 0.

Now, let us find the last value which we want rho by rho 0 and how do we get rho by rho 0, you will tell me well we have already done the rank in Hugoniot equations. What we get from the rank in Hugoniot equation, we got rho by rho 0 in terms of P by P 0 and what was the expression please go back, let us check it.

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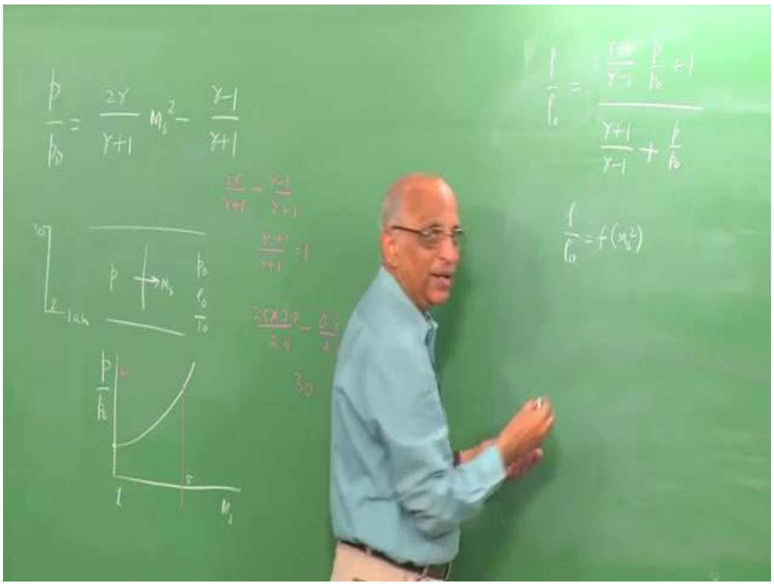

Then, rho by rho 0 was Equal to gamma plus 1 into gamma minus 1 into P by P 0 plus 1 divided by gamma plus 1 divided by gamma minus 1 plus P by P 0, this was the expression which we got. Now, I already have the expression for P by P 0 over here namely P by P 0 is given by this, I substitute it over here, therefore I get rho by rho 0 in terms of that means rho by rho 0 has a function of M s square, let us do that, I do that in the next slide.

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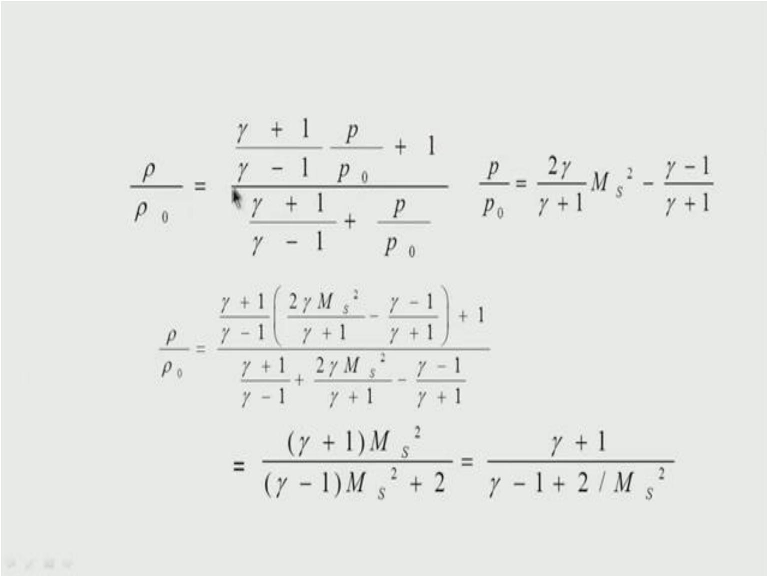

Let us go to the next one, that means I have from the rank in Hugoniot equation, which I just wrote on the board rho by rho 0 is equal to gamma plus 1 divided by gamma minus 1 into P by P 0 plus 1 divided by gamma plus 1 divided by gamma minus 1 plus P by P 0. This is the value of P by P 0, which is again on the board and therefore I substitute this over here and here and therefore, what is it I get?

I get rho by rho 0 is equal to gamma plus 1 divided by gamma minus 1 in this particular expression over here plus 1 divided by gamma plus 1 gamma minus 1 plus the value of this expression over here. I simplify this, I get gamma plus 1 into M s square divided by gamma plus 1 into M s square plus 2, I divide this equation by M s square, both the numerator and denominator. I get rho by rho 0 is equal to gamma plus 1 divided by gamma minus 1 plus 2 over M s square, therefore let me put down this final result on the board.

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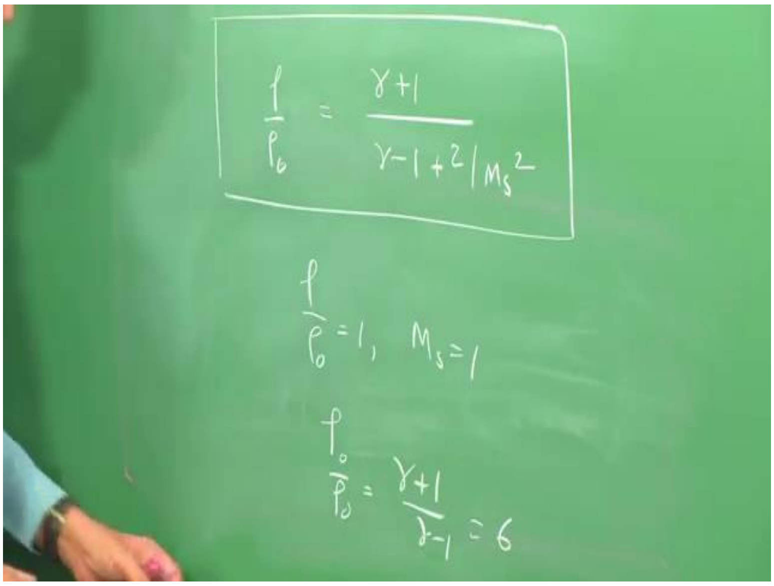

What is it I get, I get the value of gamma plus 1 divided by gamma minus 1 plus 2 divided by M s square, which is the value of rho by rho 0, therefore what is this equation tell us? Let us again discuss this particular equation, this equation tells us exactly what the pressure ratio equation told us namely when M s is 1 that is the sound speed is 1, I get 2 over here, 2 minus 1 is 1 gamma plus 1 divided by gamma plus 1.

Therefore, rho by rho 0 is equal to 1 when M s is equal to 1 when M s is very large, let us say infinite what happens the value of rho by rho 0 is equal to M s square becomes infinite this becomes 0 to over infinity. Therefore, it becomes gamma plus 1 divided by gamma minus 1, which for air for which gamma is 1.4, I get 1.4 plus 1 divided by 0.4, which is equal to 6, and therefore how does the density ratio change with the Mach number of the shock.

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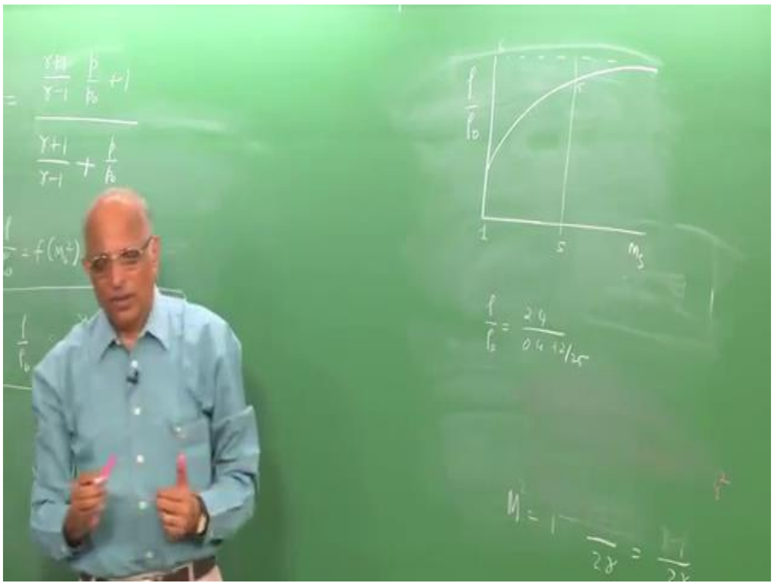

We find well the ratio of rho by rho 0 has a function of M s when M s is equal to 1, the value of rho 0 is 1, when M s becomes infinity it is around 6 over here. In fact, it does not have to go to infinity to get 6 even at a Mach number of around let us say 4 or 5, let us work out the value for 5. Let us work it out, we get rho by rho 0 is equal to 2.4 divided by 0.4 gamma minus 1 plus 2 over Mach number is 5, 2 over 25. Therefore, you know 2 over 5 are 25, therefore 2 over 25, we are talking of something like point 0.84, that is 0.48, that means that we are talking of a number something around 5.

So, even when the Mach number is around 5, that means the value increases 0 to 6 as you increase that means as the Mach number increases, the density ratio also increases. Therefore, we are now able to say something more that means it started from the Hugoniot equation or the shock Hugoniot. We talk in terms of rank in Hugoniot equation, which related the pressure and density behind the shock with the pressure and density ahead of the shock.

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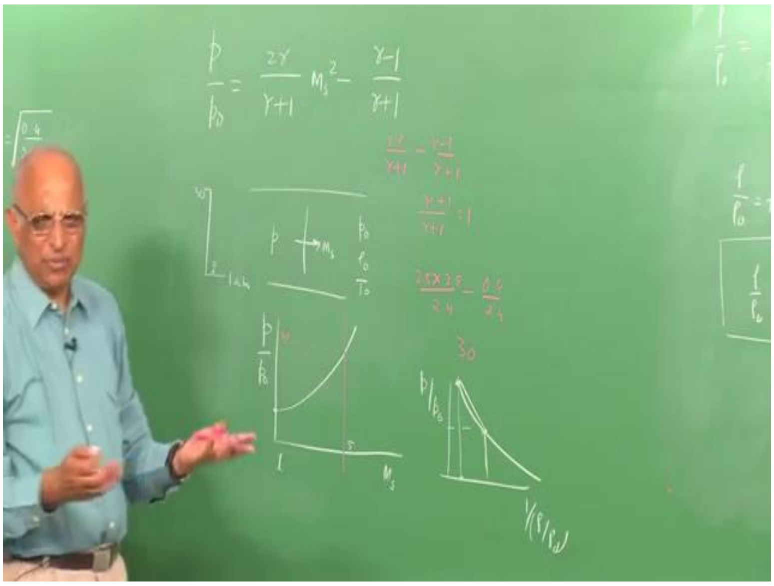

We also talked in terms of the ray length line, which helped us to fix the point namely P by P 0 divided by 1 over rho by rho 0, we had the Hugoniot, which was the initial point, you had the ray length line. Now, we are in a position to find the value of P by P 0 explicitly and also get the value of rho by rho 0, explicitly this is what we set out to do, but how would the velocity behind the shock. That is something, which I spend a minute or two and that is quite simple, let us put that thing down you know from continuity equation.

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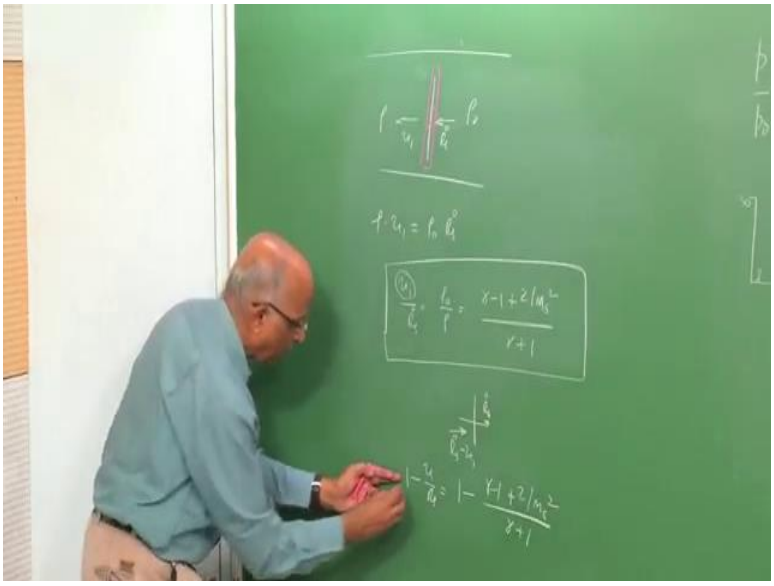

What was the continuity when I say continuity equation for a shock, we are considering the shock travelling at a speed of R s dot or else the medium. Now, travelling at the speed of R s dot d density is rho 0, the velocity behind is equal to u 1, the density is rho the continuity ratio is rho into u 1 is equal to rho 0 into R s dot. That means the mass flux ahead of the shock is equal to mass flux leaving the shock, which is the mass balance and therefore from this I get u 1 by R s dot.

That means the velocity behind the shock wave in the frame of reference of the shock wave is equal to I get now rho 0 by, but rho by rho 0. I already have the expression namely gamma plus 1 divided by gamma minus 1 plus 2 over M s square is the value of this. Therefore, u 1 by R s dot is equal to gamma minus 1 plus 2 over M s square divided by gamma plus 1 and this gives me the velocity of the gases behind the shock front.

We must remember one thing when we are solving this u 1 is the velocity of the gases in the frame of their reference of the shock being kept stationary. That means I sit on the shock and when the shock is not moving, but the gases are moving forward I see the velocity of the gases to be u 1 going away from the shock. If I consider the actual movement of the shock and we had this problem of shock moving by the velocity R s dot.

When the medium ahead is quiescent, which is not moving the value of the gases behind is now going to be when the shock is moving at R s dot the R s dot is going to be R s dot minus u. Therefore, this is what R s dot minus u 1 and therefore remembers that this value of u 1 is in the frame of reference of the shock stationary. If the shock is moving, well this value is going to be 1 minus u by R s dot rather of the right hand side. I should get minus gamma minus 1 plus 2 over M s square divided by gamma minus 1 that is gamma plus 1 over here or rather I should get the value of R s dot minus u.

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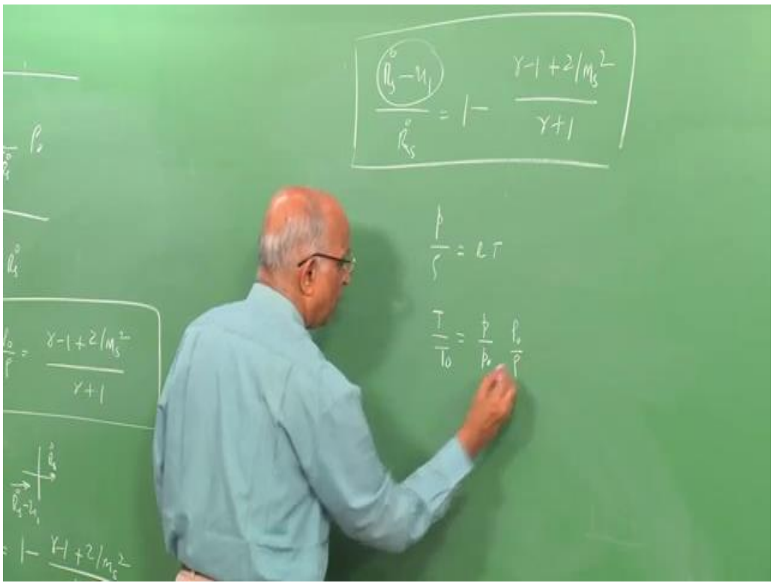

Let me put it on the other side divided by gamma plus 1, this is the velocity of the particles which are moving along with the shock, if the shock is moving if the shock is stationary R s dot. We have succeeded in therefore, telling what the conditions behind the shock are if I now want temperature and temperature for any other property. Let us say temperature what is it I do well I know I am talking in terms of perfect gas, therefore P by rho is equal to r t or P b is equal to m r t rho. Therefore, I get t divided by t 0 is equal to P divided by P 0 into something like rho 0 divided by rho, from this expression which I can simplify, which I can do in next slide let us take a look at it.

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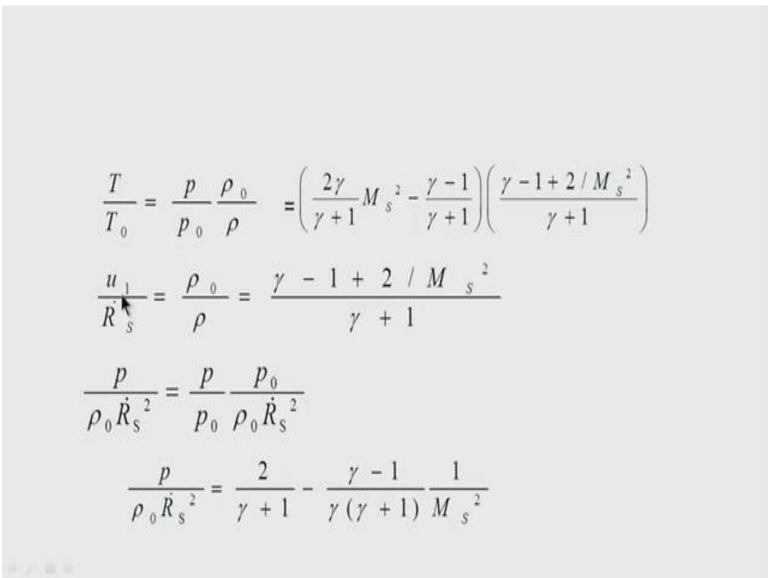

I get t by t 0 is equal to P by P 0 is equal to rho by rho 0, we have derived this expression, we know the value of P by P 0, therefore we have 2 gamma plus M s square minus gamma minus plus gamma plus 1, which is for the pressure ratio. I also get the value of rho by rho 0 given by gamma minus 1 divided by gamma minus 1 plus 2 over M s square and this gives me the value. Therefore, I know all the properties behind my shock wave as a function of properties ahead of the shock wave. We have already calculated the value of u 1 and therefore, I have been able to solve the problem, but what is the problem I have been able to solve, let us be very clear about it.

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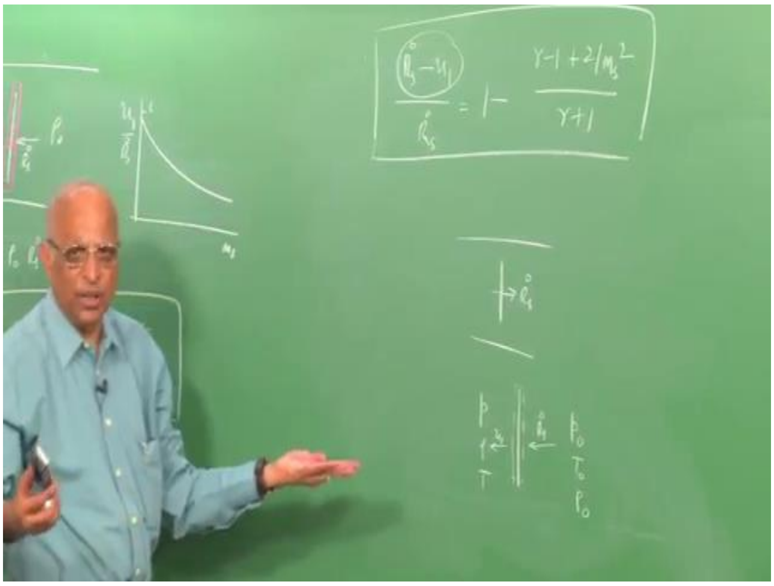

You know the problem we are solving is for a case of a constant velocity shock when the shock velocity is constant and I considered my frame of reference of the shock stationary. When the shock is travelling towards when the gases are travelling to my shock front to the R s dot if the conditions ahead is P 0 t 0 rho 0. I am able to find out the value of P rho and t and also the value of the velocity u 1 in the frame of reference or when the shock is moving I say it is R s dot minus u 1, so far so good.

Therefore, we also find the value of u 1 by R s dot it is just inverse and if I were to plot the value of u 1 divided by R s dot with m s, I start with the 0.1, well it keeps coming down inversely as the density went to the value 6. This will keep on coming down over here, that means as the shock velocity keeps increasing the value of the particle velocity in the frame of reference of the shock, but in the frame of reference of movement. That is the shock moving, well the velocity of the gases will keep increasing because it is R s dot minus u 1.

Well, this is all about shock relationships in the case of a constant velocity shock, but the problem, we are interested in is something different see we have the problem. This actually is the following the blast wave, which is not a constant velocity wave and that is where we got started today.

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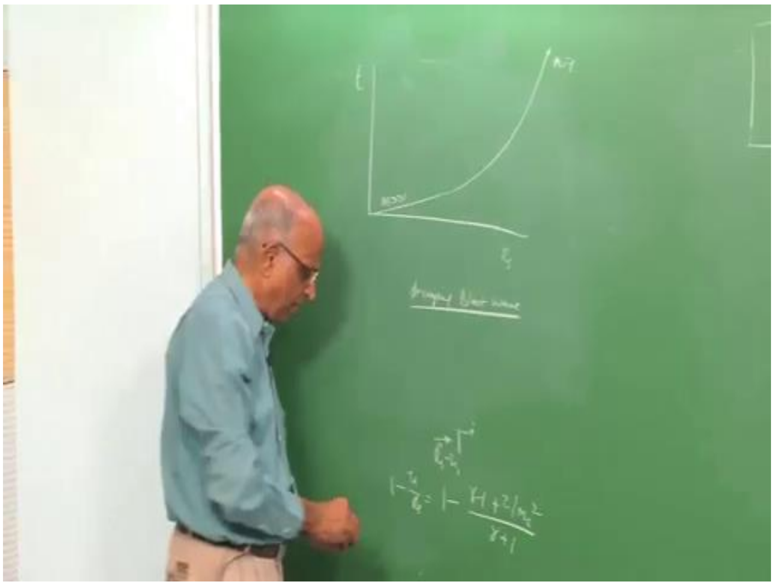

We are interested in something, let us say diagram t versus r s, we start forming with a strong shock wave travelling at high velocity and keeps decreasing and ultimately becomes a acoustic wave wherein the Mach number is equal to 1. It starts travelling with a Mach number which is very much greater, now it is a variable Mach number and how do I use the results which I have derived so far into the case. I have something like a decaying shock wave which is a blast wave, this is what I will do in the next class, that means we know now something about the conditions behind the wave, which travels at constant speed.

Now, I want to extrapolate it to the condition of a decaying shock wave and this is what I will do in the next call, but I want to remind you one last expression, which we could also think of. We put P by P 0, we write that expression again, unfortunately I rubbed it out we had the expression or let us use the board itself or the slide itself.

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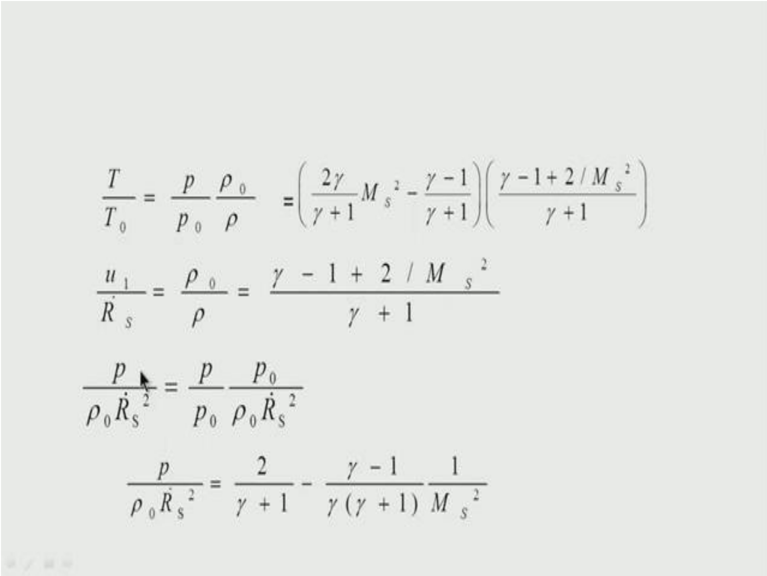

We had the expression for P by P 0 as equal to an expression in the following form P by P 0, we got it as equal to 2 gamma by gamma plus 1 into M s square minus gamma minus 1 divided by gamma plus 1. Suppose instead of having P by P 0, I am interested in the value of pressure divided by the dynamic pressure that is rho 0 is the ambient density into R s dot square. And I am interested instead of P by P 0, I am interested in P by rho 0 R s dot square.

Therefore, I can write this rho 0 R s dot square as P by P 0 into P 0 divided by rho 0 into R s dot square and I know the value of P by P 0, which I already have on the board and this the value of P by rho 0 R s dot square. If I use this it becomes 2 over gamma plus 1, I had the value of Mach number over here, I therefore, simplify it. I get 2 over gamma plus 1 minus gamma minus 1 divided by gamma into gamma plus 1 into 1 over M s square. Mainly, I will just do this on the board, because this is some expression, which I will be using, let us therefore, just complete and be done with it.

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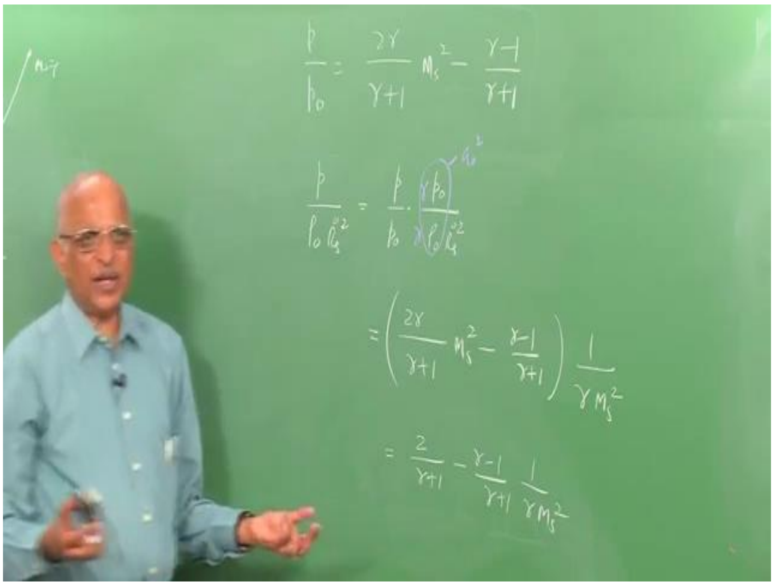

We have P by P 0 is equal to 1 plus what was the expression we had t by t 0 is equal to as Mach number increases by P 0 increases 2 gamma divided by gamma plus 1 into M s square minus u had gamma minus 1 divided by gamma plus 1. You will recall this and therefore I am interested in the expression of P 0 by the dynamic pressure associated with the shock front moving with a velocity R s dot. I write it as P by P 0 into the value of P 0 by rho 0 into R s dot square and now I substitute this value over here. I get 2 gamma divided by gamma plus 1 into M s square minus gamma minus 1 divided by gamma plus 1 into this is the value of P by P 0 into P 0 by rho 0.

If I were to multiply both numerator and denominator by let us say gamma over here gamma over here, I get gamma P 0 by rho 0 is equal to a 0 square and R s dot square by a 0 square is equal to Mach number square. Therefore, what I get is I have gamma into M s square that is 1 over gamma into M s square, please be careful gamma P 0 by rho 0 is equal to a 0 square R s dot square by a 0 square is M s square. Therefore, I get M s square I get gamma over here and therefore, the expression becomes 2 over gamma plus 1 into gamma cancels M s square cancels minus gamma minus 1 divided by gamma plus 1 into 1 over gamma M s square.

This is the expression we get and therefore we find that the value of the pressure that is the P divided by the dynamic head keeps decreasing as Mach number increases as this is what we will do when M s is equal to 1. Well, I get a value 2 gamma minus 1 divided by gamma minus 1 divided by gamma plus 1 which is the value under Mach number 1, which is equal to the dynamic pressure for an acoustic wave. When M s is very large equal to infinity it is 2 over gamma plus 1, well these are the expressions for the constant velocity shock relations.

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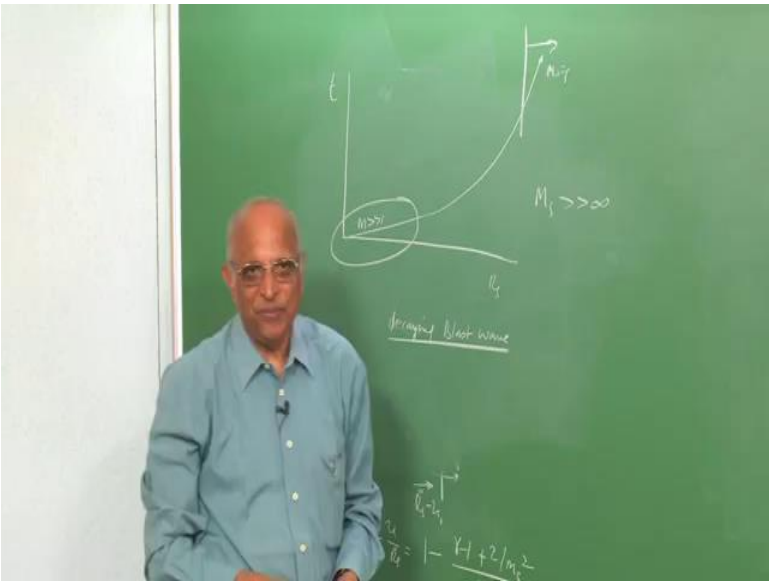

Therefore, in the next class as I said earlier, we will try to take a look at what are the properties what are the pressures across the blast wave, what are the velocities behind a blast wave and that we will do in pieces. We told well the problem is complicated because it is totally unsteady and we will try to take some limits some conditions for which we will try to do the problem the conditions, we will take care.

Let us consider the initial phase when the Mach number is very large tends to be a large number. For this, let us try to do the problem and in the limit of Mach number going to one at far in the very far field, let us do the problem and get a feel of the problem.

Well then thank you.

We meet in the next class, wherein we do the unsteady problem.