Introduction to Explosions and Explosion Safety
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Lecture - 4
Shock Hugoniot and Rayleigh Line

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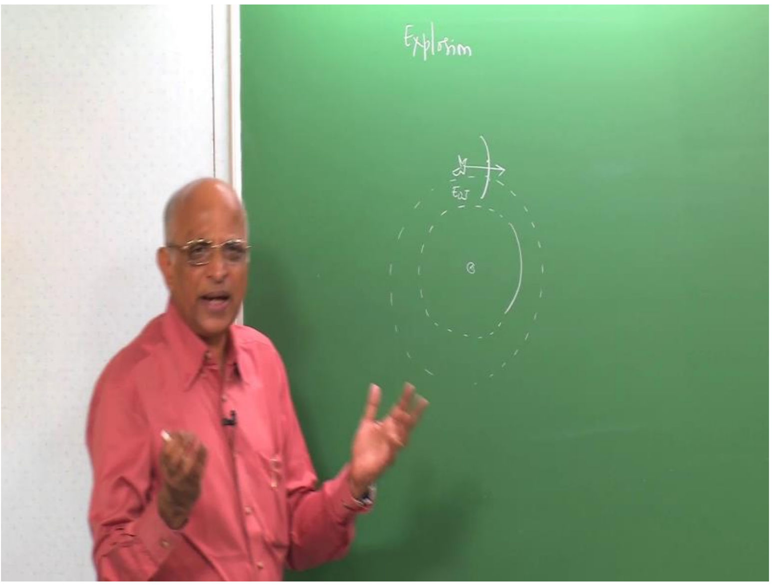

Good morning, I think let us first recap what we did in the last 3 classes, you know we defined an explosion. And we said an explosion is one, in which a blast wave gets generated, due to some energy deposition at a source. We learn how to get the magnitude or the velocity, at which the blast wave propagates due to energy deposition at the source using dimensionless analysis. There after we were interested in finding out, what is the magnitude of the pressure rise across the lead blast wave.

And also about the wind or the impulse or the change of momentum which is associated due to the movement of the matter, which is processed by the blast wave. And for that you know, before doing that we wanted to take some examples such that we are very clear about the effects of the explosion, but before we did that part on different types of explosions. We also told ourselves; when some energy is release say E 0 joules, instantaneously at some particular source. It is the blast wave which redistributes the energy, and the energy is contained within the lead shockwave and the source.

Let say that the lead shockwave is spherical; we said it could be planer it could be cylindrical. The energy distributed or deposited at the source is redistributed by the blast wave within the lead blast wave itself, and the blast wave keeps propagating outward. In the last class what we did was, we looked at the different types of explosions, such that you know we could related, and before we go any further on predicting the over pressure or the pressure rise the impulses behind it. It is necessary to have an idea of what the different types of explosions were, and we were able to catalog the different explosions into something like 8 categories.

Before these 8 categories, we talked in terms of naturally occurring explosions, we talked in terms of accidental explosions, we also talked in terms of intentional explosions. And also you know explosions used for constructive purposes and different applications, but these for accidental explosions and also some of the intentional explosions. We were able to catalog the explosions into 8 different categories.

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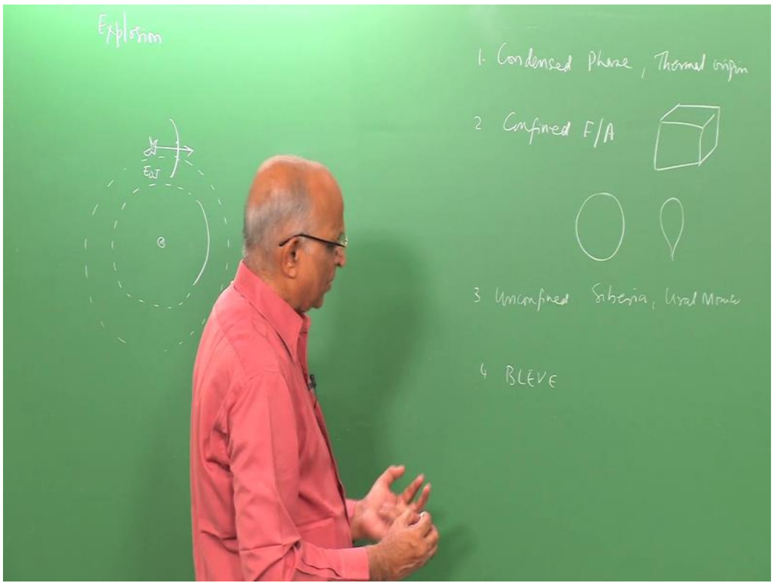

These 8 categories were may be let me start with one, we started with something like a condensed phase explosion by condensed we mean heavier substances liquid and solids. We looked at some of the examples, which released energy and caused explosions using condensed phase, the examples where the Texas City disaster in which lot of just exploded. We looked at different examples; we looked at explosions occurring at due to cracker industry handling energetic materials and so on, these we found was essentially due to the rapid chemical reactions.

We called it as may be of thermal origin, may be it could be of different origins may be when we get into some details will examine this. The second one, what we said was confined fuel air explosions or confined gaseous explosions; let me say fuel air explosions. And what happened in the confined fuel? We had some room such as a kitchen or we had an aircraft tank, in which we got a combustible mixture it ignited, and just went as a bang and destroyed the place. It was essentially confined that means in a particular confinement, you have energy release taking place.

Well it could be anything, you know in the industry like in a work shop, you have a compressor, which energizes or which pressurizes a gas bottle. And if by chance the gas bottle burst, well I have an explosion. I could have the simple example of a balloon which is bursting, which is a confined explosion involving cold gas, it could be fuel and air which causes the problem. Third, we said it could also be unconfined. In case of unconfined, we looked at spillage taking place or gas getting spilled such as the one, we saw at the in Siberia, at in the Ural Mountains.

Wherein there was this spillage, these spillage goes along the ground, two trains pass in the opposite direction they mix it with air, you have the spark and it explodes these are unconfined. The fourth one, what we considered was BLEVE B L E V E boiling liquid expanding vapor explosions. Now, what happens in this case, I think I forgot to put this while summarizing the talk, I think I omitted this, but what happened in this case was.

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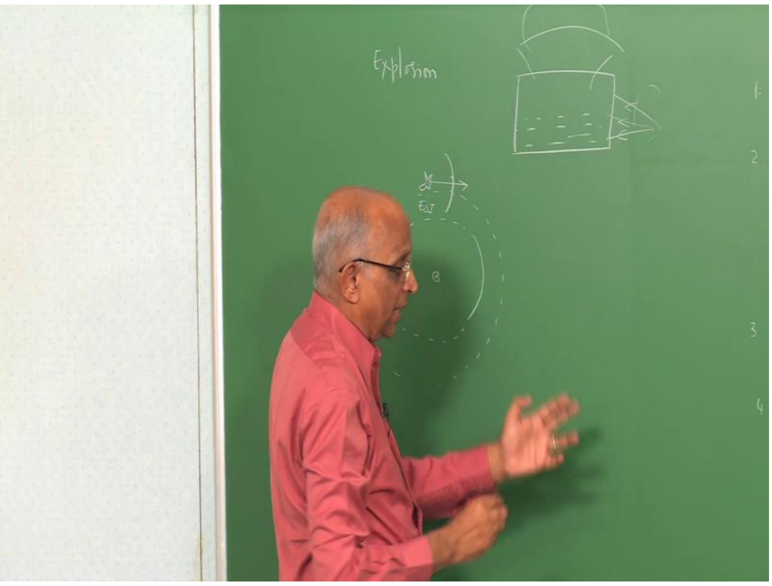

Well, we have something like particular tank, which contains some liquid fuel or some liquid, it could be volatile or could be heavy itself. Because of some fire in some other compartment or other wagons or other places I have the flame, which gets directed over here, the hot flame gets directed over here. The surface gets heated up, and when it gets heated up it boils the liquid over here generates lot of vapor, the casing also gets weaken, because it gets heated. And then it burst, and when it burst well I have an explosion, I have a huge fire ball and an explosion.

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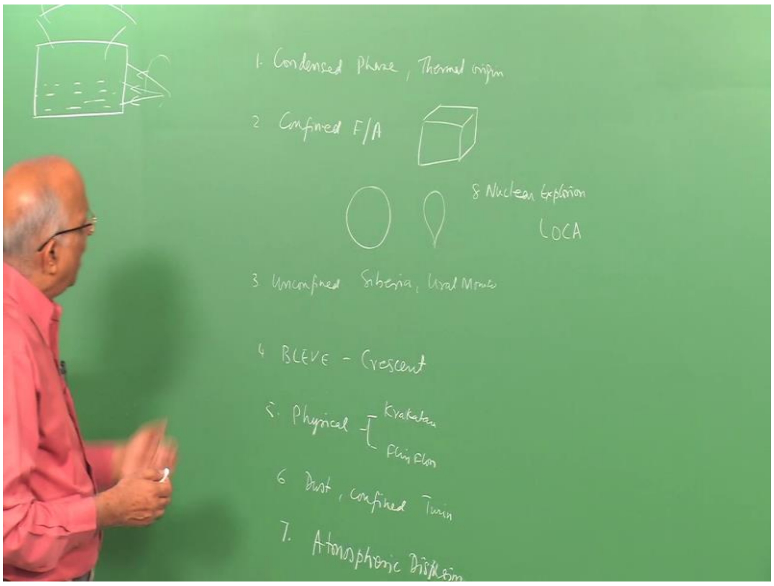

We saw the example of the crescent city at Illinois, and we also told ourselves that in petrochemical industries this do occur frequently, such as offshore platform. We said in Jaipur, we had the Indian oil cooperation having a fire in Visakhapatnam we had such tragedies taking place that is BLEVE explosion. We talked in terms of physical explosions. Physical in the sense we are not really talking of an explosive medium, but something like water which is suddenly put in a hot environment.

And it flashes into vapor, and when it flashes into vapor, you have huge super heated steam which releases energy and you have an explosion. We took the example of even we said naturally occurring explosion like, at the Krakatau volcano. Wherein you had a immense or very large explosion or when you had a pail of water at Flin Flon in Ottawa, wherein you had such physical explosions. We also told ourselves well even the dust, such as what the food stuffs we eat may be wheat powder it, may be corn flour, wheat flour, may be sugar powder could be explosives when mixed with their especially in a confined geometry.

We talked in terms of the explosion taking place at Turin in Italy, wherein an young boy he takes corn flour in one hand, mixes it he has a candle on the other hand it just burst into flame and an explosion. The seventh one was wherein we said well, you could have the atmosphere play a role in mixing the gases, we also said atmospheric dispersion. This atmospheric dispersion could come either the fuel which spills over, and it vaporizes it moves by the atmosphere.

It could have that or else may be even the toxic gases like we had at the Bhopal gas tragedy, wherein you had people dying because the you know cold day when there is temperature inversion, you have the gases which cannot really escape into the atmosphere, it causes a problem. We also talked of the great smog of London. And the last one which was number 8 was nuclear explosions. In nuclear explosions we talked of loss of coolant analysis, loss of coolant type of explosions we call loss of coolant type of accidents.

Wherein maybe because of the hot metal coming contact with water, which is used for cooling it generates hydrogen, and this hydrogen mixing with air causes an explosion. We also said well fission and fusion are cases wherein intentionally these are exploded, but we will not consider fission and fusion type of accidents in the course which we are doing. And this where the type of 8 different categories of explosions, you know this gave us a picture of what happens in an explosion we also known that a blast wave is generated in an explosion.

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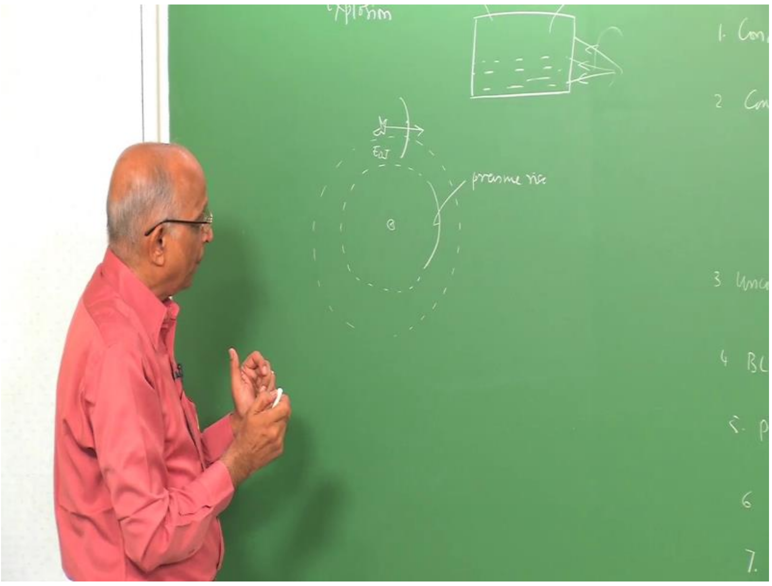

And now, our next aim is well can we sort of predict, the pressurize across a blast wave point one. And also what is the impulse or the blast wind or the wind which is associated which causes momentum, and disrupts the things from the scene of the explosion, and this is what I want to do today. Therefore, let us put this schematic again, what we want to do in this particular talk today.

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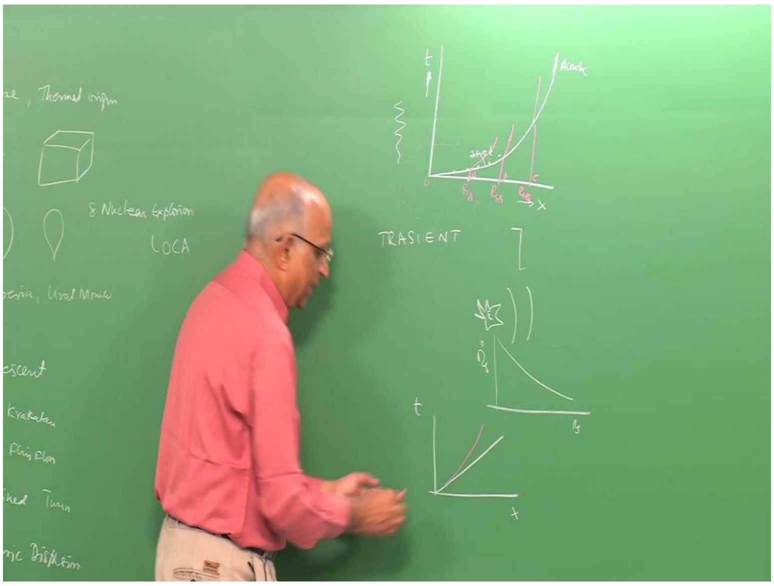

In other words we told ourselves well, I have on a street diagram in which the y axis is the time, the x axis is the distance. We said well, and when I release energy spontaneously at a point, a strong shock gets generated, it keeps weakening it becomes an acoustic waves in the far field this is acoustic. Here it is strong shock, as it were and what was the condition in a shock there is an abrupt rise in pressure. Well, this is what happens? A shock attenuating and we discuss this in terms of lightening, if we are very near to the source of lightening what happens?

Well, very near to it I hear a very loud bang, if I am far away I hear a rumble like an acoustic signature. Therefore, see we are interested in finding out, what is the over pressure generated at different points. Now, to be able to do that, we immediately tell ourselves well the problem is little complex, because the lead shock. I have an explosion taking place, let us say some energy getting released here, the front the wave front is continually decaying. In other words, if I have to plot the velocity which we did do, as a function of distance we said R s dot goes as a constant into R s to the power minus 3 by 2.

The velocity gets decaying the blast wave keeps decaying as the distance proceeds point one. We also know that if I have to look at this figure a little more closely. Let us try to follow the path of the particles which are process by the shock. To do that, I take let us say at this when the shock wave comes out this particular point, may be it picks up a particle here. Let us say that the identity of this particle, is let us say at a distance A from the origin over here, I call it as the distance x is equal to R s A.

Well the particle enters the shock over here, it rises in pressure, then it expands out and therefore this is the type of trajectory which this particle entering at A would have. In other words, I am defining this particular particle therefore, I am working in I am looking at the Lagrangian point of view namely following the particle, as it moves behind the shockwave, it is ahead of the shockwave it is stationary. Well, a particle which is away from it at B is process by the wave, when it has decayed out in strength. Therefore, the over pressure here is low, it also follows the particle, it also follows after the shock it has a particular trajectory and so on.

A particle let us say this is at R s B, I have another particle at C, let us say R s distance R s C from the source over here, it again goes behind. Therefore, when I look at the properties of this particle A, it is also influenced by the blast may be at this particular time. The blast is here, it is influenced by the blast at this particular time, the decay of the properties. Therefore, the problem becomes very complex it is not only it is decaying, but the properties of the medium enclosed by this or also governed by subsequent motion of the blast wave.

And therefore, the problem becomes highly, what we said earlier was highly transient. And not only transient, terribly unsteady and terribly non linear things are decaying, things are affecting one which affecting the other highly non linear. And predictions become a little more difficult, but we should be able to predict it, we should be able to write the governing equations and predict it, but before I get into some detailed predictions. Can we make the problem a little simpler; understand the problem a little better. And how do we do that? Instead of looking at the decaying shockwave, can I sort of presume or look at the properties, let say on the streak diagram t versus x.

Now, I say well if the shockwave to move at constant velocity, can I predict the properties behind it. And therefore, let us therefore start with this, and then put the decaying wave a little later, and then look at the decaying affect, therefore let us do the problem for a wave travelling at constant velocity namely the shockwave travelling at constant speed, predict the properties behind it, and then extrapolate it to the case of a blast wave.

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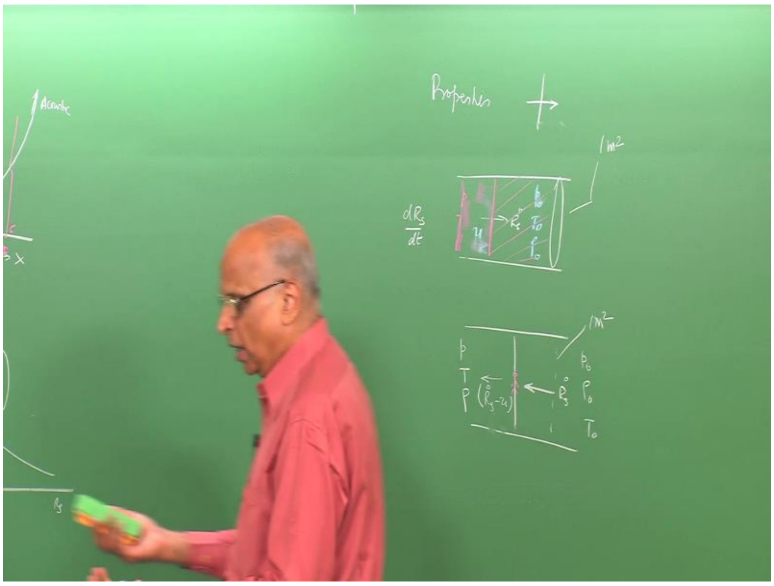

Therefore in today's class, I will specifically look at may be the properties, behind a shockwave moving at constant speed. Therefore, let us take a look what is it I want to do lets be very clear about the problem. Let us say I have something like this a pipe or something, let us consider it is a unit's area, cross sectional area is a unit over here, 1 meter square such that I do not need to carry my area cross section into it. I presume well I have a shockwave, which is travelling at constant velocity R s dot.

When I say R s dot I mean the velocity of the shockwave and it is equal to d R s by d t over here, which is R s dot. Let us also presume that the shock is moving into a medium, and this medium has properties let us put it in a slightly different color. The properties of this medium are let us say initial pressure is p 0, let get the atmosphere in which case the pressure is equal to 100 kilo Pascal or in to the power 5 Pascal. Let it be at initial temperature T naught, let the property let me let it have no air, let it be stagnant, it does no velocity, let it have a density let us say rho 0.

And then behind the shock, because it is getting process the pressure could be let say p over here, the temperature could be T, the density could be rho. And because the shock is moving it also drags the particle, let the velocity behind it be let us say u over here, velocity of the particle. Now, when the shock is moving at constant velocity at least I neglect the effect of decay, I want to know what are the what is the value of p rho T and u for this constant velocity shock, and this is what I want to do today. Therefore, what I do is I have to write the equations of motion relating sort of the portion over here, which is the free unaffected zone into which the shock is traveling.

This is the zone in which the pressurizes taken place, and I want to be able to predict p T and rho over here. You know to be able to do a problem with wave moving is difficult, and what we do is? Maybe we keep the frame of reference of the shock to be stationery. In other words, I sit on the shock waves such that the my plane of reference is the shock itself, and when I look at this, what is going to happen? Well the medium is going to move towards it at velocity R s dot. Well the pressure of the medium is p 0 rho 0 and T 0, and what happens to the process gases.

Well, it is at pressure p, it is at temperature T, it is at density rho. And it is now moving, since I am having the shock stationery, in other words earlier the shock was moving, the velocity behind is equal to R s dot minus u over here. And this is the frame of reference I do well I consider unit surface area, such that I do not need to bring area into account. And for this particular problem in the frame of reference of the shock, I would like to write the continuity equation, the momentum equation and energy equation.

And try to solve for the shock properties that is the properties I want to determine a p, rho, temperature and the value of the velocity behind the shock. Well let us do it, let us now write the equations down see so far we have not said what is the medium? The medium could be a solid substance, could be a liquid substance, could be a gaseous substance, could be anything.

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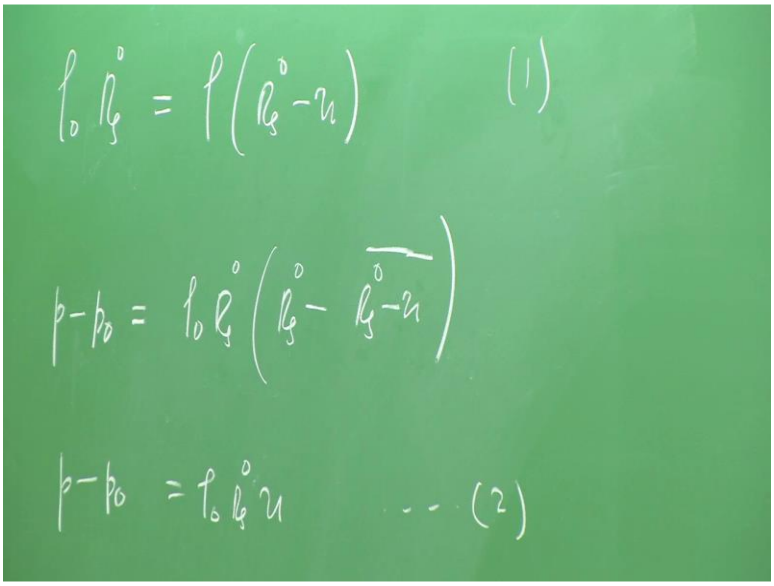

And therefore, let us write the continuity equation, well continuity equation says that the mass is conserved, rather the mass which is entering the shock we are studying in the plane of reference of the shock, rho 0 into R s dot into unit surface area is equal to rho into R s dot minus u over here, what is happening? Well, the mass flux which is entering is equal to rho 0 into R s dot, mass flux which is leaving is rho into R s dot minus u which is the continuity equation, call it as continuity equation. Equation 1 what is the momentum equation, well rate of change of momentum is equal to the impressed force, we are considering unit surface area therefore force is pressure.

And the pressure is equal to pressure increases is equal to p minus p 0, where does it come from? It comes from the mass flux that is rho 0 into R s dot which is the mass flux, it could have been I could have been written this over here, into the change in velocity, change in velocity is equal to R s dot minus, I have R s dot minus u which is the velocity behind. And therefore, this gives me the value as rho dot R s dot into R s dot minus this, this becomes plus here because this is the velocity behind is equal to u is equal to p minus p 0, which is the momentum equation.

Well, I have to write the energy equation, but before writing the energy equation. Let us see whether we can make out something from these 2 equations, before I write the energy equations and I have some reason for doing this, so far we have not said what is the type of substance, the properties of the substance. We said it could be a solid, it could be a liquid it to, it could be a gas into which the shockwave is moving. And if I want to solve these equations well, I could write R s dot minus u, let us go back to this figure.

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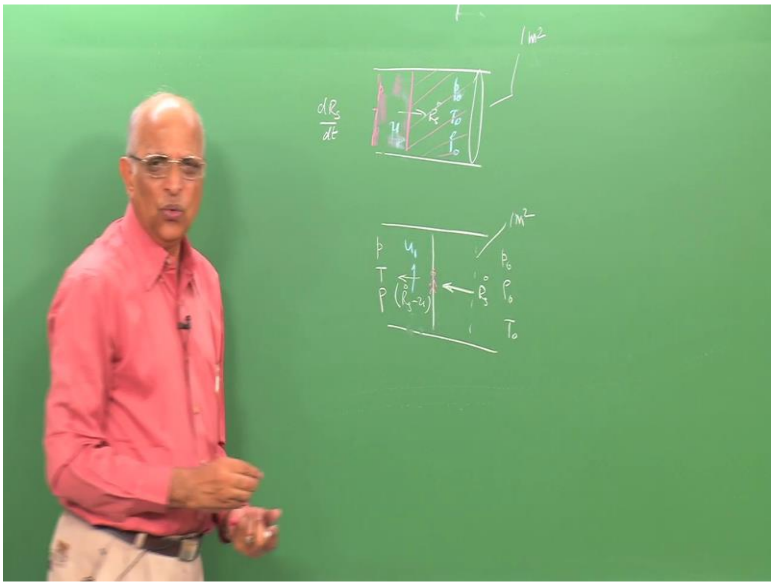

You have the velocity behind the shock as R s dot minus u, let us call R s dot minus u as u 1, you know it is simplifies instead of carrying this term, I just put it as u 1.

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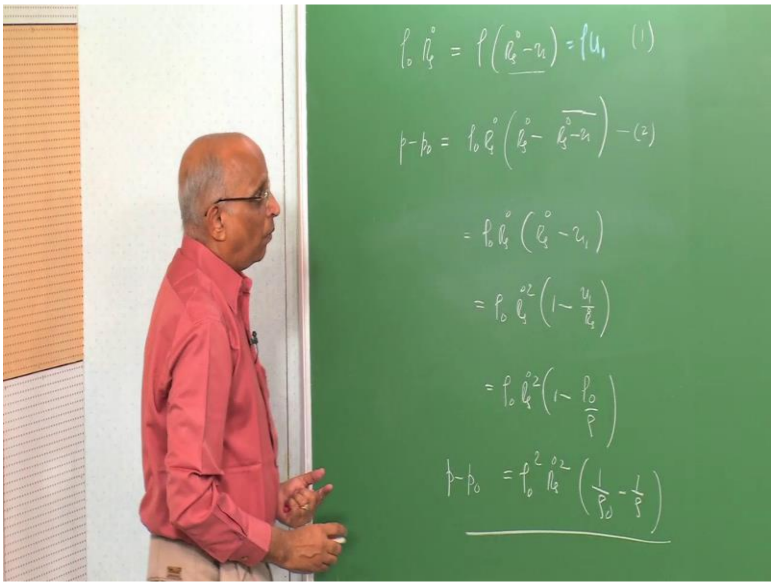

In which case what I get here is, I get this is equal to rho into u 1 over here, you know I have the momentum equation here, which is equation 2 over here. And now, I write R s dot minus u is equal to u 1, therefore I can write this as equal to rho naught into R s dot into R s dot minus u 1 over here. And therefore, I could again write this as equal to rho naught into R s dot square into 1 minus u 1 divided by R s dot. And if we write the continuity relation, wherein I say R s dot minus u as u 1, I get the value of u 1 over R s dot is equal to rho naught by rho.

And this I can write as rho naught into R s dot square into 1 minus u 1 is equal to rho naught by rho into R s dot that means, it is equal to u 1 divided by R s dot is equal to rho naught by rho. And if I have to take rho naught outside, this becomes rho naught square into R s dot square into since I have taken rho naught outside, this becomes 1 over rho, this becomes 1 over rho naught, it becomes 1 over rho naught minus 1 over rho over here. And this becomes the value of p minus p naught or rather what is it I get?

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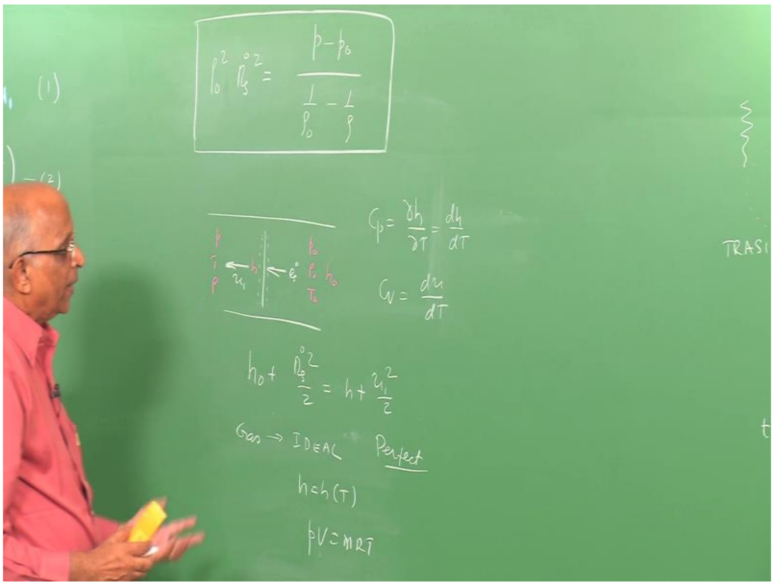

I get the equation as the value I get as rho naught square, that is the initial density square into R s dot square is equal to p minus p naught into divided by 1 over rho naught minus 1 over rho. And this is what I get by solving the continuity equation 1 with the momentum equation 2. Now, see I have been able to get some relation between pressure and density, in terms of the shock speed over here, but I want explicitly the relation between p rho with p 0 rho 0. And therefore let us try to get rid of this term, and get the general types of solutions for which I look at the energy equation.

Now, to be able to solve the energy equation, well I have to go back to my scheme again. Well I have the gas coming at a velocity R s dot, it is leaving with a velocity now I say u

1, because u 1 was equal to R s dot minus u. We have the properties here p rho T, we have p 0 rho 0 T 0 over here, I want to write the energy equation. And let us further assume, that in addition to properties being p 0 rho 0 and T 0.

And let the initial enthalpy of the gas here be h naught, let the enthalpy of the gas in this region be h in addition to pressure p T and rho over here. Therefore what is it? Now I consider this shock which is steady as a control volume, it is a steady shock moving at constant velocity, and therefore when the shock is stationery gas is coming at constant velocity leaving at constant velocity u 1. And the equation for this control volume is well, I have h 0 plus the kinetic energy per unit mass.

This is enthalpy so much joules per kilogram per unit mass into R s dot divided by 2 is equal to h plus I have u 1 square divided by 2 u 1 is equal to R s dot minus u is the value of u 1, this is the energy equation. And now, if I want to solve this, I have to express enthalpy in terms of these particular properties. And depending on the substance the enthalpy could be a function, if it is a non ideal gas well, h could be a function of temperature and pressure, if it is a solid or liquid substances. Well it is very cohesive, if the enthalpy is more complicated function of pressure, temperature and density.

And therefore at this particular point, we would like to introduce some simplification for the substance which is involved. Let us take the case of the substance being let us say a gas. Therefore now, even though the momentum equation what we derived here, is irrespective of the substances which are associated in which the shock is travelling. When I look at the energy equation it becomes necessary for me, to specify whether the substance is going to be a solid, liquid or gas.

You know we know for substances such as solids and liquids the enthalpies is much more complicated and the relation between pressure, temperature and density is through the equations of state are much more complex. Therefore I take a gas, and in this particular gas I say, let us also further assume that this gases ideal, what do we mean by a gas is ideal? We say, well for an ideal gas it is a pure substance, enthalpy is only a function of temperature, internal energy is only a function of temperature.

And therefore, h is a function of temperature alone for an idea gas, and the moment we say h is a function of temperature internal energy is a function of temperature. We also know that for an ideal gas, the equation of state become p V is equal to m R T mass of the gas having a volume v. Therefore we start with an ideal gas and we also say in addition to being ideal, let the gas be perfect, what do we mean, what is the difference between a prefect gas and an ideal gas?

For a perfect gas, well d h by d T which is the specific heat at constant pressure. Well C p is equal to del h by del T at constant pressure, and C v is equal to del u that is the internal energy by d T by del T at constant volume or we do not even need to do this, because h is only a function of temperature. Therefore I can as well write is as equal to d h by d T, is only a function C v is equal d specific internal energy by temperature. And therefore, for a perfect gas I can simplify this equation, and write it in the following way.

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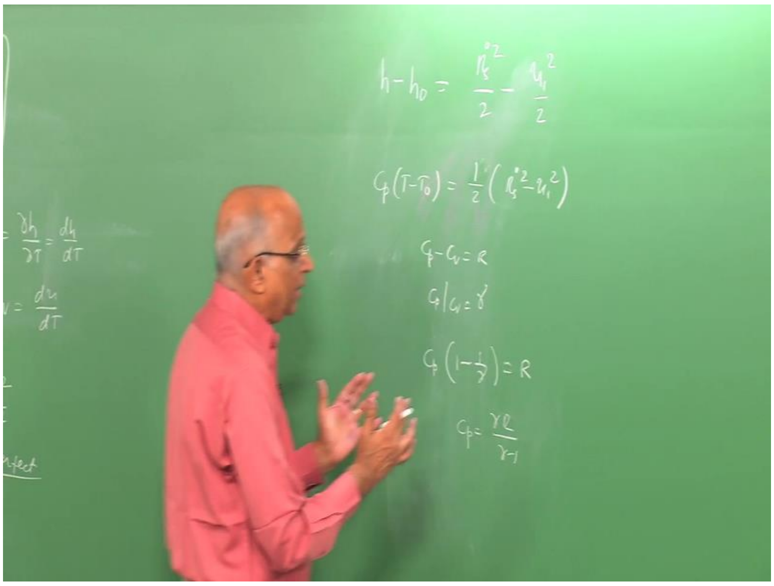

Let us write the energy equation, now we say that the gas is a perfect gas, and therefore I write h 0 h minus h 0 is equal to I bring h 0 on this side is equal to R s dot square divided by 2 minus u 1 square divided by 2. And h minus h 0 I can write as C p, h is equal to C p T, and therefore it is equal to T minus T 0 is equal to half into R s dot square minus u 1 square over here.

Now, C p can I express it in terms of the gas constant, well we know C p minus C v is R, and also we know that the ratio of specific heat C p by C v is equal to the specific heat ratio gamma. And therefore, I can simplify with this by taking C p into 1 minus 1 over gamma is equal to the specific gas constant R. Well, we are talking of specific gas constant, which is unit is joule per kilogram Kelvin. And therefore, C p is equal to gamma R divided by gamma minus 1, gamma minus 1 by gamma, gamma R by gamma minus 1. Therefore the energy equation when we put some restrictions, and tell ourselves we are only interested at present, perfect gas over here into which a shock is moving, what is it we start getting.

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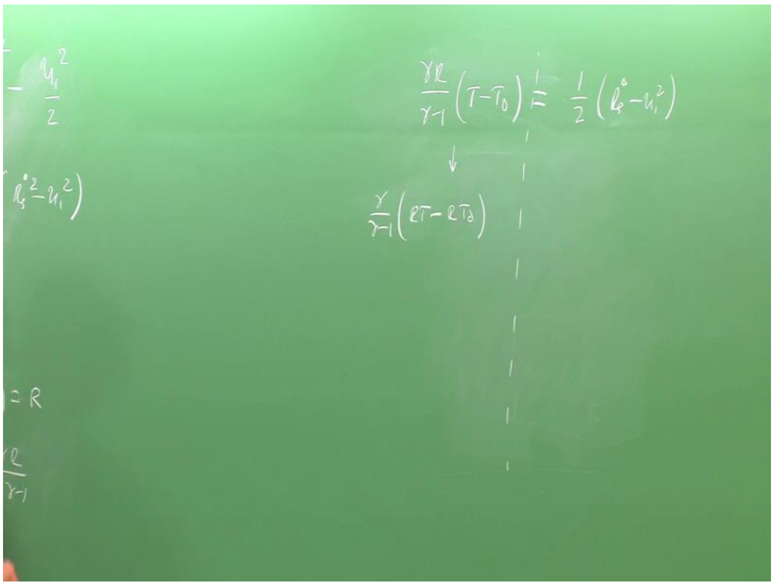

We get the equation as gamma R by gamma minus 1 into T minus T 0 is equal to I have a half into R s dot square minus u 1 square over here. Now, we look at this a little more closely; let us first take a look at the left hand side. Well, for the left hand side, I have gamma by gamma, gamma would divided by gamma minus 1 into R T minus R T 0.

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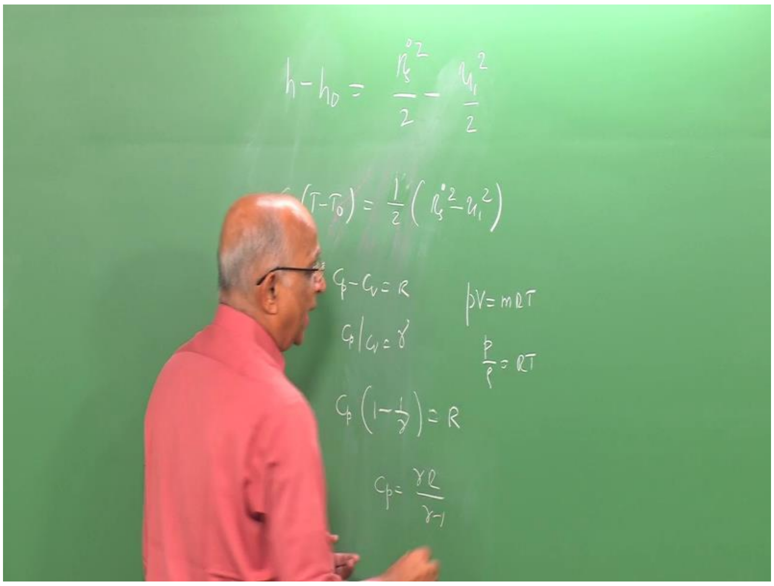

Well, p V is equal to R T m R T or we have p V. Let me use this part of the board, P V is equal to m R T or p by rho is equal to V by m is rho, m over V is rho, p by rho is equal to R T.

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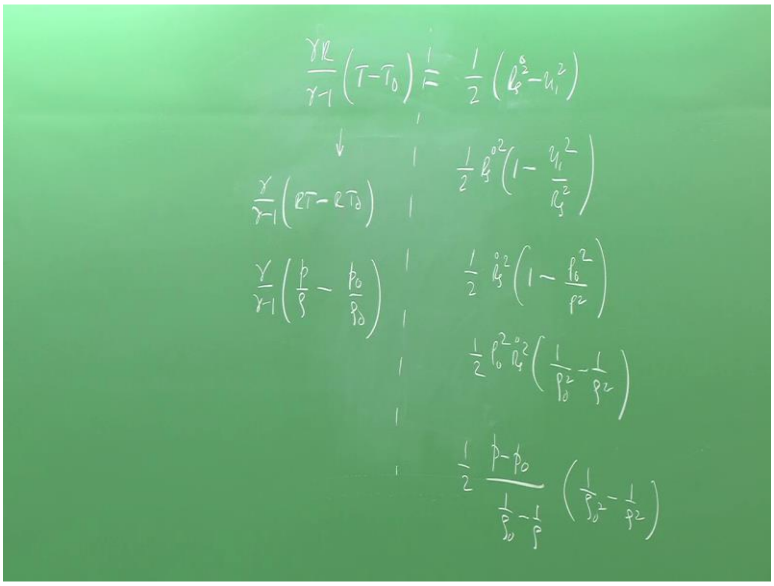

And therefore, this I can write as equal to gamma by gamma minus 1 this is the short gas, I have the pressure and density given by this. This is the initial gas R T 0 is p 0 by rho 0, and this becomes my left hand side. On the right hand side if I have to do a similar simplification, what I get is half into R s dot square into 1 minus u 1 square divided by this would have been R s square R s dot square, we have a R s dot square here, we should have been R s dot square 1 minus u by R s dot square.

And therefore, this become half into R s dot square, this is what I have taken outside, therefore R s dot square into 1 minus u 1 by R s from the continuity equation namely, u 1 by R s dot is equal to rho dot by rho. And therefore we get the expression has coming over here is equal to rho dot square divided by rho square. Let us be clear, this is equal to rho into u 1 is equal to rho naught into R s dot, therefore u 1 by R s dot is equal to rho 0 by R s dot.

So, that we get the value of this over here, and therefore now if I take rho dot square outside, I get half into rho dot square into R s dot square into 1 over rho dot square minus 1 over rho square. And now, we have really got an expression for rho dot square into R s dot square by solving the momentum equation and this is where it is, we have rho dot square into R s dot square is p minus p 0 into 1 over rho 0 minus 1 over rho.

And therefore, this I can now write as half into p minus p 0 into divided by 1 over rho 0 minus 1 rho, this was the expression we got for rho dot, and this is multiplied by 1 over rho 0 square minus 1 over rho square. And this we find is equal to a square minus b square is equal to a plus b into a minus b, and therefore this is equal to a minus b, therefore this expression becomes let me write it on the other side of the board here. I can now delete the continuity equation; I can delete the momentum equation.

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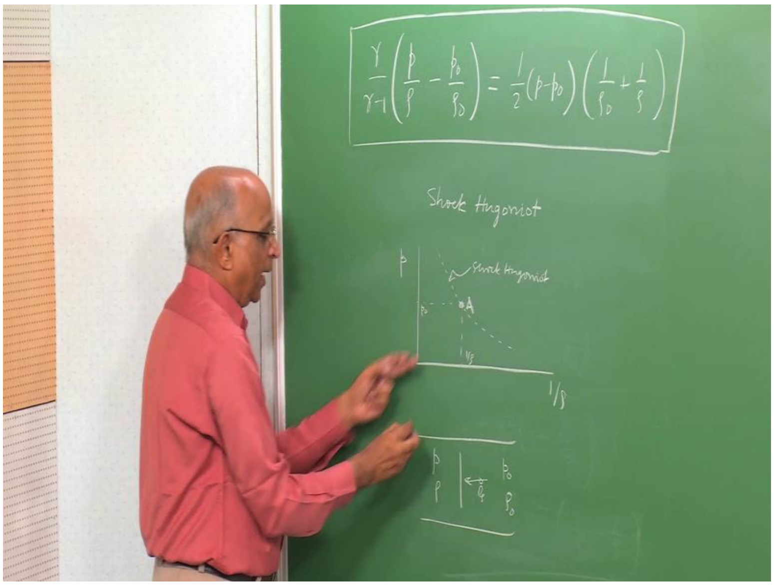

And write the final equation as equal to gamma over gamma minus 1 into, let me write it p by rho minus p 0 by rho 0 is equal to half what I have here, p minus p 0 into all what I get is, I get this to be cancel I get 1 over rho 0 plus 1 over rho over here. And this becomes the equation, we obtain by solving the continuity momentum and the energy equation, and what does it tell? It tells what are the properties behind the shock p and rho, as a function of properties ahead of it p 0 and rho 0, and the only other factor which comes is the specific heat ratio of the gases.

Well, this equation is what is known as a Hugoniot, and since we are using it for a shock it is known as a Shock Hugoniot. Let us see what does it give? Let us say I am interested in the properties behind the shock, which is p. I look at the properties ahead of the shock may be let me sketch it again; I have the shock which is stationery. I have which we kept stationery the gas is moving with a ahead of it with a velocity R s dot, the pressure is p, the density is rho, the initial pressure is p 0, the density is rho 0. And what does it give?

It gives a variation of p as a function of 1 over rho, and it depends on the initial condition. Well p 0 could be somewhere here, the 1 over rho this is let us say p 0, this point could be 1 over rho 0 this is the initial point. And for this initial point, how do the state change? And this becomes what we call as the Shock Hugoniot. The Hugoniot equation tells what are the properties behind the shock? As a function of the properties ahead of the shock, that is pressure and density behind as a function of pressure and density ahead of it.

And this is the point initial point let us call it as A and for all these points are the properties corresponding to the shock, and see why do you get so much multitude of properties. Well we are not specified in this equation the velocity at which is the shock is moving, depending on the velocities I could have a number of properties which are there, and this is what we call as a Shock Hugoniot. Let us go further, see we want specifically the relation between the value of let say p as a function of rho. Therefore, let us try to plot p by p 0 as a function of 1 by rho divided by rho 0. And let us therefore simplify this Hugoniot Shock Hugoniot equation further, and let us do it.

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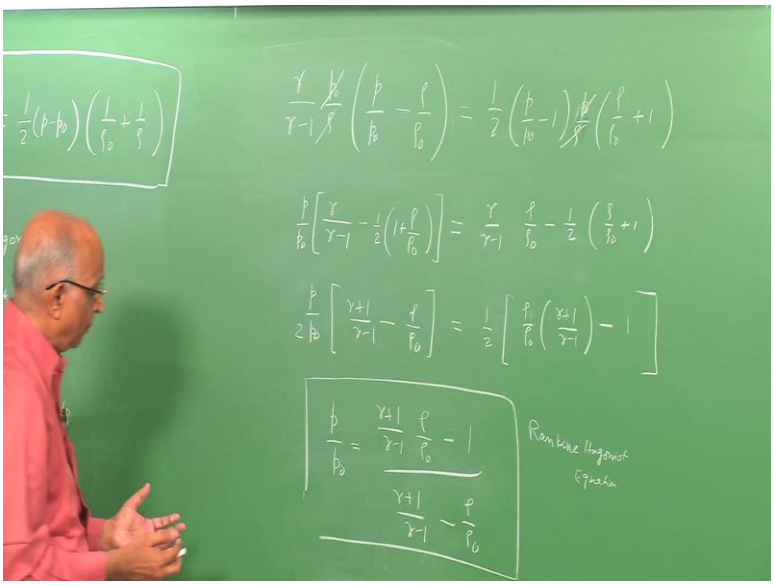

I want to specifically write p by p 0, therefore I simplify the Shock Hugoniot equation as gamma divided by gamma minus 1, I take p 0 outside, I let me also take rho outside, and therefore I get over here p by p 0 have taken rho outside, therefore it becomes rho by rho 0. Therefore this will be clear I have taken p 0 outside, therefore p comes over here, I have taken rho outside, rho gets out here rho comes on the top minus rho by rho 0 is equal to, on the right hand side I have half into, I again take p 0 outside, I have therefore p by p 0 minus 1 into p 0, and I also take rho in the denominator, let us say p 0 by rho over here and if I take rho over here, I get rho by rho 0 minus plus 1 over here.

Therefore, you know in this equations I am wanting a relation between p by p 0 and rho by rho 0 and that is the reason I am doing this, I find since I have taken these two outside, these two cancel on the left side and the right side. And therefore, now if I were to write the expression for p by p 0 as a function of rho by rho 0 I take it on the other side, I have p by p 0 into gamma by gamma minus 1 on the left hand side. Now, I have p by p 0 with a half, and therefore I get a minus half, because I am still keeping it on the left hand side I bring it here minus half, and minus half of what? It becomes 1 plus rho by rho 0.

And now I go to the other side, I have this value, therefore I get gamma by gamma minus 1 into the value of rho by rho 0, and what is it I am left with have taken this, now I have minus half I brought p by p 0 I still have minus half over here into what happens? p by p

0 have taken therefore, this becomes p by p 0 into this, now I have minus half into rho by rho 0 plus 1 over here. This I simplify it I want to right it, let us check this again I have p by p 0 into gamma minus 1, and then I brought p by p 0 on this side minus half into rho by rho 0 plus 1 over here, then I take rho by rho 0 on this side it becomes gamma by gamma minus 1, minus sign becomes plus rho by rho 0, then I have minus half into rho by rho 0 plus 1.

Therefore now I want to simplify this term, I have p by p 0 into now I write, well it becomes minus half, gamma by gamma minus 1 minus half which is equal to 2 gamma it becomes gamma plus 1 divided by gamma minus 1, minus I have the value of half that is equal to rho by rho 0 that means, I have 2 by gamma minus 1 over here, that means 2 gamma by 2 gamma minus 1, that means 2 gamma minus gamma minus 1, which becomes 2. Therefore I have 2 over here is equal to the same thing I write on the other side, I have a half over here.

And therefore, I have rho by rho 0 which is here, therefore I write now gamma by gamma minus 1 minus half which is equal to gamma plus 1 divided by gamma minus 1, and the half I have already taken outside, therefore I have minus the value of this is rho, this becomes minus 1 over here. And therefore, what is it I get? I get the value of p by p 0 is equal to gamma plus 1 divided by gamma minus 1 into this was rho by rho 0 into rho by rho 0 minus 1 divided by, I have gamma plus 1 divided by gamma minus 1 minus rho by rho 0.

And this becomes an explicit relation, which relates the pressure ratio that is the pressure behind the shock, with the pressure ahead of the shock, in terms of the density behind the shock divided by density, and this equation is known as the Rankine Hugoniot equation. We could similarly have an equation, for instead of I could put rho by rho 0 and find it in terms of pressure, and if you do this you will find that the expression comes out to be very similar, even though the signs will be slightly different on the right hand side.

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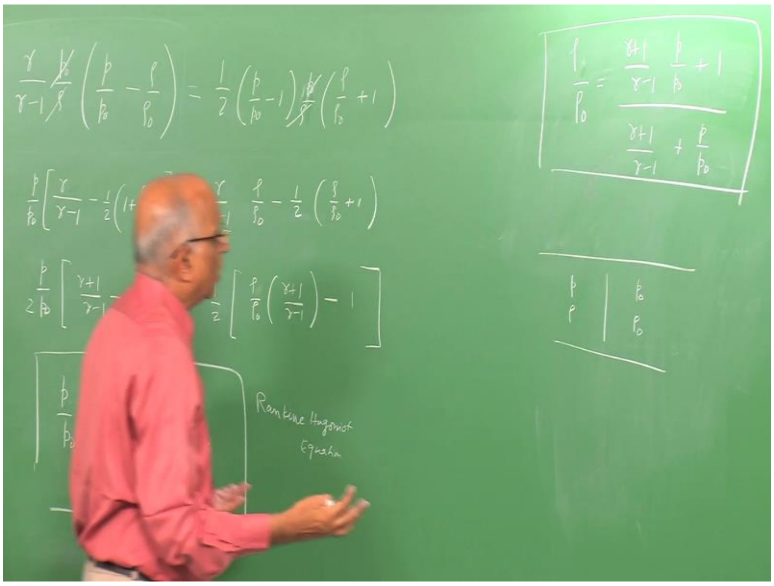

You will get rho by rho 0 is equal to gamma plus 1 divided by gamma minus 1 into p by p 0, in this case it will be plus 1 divided by gamma plus 1 divided by gamma minus 1 into plus of p by p 0, this is also Rankine Hugoniot equation. Therefore, these 2 equations are known as Rankine Hugoniot equations, and described what is the value of the pressure ratio in other words, all what we are talking is your shock moving in a medium, in which the pressure is p 0, the upstream value is rho 0 the undisturbed medium.

This is the disturbed medium p and rho, we are relating the value of p by p 0 p by p 0 and expressing it as a function of rho by rho 0, this is what we call as Rankine Hugoniot equation. And the Rankine Hugoniot equation is just an extension of the Shock Hugoniot, in which explicitly we express the pressure ratio and the density ratio, I should have been rho by rho 0, just like I write p by p 0 p by p 0 p by p 0 rho by rho 0 over here, and this is what is the Rankine Hugoniot equations. Therefore, let us now summarize at this particular point of time before we proceed, let us take stock of the problem what we wanted to do, and then proceed further.

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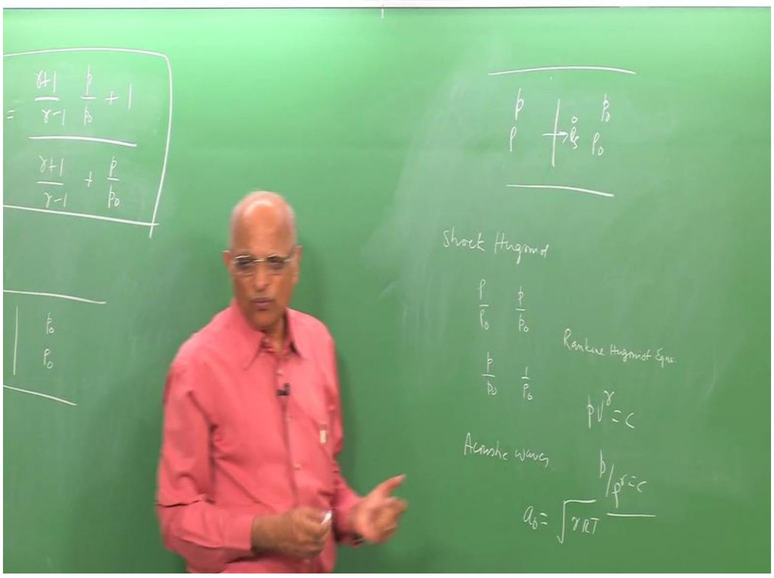

Therefore, what is it we wanted to do and where are we now, we started off by saying. We wanted the properties behind the shock wave, which is moving at constant velocity like we have a pipe, the shock is moving at R s dot into a medium whose pressure is p 0 rho 0, we wanted the value of p and rho. Once I know p and rho, if the gas is an ideal gas I know p is equal to rho R T I can also find out the temperature, what did we get? We got the Hugoniot, that means I write the continuity momentum and energy relation relating this and this, I took the condition of the plane of reference as the shockwave.

I got the Hugoniot equation which I called as the Shock Hugoniot, because I am specifically looking at this particular wave as a shockwave, I got the Shock Hugoniot. And then I related the value of rho by rho 0 as a function of p by p 0, and also p by p 0 as a function of rho by rho 0, which we called as the Rankine Hugoniot equations. Now, our next step is, well I am writing the equation for this discontinuity, see mind you but in the first class, when we looked at the waves.

We also talked in terms of acoustic waves or rather the sound waves, can we say in what way this wave which is a shock wave is going to be different from an acoustic wave. After all you know for acoustic wave also you remember, we solved it we said it is isentropic. We also told ourselves well, if it is isentropic I can write is p V to the power gamma is a constant or rather p by rho to the power gamma is a constant.

And therefore, we said and we also derived an expression for sound velocity, and we said sound velocity is given by under rho gamma R T, can we now relate the pressure and density behind the shockwave, to pressure and density behind an acoustic wave what happens if the process is isentropic like in sound wave. Let us spend a couple of minutes on it before proceeding further with this particular problem.

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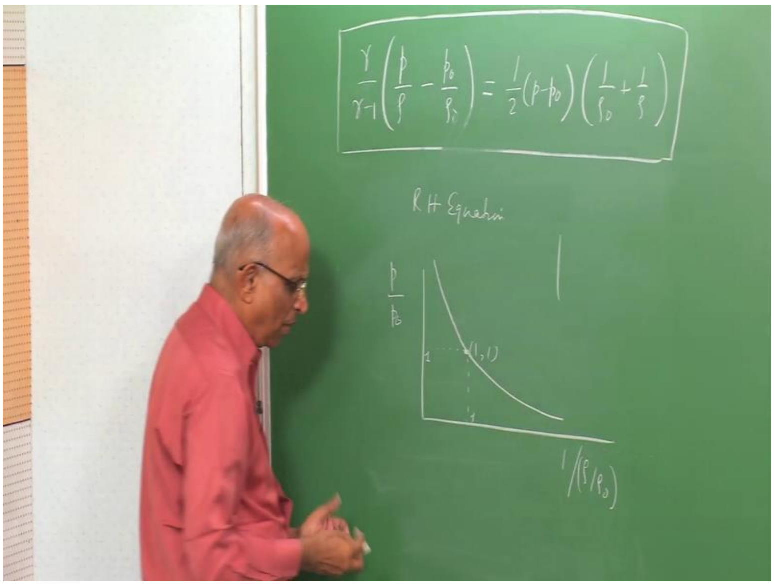

Let us now say yes, I have been able to relate the pressure, let us say I look at the Rankine Hugoniot equations, and what is the Rankine Hugoniot equation tells me. Well have p by p 0 as a function of 1 divided by rho by rho 0, and what is it I get? My initial point is p 0, my initial point is rho 0, which in this non dimensional plot becomes 1, it becomes 1 because I am p by p 0 this is point 1, this is point 1 this is my initial point over here.

And the Rankine Hugoniot equation tells me well the properties are all like this; these are the different values of p by p 0 as a function of rho by rho 0. And if I have to plot the isentropic solutions on this particular plot, you know where is isentropic or where is the sound wave or if I have something like the process of shock being isentropic will I get the same value or not, let us try to do that, therefore now I erase this equation.

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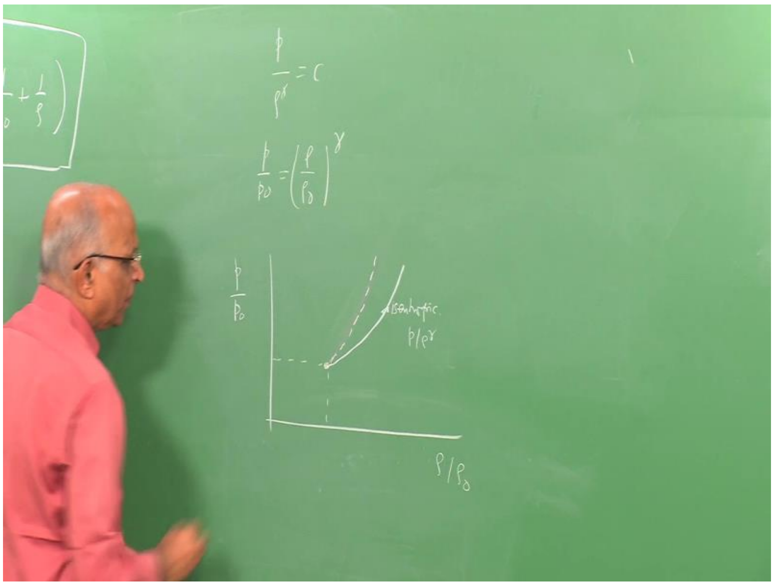

Well, I say well for a sound wave p, p by rho to the power gamma is a constant. And therefore, now I can write p by p 0 is equal to rho by rho 0 to the power gamma. And now if I have to plot it in the C instead of plotting this, why not plot it as a function of rho by rho 0, so that you know the thing gets diverted. If I plot it as a function of rho by rho 0, what is this figure going to be instead of being like this, is going to be like this over here. Therefore let us plot that, because we are looking at a compression process which is a shock.

And therefore, let us now may be make it plot it as rho by rho 0 as a function of p by p 0, and what is it? This is my initial point one, and therefore an isentropic process I have this equation. Well this is the type of the solution what I get if the process is isentropic, if I have a shock and if I have to plot this particular equation it is a Rankine Hugoniot equation, what is it I will get? I will get the solution to be something like this. In other words, the pressure ratio is higher for the same density ratio, and the reasons it is not isentropic there is some dissipation, there is some it is no longer adiabatic, and that is and no longer reversible.

And therefore I get a higher pressure ratio compared to the isentropic assumption of p by rho to the power gamma is constant, in fact on the state properties I cannot show this by a continuous line, because it is something which is not reversible. And therefore normally we show it by a dash line and we say well, the shock properties are denoted by this particular dotted line across a shock. Therefore, we are now very clear about Hugoniot that is the Shock Hugoniot.

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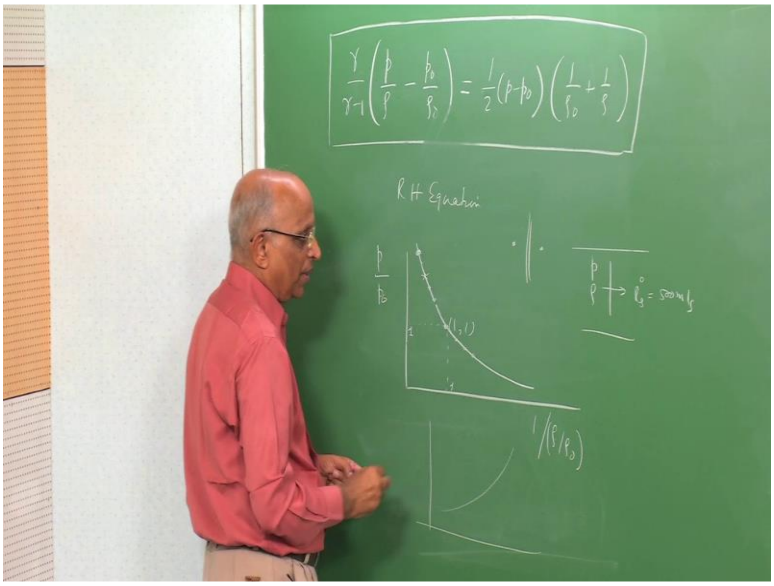

We are also clear about the Rankine Hugoniot equation, which comes by for solving for the properties behind the shock as a function of properties ahead of the shock, but we have also forgotten what we wanted to do. We wanted explicitly, when the shock is propagating in a particular medium, what is the value of pressure? What is the value of density? And now what we have got is the family, we have got may be number of pressure ratios are possible, but which is the pressure ratio which is really going to be there when the shockwave is R s dot.

Let us say R s dot is let us say 500 meters per second, is it going to be this point, is it going to be this point, that means into this plot I have to put a velocity. If I can put a velocity and say for this velocity it is going to be here, for this R s dot it is going to be here then I am in business, otherwise I do not know what to do. Therefore, it is necessary for us to look at the Rankine Hugoniot equations again or look at the Shock Hugoniot equation again, but we have to define the velocity on this plus. And let see, how to define a velocity such that I can say, if the velocity is here it is over here, the velocity is say 600 meters per second it is over here.

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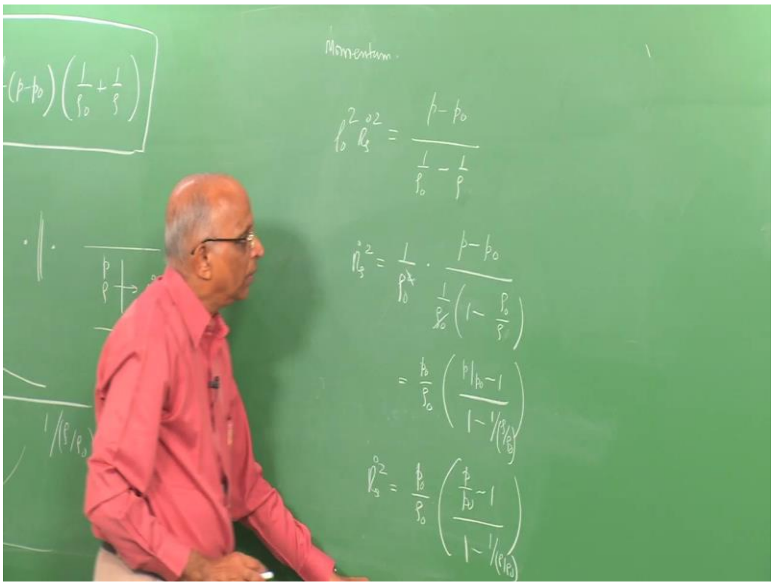

And therefore, the second part is let us take a look at velocities, and how to represented on the plot of p by p 0 versus 1 over rho by rho 0, but let us take a look at the momentum equation which we have solved, some time ago what did we get? We got rho 0 square into R s dot square is equal to p minus p 0 divided by 1 over rho 0 minus 1 over rho. Please check, we have got this we use the continuity equation and momentum equation to give this form.

Therefore now I say, well I have R s dot square is equal to 1 over rho 0 square into I get p minus p 0 into now 1 over rho 0 minus 1 over rho about this, it is the final minus initial. Please check it again, this is the value we have 1 over rho 0 minus 1 over rho. Therefore, now I take 1 over rho 0 outside, and now I get 1 minus rho 0 by rho over here. Now, I find well, rho 0 and this gets canceled over here, I can also take p 0 outside. And therefore this becomes p 0 by rho 0 into p by p 0 minus 1 divided by 1 minus 1 over, I have this becomes 1 minus rho 0 by rho or rather this becomes 1 minus rho by rho 0.

Let us be very clear 1 over rho 0 minus 1 over rho, I take rho 0 outside, therefore if I take rho 0 outside rho 0 comes on top or rather it comes in the bottom over here that is rho 0, and therefore I write 1 minus 1 over rho minus rho 0. And therefore, what is it I get? I get the value of R s dot square is equal to p 0 by rho 0 into this particular value of p by p 0 minus 1 divided by 1 minus 1 over rho by rho 0 over here in the denominator, what this tells us, let us take a look form the plot.

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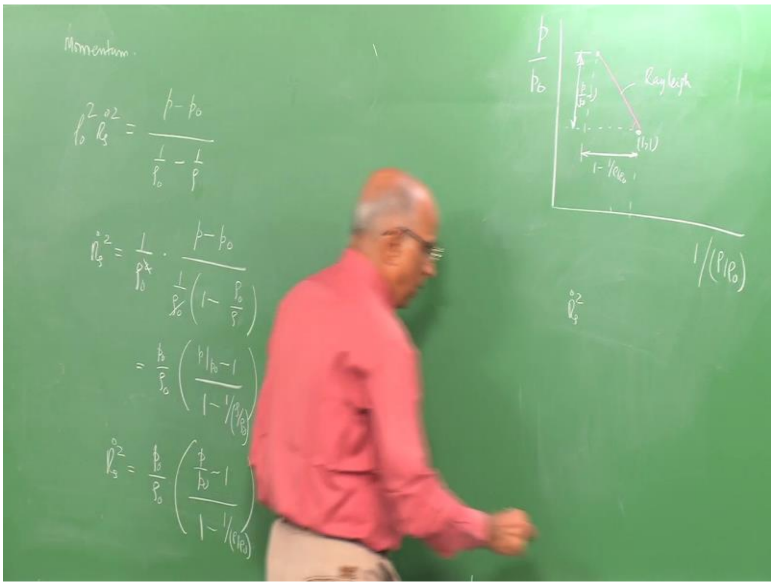

Well again I have to plot the value, I to put the ordinates and the abscissa, let us put it down. Let us say that the initial point is here p 0 rho 0 this is the point one, initial point this is 1 into 1, because pressure is p 0 by p 0 is 1, rho by rho 0 is 1. And let us say that the final point is somewhere over here, corresponding to this what does this tell us it is p by p 0 minus 1, that means we are looking at this particular value p by p 0 minus 1. You are looking at this particular value divided by 1 minus rho by rho 0, this is the rho 1 minus rho 0, 1 minus 1 over rho by rho 0 this is equal to p by p 0 minus 1 over here.

And therefore, if I have to join these two points, the slope of this particular curve is this particular bracket over here, and if it is multiplied by the value of the initial pressure to the initial density. Well, it gives me the value of R s dot square. In other words, this line gives me an indication of the initial velocity, when I multiply the slope by initial pressure and the divided by initial density and this line is known as the Rayleigh line. It gives me an idea of what the initial velocity is, and therefore if I say well from the momentum equation, I am able to get the Rayleigh line which is given by this expression.

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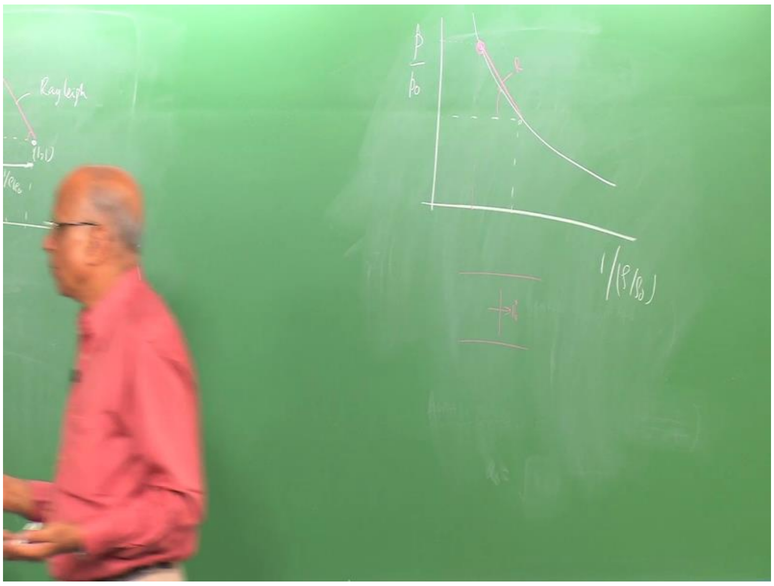

And now, if I look at the net problem what is my problem? My problem was to be able to determine the value of the final pressure and the final density. Therefore all what I have is, I have a Shock Hugoniot, initial point this is the Shock Hugoniot which is passing. Now, I have the shock velocity, which is going at a given value of R s dot for a given value of R s dot. Well I have only a particular value of velocity which comes and hits here, and therefore this is my value of pressure and this is my value of density.

Therefore, by solving the Rayleigh line along with a Rankine Hugoniot equation or the Shock Hugoniot equation both are same. I am able to get the final point, and I can get the final pressure and the final density, because initial density is known I get the value, and I can get the properties of my medium. This is how we calculate the properties behind, but I think why not put it in a proper form, see the point is can I get an explicit relation also. Therefore, let us quickly repeat what little we have done, and set course for how to get an explicit relation, see I would like to know.

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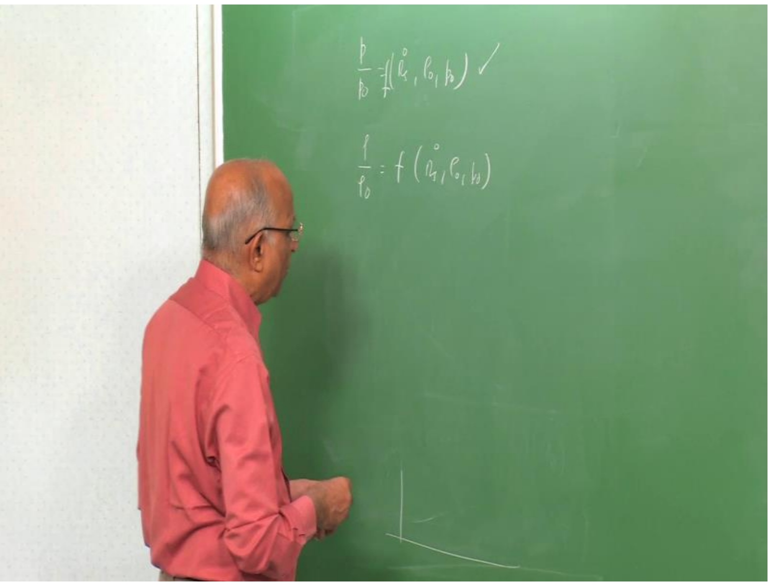

Well, p by p 0 as a function of shock velocity, as a function of rho 0, as a function of p 0 is what? This is what I am aiming at, I tell myself well, from the Shock Hugoniot and the Rayleigh line I am able to get p by p 0, and I am able to get p by p 0 I am also able to get rho by rho 0 as a function let us say of let us say R s dot, rho 0 and p 0. This I am able to get, but this is graphically using the Shock Hugoniot and the Rayleigh line.

Let us try to solve for it explicitly an expression to be able get this back, to be able to get that expression we need to do a little more homework, and let us see what little we will have to do in that, we tell ourselves. Well, I am able to get the Rankine Hugoniot equation may be from Rankine Hugoniot equation, can I put it the Rayleigh line into the Rankine Hugoniot equation, and solves specifically for the pressure and the density behind the shock.

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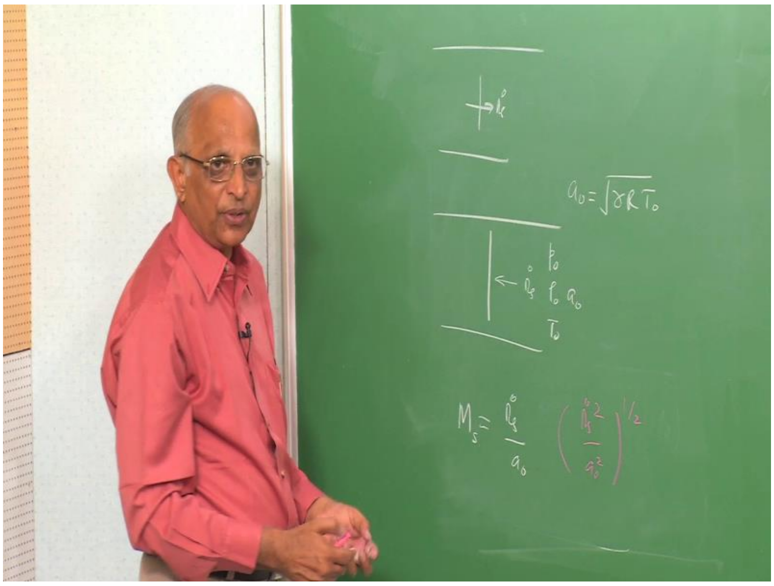

To be able to do that, it is better to do it in a non dimensional case. In other words, instead of writing that the shock progresses with a velocity R s dot or rather in the plane of frame of reference, that means the shock stationery, the gas is moving towards it with a velocity R s dot. Instead of carrying the term R s dot can I non-dimensionalize, and put all the expressions in terms of p by p naught, so also I have do not carry the units of meter per second or kilometers per hour and all that in the shocks speed. Therefore the non- dimensionalization we do is may be with respect in the free medium, we have p 0 rho 0 T 0 which gives me a sounds speed a 0 in the free medium.

And we already know, well a 0 is the function of the initial temperature or rather we have derived this expression a 0 is equal to gamma specific gas constant into T 0. And if I write Mac number as equal to R s dot divided by a 0, well it becomes non- dimensional, I express it in terms of Mac number, what does the Mac number represent? It tells me, the ratio of the shocks speed or this say this is the shock Mac number, the velocity of the shock divided by the density of the medium upstream or rather what does this really denote? Let us spend a moment on this, I can therefore write it as equal to R s dot square divided by a 0 square to the power half.

Now I say well, R s dot square is an indication of the kinetic energy associated with the shock or associated with flow of gases over here, a 0 square we found a 0 square is equal to gamma R T or the initial temperature gamma R T 0. T 0 tells me what is the temperature of gases or what is the level of the molecular energy which is available or something like an internal energy of the gas which is available. Therefore, Mac number is a representation of the kinetic energy of the gases to the molecular or the internal energy of the gases, and as the internal energy increases for a given velocity.

Well the shock Mac number comes down, it just the ratio of the kinetic energy to the internal energy. And therefore we will use the Mac number, instead of the shock velocity to be able to derive an explicit expression. Let us do that, let us get started. You know for that we again revisit the problem, we reformulate the problem, let us look take a look at the continuity equation and momentum equation and the energy equation again, what is the momentum equation?

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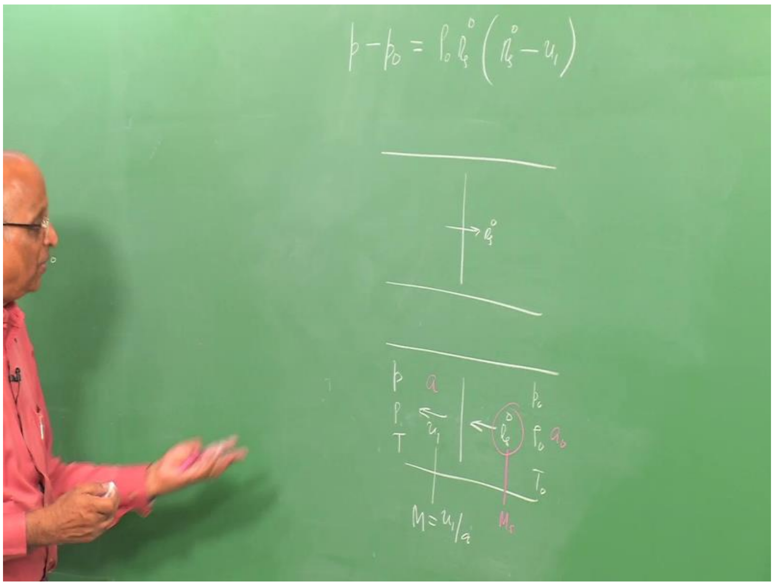

Let me start with the momentum p minus p 0 was equal to, we had rho 0 into R s dot which is the mass flux which is coming. And there was a velocity change, and what was the velocity change, initial velocity was R s dot, the velocity behind the shock was u 1 over here this was my momentum equation. And now, we would like to express it in terms of let us say, we will get rid of R s dot, we will get rid of u 1, we will put it in terms of Mac number.

Therefore the problem which we consider is, well I have now this shock which is travelling at velocity R s dot or equivalently in the frame of reference of the shock. The gas is moving at a velocity R s dot, it is moving here with a velocity u 1, the properties here are p 0 rho 0 and T 0, the properties behind are p rho and T over here. Instead of R s dot, I put in terms of M s over here; the sound speed in this medium is a 0. The sound speed in the medium process by the shock is at higher temperature, therefore the sounds speed is a over here.

And therefore, the Mac number behind of the gases which is leaving the shockwave in the frame of reference of the shockwave, is equal to M which is equal to u 1 divided by the sounds speed of this medium. And therefore, I would rather translate this momentum equation into an equation which contains the shock Mac number M s over here, the Mac number of the gases behind the shock. And this is what I will do now, let us quickly derive it, and then leave the final expression for the next class. Let us just simplify the momentum equation, let us see where we could end up with.

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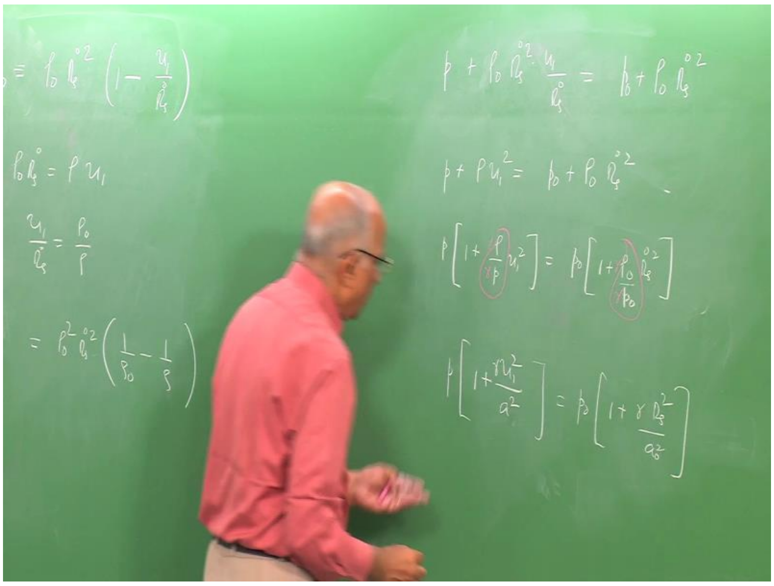

Therefore R s dot square over here, let us take a look at this rho 0 into R s dot is equal to rho u. And therefore, I get p plus rho into u 1 square is equal to p 0 plus rho 0 into R s dot square, how did this come? rho 0 R s dot is equal to rho u 1. And I have u 1 it becomes u 1 square R s dot is left, R s dot and R s dot get cancels this is my equation. And now can I put this in this particular form, I take p outside, I get p plus I have rho by p into u 1 square is equal to I take p 0 outside, I get 1 plus rho 0 into R s dot square over here, and since I have taken p 0 outside it should be rho 0 by p 0.

Now in this particular expression, if I have to multiply on this particular fraction by gamma here, and gamma over here, and also over here gamma and gamma. And now, I look at this particular expression gamma p by rho, here also gamma p 0 by rho 0, what did we find earlier? We found that when we derived an expression for sound speed a 0 square is equal to we said is equal to d p by d rho or del p by del rho, and that was equal to gamma p by rho or this what gave as gamma R T. And therefore, if sound speed is given by gamma p by rho, well gamma p by rho is the sound speed behind the shock, gamma p 0 by rho 0 is the sound speed ahead of the shock.

And therefore, I can write this expression as equal to p into 1 plus I have u 1 square, and I have still gamma over here into a square behind the shock which I call as a 1 now here, because it is gamma p by rho it is behind the shock corresponds to 1, which I could either call as a or a 1 distinct from the other side. Wherein I get p 0 into 1 plus, I have gamma into R s dot square divided by a 0 square. And now we know u 1 by a is the Mac number behind which is M, R s dot and this is R s dot divided by a 0 is equal to M as shock Mac number. And therefore, the momentum equation now becomes.

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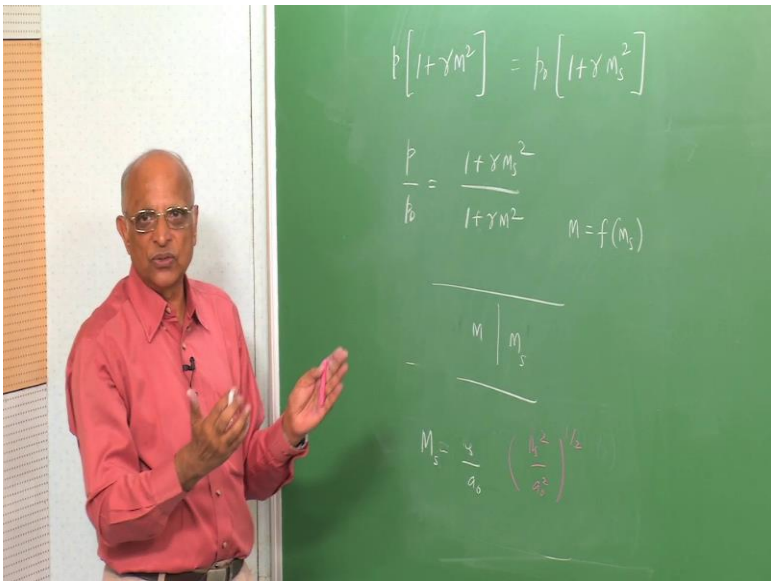

Let us write it, I have p into 1 plus gamma M square, why M square? We said well u 1 by a is M, therefore u 1 square by a 1 square is m 1 square is equal to p 0 into 1 plus gamma into M s square of Mac number. Therefore we now have a relation between M and M s, and our affect is we can write this equation as p by p 0 is equal to 1 plus gamma

M s square divided by 1 plus gamma M square. Therefore what is it we do? We will proceed with this in the next class, and we will get an expression linking the Mac number behind the shock, with the Mac number ahead of the shock. And once we do that will be able to correlated with the properties.

Therefore to summarize in this particular class, we started with the discontinuity moving, we looked at the properties ahead of the shock and behind the shock. And we got an expression namely the Shock Hugoniot which link the properties. Then we said that the shock moves at a particular velocity, which was defined by the Rayleigh line, the intersection of the Rayleigh line with the Shock Hugoniot or the Rankine Hugoniot relations gave you the properties namely pressure and density, dependence with respect to ahead and behind the shock.

And then we wanted to get an explicit relation, and for that we introduce the Mac number M s of the shock, and the Mac number behind the shock. And we are now trying to solve the relation between M and M s, we got the momentum equation in this form. We will again get the value of rho by rho 0 and put it together will be able to get the Mac number behind the shock, as a function of Mac number ahead of the shock. This is what I do in the next class, and with that we will be able to get this shock relations, and use it for predicting the pressure and impulse for a blast wave.

Well, thank you.