Introduction to Explosion and Explosions Safety
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Lecture - 10
Blast Waves: Interaction with Objects, Reflection
Transmission of Blast Waves, Impedance

Good morning, in today's class we will take a look at the interaction of a blast wave, which is formed from an explosion. How it interacts with objects placed in the path of the blast wave?

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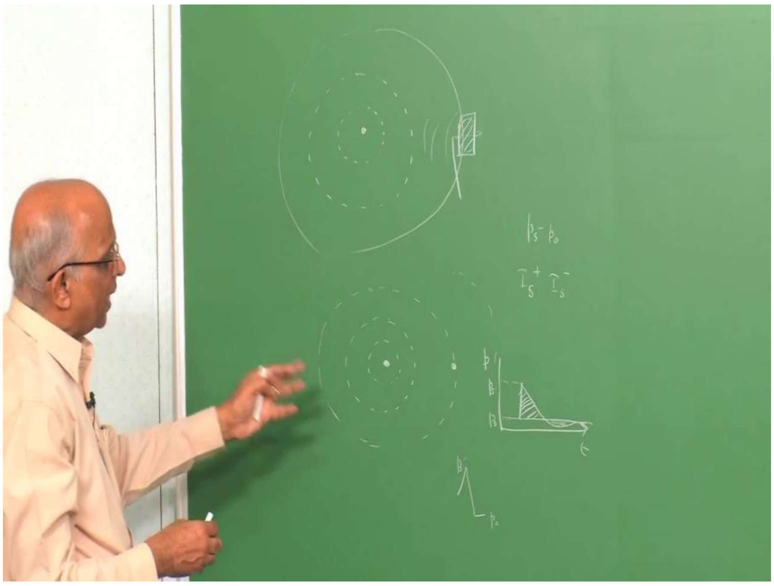

Let us take this example, I form an explosion here, I form a blast wave which travels out with distance. This is the lead wave, which is moving out as time increases it moves forward and now if I place an object here, let us say I put an object here. How does the blast wave interact with the object? That is what I want to do, what happens literally is we will expect the some reflection here, some transmission here. I want to take a look at this problem, but to be able to do this problem, we go back to what we have done so far.

We have looked at the problem of let us say I have an explosion at a particular place over here, I have the blast wave which moves out. That means, it moves out with distance may be at some time here I am interested in putting an object here at this particular point. At this particular point, we have been able to calculate what is the type of the, if this is the ambient pressure if this is the pressure behind the wave.

We have been able to calculate p s minus p naught, we have been able to determine for strong blast assumption in the near field and in the far field. Wherein the blast becomes weakened, we have been able to get the over pressure which is p s minus p naught we also tell ourselves at this particular point may be the blat comes at the time t a. Therefore, I have the time axis over here at the time t a till then the pressure is ambient, then at the time ambient well I have the pressure jumping from p 0 to the value of p s this is my p axis, that is my p axis, this is the time axis.

Then, well the blast wave keeps propagating away and then what happens? The pressure falls, the pressure keeps falling and then it comes back and may be it reaches the p 0 value. Therefore, I have a region of pressure, which is an excess of ambient which contributes to a positive impulse and I have a region of let us say a negative value which is over here, which corresponds to negative impulse. I know how to calculate the at a particular point, when I look at the wave travelling I know how to calculate the positive and negative impulses, I also know how to calculate the over pressure.

Now, when it interacts with an object placed in the path, well I have to consider the reflected wave, I have to consider the pressure behind the reflected way, which is the actual pressure which is acting on the object. Therefore, well the problem becomes a little more complicated, but still doable. Let us try to see how to do that, let me therefore go back to the example of the crater, which we talked of in the last class.

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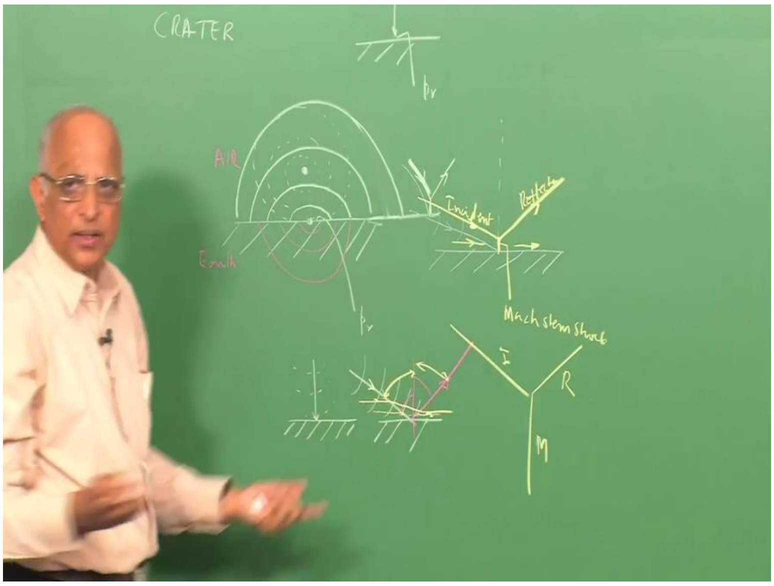

Well, you know we told ourselves, well I have the surface of the earth over here. We said well I could have may be a blast wave originating from the surface may be a buried explosive. I could have, well I could also have an explosion, which originates from a distance above the surface. Then, what happens? The blast wave propagates down, it propagates down, reaches the surface of the earth over here may be at some particular time over here. Now, what is going to happen? Part of this wave which reaches is going to be transmitted into the earth it gets further transmitted over here, but then you know at the surface I have air over here, I have the earth over here.

The earth could be loose and it could be rocky soil it could be anything and depending in that may be what is going to happen is may be some part gets reflected over here. Therefore, I have something like a reflected wave form the initial thing, which is propagating out with time that means for increasing time this is propagating out. Therefore, now I get a slightly different picture, now at this place originally I calculated what is the over pressure, what is the side view? That means at this particular point, now because of this surface and the reflected wave. The pressure here that is the pressure behind the reflected wave will be quite high because it is to be expected because even if I consider something like a sound wave in which there is some velocity disturbance, in which case the velocity changes, therefore the pressure gets doubled.

I want to be able to calculate what the reflected pressure is, but then the problem is not so simple it becomes even more complicated in the sense like if I have a surface over here. Mind you I am not talking of the nature of the surface just taking a surface here, I could have the blast wave which travels in this particular direction the blast comes like this and hits the surface. I could also have blast wave, which comes at an angle maybe the blast wave comes like this and hits the surface over here n. When I have the blast wave coming in this direction, I expect the blast wave to get reflected, but this just does not follow the sign of what we call the like the angle of incidence is equal to angle of reflection, because the condition which needs to be fulfilled is along the surface.

Well, the flow can take place only along the surface at the at the interface and therefore what will happen is maybe I have a flow path like this, maybe it bends like this and the angle over here and the angle over here are not the same. This is what we call is a reflection of a blast wave from a particular surface, but then there is another problem if I have let us again sketch it out, let us say I have the surface over here. Let us say this is normal, well supposing the blast wave comes at grazing at an angle which is less than let us say a large value say in practice is greater than around 40 degrees.

Suppose, the blast wave comes and hits the surface, well what is going to happen? I need to be able to get the boundary at the surface namely I should have may be a velocity which goes along the surface because that is the condition of the surface over here. Well, after the wave the thing must be in the same direction, well it is not possible to have this condition when it is at grazing incidence. And what happens is the incident wave gets raised off.

That means it gets pulled away and I have the incidence angle like this and I have the reflected one going like this. I have another shock which is formed over here, which means when I have grazing incidence it is not possible for the condition at the surface to be met. Therefore, I have this getting lofted up and I have another shock here, which we call as a mach stem shock. Therefore, depending on how the blast wave comes and hits the surface.

Well, I could have reflected one which obeys the condition at the surface namely the velocity which is induced by the wave is has to be parallel to the surface at the surface or if I have razing incidence. Well, I have a secondary shock, which is sort of normal to this particular surface I have the incident one over here, I have the reflected one over here. Therefore, I have a three shock pattern something like a mach stem shock over here, I have the reflected shock over here, I have the incident shock over here. This is what the mach stem the mach stem shock is quite strong and therefore I could have different configurations.

If I were to put it in this particular figure, let me slightly change it, because here it is like this when it is at an angle the value will be something like this if it becomes at a grazing incidence over here at the surface. Well, what could happen is it could have, it could have the incident shock like this I could have the reflected shock like this I could have a mach stem shock like this. We have to consider these reflections also, maybe we will summarize it tomorrow or in the next class.

What we do today is, let us take a look at what are the pressures which are formed behind the reflected wave we can immediately tell ourselves when we have these things happening in a direction which is not normal. Well, the pressures will be less, because you know only a component acts like this we should expect when I have a surface. Supposing the wave comes normally, in this case the reflected pressure will be higher, therefore let us do the problem for the pressure behind the reflected wave and then go about conjuctioning for the other cases and this is what I will get started with. Therefore, the problem which we will consider is well I have a surface, let us define the surface.

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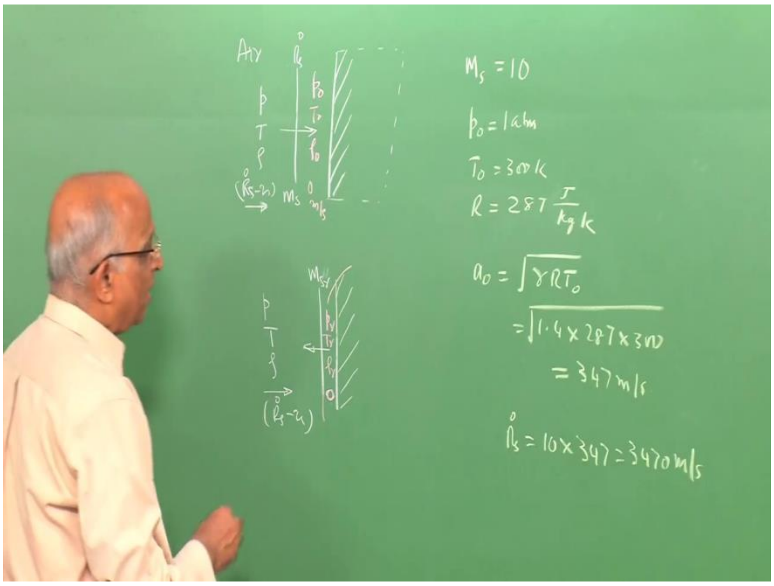

Let us say I consider air over here this is some object which is placed I am looking at the surface of this particular object I am not going to take a look today at the transmitted wave into the surface. All what I say is well a blast wave approaches the surface, let us assume that it is plainer. Let us also assume for the present that it is travelling at a mach number M s, therefore what is it? I have ahead of the wave, I have the pressure let us say the ambient pressure p 0, I have the temperature t 0, I have the density rho 0 and the velocity ahead is coerced which is 0 meters per second.

This particular wave which travels at Mach number process, it reaches the surface and then what happens well at the surface, let us look at what is going to happen when it gets reflected. Then, the wave gets reflected back let us assume I said it is travelling at Mach M s, well it gets reflected back and what does it do? It gets reflected, it travels in the opposite direction, let us say it is the reflected wave. Let us say M s r, it is M s, which is incident M s r is reflected and the medium in which it is getting reflected is not the coercion medium, but it is the medium let us say p here t here rho here.

The velocity behind a moving shock if this is M s and corresponding value is r s dot in the frame of reference of the shock the velocity is u. Therefore, it is moving with the velocity r s minus u over here in this particular direction, therefore this reflected shock now moves in a medium which is at pressure p which is at temperature t which is at density rho compared to p 0, t 0, rho 0 over here. The medium is also moving in this direction that is it is the medium is moving its coerced no more, it is moving at a distance r s dot minus u over here this is r s dot minus u.

Therefore, what is the condition behind over here? Well, this is this shock is going to process this and therefore the pressure is going to be the reflected pressure. The temperature is going to be reflected the density is going to be reflected and hat should be the velocity behind this traveling wave. Well, at the wall it is the wall is stationary, it is rigid it does not move, therefore the velocity at the wall should be 0 and therefore we say the velocity here is 0.

Therefore, with these boundary conditions i have to determine what must be the type of pressure what bi get and let us do this problem you know the as long as we are clear about the problem may be we can try to do it. As we do it, maybe we will make some assumptions we will go back and see what, how it is stated in the case of hitting the objects with different angles a little later, but for now let us takes a look at this at this particular phase. Let us do a small numerical problem such that we are able to understand the physics of the problem, let us say that the shock or the blast wave. We model it as a constant pressure wave constant shock wave, let us say it moves with a M s is equal to 10.

Let the ambient pressure p 0 be equal to 1 atmosphere that is 10 to the power 5 Pascal, let the temperature be equal to 300 Kelvin once I know the temperature, I know it is air the value of specific gas constant is equal to 287 joules per kilo gram Kelvin. Therefore, I can say that the speed of sound in this coercion medium a 0 is equal to under root gamma r t, which is equal to at coercion medium t 0, which is equal to 1.4 into r is 287 into 300 Kelvin, which comes out to be something like 347 meters per second. Therefore, I consider this incident shock at Mach number M s, which is equal to 10 moving into this coercion medium, which has these particular properties.

Therefore, what are the velocities of this particular shock? Therefore, r s dot Mach number is ten into 347 which is equal to 3470 meters per second. Now, we have we have the shock moving at this velocity into this medium, I am interested in calculating pressure temperature density and the velocity with which the particles follow the shock and therefore now I calculate those values.

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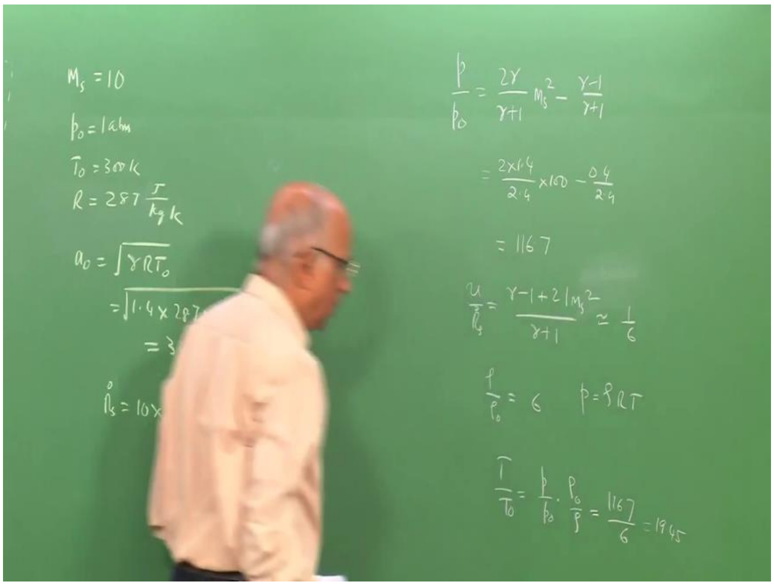

What the value we know you have derived? The value p by p 0 is equal to 2 gamma divided by gamma plus 1 into M s square minus gamma minus 1 divided by gamma plus 1. Now, I substitute the values, well we say mach number is 10, the shock speed is 3470 meters per second and therefore, if I substitute it, I get 2 into 1.4 divided by 2.4 into M s square is 100 minus 0.4 divided by 2.4 and this will work out to be equal to116.7. That means when the ambient pressure is equal to 1 atmosphere, the pressure behind the incident shock is equal to 116.7 atmospheres.

Let us now calculate in the frame of reference of the shock, that means shock stationary I have u behind it that means the shock is stationary medium is moving at a speed r. what is the value of u? Which is what we put over here as equal to r dot minus u, I am interested in u over here.

We know u over r s dot is equal to we again look at what we derived earlier gamma minus 1 plus 2 over M s square divided by gamma plus 1, M s is equal to 10 and compared to 0.4, this is 100 M s square is 100 this becomes 0.4 is almost negligible. I can say is equal to 1 over 6 and the value of density by rho naught that is density ratio is equal to gamma plus 1 this is just inverse from the mass balance equation this is equal to 6. Now, I know the value of p by p naught, I know the value of rho by rho naught from the equation of state for air which is equal to p is equal to rho p r p.

I get the value of p by p 0 as equal to I have p by p 0 into the value of rho 0 by rho and if I take this value 116.7 multiplied by rho 0 by rho 6, which is equal to I get the value 116.7 divided by 6 and which equals 19.45. Therefore, let us let us put some of these values down over here, what is it I get?

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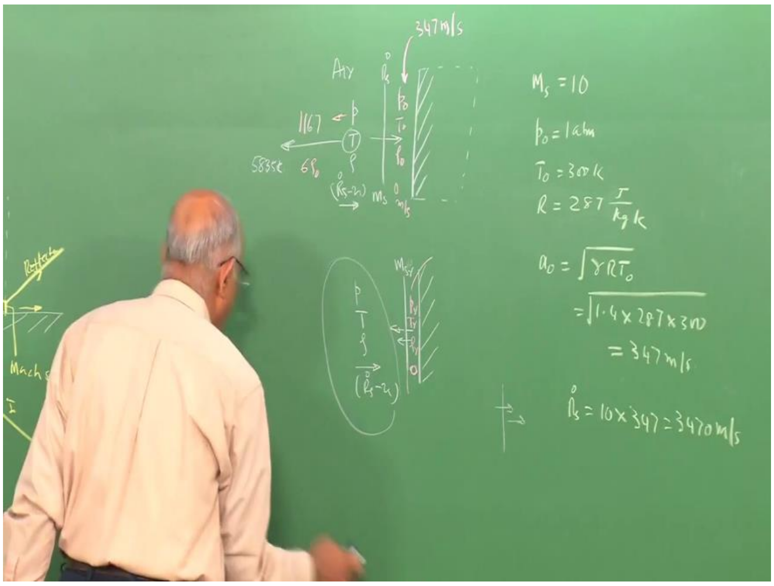

I get the value of the sounds speed in the undisturbed medium before the incident shock comes over here is equal to 347 meters per second and the value of pressure temperature and density behind the pressure is something like a compared to 1 atmosphere pressure. It is something like 116.7, the density is 6 times the value of rho 0 over here.

Well, the temperature now I can put the value t by t naught is 19.45, I am taking the temperature as 300 Kelvin, the value of t is equal to 300 Kelvin into 19.45 is the temperature which comes out to be 5835 Kelvin. That means the temperature is quite high that means the temperature over here is high at a value around 5835 Kelvin. Now, this is for the incidence case, now what happens? The shock now travels at this particular speed which is equal to 3470 meters per second hits the surface and gets reflected and into what medium does it move? It moves into this medium in which the pressure is high the temperature is high at around 5835 the density is 6 times the original density and also the gases are moving towards this particular shock as it is moving head.

What we derive so far is only for a coercion gas we have not considered the case wherein I have the medium in which gas is moving and the shock is moving we did this, but we can always transform the coordinates into shock stationary and do this. Therefore, let me consider the case wherein this reflected shock is moving into this particular medium. Let me sketch it out again such that we are able to get the reflected values reflected pressure values. What do I mean by reflected pressure? The pressure behind the reflected shock.

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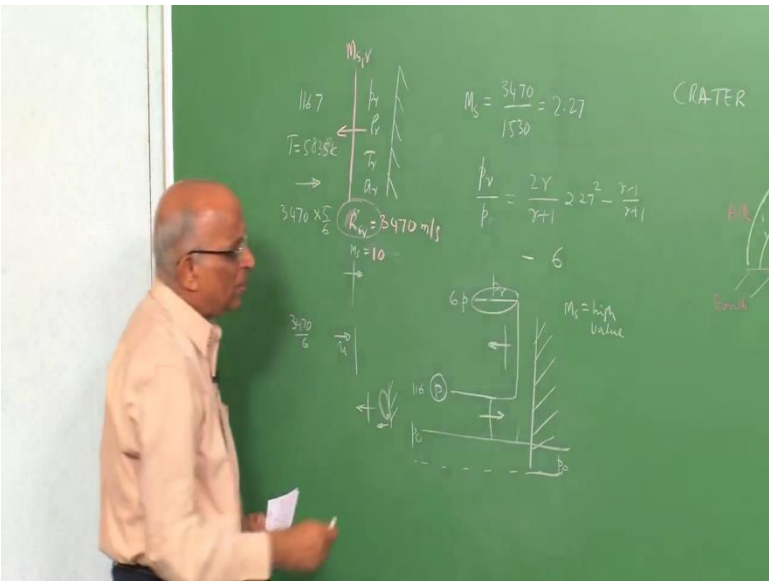

Therefore, again we draw the surface over here, well I have the reflected shock over here this is travelling with, let us say M s corresponding to reflected what is the medium into which this shock is travelling. Well, we said that the pressure is high something like 116.7 we said that the temperature is high something like 116.7. We said that the temperature is high around 5800 or something 5835 Kelvin and you know we also yes told ourselves, it is moving with some velocity. This velocity with which it is moving is equal to the shock velocity the original value of the shock velocity with which it was moving was equal to 3470 minus the particle velocity.

What was the particular velocity? The particular velocity was u by r s is equal to 1 by 6, therefore the value of r s dot minus u is equal to 5 by 6 into is the velocity with which it is moving. Now, you know in this frame of reference is somewhat difficult to do the problem, let us make some simple assumptions. Let us assume yes I had the incident shock that is M s, which was originally travelling at a speed of 3470 meters per second, what does it do in the frame of reference of the shock? It pulled the particles with a velocity u and this velocity u is equal to 3470 divided by 6, then what happens? It gets reflected from the surface and now when it gets reflected from the surface I must get 0 velocity here.

Let us assume that it has to pull the gas with the same velocity such that I have, I get 0 velocity condition over here and since I am considering some of these shocks which are quite strong. Let me assume that therefore the value of r s dot, which is reflected will be same as r s dot which is incident the value of r s dot in the reflected shock wave, which is travelling in the opposite direction is same as the incident wave, which is equal to I get 3470. In this case, M s was equal to 10 M s was r s dot is equal to 3470 meters per second. Now, therefore this shock is travelling in this medium whose temperature is is equal to 5835, yes 5835 Kelvin.

Therefore, I find well the shock is travelling at a velocity of 3470 meters per second into a medium whose temperature is high at this values and what is the sound speed of this medium. Well, a is equal to under root gamma r t which is equal to gamma is 1.4, R is equal to 287 into the temperature which is equal to 5835 and if I calculate this number the value of sound speed comes out to be equal to 1530 meters per second.

If this sound sped into the medium into which this shock is traveling is 5835, well I can also consider this, therefore the mach number of this shock is equal to the velocity that is 3470 divided by 1530, which is equal to 2.27. Well, if the shock is traveling at two unit two seven, what is the value of the reflected pressure, well let us put down here the condition is the reflected pressure infinity is reflected temperature is reflected the sound speed in the reflected medium. What the value is of reflected to the value, which is upstream? This is p, which is equal to 116.7 is equal to two gamma divided by gamma plus 1 into M s squared that is 2.27 square minus gamma minus 1 dived by gamma plus 1.

If I substitute the value of gamma is equal to 1.4 in this particular expression, I get the value of p r by p is equal to something like 6. In other words, when I have the shock which is incident on the surface at a mach number is equal to 10 and I want to calculate what the pressure in the medium behind the reflected shock. I find that the pressure jumps up again by a factor six in other words let us let us put this down diagrammatically. That means I have the surface over here may be let us say the in value of the initial pressure is p 0 over here, this is the value of p 0, I have when the incident shock is traveling.

The pressure jump is equal to p this is equal to something like 116 times the value of p 0, this is the value of p which is 116 times p 0 and then what happens this is traveling in this direction when I have the reflection coming over here. This p jumps to a value around according to the calculation what we have done it is something like 6 p over here. Therefore, I find that the magnitude of the reflected pressure is something abnormally high something like 6 times or, so the value of the incident pressure and therefore, I get phenomenally high pressures when the mach number was equal to a high value.

Let us do the same set of calculations for the case when M s is small, let us say when M s is equal to 1.5 since we have done this particular calculation. I can erase this and just repeat what are what are the numbers we get let us see what is the magnitude of the reflected pressure.

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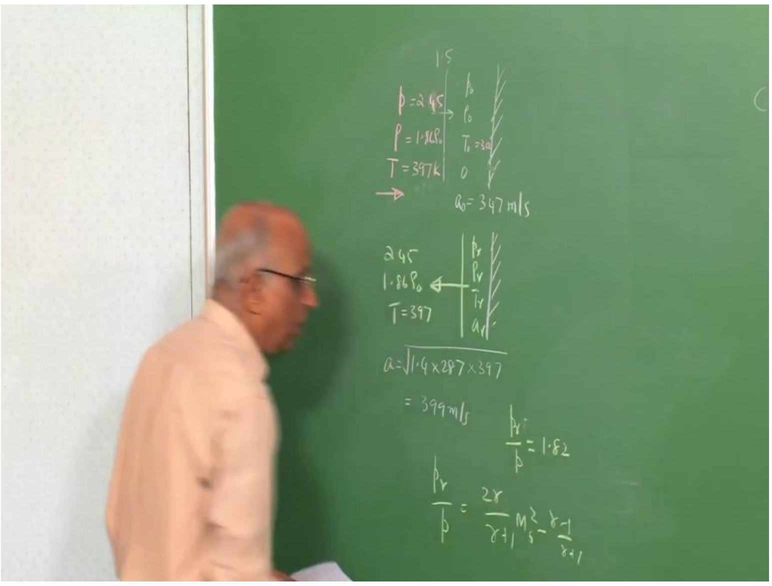

Therefore, now again I consider the surface over here let us quickly go through it I have the incident shock travelling at a mach number of 1.5. The conditions here are p 0 rho 0 t 0 it is coercion, well the conditions here should be let us use a colored chalk over here p rho p and it has a given velocity over here i use p by p 0 with 1.5 I get the jump condition here as equal to p by p 0 is 2.45. The value if it is 1 atmosphere, it is 2.45 atmospheres, the value of rho is equal to 1.86 times the value of rho 0, the value of temperature if this is 300 taking the this divided by this is equal to t by t naught. The temperature works out to be equal to 1.32 times the initial temperature and that multiplied by 300 is equal to 397 Kelvin.

Now, what is going to happen at this surface, well the shock units it and now I have the reflected shock which moved into this medium. Well, the properties of this medium are same namely it is now 2.45 atmospheres the density is equal to 1.86 times the initial density the temperature here is equal to 397 and if it 397, the sound speed in this medium compared to sound speed here which we said is gamma r t. We calculated as 347 meters per second in this case the value of a is equal to gamma p1.4 into specific gas constant 287 joules per kilo gram Kelvin into 397 under root and this comes out to be 399 meters per second into this reflected shock is propagating.

Well, the conditions here will be the reflected shock pressure the density behind the reflected shock temperature behind the reflected shock sound speed behind the reflected shock. Now, if I want to calculate the value of p r and I sue the same type of simplification which we did earlier and what do we get we get p r by the value of p 1 over here that is p over here this was equal to p 0. This was p is equal to again I write 2 gamma plus one into M s square minus gamma minus 1 divided by gamma plus 1. Now, I get the value of reflected pressure as equal to if I use this equations divided by p I get it get the value as something like 1.82. I get the pressure rise be behind the reflected shock to be 1.82 times that what has been processed already.

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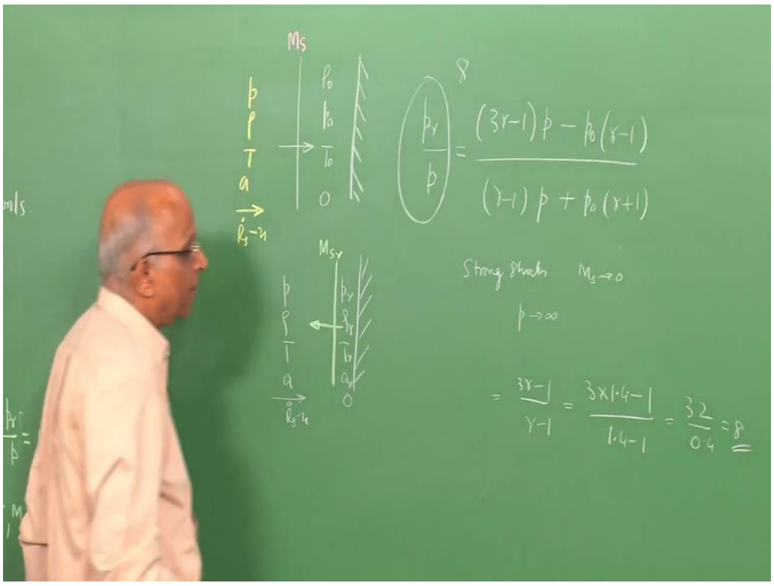

If I have a strong shock it is something like 6 times or so but in the actual problem if I now consider the actual problem what is it I get well I have the surface over here. I have the shock moving in this medium let us say the medium is rho 0 the pressure is p 0 temperature is t 0, its coercion the conditions behind this particular shock which let us say is moving at a value M s is given by let us say p rho t. The sound speed is a in this case in this case, it was a 0 and it has a particular velocity over here what is the velocity r s dot minus u and therefore, when at the surface.

The shock gets reflects and we have the reflected shock which is over here the reflected shock is moving in this particular direction. The mach number of the reflected shock is let us say M s is reflected the condition into which it is moving is these conditions namely let us write it down is equal to p rho t which sound velocity in the medium of this reflected shock is a over here, it is also moving at a speed r s dot minus u. What should be the condition here? At the wall the velocity must be 0, the condition here is p r the condition is rho r the condition is t r over here the reflected is a 0, it has been done by Kinney and Graham in his book on explosions in air the expression.

What we get is, we get the expression for p r that is the reflected value divided by the pressure into which this reflected shock is moving as equal to 3 gamma minus 1 into the pressure. This reflected shock is moving minus the ambient pressure into which the incident shock is moving into gamma minus one divided by the value of gamma minus 1 into the shock which has been processed already by the incident shock. That is the pressure plus I have p 0 into gamma plus 1, that means the reflected the magnitude of the reflected pressure to the pressure. This got increase because of the instant shock has processed the gases from p 0 to p is given by 3 gamma minus 1 p minus p 0 into gamma minus 1 divided by gamma minus 1 p plus p 0 into gamma minus 1.

In the limit of very strong shocks, what do you mean by strong shocks? We say well M s tends to infinity and if M s tends to infinity well the value of p divided by p 1 goes as M s square p tends to infinity when p tends to infinity what is it I get over here? If I were to simplify this expression that means I have the condition of p tending 2 infinity I take p over here, I get three gamma minus 1 divided by p 0 by p which gets 0 in the limit, I get three gamma minus 1.

I get p 0 by p p is tending to infinity that mans I have divided numerator by denominator by the value of p I get gamma minus 1 which for air comes out to be 3 into 1.4 minus 1 divided by 1.4 minus 1, which is equal to 4.2 minus 13.2 dived by 0.4, which is 8 times. Therefore, what we did was just a feel you know we got a value around 6 in practice for a strong shock the value of the reflected shock pressure divided by the pressure, which is behind the incident shock is of the order of 8 times.

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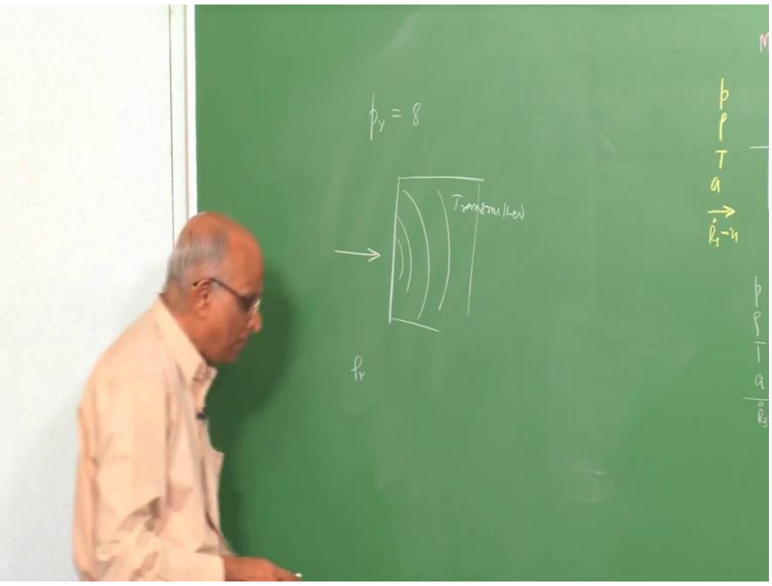

We tell ourselves I have been able to get the reflected shock pressure which is around 8 times, but you now in the problem we also had maybe I have the incident shock, it also gets transmitted into the medium into the medium into the object. This transmitted shock also I need to be able to get the value of the transmitted shock, I need to do some more exercise. All what we have been able to do is we calculated the value of the reflected shock how do I get the value of the transmitted shock? For this, you know we have to do some more work let us try to figure out how to go about it.

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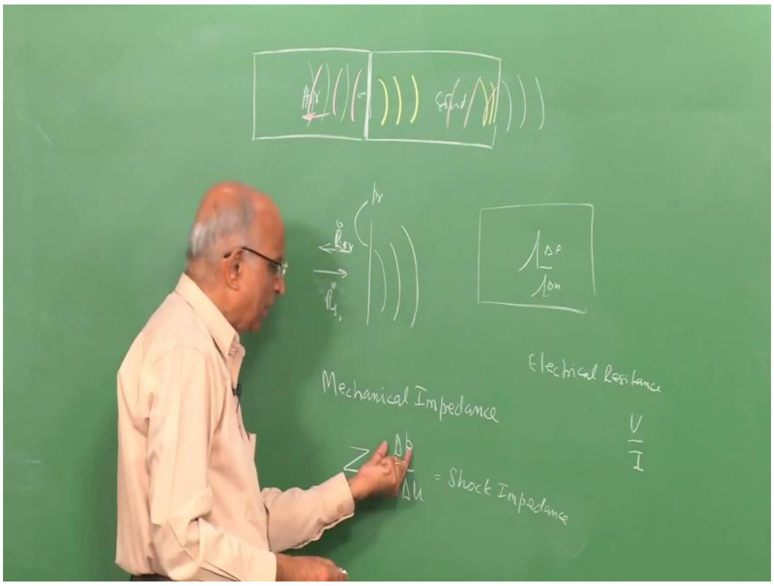

Let us consider a particular medium let us say air and I also consider another medium over here let us say a solid. Now, all what I am interested is in may be the black wave comers over here gets reflected when it gets reflected, I have the reflected waves which are travelling in this particular direction also into this medium. I get the transmitted waves which are coming in if the transmitted waves if this solid is again going into the third medium here. I have the weaves which are coming well it will get reflected over here and the transmitted waves will still go further.

That means I am interested at this particular interface at which maybe b the medium changes form air to a solid and I know that if I consider the normal case of reflection. I am able to calculate the value of the pressure behind the p that is the reflected the pressure behind the reflected wave that is this is the incidents wave this is the reflected wave i have this travelling at a speed let us say r s dot r. This is travelling at a speed r s dot incident over here I am able to calculate the pressure behind this, but I also want to know what is the type of marked number which I have into the transmitted medium. To be able to do that, well I need to have some more definitions and I define a quantity known as mechanical impedance.

What is this mechanical impedance? You know we characterize a medium such as air a solid or a liquid or any substance as a as having a mechanical impedance and I call this is z it is defined as may be the pressure change in the medium divided associated with the equivalent velocity change in the medium. What is it I am talking of? Well, I have a medium let us say air over here, I have some medium over here supposing a wave with a pressure delta p travels into it. You know the delta p also is associated with a particular delta u. The pressure change in the medium is associated with a equivalent velocity change in the medium and the ratio of the delta p divided by delta u is what I call as a mechanical impedance, which some people also call as shock impedance.

It is the shock impedance, which tells us when the characteristic of one medium changes into some other medium. It is the impedance changes which tell us how much it gets transmitted how much gets reflected because if I assume reflection I am able to calculate the pressure value over here, I would like to calculate the same thing on what is transmitted. Therefore, this shock impedance is something or mechanical impedance is something like electrical resistance, which we define as equal to voltage divided by current is similar to this voltage is equal to equivalent to pressure and equivalent to electrical resistance r. We define something like an impedance is at a shock impedance over here.

In general, the value of the velocity may not be in phase; therefore impedance might be a complex number, but in the case of shock wherein we are considering the pressure changes and velocity changes. We just consider the value of impedance as it is we do not consider it as a complex number. Therefore, let us try to see how this impedance what how it depends on the medium and whether I can whether I can sort of determine an expression for this particular impedance, let us do that.

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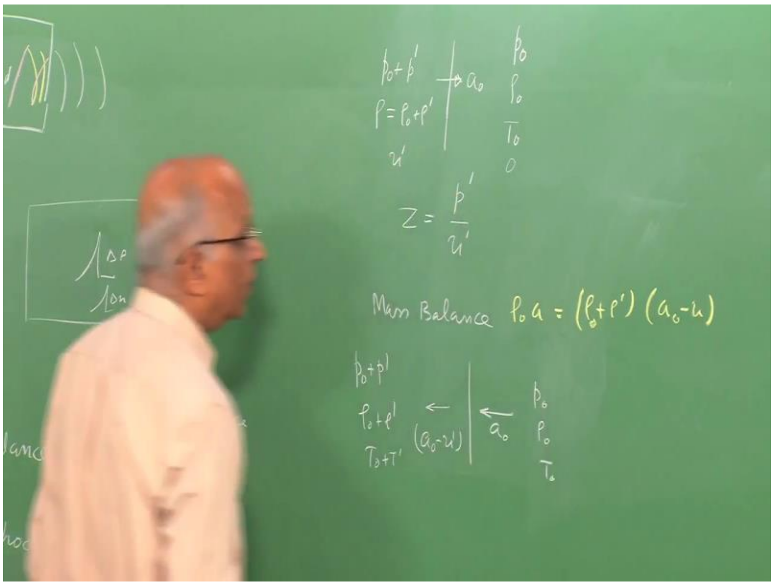

Let us consider a case any substances we take let us take a way propagates into this particular medium. Let the wave propagates, let us say at the sound speed may be when the sound speed is high it travels into the medium whose pressure is p 0 density is rho 0 let us say that the temperature is t 0. Behind the medium in which it travels, the pressure slightly increases to p plus let us say p prime a small increment over here because I am talking in terms of a weak wave which is a 0, the density is equal to rho plus rho prime which is a small increment over here. Well, the temporize also changes slightly and therefore let us try to determine from this whether I can determine the impedance which we calculated as equal which we defined as equal to p prime divided by U prime.

The velocity changes associated with this when a wave travels it is a coercion medium zero velocity ahead have a small disturbance in velocity u prime and this is the value of p prime by u prime. I want to calculate this value of z and therefore what I do? I write the mass balance I write the momentum balance and try to see whether I can calculate the value of z, let us write the mass balance. When I write the mass balance, you know what is it? I cannot write when the wave is moving, therefore I keep the wave stationary, let us also assume I am considering a unit surface area that means my the height of the wave or the area of the wave is one meter square.

Therefore, if I keep the wave stationary, the medium which is at p 0 rho 0 t 0 coercion moves towards me with a velocity a 0, these the sound speed here is a 0. The medium behind it now moves it has moving with the velocity u aero following this is equal to a 0 minus u prime over here. The conditions or the medium here are same p 0 plus p prime it is equal to density is equal to rho 0 plus rho prime and the and I have taken the velocity here let us say that temperature is equal to t 0 plus p prime. I want to write the mass balance and momentum balance equations, let us write first write the mass balance equations.

The mass balance equation is the mass flux which is coming that is per unit surface area is equal to rho 0 into a and mass, which is leaving is the same as the mass which is entering which is equal to rho plus rho prime rho plus rho 0 plus rho prime. That is the density leaving into equal to a 0 minus u which is the velocity which is leaving and what is it? Let us solve this and then go to the momentum equation.

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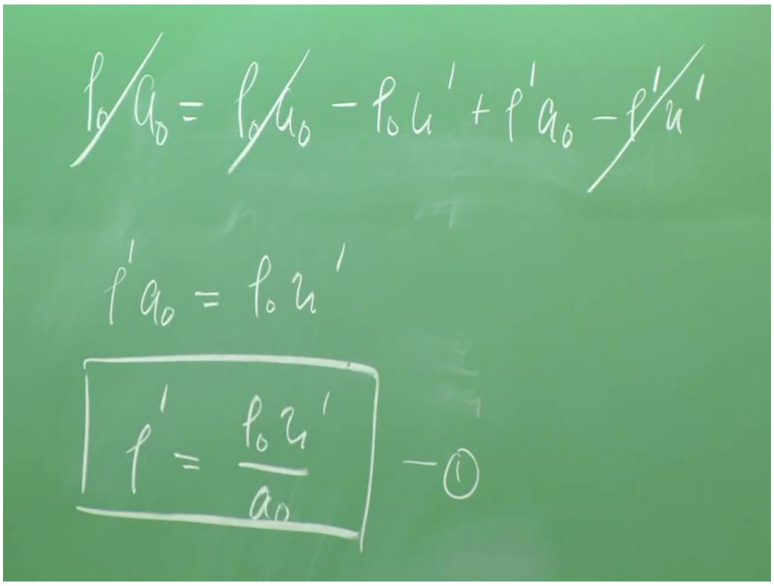

I get solving this I get rho 0 into mind you this is which is approaching that means this is equal to a 0. That means I get rho 0 a 0 is equal to, if I simplify this particular term I get rho 0 a 0 minus rho 0 u and this u is actually the velocity behind it that is it is equal to u prime over here because the velocity behind it is a 0 which is u prime.

Then, i get rho prime into a 0 minus I have rho prime into u prime, now when I look at this expression this is the small quantity because I am talking writing for an acoustic wave u prime is again for an acoustic wave product of two small quantities I can neglect. I also have rho 0 and rho 0 a 0 and rho 0 a 0, which cancels of on both the sides.

Therefore, I get minus rho 0 u prime plus rho prime into a 0 is equal to 0 or I get rho prime into a 0 is equal to rho 0 into u prime or rather I get rho prime is equal to rho 0 into u prime divided by a 0.

This is one equation which I can get, I call this is as equation one coming from mass balance. Now, I want to do the momentum balance, what is the momentum balance? Rate of change of momentum is equal to impress force. I am talking unit surface area impress force is equal to the pressure, and therefore the change in pressure is equal to right, let us say momentum balance.

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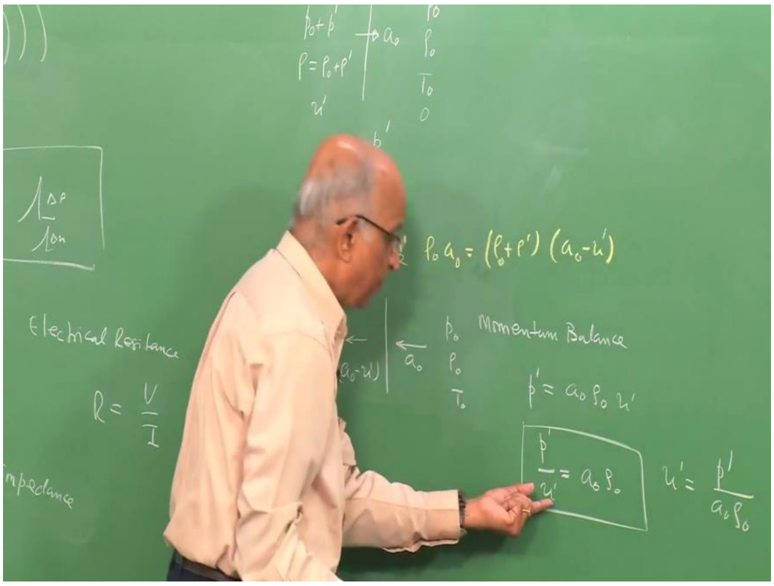

The change in pressure is p prime and the change in momentum rate of change of momentum. The rate of change of mass that is mass flux is equal to a 0 that is in this frame of reference of the wave stationary a 0 into rho 0 is the mass which is approaching per unit area. Therefore, I am talking of force per unit area over here, the change of velocity is u prime and now the velocity changes.

Therefore, the pressures in the opposite direction therefore, I get this as the expression or rather I get the value of p prime divided by u prime is equal to, mind you here we must not the change in velocity is a 0 minus u prime minus a 0 which is equal to minus u prime. Therefore, the change of momentum is equal to a 0 rho 0 into minus u and therefore, p prime is equal to the change of momentum over here. Therefore, p prime by u prime is equal to a 0 into rho 0 over here, now if I were to call this as the momentum equation u prime and I go and substitute the value of prime. Therefore, u prime from this equation comes out to be equal to p prime divided by a 0 into rho 0 p prime divided by rho 0.

I substitute it over here what is it I get? Rho prime is equal to let us write it out rho 0 divided by a 0 into the value if p prime that is substitute the value over here a 0 into rho 0 over here. Therefore, I get rho 0 and rho 0 gets cancelled, I get rho prime divided by p prime is equal to 1 over a 0 square.

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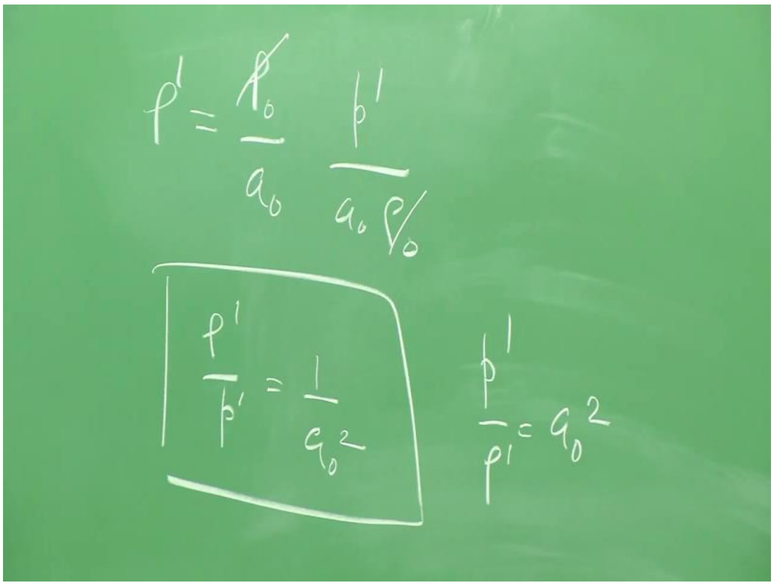

This is what we have been telling you know d p by d rho is equal to a square or rather p prime by rho prime is equal to a 0 square. Now, let us use this equation and this equation into my impedance equation, let us do the last part of it, therefore what is it I get? I get the value of z that is the impedance which we define.

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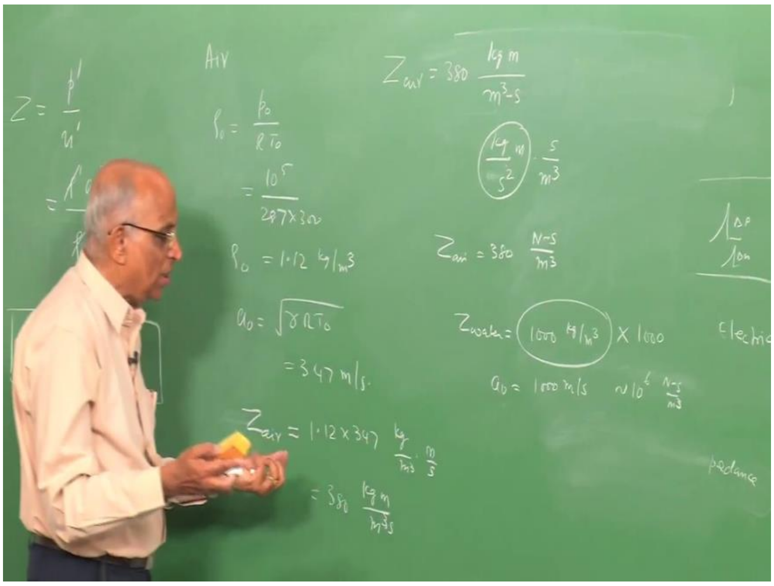

Mechanical impedance z is equal to pr prime divided by u prime, therefore I get the value of p prime from this equation as equal to. Let us put it down p prime is equal to rho prime into a 0 square u prime u prime from the mass balance equation. Let us write it out yes from u prime from mass balance equation is equal to rho 0 a 0 by rho 0, which is equal to a 0 a 0 gets cancelled. This gets cancelled, a 0 gets canceled into a 0 and therefore I get z is equal to rho 0 into a 0. Therefore, we get may be when we consider the waves in the limit of being weak waves like we have sound waves.

I get the impedance of the medium is equal to product of density if medium into the sound speed in the medium. What does it really denote? See, after all when we say impedance all what we are talking is p prime divided by u prime and this z by u prime we are talking in terms of when I have the medium, which is having some velocity disturbances. What is the actually pressure differences which are associated with it? This is the way we have to look at the value of the impedance. What is the value? Let us calculate the value such that we can put some units onto it.

We can also work with it if you have to consider air let us say for air rho 0 is equal to typically is equal to p 0 by r into t 0 that is p v is equal to m r t rho is equal to p by r t which is equal to p 0 is 10 to the power 5 Pascal r is equal to 287 temperature is 300.

Therefore, it will be something like 1.12 kilo gram per meter cube is the value of the density of the air at atmospheric pressure. If I calculate the value of sound speed, we said sound speed is equal to under root gamma r into t 0 which we saw in the previous problem was something like for a temperature of 300 Kelvin for air it is equal to 347 meters per second. Therefore, the value of z for air is typically equal to something like density is 1.12 into something like 347. Let us put the unit together kilo gram per meter cube into meter per second, something like we are talking of something like 1.12 means another 34 to it something like 380 kilo gram meter per meter cube second.

Now, this unit is little difficult to carry let us see whether we can simplify the unit and put it in some other form. Therefore we say that the mechanical impedance of air can be written as something like approximately 380 kilo gram meter divided by meter cube second. I write this as kilo gram per meter square into kilo gram meter per second square kilo gram meter per second square then I am left with a second over here. I am left with a meter cube kilo gram meter per second is Newton and therefore, I say z for air is approximately equal to 380 Newton second by meter cube.

You have the impedance expressed in this particular units of Newton second by meter cube and the impedance for air is so much, if I look at a solid let us let us say like let us say let us say I consider water what is the impedance of water, let us calculate again. Well, the density of water is around thousand kilo gram per meter cube, we know that the sound speed in water is quite high because it is coercive molecules are nearby and they transmit the sound speeds much higher. Let us say that the sound speed, we will assume as equal to another, let us say is equal to of the order of 100 meters per second, it is around 900 meters per second.

Therefore, we say z for water is equal to density into something like sound speed in water which is equal to 1000 which is of the order of 10 to the power 6 Newton second per meter cube. Therefore we can have the impedances for different mediums put together and if we tell this we find that for air the impedance is of the order of say 400 for water, it is around 10 to the power 6 for a substance like steel, which reflects more. It could be around if it is really hard steel, it could be around 10 to the power 7.

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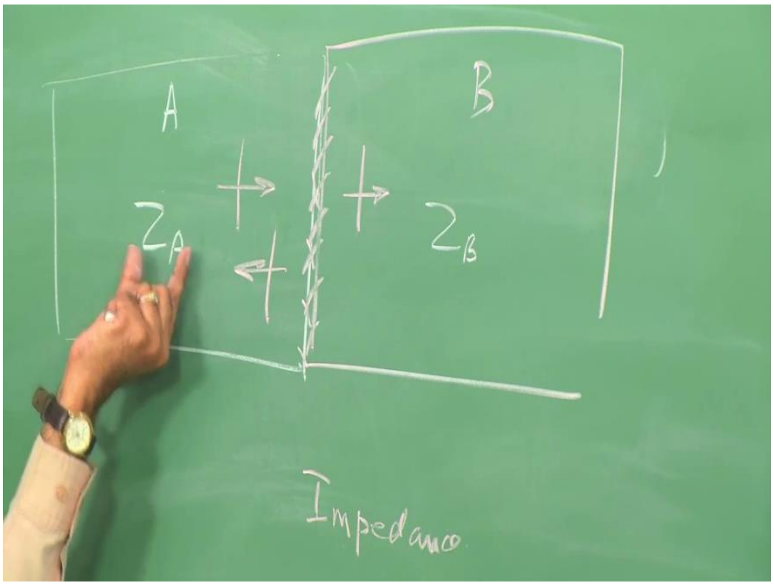

Therefore, the problem which we consider is when we look at interaction of let us say a blast wave is surfaces we say yes I have one medium which has let us say medium a which has impedance z a. I have another medium b, which has impedance z b, now I look at the interface between these two medium is given over here is the interface I look at the wave as it is propagating in medium a which is characterized by impedances z a. If the impedance of medium two is different from this well you know it will tell us how much of the short wave gets transmitted into it, how much of it gets reflected into it. Therefore, we characterize the medium to be able to solve the interaction problem of an object hitting the wave of an object being hit by a blast wave in terms of impedances.

I will proceed with this in the next class and we will see well it is not that difficult after all we can find out after characterizing a medium in terms of its mechanical impedance. We will be able to find out how much is what is the magnitude which is reflected what is the magnitude which gets transmitted. Therefore, in this class today, we first took a look at a simplified way of determining when a blast wave is moving and it hits an object what is the magnitude of the reflected pressure. If you considered a constant short velocity we found that when the shock speed is very high mach number is very high, the reflected value jumps to a value 8 times the shock pressure behind the incident.

That is the incident blast wave that is if the over pressure of the blast wave is let us say 20 the reflected pressure is something like 160 times, 160 atmospheres. If it is 8 times the value if the blast wave is weak, well it is only twice the value. Then, we were looking at the how to characterize a medium to be able to find out how much gets transmitted into the medium and how much is reflected back into the original medium and this is what we do in the next class.

Well, thank you.