Introduction to Explotions and Explosion Safety
Prof. K. Ramamurthi
Department of Mechanical Engineering
Indian Institute of Technology, Madras

Lecture - 25
Detonations: Realizable States in a Reaction Hugoniot
Chapman Jouguet Detonation
Overdriven Detonation
Properties

Good morning. You know we said that a detonation consist of a shock front followed by chemical reactions.

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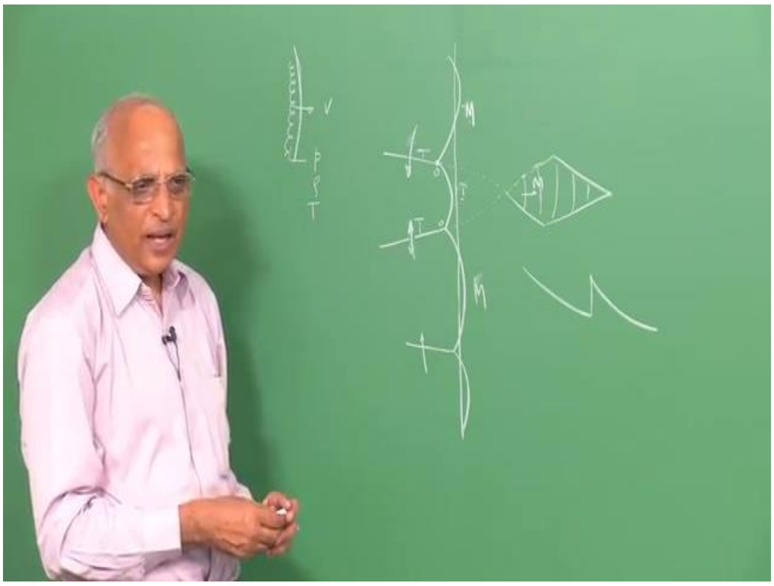

In fact what we said was you have something like a shock which causes chemical reactions to occur in the zone which is heated and you have high pressure behind the shock. And it is this chemical reactions which maintain the strength of the shock, that means it is the chemical reactions are coupled to the shock.

In today's class, we will try to determine the velocity with which this, particular detonation wave comprising the shock and chemical reactions couple together move that is 1 velocity. We would also like to determine what is the pressure behind a detonation, what is the density behind a detonation, and also let us say the temperature behind the detonation.

To be able to formulate this particular problem, we need some more thinking because in the last class, we again said that the front of a detonation is not planer surface like this, but it consists of multiple shocks. It consist of shocks number of them together, and not only does the front consist of multitude of shocks, but also you have transfer shocks behind, like you have shocks like this which are transversely moving something like this over here, this over here, this over here, the shocks are moving.

And the point of intersection of these three shocks here, let us say this is a mark shock, this is an incident shock, this is the transfers shock over here, you have a transfer shock and incident shock over here. Well, the front consists of these shocks, and these shocks are also the front shocks are also in a state of continuous decay. Like for instance, the triple point here o the triple point here o traces something like a, like a trace which is something which we saw was a characteristic shell of a detonation.

In other words, when the transfer wave come and hit together, the shell gets rejuvenated and another shell is born formed. You know this should have been an incident shock, this would have been a mark shock over here. If it is an incident shock, the transfers shock should have travelled like this. When the transfers shock hit together over here, another shell is rejuvenated, a mark stem shock is form, you have a mark shock over here. That means, the front shock consist of a mark stem shock, an incident shock, a mark stem shock that means, it has a series of these particular shock waves, and the interaction point traverses or form something like a shell of a detonation.

And within the shell of a detonation, the shock waves are in a state of continuous decay therefore, it is not that these the detonation wave within this shell is travelling at constant speed. But it is travelling at variable speed always decaying, said that means, it is travelling in sprats. If it is travelling in sprats, it is difficult to imagine how I can talk in terms of a single velocity for a detonation. But when the shell size is small and also if I use the front of a detonation to be given as planer I get velocities which are quite meaningful which are very representative or quite accurate for the combine shock and chemical reactions for a detonation and this is what we will do today.

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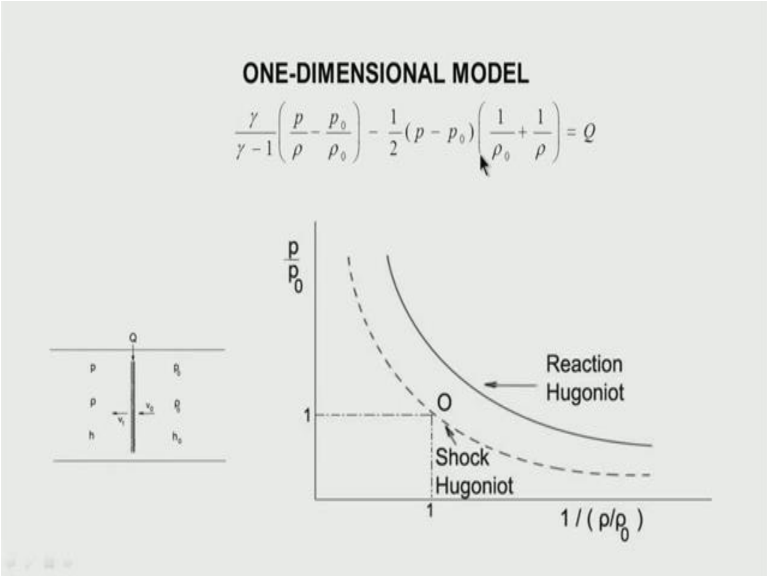

Having said that let us take a look we already saw what was the equivalent 1 dimensional model. And therefore, what I take a look is I come back to the slide over here, I say well I have a detonation here you have a front of a detonation, I have chemical reactions occurring behind it. Well, the front travels in the unburned gas medium with a velocity v 0 therefore, in the frame of reference of the front itself that means, I sit on the wave over here the gases approach me with the velocity v 1, the gases leave me velocity v 1, the gases approach me with a velocity 0.

We wrote the energy equation, and we found this energy equation is very similar to the energy equation for a shock wave. That means, a discontinuity here accept that heat is released over here, and we got the 1 dimensional equation as gamma by gamma minus 1 p by rho, minus p 0 by rho 0, minus half of p, minus p 0. The pressure behind the wave or behind the detonation is p ahead of it is p 0 p minus p 0 into 1 over the density ahead of it plus 1 over rho behind it is equal to the heat release.

Therefore, compared to the case of a shock wave wherein we had the Shock Hugoniot, when heat releases there you get the reaction Hugoniot, and what does the reaction Hugoniot denote? It denotes the condition p and rho that means, it is it gives the state of p and rho behind a detonation. That means for all possible velocities with which maybe the detonation can propagate into the medium with v 0, I get the reaction Hugoniot, it gives all the states p and rho behind it for any initial point which is at this point of which is, let us say p 0 and rho 0. Since, it is p 0 since, the y axis is p by p 0 it is 1 over here, the x axis is rho by rho 0 is 1 point, that is the initial point is here.

For this initial condition, when I have exothermicity cube, all the possible states are given by the reaction Hugoniot over here which is the particular equation. Now, in the last class we also saw, that you know it gives all possible states, but for a given velocity the velocity is given by the rally line and when we talk in terms of the rally line you know I could have a rally line which has this slope a smaller velocity, or it could have a slightly higher velocity, or still a different velocity over here.

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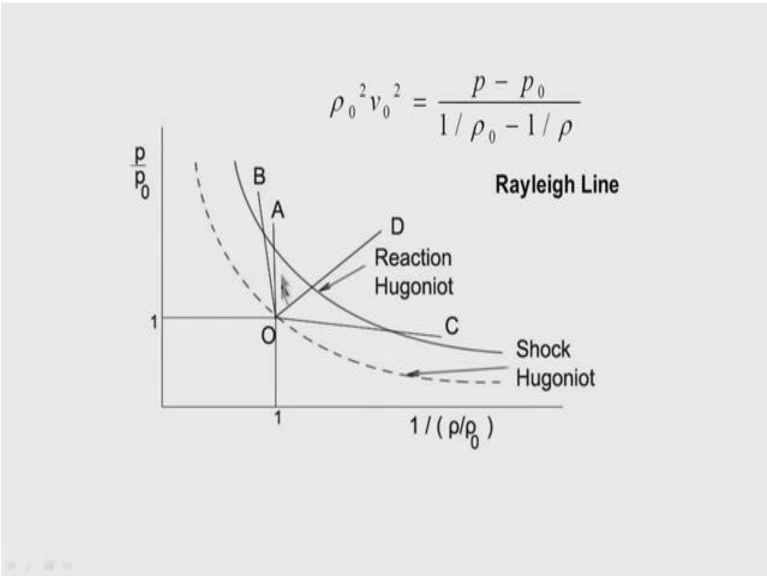

And the rally line is given by rho 0 square into v 0 square is equal to p minus p 0 divided by 1 minus rho 0 minus 1 over rho which is the rally line. And this gives an indication of the velocity and therefore, for a given velocity. If I say well, the velocity is given by this particular slope, the slope of the line O B with respect to the axis this is the value, for this particular velocity well, the condition of pressure is over here p by p 0 is over here, and the density corresponds to 1 over rho that is this corresponds to the density, this corresponds to the pressure. Therefore, for different velocity lines, I can find out the different points that means, for a given velocity I know what is the value of p and row.

Having said that you know, we also took a look at the different states which are possible we found that, when the states are above the 0 point well, the pressure increases when the states are below this particular point the pressure decreases. Therefore, the compression solution should be a, should be above the point 0 and the, that means, for all these points are the compression solution, and when the pressure is lower I said well, the expansions solution should be here.

Therefore, since we are talking of shock, we expected the solutions to be in this particular region on the reaction Hugoniot, and the expansion solution to be in the part between this and onward. Therefore, let us take a look what are the possible states behind the in the reaction Hugoniot.

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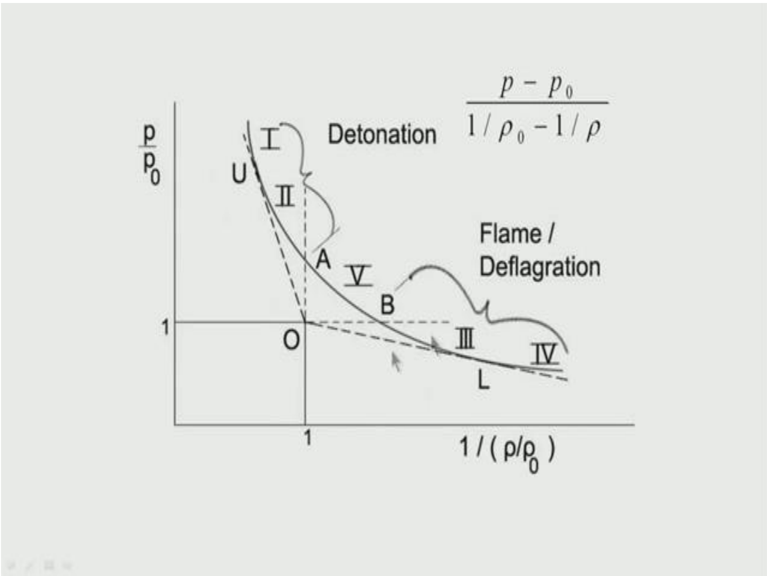

We also found out that if I draw a vertical line from the initial condition, and the horizontal line from the initial condition. The states between this and this that is between A and B are such that, the pressure is increasing, but when I look at density you know 1 over rho by rho 0 therefore, the density is decreasing.

Therefore, between A to B well the pressure increases, but the density decreases therefore, this is contradictory, pressure gives a contradictory message to the density. Therefore, these states are not possible and in fact if I take these states here and substitute it, I get p minus p 0 divided by 1 over rho 0 minus 1 over rho. This gives me a negative value of B 0 square, which is just not possible therefore, we say states between A to B cannot exist.

Therefore, only states A and above on the reactive Hugoniot line can exist, and states below let us say B on the reactive reaction Hugoniot can exist. Therefore, now let us take a look at this, you know when I talk of a state in this particular line in this particular line A and going towards U and beyond, what is happening is the pressure keeps increasing.

The density is to the left of the initial point 0 therefore, the density of ((Refer time: 09:05)) also increases therefore, the branch of the reaction Hugoniot from A as it extends above is the compression solution or the shock solution. Therefore, we expect the detonation to give the final solution to be over here, and we are interested in finding out the velocity of a detonation. While doing this lets also take a look at the expansion effects over here, we find well between B L and beyond you know the curve is over here, you know you find that the pressure decreases.

Well, the density is also decreased over here and therefore, since the pressure is lower at the density is lower it corresponds to the expansion solution. And therefore, the region between this and this should correspond to a flame all a or a deflagration. Now, we want to find out little bit more about the states in that detonation we want to find out, what is the velocity of a detonation? Is it that a detonation will have a unique velocity like a flame laminar flame, or the burning velocity was unique you said it is some something like 0.2 metres per second or 1 metre per second.

What is the velocity of a detonation can I find out from this particular curve over here? To do that I go to the next slide over here well, I tell myself well if I, if I go back to the previous slide, I can draw a tangent from the initial point which is tangential to the reaction Hugoniot at the point U. When I say tangent to the point well this velocity is a minimum because, if I have other velocities you know the velocity is higher because the slope is higher therefore, you say well U has a minimum velocity, and let us examine this point U before I can say anything about which are the states in this particular curve between A U and beyond for which I can have detonations.

Therefore, let us examine the point U first, and when I examine the point U, what is it I get? I get that the slope of the rally line is tangent is tangential to the reaction Hugoniot therefore, let us go back and find out what is the...

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RAYLEIGH LINE: $\dfrac{p - p_0}{1/\rho_0 - 1/\rho}$

SLOPE OF REACTION HUGONIOT:

$$\frac{d}{d(1/\rho)}\left[\frac{\gamma}{\gamma-1}\left(\frac{p}{\rho}-\frac{p_0}{\rho_0}\right)-\frac{1}{2}(p-p_0)\left(\frac{1}{\rho_0}+\frac{1}{\rho}\right)-Q\right]=0$$

$$\left[\frac{\gamma}{\gamma-1}\left(p+\frac{1}{\rho}\frac{dp}{d(1/\rho)}\right)-\frac{1}{2}(p-p_0)-\frac{1}{2}\left(\frac{1}{\rho_0}+\frac{1}{\rho}\right)\frac{dp}{d(1/\rho)}\right]=0$$

slope of the reaction Hugoniot and equated to the rally line to get the point U, and if I do, I am interested in this slope of the reaction Hugoniot well, the reaction Hugoniot had the equation gamma by gamma minus 1 into p by rho minus p 0 by rho 0 minus half of p minus p 0 into 1 over rho plus 1 over rho 0 minus Q and therefore, I was equal to 0. Therefore, I differentiate this or I differentiate it d by d of 1 over rho to get the slope because, my x axis if you saw earlier was given by 1 over 1 over rho 0 is the reference value therefore, if I differentiate it what do I get?

I get gamma by gamma minus 1, I have p by rho therefore, if I differentiate first with respect to 1 over row, I get p and then I get 1 over rho into d p divided by d d p by d 1 over row, and this takes care p 0 rho 0 are the initial states which are constant. Therefore, it becomes 0 here d by d 1 over rho of d these two terms are 0 minus, I get d of d 1 over rho of p minis p 0, I take out I differentiate, I differentiate with respect to 1 over rho I get 1 therefore, it is minus half of p minus p 0 minus, I take this to be the term and differentiate with respect to p, I get d p by d 1 over rho is equal to 0. Therefore, when I differentiate this equation, I am able to get terms of d p by d 1 over rho and I would like to determine d p by d 1 over rho from this equation and when I do that, what is it I get? d p by d 1 over rho, let us go back to the previous one.

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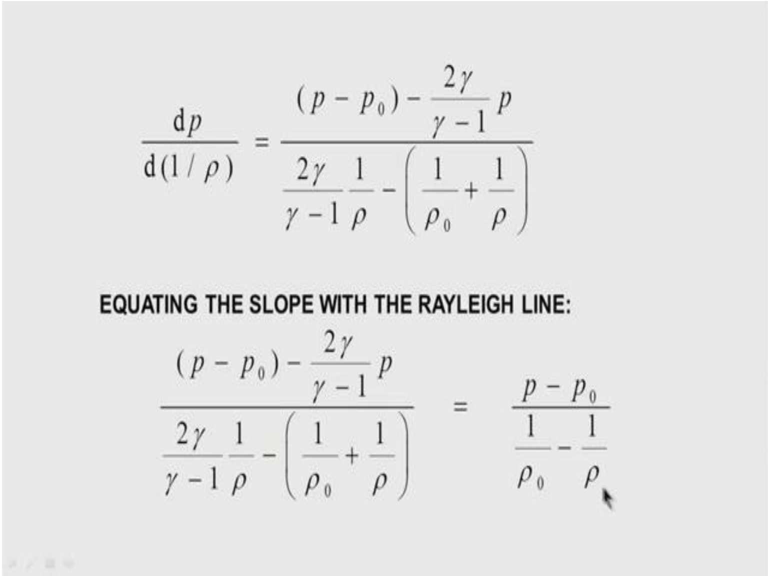

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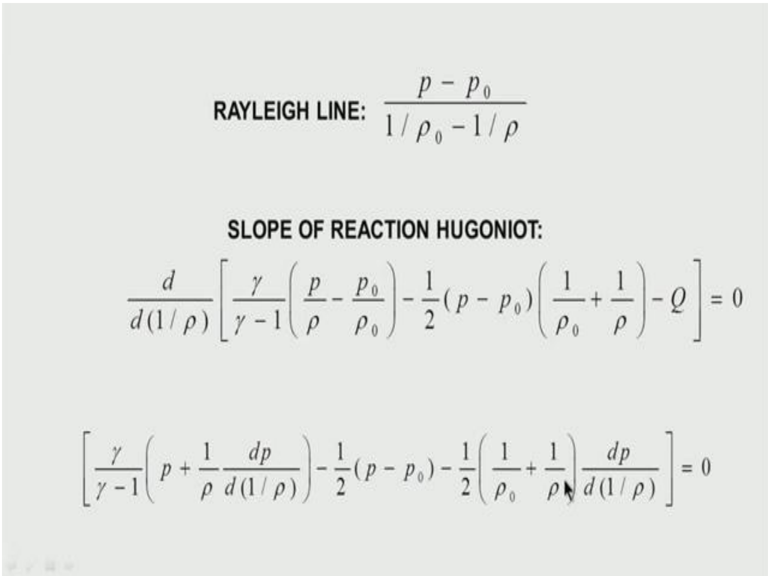

I get well, I have gamma by gamma minus 1 p, I also have half into p minus p 0 which is independent of this factor. And therefore, I have these two which come on the right hand side, I have d p by d 1 over rho into 1 over rho and I have this particular term minus half over here.

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$$\frac{dp}{d(1/\rho)} = \frac{(p - p_0) - \frac{2\gamma}{\gamma - 1} p}{\frac{2\gamma}{\gamma - 1}\frac{1}{\rho} - \left(\frac{1}{\rho_0} + \frac{1}{\rho}\right)}$$

EQUATING THE SLOPE WITH THE RAYLEIGH LINE:

$$\frac{(p - p_0) - \frac{2\gamma}{\gamma - 1} p}{\frac{2\gamma}{\gamma - 1}\frac{1}{\rho} - \left(\frac{1}{\rho_0} + \frac{1}{\rho}\right)} = \frac{p - p_0}{\frac{1}{\rho_0} - \frac{1}{\rho}}$$

When I add these two things together, I get d p by d 1 over rho is equal to p minus p 0 minus 2 gamma gamma minus 1 into p divided by the term which multiply d p by d 1 over rho which gave me 2 gamma gamma minus 1 into 1 over rho minus 1 over rho 0 plus 1 over rho this is the slope of the rally line which was tangent to the point U that means, the slope of the line 0 U is given by the particular expression which we have derived.

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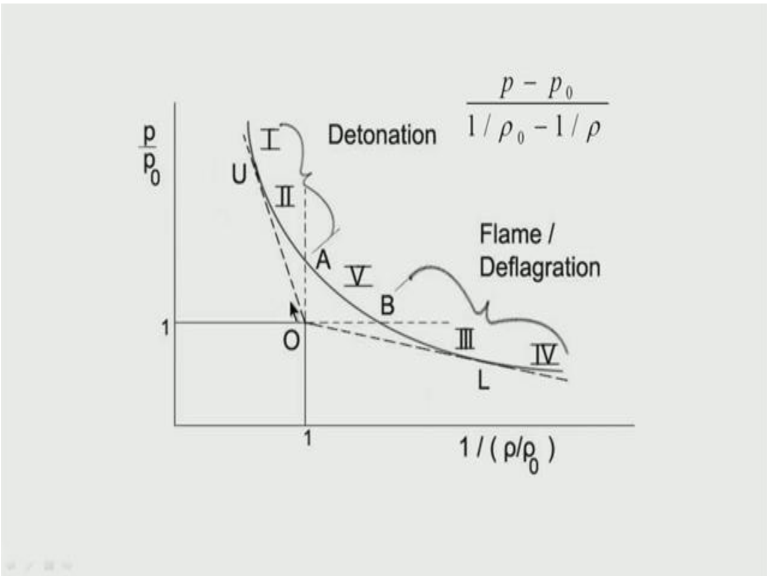

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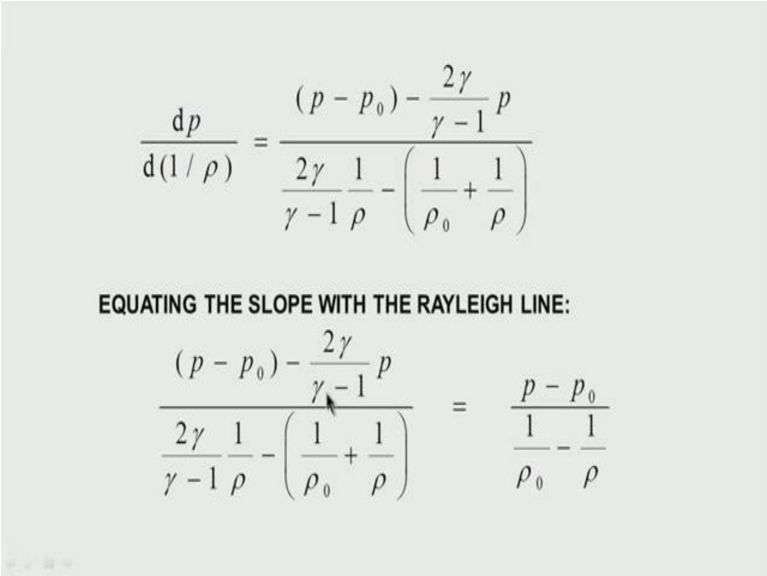

Now, equating this slope to the slope of the rally line mind you this is the slope of the rally line, this is also the slope of the rally line, and slope of the rally line is p minus p 0 divided by 1 over rho 0 minus 1 over row, why is that? If you go back and see well, the slope of this is this equation p minus p 0 divided by this particular value which is equal to this minus this, which is the value.

And therefore, I equate the two expressions together, I equate this equal to p minus p 0 divided by 1 over rho 0 minus 1 over row, and this will should be able to define the properties at the point p. Therefore, I simplified further and, how do I simplify it?

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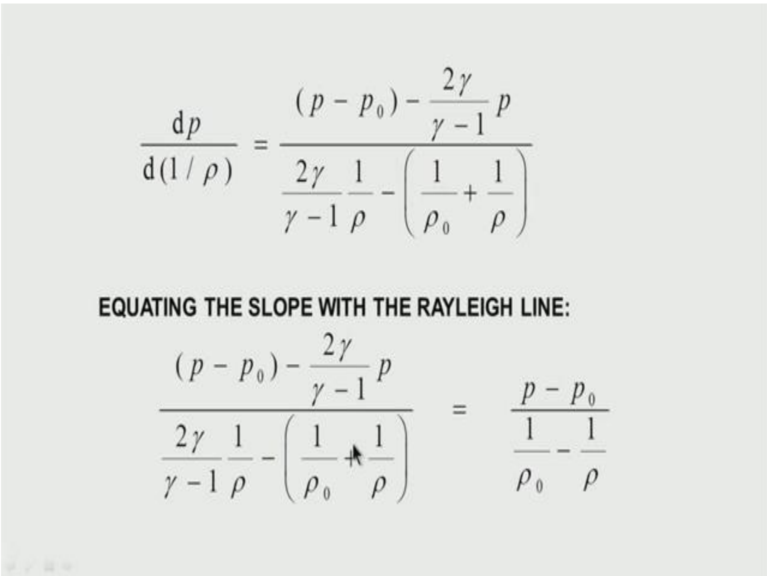

Well, you know I have in this particular equation. If I say, I have p minus p 0 minus 2 gamma gamma minus 1 p, this is multiplied by 1 over rho 0 and I have p minus p 0 multiplied by the denominator over here 2 gamma gamma minus 1 over rho minus 1 over rho 0 plus 1 over rho.

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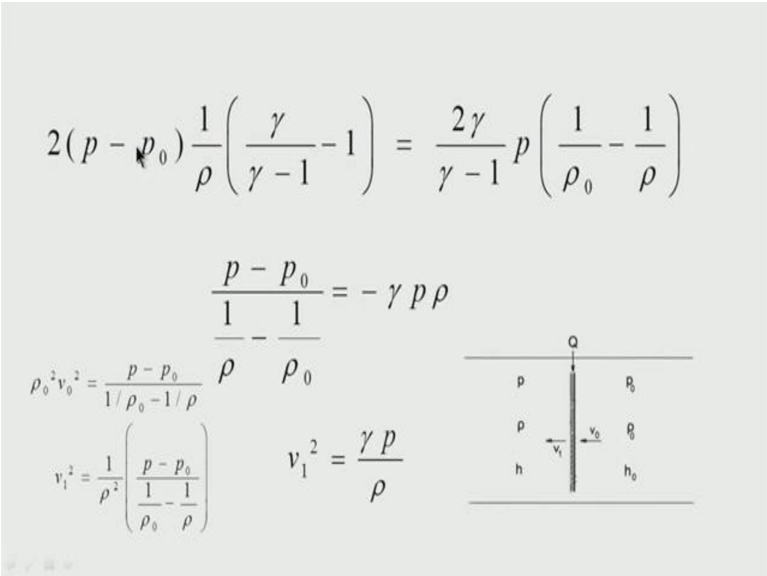

And then equating the two together... Well, I get this particular equation over here. It is just multiplication of the left hand side numerator with denominator on the right hand side, and the numerator on the right hand side multiplied by the denominator. I simplify this when I look at this particular expression, you know what is it I get gamma by gamma minus 1 minus 1 is equal to gamma minus 1 divided by gamma minus gamma minus 1 that means, 1 over gamma minus 1.

And therefore, if I have 1 over gamma minus 1 here and we have 1 over gamma minus 1 it cancels off and therefore, and in the therefore, what is it I get? I get p minus p 0 divided by 1 over rho minus rho 0 there is 1 particular gamma left over here, gamma p into rho from this left hand side therefore, what is it I get? I get the value of p minus p 0 divided by 1 over rho minus 1 over rho 0 is equal to minus gamma p rho which is the solution of the value of pressure and density at the point U.

Now, if I look at this particular expression, you know it tells me, but I also know that p minus p 0 divided by 1 over rho minus 1 over rho 0 was my rally line which was equal to rho 0 into v 0 square. And also I know that from continuity that, rho 0 v 0 because this is my plane of reference rho 0 v 0 is equal to rho into v 1. And therefore, I can write this rho 0 v 0 square rho 0 square into v 0 square as equal to rho square into v 1 square or rather v 1square is equal to 1 over rho 1 square into p minus p 0 divided by 1 over rho 0 minus 1 over rho.

And therefore, this p minus p 0 is given minus given by minus gamma p into row. I substitute the value of minus gamma p rho over here, and what is it I get? I get it is equal to now, I have here 1 over rho 0 minus 1 over row, here I have 1 over rho minus 1 over rho 0. Therefore, minus and minus I substitute plus gamma p rho divided by rho 0 square and therefore, v 1 square comes out to be gamma p by row. Therefore, I get the value of velocity in the frame of reference of the detonation behind the detonation namely v 1, is equal to gamma p by row. What is the implication of this? Let us debate this therefore, let me go to the board and see what is the final expression what I get, and how do I interpret this particular point?

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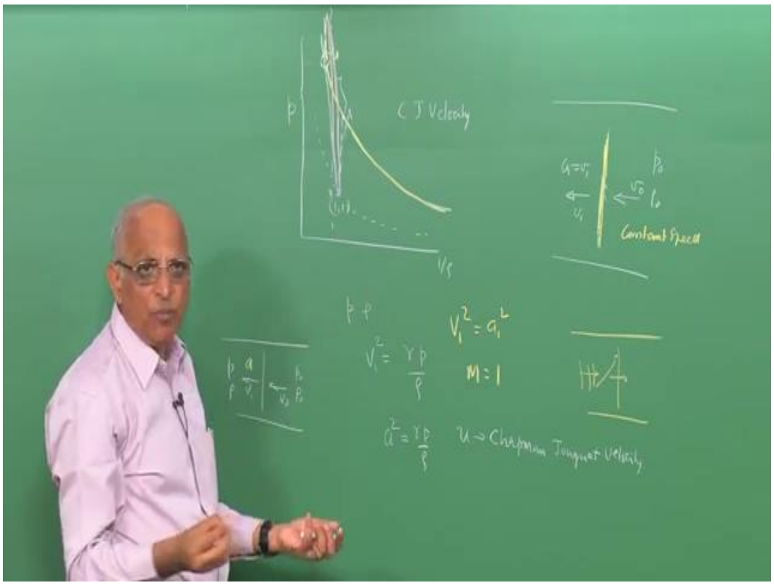

Therefore, let us come back over here what is it we have? We said I have the pressure over here, have 1 over density over here, I had the initial point which is 1 1 which we called as O over here. We had the shock Hugoniot over here, we had the reaction Hugoniot over here. We draw a tangent from here from the initial point which was tangent over here maybe this was your reaction Hugoniot.

Let me put a colour chalk on the reaction Hugoniot that means, the reaction Hugoniot was is shown by the yellow line. I want to determine this particular point U, what is it I get? I get the value of p at U that is along this I get the value of rho at this, rho over here and the relation I find is the velocity v 1 square. That means, I come back to the point namely I get the in the frame of reference of the detonation well the particles are moving behind it with a velocity v 1 ahead of it.

The unburned gases at p 0 rho 0 are moving with a velocity here it is p and rho over here, I get v 1 square is equal to if I if I take a expression I get as equal to gamma p by rho is equal to v 1 square. And what does this represent, you know what does gamma p by rho represent? It denotes we would have seen earlier that when we wrote the expression for the sound speed, we said sound speed is equal to gamma p by rho in any medium. A 0 square is equal to gamma p 0 by rho 0, the sound velocity behind the medium is equal to gamma p by rho.

That means, the sound speed in the medium process by the detonation is A, which is equal to this that means, v 1 square is equal to that means, the velocity of the gases in the frame of reference of the detonation itself v 1 square is equal to A 1 square or rather the mark number behind the of the particles process by the detonation is unity. What is the implication of this? Let us again take a look at this particular expression namely let me, let us plot it over here, you have a detonation over here in the frame of reference of the detonation.

That means, the observer is on the detonation well the velocity coming towards them is v 0 from the condition p 0 and rho 0. The velocity with which the gases are leaving the detonation is v 1 well, the sound speed here is equal to A 1 which is equal to v 1. Let us say or A is equal to v 1 over here we called it as A 1 we will keep A 1 is equal to v 1, what is the implication of v 1 being equal to A 1? You know you know behind the detonation, you know we expect to have a compression of the gases that means, high pressure over here.

Once the detonation moves, there will be some sort of expansion that means, in the gases here there will be expansion and because of expansion the disturbances will try to catch up with the detonation. That means, in the reference in which maybe a detonation wave is travelling, we have high compress gases over here. Therefore, there is expansion of gases therefore, the disturbances here will try to meet the detonation. But since, the velocity of the particles here is moving with a velocity v 1, which is same as this the disturbances can only meet only travel at the sound speed.

And since, the gases are moving away at v 1 well, this disturbances can never really catch up and therefore, a detonation at the, at the point U will always travel at constant speed. Why it will travel at constant speed? The disturbances behind the detonation can never really catch up over here because, the velocity behind the detonation is same as the sound speed. And therefore, disturbances cannot catch up and therefore, it will just go undisturbed or un decayed into the medium as it were.

Therefore, and the speed corresponding to U is spoken off as Chapman Jouguet velocity. It is the constant velocity with which a detonation travels, and it is the point of tangency between the reaction Hugoniot shown by the yellow line, and the rally line which are tangent at the point U. Therefore, we tell well for a detonation I would like to calculate the Chapman Jouguet velocity. And therefore, I would like to calculate the velocity for a detonation which I presume, will travel at constant speed here because, this sound sped behind cannot really catch up and retard the motion of the disturbance.

That is no disturbance can catch up with the detonation front, but then we also said well that detonation. If I draw a vertical line from the initial point that means, I draw a vertical line over here, this is the point. I find well, detonation can be anywhere between let us say, we said this is point A, this is point U, the detonation can be anywhere between this and this, or also anywhere between this. Why do I have to have that detonation only for the tangent? I could have a series of lines maybe I could draw a series of lines here, maybe I could draw a line like this, this could be 1 detonation velocity this could be another detonation velocity. Are all these detonation velocity is possible?

Therefore, we will try to find out what are the regimes in this particular compression solution for which detonations are possible. Let us do that and that I show in the next slide over here.

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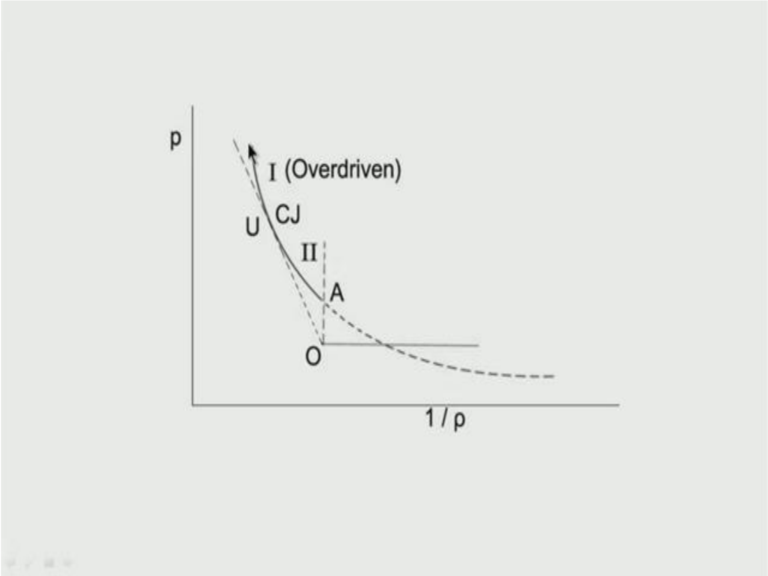

You know if you look at this particular figure, I take the point from A and this is the reaction Hugoniot, U is the Chapman Jouguet point which we say for which the conditions behind the detonation, the velocity is such that, the velocity is same as sonic speed at the locally under those conditions.

And therefore, when I talk of this is it possible if I say I divided into two regions. This particular reaction Hugoniot I divide it into region I and region II. In region I, I consider those regions which are above you, you know in this particular region I, the pressure is higher than at the point U. And therefore, since the pressure is higher the velocity behind the detonation in those regions is going to be lower, p 1 is going to be lower and since, it is going to be lower than therefore, the sound speed well the sound speed can catch up with the front.

And therefore, what do you get is, you get a strong detonation in which the pressure is higher, but the sound speed behind will cause the disturbances to catch up with the front and the detonation will keep on decaying. Therefore, we say well, in I you have something like a like a detonation which has higher pressure or sort of over driven, but then the detonation velocity cannot be constant because disturbances behind the over driven detonation catch up with it and cause it to decay.

And therefore, you have a decaying detonation which has pressure higher than the Chapman Jouguet velocity, which Chapman Jouguet pressure or Chapman Jouguet velocity and since, this particular and since, it is a higher pressure we call it as a over driven detonation, but this cannot be a steady detonation propagating at constant velocity. When we look at the point between U and A well, what is what is happening? You have the velocity which is higher than at U point 1, but then we find that the pressure is lower.

Since, pressure is lower, the velocity behind the detonation is going to be higher than the velocity behind the point U that is the Chapman Jouguet condition and since, the velocity behind the detonation is higher is going to be higher than the sound speed. This is not possible in practice because in a constant area, see there is no way I can accelerate particle velocity from subsonic to supersonic. It is just not possible and therefore, we say well the region to which corresponds to the region between A and U cannot exist in practice.

Therefore, for a detonation only the overdriven region wherein I cannot get a constant velocity detonation, but I get a high velocity detonation velocity being higher than CJ. It continually keeps decaying until it reaches the CJ value, this is the detonation branch. Since, we are talking of this we can as well talk of the expansion solutions also.

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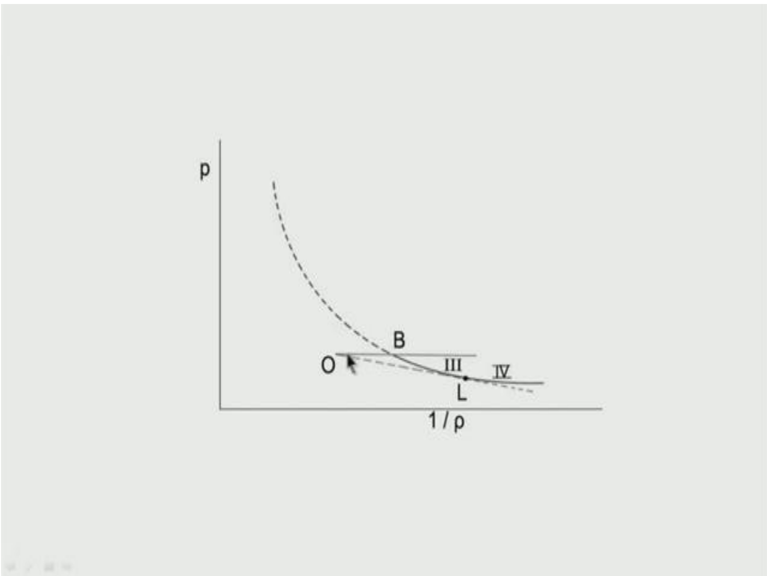

And we look at the expansion solution well, from the initial point I draw a tangent the tangency is at point L, similar to the tangency at point U in the detonation region and over here again the sound speed in the particulates behind the flame or behind the discontinuity is sonic. And what happens in the region III, I can divide the expansion region B, L and so on over here.

And in the region between B and L, what is going to happen well the pressure is higher the velocity behind the discontinuity is going to be lower. And therefore, this is the case of a flame which is continually getting decelerated because of the disturbances catching. In the region L and beyond well the velocity comes out to be greater than sound speed which cannot be met with in practice therefore, this cannot exist well, this is the region of the decaying flame.

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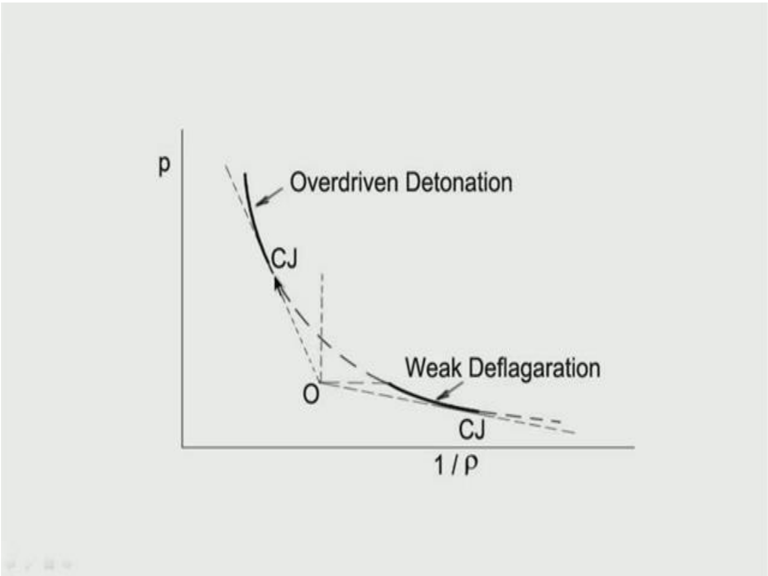

Therefore, putting these things together we tell ourselves on the Hugoniot well, this part is not possible for reasons that the velocity behind the detonation is greater than sound speed. This region is not possible because the velocity exist, a velocity is again higher than the sound speed therefore, these two regions are not possible, only the overdriven and CJ is possible for a detonation. So, also for a flame or a weak deflagration, only the region between this point to the point L which is the CJ point is possible. Point U and above corresponds to overdriven detonation, U corresponds to the Chapman Jouguet detonation.

Similarly, here the point L which is tangent is the Chapman Jouguet deflagration and well you have lower velocities over here which corresponds to the weak deflagration. Well, this is about the possible states on the reaction Hugoniot, but what did we set out to do? We wanted to calculate the velocity, and we find that the velocity of a detonation if over driven is going to be higher, but if it is higher it keeps on decaying and reaches the Chapman Jouguet value.

Therefore, let us go back and calculate the velocity at the Chapman Jouguet point that means, we would like to calculate the Chapman Jouguet velocity of a detonation. Let me put a problem again on the board what we want to do is we want to calculate the velocity, the study state velocity of a detonation.

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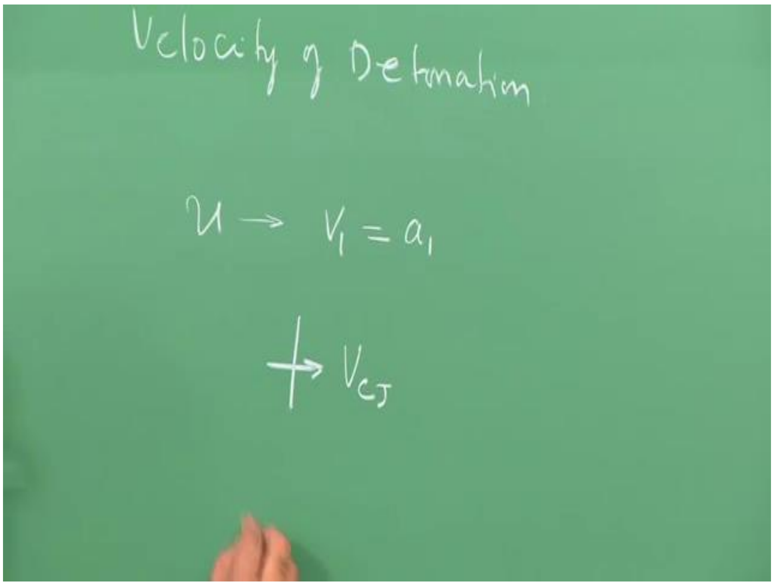

And this we find is only possible at the point U on the reaction Hugoniot for which we just said that the slope of the rally line and the slope of the reaction Hugoniot are the same. And for which we found out that v 1 behind the detonation is equal to the value of the sound speed.

Now, for this the disturbances are enable to catch up together the detonation travels at the value V cj which is possible. When the pressure is higher you have overdriven detonation, but the over driven detonation has a velocity greater than V cj, and in the region when we looked at the particular Hugoniot over here.

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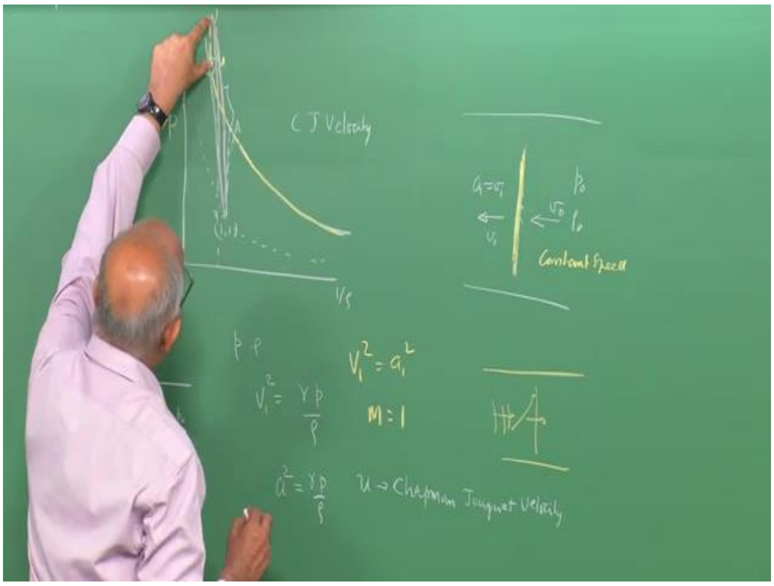

We are talking of points which are greater than A and in this region the velocities are higher the pressures are higher, we say it is over driven and since, it is over driven the velocity of a detonation is greater than the V CJ, but it asymptotically decays till it reaches the CJ value.

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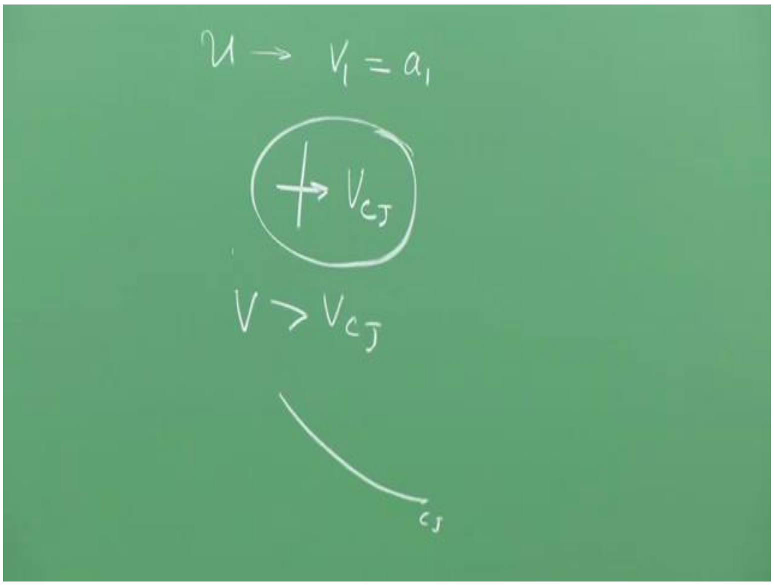

Therefore, an over driven detonation is not steady we would like to calculate the study state Chapman Jouguet velocity for a detonation. Therefore, how do we calculate it? Let us just proceed we have all the equations with us to calculate, let us again go back to the Hugoniot, reaction Hugoniot and what is the reaction Hugoniot?

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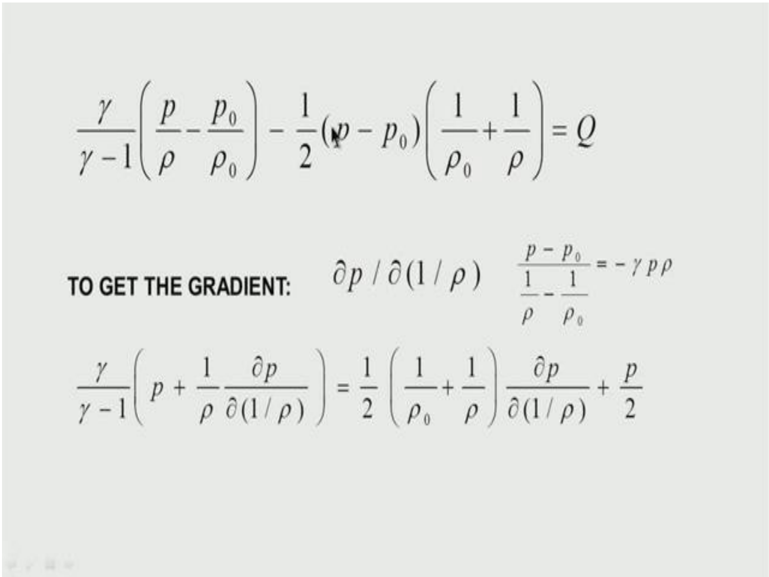

Well, you have gamma by gamma minus 1 into p by rho minus p 0 by rho 0 minus half into p by p 0 into 1 over rho 0 is equal to the heat release. The left hand side corresponds to the shock Hugoniot, the right hand side you have Q therefore, it becomes a reaction Hugoniot. Again I am just interested in getting the value of d p d 1 over rho we have just done it, and what did we tell?

Well, we if we look at it we have already calculated the value, we said p minus p 0 divided by 1 minus rho divided by 1 minus rho 0 is equal to minus gamma p row, we had 1 over rho 0 minus 1 over rho giving gamma p rho. Therefore, I inverted because, I am looking at the change d rho over here is equal to minus gamma p row.

And we want to substitute the value of d p by d 1 over rho when we, when we differentiate this particular reaction Hugoniot and when we differentiate it, what is it we get? Gamma by gamma minus 1 over rho therefore, 1 over rho when differentiated by d d 1 over rho becomes 1 therefore, I have p plus 1 over rho into d p by d 1 over rho. Similarly, on the right hand side, I take it on the right hand side Q is constant heat release well, it becomes d Q by d 1 over rho becomes 0 and I have 1 over rho 0 plus 1 over rho over here.

The value of half is still lingering on d p by d 1 over rho plus, I have d by d 1 over rho is 1 by this p by 2 this becomes the equation from which I get the value of d p by d 1 over rho this is same now, as what we did earlier. Now, I substitute the value of d p by d 1 over rho is equal to p minus p 0 into 1 minus divided by 1 over rho minus 1 over rho 0 and this is equal to minus gamma p row. I substitute the value of this as minus gamma p rho the value of this is minus gamma p row, and what is it I get?

(Refer Slide Tim: 33:14)

$$\frac{\gamma}{\gamma-1}\left(p+\frac{1}{\rho}(-\gamma p\rho)\right)=\frac{1}{2}\left(\frac{1}{\rho_0}+\frac{1}{\rho}\right)(-\gamma p\rho)+\frac{p}{2}$$

$$\frac{\gamma p}{\gamma-1}(1-\gamma)=\frac{1}{2}\left(\frac{1}{\rho_0}+\frac{1}{\rho}\right)(-\gamma p\rho)+\frac{p}{2}$$

$$-\gamma p=-\frac{1}{2}\gamma p-\frac{1}{2}(\gamma p\rho/\rho_0)+\frac{p}{2}$$

$$\frac{1}{2}(\gamma p\rho/\rho_0)=\frac{p}{2}(\gamma+1)$$

Let us substitute well, here I had the expression for d p by d 1 over row, I had the expression for d p by d 1over rho. I am substituting as minus gamma p row, and when I substitute it, I get 1over rho into minus gamma p row, rho and rho gets cancelled that is, p minus gamma p over here. And similarly, I have d p by d 1 over I substitute over here, I substitute over here, let us take a look at the left hand side well, rho and rho get cancelled I get p over here, I get 1 minus gamma is equal to half into 1 minus 1 over rho 0 plus 1 over rho into this value plus p by 2.

And therefore, if I further simplify this expression, I get now, I expand it out I put gamma p rho into 1 over rho and I have 1 rho 0, I find yes if I take a look at 1 over rho 0 over here, that is p by rho divided by rho 0, I have p over here, I have p by 2 over here. Therefore, I get 1 over 2, let us just expand it out, I get minus gamma p into gamma minus 1 comes over here into p by 2 or rather you find that 1 minus gamma you have gamma minus 1, you have minus 1. Therefore, minus gamma p on the left hand side, minus half gamma p because, I take the value of rho over here minus gamma p over here, minus 1 over rho 0 into minus gamma p rho divided by rho 0 over here plus p by 2.

And further, if I look at this, I bring this on this side, I get gamma p on this side because this becomes plus and I take this in the right hand side half of gamma p rho by rho 0, this gives me gamma p by 2 plus p by 2 that is, p by 2 into gamma plus 1 and this becomes my equation. And when I take a look at this particular equation, I find I have p on the left hand side, I have p on the right hand side, I have 2 over here, I have 2 over here. Therefore, I can cancel p and p, I can cancel 2 and 2, and what is it I get? I get gamma rho by rho 0 is equal to gamma plus 1 or rather rho by rho 0 is equal to gamma plus 1 divided by gamma, rather what is it I get?

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$$\frac{\rho}{\rho_0} = \frac{\gamma + 1}{\gamma}$$

For a strong shock $\frac{\rho}{\rho_0} = \frac{\gamma + 1}{\gamma - 1}$

$$\frac{(\rho / \rho_0)_D}{(\rho / \rho_0)_S} = \frac{\gamma - 1}{\gamma} \qquad 0.3$$

I get rho by rho 0 is equal to gamma plus 1 divided by gamma or rather I am able to get the density behind the detonation. That means, for the CJ condition I get the density behind my detonation to density upstream is equal to gamma plus 1 divided by gamma.

Now, you know in this case you know I also presume that the shock is sufficiently strong. We had, we did not consider the effect of weak shocks because, the weak shocks cannot promote chemical reactions and form and give heat release. But if you are considering a strong shock in which chemical reactions are not occurring, we had the expression rho by rho 0 is equal to gamma plus 1 divided by gamma minus 1 and therefore, what is it we see? Let us examine this expression before we proceed any further.

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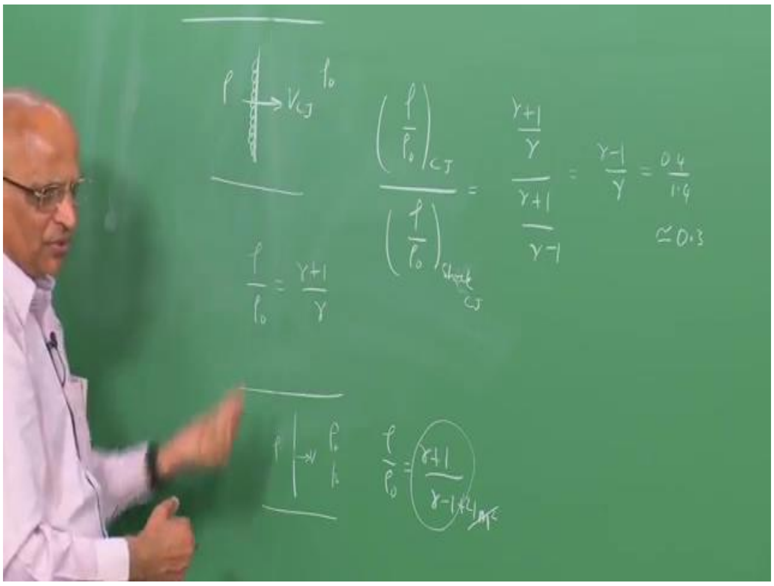

We are finding that whenever you have a detonation which is propagating at the Chapman Jouguet velocity. The initial condition is rho 0, the condition behind the shock is rho behind the detonation in which let us say chemical reactions are also occurring is row. We get the value of rho by rho 0 is equal to gamma plus 1 divided by gamma.

Earlier we have done, we have seen the case if I have a shockwave, let us say I have a shock wave which is which is propagating in an inert medium of properties like the physical properties are similar to the gas mixture in which it is propagating. But there is no reaction happening that means, it is propagating with a velocity v in a medium of rho 0 p 0. And what is the condition behind the shockwave over here?

It is without reaction, we find that rho by rho 0 is equal to gamma plus 1 divided by gamma minus 1 plus 2 over the mark number of the front square, we said it is sufficiently strong because, it is promoting chemical reactions we say V cj is equal to v. We found out that maybe for a shock wave the density is gamma plus 1 divided by gamma minus 1 whereas, when we are talking of a detonation the density ratio is gamma plus 1 divided by gamma minus 1.

Therefore, when I say rho by rho 0 for a Chapman Jouguet detonation divided by rho by rho 0 for a shockwave travelling with the same speed as the CJ that means, for a shockwave travelling with at the same CJ value, you find that the value is equal to you find well, it is equal gamma plus 1 divided by gamma, divided by gamma plus 1, divided by gamma minus 1, or rather because this is negligible because, mark numbers are quite high it gives me gamma minus 1 divided by gamma or rather we find that the density behind the detonation is much smaller.

That means, if I take gamma as 1 point 4 it gives me 0 point 4 divided by 1 point 4 which is of the order of 0 point 3 that means, the density behind a Chapman Jouguet detonation is something like 1 third the value of the density of an of a shock moving at the same speed in a similar medium. Therefore, the density decreases from behind the shock to the zone of chemical reactions. Now, in a similar way I can also calculate the pressure. I can also calculate the other parameters let us just calculate the pressure and then, let us calculate the value of the, of the CJ velocity.

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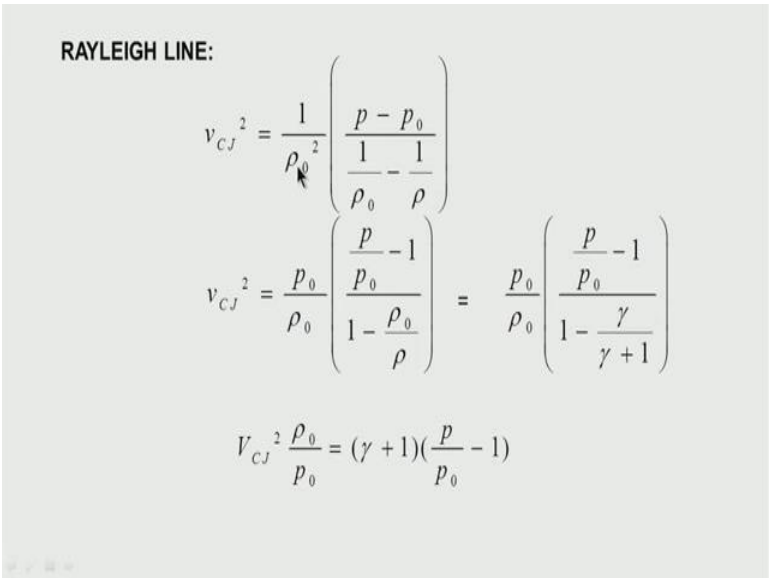

You know from the rally line we also find that rho 0 v v 0 square, that is the Chapman Jouguet velocity square is equal to p minus p 0 into 1 over rho 0 minus 1 over rho this is the rally line. Therefore, I have V cj square is equal to now, I take p 0 outside I take rho 0 outside and I get p 0 by rho 0, I this rho 0 cancels with this rho 0, I get p 0 by rho 0 divided into p by p 0 minus 1 into 1 minus rho 0 by rho which is equal to the value which I have written over here.

The thing what I have written is I take the value of rho by rho 0 behind the detonation. We just now calculated as equal to gamma plus 1 divided by gamma. And therefore, I substitute for rho by rho 0 over here and therefore, now when I look at this equation, I get p by p 0, I get p 0 by rho 0 as the initial condition, I get the velocity of a detonation that is the Chapman Jouguet velocity. Therefore, I solve this particular equation, and what do I get? I move the value of rho 0 on top over here, p 0 in the denominator over here, I get V cj square into rho 0 by p 0 is equal to, I get the value of gamma plus 1 minus gamma divided by gamma plus 1, it gives me 1 over gamma plus 1.

Therefore, gamma plus 1 into p by p 0 minus 1 over here, which means, I should be able to get the value of p by p 0 from here. I can solve this equation, and what is the value I get? I get p by p 0 minus 1 is equal to gamma by gamma minus 1 into the V cj square by a square, how did that come?

We have p 0 by rho 0 if I put gamma p 0 by rho 0 it is equal to a 0 square that means, I multiply numerator and denominator by gamma. And therefore, I get gamma V cj square by a 0 square by V cj square divided by a 0 square is equal to the mark number of the detonation, that is M CJ square therefore, I get gamma M CJ square and therefore, what is it I get?

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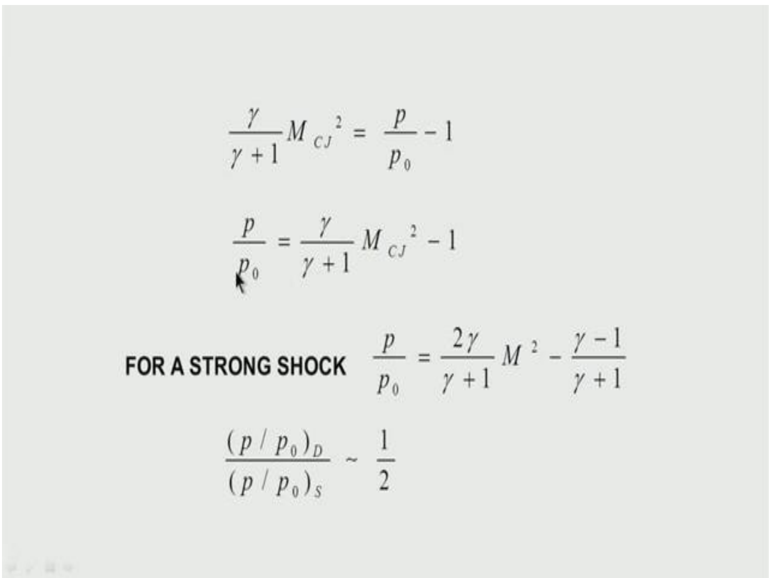

M CJ square is equal to p by p 0 minus 1 divided by gamma by gamma minus 1 from the previous expression or rather p by p 0 is equal to gamma by gamma plus 1 into M CJ square minus 1, this gives me my pressure ratio.

If I were to look at the pressure ratio as the mark number of my CJ detonation, that is the mark number of the of the detonation keeps increasing my pressure ratio also goes up. But if you compare a strong shock travelling at the same velocity at the same mark number as the detonation, you know that we have derived this expression earlier p by p 0 is equal to 2 gamma gamma plus 1 into M square minus 1 for a unreactive shock. Therefore, looking at an unreactive shock travelling at the same mark number as the mark number of a Chapman Jouguet detonation, we find well, p by p 0 is 2 gamma gamma plus 1.

In this case, it is gamma by gamma plus 1 that means, and you know M is a large number therefore, we find that the pressure behind a detonation is half the pressure behind a shock travelling at the same speed. Therefore, there are two conclusions that we draw from this namely, one is the pressure behind a detonation is less than the pressure behind the shock travelling at the same velocity. Second, we also found that the density behind a detonation is something like one-third the density behind a shock travelling at the same mark number as the detonation itself. Having said these two you know our aim was to be able to calculate the Chapman Jouguet velocity rather the mark number of the detonation.

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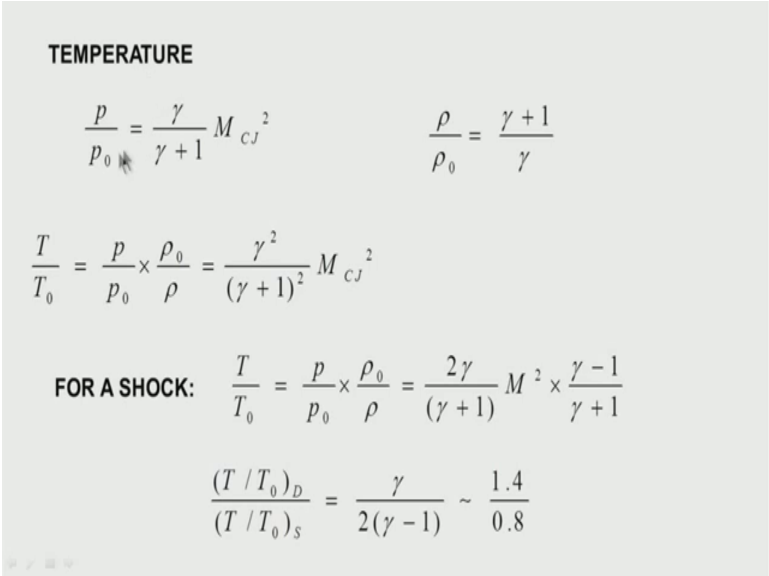

Therefore, but before we do that let us take a look at temperature, we know we have just now derived the value of p by p 0 is equal to gamma by gamma plus 1 into the mark number square minus 1, 1 is a small number. I can neglect it compared to the high velocities high mark numbers over here. We said rho by rho 0 is equal to gamma plus 1 divided by gamma.

Therefore, T by T 0, I can express in by using the gas equation p is equal to rho r T has T by T 0 is equal to p by p 0 into rho 0 by rho this gives me the velocity as gamma square by gamma plus 1 square into M CJ square. And if I look the same thing for a shock that means, I unreactive shock at the same mark number, I get p by p 0 is equal to 2 gamma by gamma plus 1 into M square rho by rho 0 for a unreactive shock is equal to gamma plus 1 divided by gamma minus 1. Because here rho by rho 0 and the value of the temperature rise in a detonation T by T 0 T 0 is ambient, temperature behind the detonation divided by the temperature rise behind an equal and shock wave travelling in a unreactive medium of similar properties.

But without heat release is equal to gamma by 2 gamma minus 1 which is almost twice. In other words, the temperature behind a detonation is much higher than an equal and shock travelling at the same speed, but the pressure is much lower at around half and the density is around 1 by 3. Having done these two things that means, we have determine the density, and the pressure behind the detonation we still want to determine the velocity of the of Chapman Jouguet detonation and let us do that now.

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$$v_{CJ}^2 = \frac{1}{\rho_0^2}\left(\frac{p - p_0}{\frac{1}{\rho_0} - \frac{1}{\rho}}\right)$$

$$\frac{dp}{d(1/\rho)} = \frac{p - p_0}{1/\rho_0 - 1/\rho} \qquad v_{CJ}^2 = -\frac{1}{\rho_0^2}\left(\frac{dp}{d(1/\rho)}\right)$$

$$\frac{dp}{d(1/\rho)} = -\gamma p \rho$$

$$v_{CJ}^2 = -\frac{1}{\rho_0^2}(-\gamma p \rho) = \frac{\gamma p \rho}{\rho_0^2}$$

You know we find that V cj square. That means, again we are looking at the rally equation V cj square is equal to 1 over rho 0 square into p minus p 0 into 1 over rho 0 minus 1 over rho. And this we could also write as equal to p minus p 0 into 1 over rho 0 minus 1 over rho is equal to something like minus d p by d 1 over rho. Because this should have been 1 over rho minus 1 over is equal to d p by this.

And therefore, I can say V cj square is equal to minus 1 over rho 0 square into d p by d 1 over rho because here it there should have negative sign because I am considering the density d rho is equal to d 1 over rho is equal to 1 over rho minus 1 over rho 0. Therefore, I have d p by d 1 over rho is equal to gamma p rho. We have used this earlier and therefore, I can write V cj square is equal to 1 over rho 0 square into minus gamma p rho which is equal to something like minus gamma p rho divided by rho square.

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$$v_{CJ}^{2} = -\frac{1}{\rho_0^{2}}\left(-\gamma p \rho\right) = \frac{\gamma p \rho}{\rho_0^{2}}$$

$$\frac{p}{\rho} = V_{CJ}^{2}\,\frac{\rho_0^{2}}{\gamma \rho^{2}} = \frac{\gamma}{(\gamma+1)^{2}} V_{CJ}^{2}$$

SUBSTITUTE IN REACTION HUGONIOT:

$$\frac{\gamma}{\gamma-1}\left(\frac{p}{\rho}-\frac{p_0}{\rho_0}\right) - \frac{1}{2}(p-p_0)\left(\frac{1}{\rho_0}+\frac{1}{\rho}\right) = Q$$

$p >> p_0$: $$\frac{\gamma}{\gamma-1}\frac{p}{\rho} = \frac{1}{2}p\left(\frac{1}{\rho_0}+\frac{1}{\rho}\right) + Q$$

That means, I have an expression for V cj square in terms of, in terms of p and rho we have determined these quantities and therefore, let us substitute them again and try to find out. That means, we have V cj square is equal to gamma p rho by rho 0 square. We also had the expression of p by rho when the Chapman Jouguet velocity is high, it has, it is anyway supersonic M CJ is high number is equal to gamma by gamma plus 1 square into V cj square, you substitute it in the reaction Hugoniot, and what was the reaction Hugoniot?

Gamma by gamma minus 1 into p by rho minus p 0 by rho 0 minus half into p minus p 0 into 1 over rho 0 plus 1 over is equal to Q. Let us consider this expression because the pressures behind the detonation are rather high as we saw on the, on the reactive Hugoniot which was sketched on the board of few minutes ago. And therefore, since, p is very much greater than p 0, I can neglect p 0 over here, I can neglect p 0 over here. And therefore, the equation becomes gamma p by gamma minus 1 rho on the left hand side, and taking this on the right hand side, I have half p into 1 over rho 0 plus 1 over rho plus Q, same equation for when p is very much greater than p 0. And now, what I do is...

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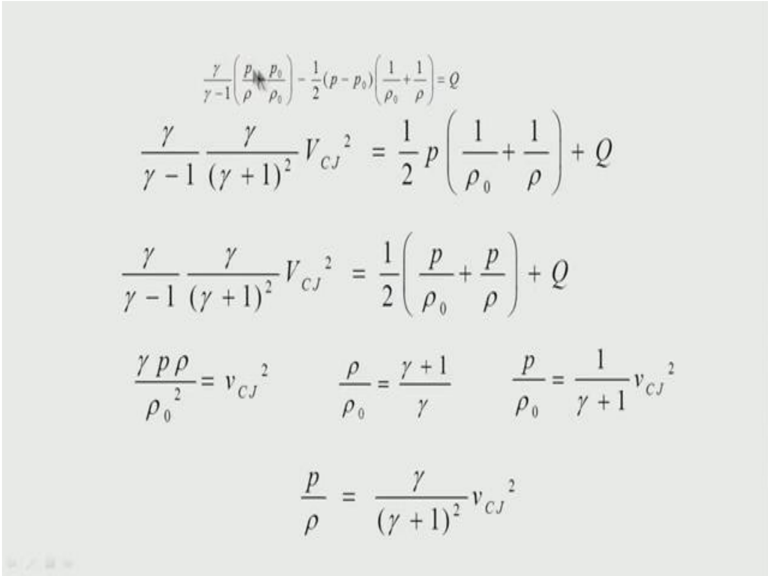

I just have this equation in which this term is not there, this term is not there, I substitute the value of p by rho which I found is equal to gamma by gamma plus 1 square into V cj square this becomes my left hand side. And now, on the right hand side, I have half p into 1 over rho 0 plus 1 over rho plus Q, and this I have p by rho 0 plus p by rho on the right hand side.

Well, I already have the expression for the value of rho by rho 0 is equal to gamma plus 1 by gamma and we just now saw that gamma p by rho is equal to rho 0 square is equal to V cj square. And therefore, if I were to substitute rho by rho 0 here in terms of gamma plus 1, I get the value of p by rho 0 is equal to 1 over gamma plus 1 into V cj square and I already know the value of p by rho. From the previous expression on the last slide we got it as equal to gamma by gamma plus 1 square into V cj square. I substitute the value of p by p 0 as equal to 1 over gamma plus 1 into V cj square I substitute the value of p by rho as equal to gamma divided by gamma plus 1 square into V cj square over here, and what is it I get the final expression?

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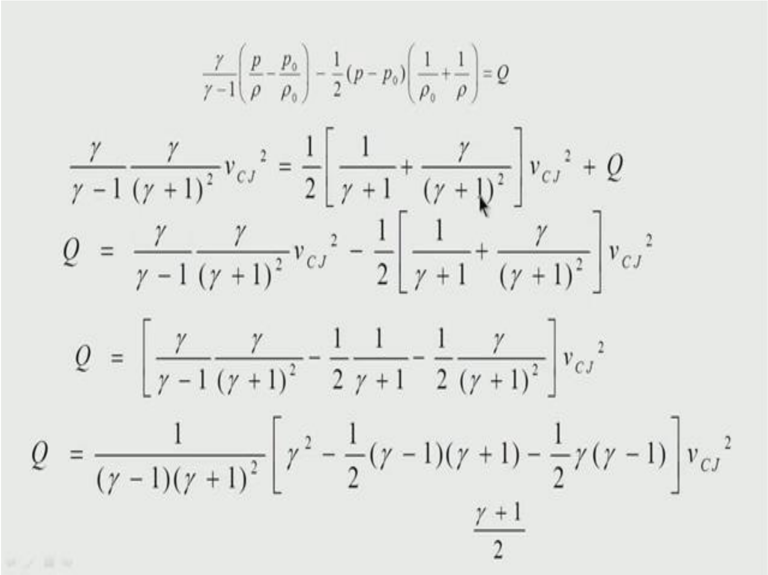

When I substitute this, we have seen the left hand side, I substitute 1 over gamma plus 1 plus gamma by gamma plus 1 square into V cj square is equal to Q, this becomes my Hugoniot. Or rather Q becomes equal to gamma by gamma minus 1 into gamma divided by gamma plus 1 square into V cj square minus half into 1 over gamma plus 1, plus this particular term into V cj square. I take V cj square common over here, I take all the factors of gamma over here, gamma by gamma minus 1 gamma by gamma plus 1 square minus half into 1 over gamma plus 1 minus, you again have minus half into gamma into gamma plus 1 square.

Now, I take gamma minus 1 gamma plus 1 square outside, I get gamma square over here minus, you have gamma plus 1 therefore, I multiply the numerator by gamma minus 1 into gamma plus 1 and here I have gamma plus 1 square. Therefore, I have numerator gamma I multiply numerator by gamma minus 1. I simplify this expression, if I simplify this becomes gamma square minus 1, this becomes gamma square minus gamma the net result I have gamma plus 1 by 2 for this term within the square bracket.

And therefore, Q becomes equal to 1 over gamma plus 1 into gamma plus 1 square into gamma plus 1 divided by 2 into V cj square, gamma plus 1 cancels with this particular gamma I get gamma minus 1 into gamma plus 1 that is gamma square minus 1 into V cj square and of course, I have 2 over here.

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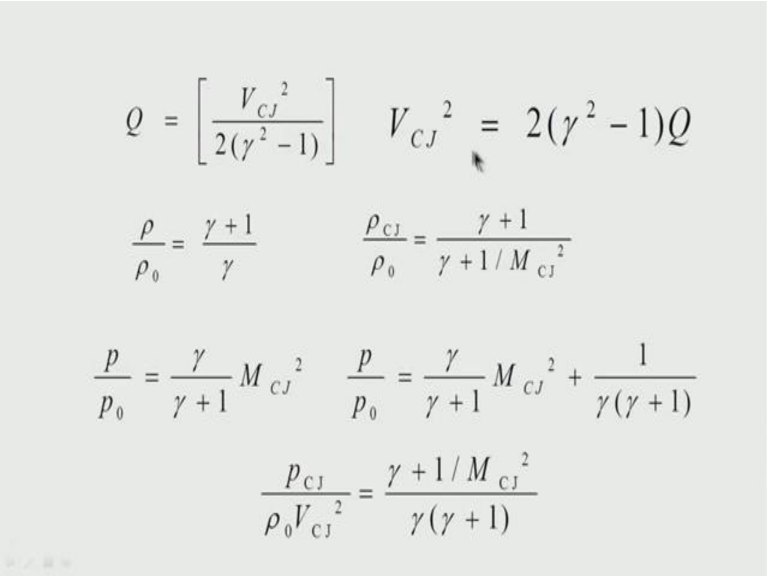

And therefore, the expression I get is Q is equal to V cj square divided by 2 into gamma square minus 1, or rather the Chapman Jouguet velocity square is equal to 2 into gamma square minus 1 into Q. That means, I am able to get the expression for V cj square, and what does it give? Let me write the expression on the board. I get through the manipulations of my reaction Hugoniot at the point u, I get at the point u the value of V CJ square is equal to 2 into gamma square minus 1 into Q.

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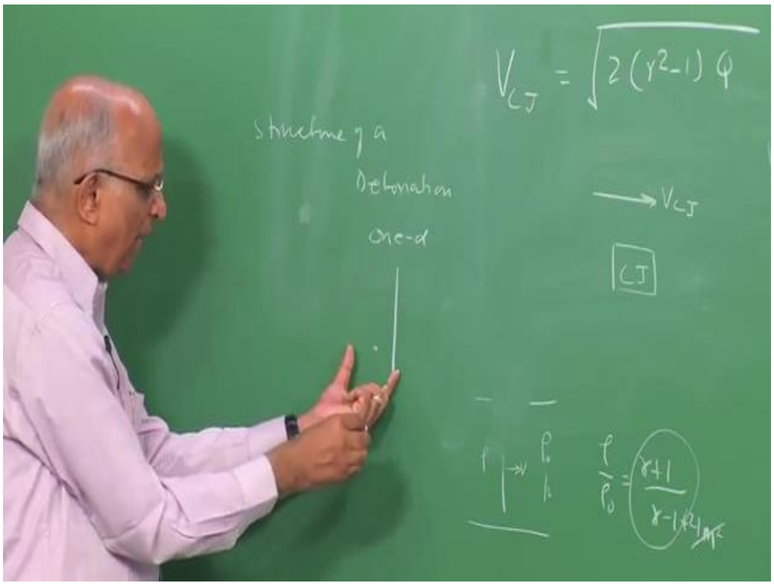

Or rather V cj is equal to under root of this. Therefore, once I know that the heat release from combustion, I also take the value of gamma which for the present I have assumed to be the same ahead of the detonation and behind the detonation.

I can calculate the unique velocity with which a detonation propagates at steady conditions which is the V cj value. And so we are able to get the value of V cj, we are able to get the value of rho cj by rho 0, which we saw was equal to gamma plus 1 divided by gamma. We are also able to get the value of p cj, that is the velocity behind the Chapman Jouguet detonation divided by p 0 also in terms of the values over here which I just ((Refer time: 51:50)).

Therefore, we are able to get the properties of a detonation corresponding to the Chapman Jouguet point, if the detonation is overdriven well, the pressure is higher, if the pressure is higher well, the density is higher. But it cannot propagate as a steady state detonation, it continually decays in strength till it asymptotically reaches the cj value. Therefore, in this class, we have been able to calculate the properties of a Chapman Jouguet detonation which propagates at steady values.

In the next class, we will take one or two examples on how to calculate these values and also we will see what could be the structure of a detonation, when we are considering the 1 dimensional wave which propagating. Because we have done something you know we find that, the density behind the detonation is less than that of a shock, the pressure is less, only thing is that temperature is higher. We would like to plot and see how these things vary in a detonation.

Well, thank you.