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Video Lectures on**

Convective Heat Transfer

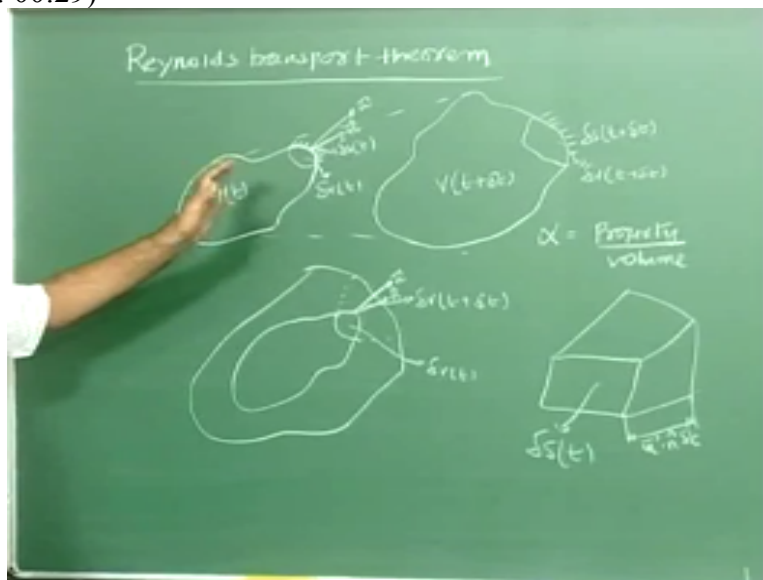
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**Lecture 6
Reynolds Transport Theorem**

So good morning again so we will continue on the aspect of the revision of the continued continuity momentum and the energy equation, the governing equations by means of another approach .

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Where you are completely working with the arbitrary shape control volume, which is not fixed to any coordinate system , okay and this is a very interesting approach. So here what we do is we consider a control mass essentially a system that means, we are considering fixed packet of fluid with a given mass and tracking it with respect to time. So if you assume that there is a

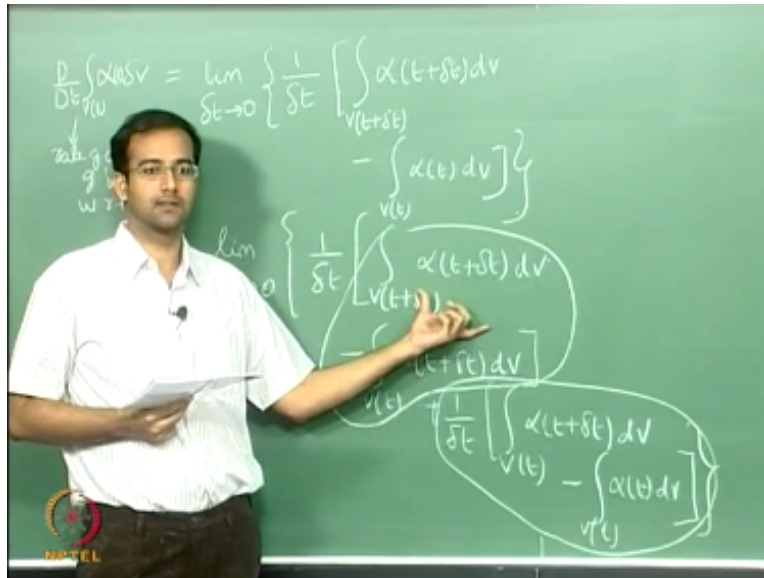
fixed mass of fluid with the volume V at a time T and then after some time $T + \Delta T$ you see the volume changes but the mass content is the same and if you define any property α as a property per unit volume and this property is supposed to be a function of time. okay so this is this is the traditional Lagrangian approach so where you just the observer travels along with the frame of reference of moving frame of reference of the the packet of fluid so and then therefore the property changes are all happening over time right rather than the Eulerian approach.

Where the observer is stationary the control volume is stationary and then the fluid crosses the control volume boundaries and there is a corresponding change of mass, and mass flux quantities of this particular property okay. So that is a different frame of reference so now so this Lagrangian frame of reference can be used to calculate what is the rate of change of this particular property with respect to time okay so now for that we have taken a small differential volume ΔV corresponding to this time T and seeing how it evolves over time okay the same differential volume what happens time $T + \Delta t$ so if you overlap these two control volumes essentially you see that this Δv has probably grown to this size now ΔV at $T + \Delta T$ okay so therefore what we can do is.

We can calculate the change ΔV at $T + \Delta t - \Delta V$ at t from the velocity vector basically if you take this particular differential volume so this is ΔV at T and this entire thing is ΔV at $T + \Delta T$ so this change is basically the volume of this element that I have drawn here okay.

So therefore if this surface is actually sweeping a distance of U vector dotted with the normal okay so that much displacement is happening to that the differential surface element at time T and it is sweeping that much of distance so that should be = to this difference change in the differential volume okay so this is nothing but $\mathbf{u} \cdot \mathbf{n} \Delta T \Delta s$ right so this is this is just how we are representing the change in the volume if you want to calculate from the Eulerian point of view what is the rate of change of this property.

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With respect to time okay from from the control mass system so the mass doesn't change so you are just tracking this property change with respect to time so this is the total derivative okay This single total derivative okay so that you have to say suppose you have a particular value of α for this differential volume so you will have different values of α for differential different differential elements and you have to integrate all of them okay that will give you the total property of that particular control volume okay so and you want to calculate the rate of change of this particular property now this can be written as your limit ΔT goes to zero you can split it up into the property at $T + \Delta t$ okay - the property at time T okay so these are the two integrals that you see here and what I am going to do a little bit of mathematical jugglery .

I am just adding and subtracting in this particular term right here. So this is integral at V of $T + \Delta T$ αdV so I am just subtracting and adding the same term here so now if you look at this particular term okay. So this is nothing but you are keeping the integral in integrand the same okay and you are changing the volume okay we get V of $T + \Delta t$ - V of T so essentially that is like you are keeping the integrand same and you are integrating with respect to a change in the differential volume so which is nothing but this particular term right there okay so this can be combined together and we can write this as α at $t + \Delta t$ dV where your V changes from $t + \Delta t$ to the earlier volume T okay so that can be replaced by this particular term right here and this term that you see so in this case . You are keeping the volume fixed okay so this volume is at time T and the integrand is changing okay from α at $T + \Delta T$ okay - α at T okay.

So this describes what is happening to the change in the property from time T to $T + \Delta T$ for the same control volume okay so this is nothing but the material derivative okay so now what you are essentially doing this you are changing the frame of reference from a moving frame to a stationary frame okay.

So left hand side is the moving frame of reference, where you are tracking a control mass, with a fixed amount of mass and you are tracking the evolution of the property, with respect to time and when you go to a stationary frame of reference, so that should involve partial derivative with respect to time that is changing change of property with respect to time for fixed control volume so that is you are converting a control mass to a control volume okay + now once you convert that to a control volume there should be some flux of this property which is crossing the control volume boundaries so that should come in terms of this.

So we will see how it comes okay so in the next step I am just going to replace this particular terms right here okay.

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$$\frac{D}{Dt} \int_{V(t)} \alpha(t) dv = \lim_{\Delta t \rightarrow 0} \left\{ \int_{S(t)} \alpha(t + \Delta t) \vec{u} \cdot \vec{n} ds + \int_{V(t)} \frac{\partial \alpha}{\partial t} dv \right\}$$

Gauss divergence theorem

$$\int_{V(t)} \nabla \cdot (\alpha \vec{u}) dv \rightarrow 0$$

$$\frac{D}{Dt} \int_{V(t)} \alpha(t) dv = \int_{V(t)} \left[\frac{\partial \alpha}{\partial t} + \nabla \cdot (\alpha \vec{u}) \right] dv$$

So this will give me $\frac{D}{Dt} \int_{V(t)} \alpha(t) dv$ should be = to limit going to zero I am saying $\alpha(T + \Delta T)$ and this is nothing but your change in the volume from $V(T)$ to $V(T + \Delta T)$ and that I am going to replace by $\vec{u} \cdot \vec{n} ds$ all right so the $\frac{D}{Dt}$ cancels there so this will be $\vec{u} \cdot \vec{n} ds$ so you are converting the volume integral to a surface integral okay so this also will be corresponding to the surface at time T okay + the other term right here is nothing but the rate of change of this property for the same volume with respect to time so this is the partial derivative integral $\int_{V(t)} \frac{\partial \alpha}{\partial t} dv$ this is $\frac{D}{Dt} \int_{V(t)} \alpha(t) dv$ okay so it is clear.

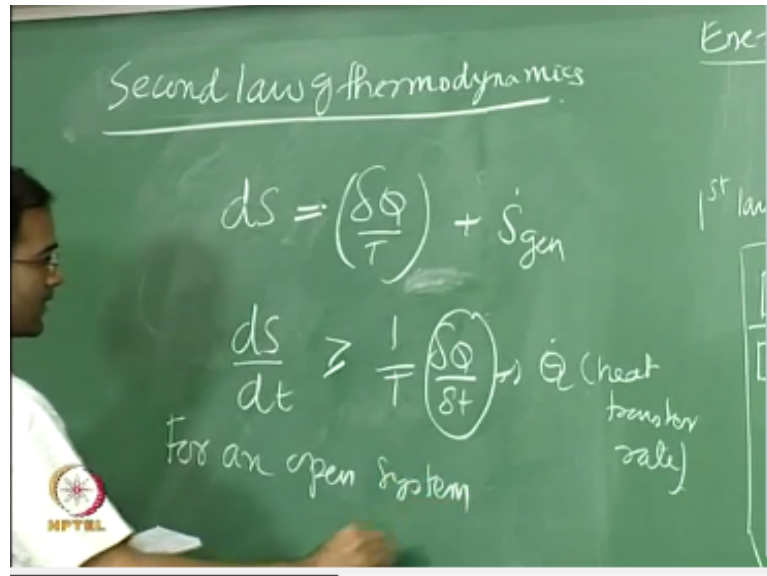
Because this is for the control mass or the system which is essentially a closed system so it doesn't allow any mass transfer so essentially the property can change only with respect to time spatial it cannot change yes but this is with the reference frame where you are moving along with the particle so if you convert that to a stationary reference frame that will have the spatial derivative so that is what essentially the Reynolds transport theorem does this at it essentially transforms it gives you a relationship to transform a control mass to a control volume okay.

So if you know that I think after that working out for a control mass will be very straightforward because if you write the energy conservation for control mass you do not have things like reflux right because it is impervious to mass okay so much it is much easier to write the conservation for a closed system rather than the open system and all you need to do is the link between the total derivative and the partial derivative so that you just substitute and then you will bingo finally you reach the conservation equation we have derived earlier which is for a control volume okay so very shortly we will see that you know but this is just a routine way of no substituting now we are looking at two terms here

One we are fixing the integrand and changing the volume okay so that is being substituted by this particular term here okay and the other we are basically fixing the volume V of T and trying to track the change with respect to time so that is the partial derivative with respect to time okay. So now we can apply Gauss divergence theorem you all know that the surface integral can be converted to a volume integral okay so how this can be done so this can be integrated over volume then fair play class divergence theorem to this divergence of this particular term that is Del dot okay you have your α and u vector and integrated over the wall okay.

So therefore I am just I am just going to replace these terms in terms of the divergence operator and this becomes D by DT of $V \alpha DV$ should be $=$ to integral so the entire right hand side is having a common integral with respect to volume T so this can be $D \alpha$ by $DT + \Delta \text{ dot } \alpha u$ vector and this is all integrated over the differential volume DV okay so this essentially is the Reynolds transport theorem. I will call this as one okay so this is the Reynolds transport theorem which says that you are converting some property variation from a control system mass which is a closed system to a control volume which is the open system.

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So it is nothing but you are converting a total derivative writing a total derivative in terms of partial derivatives okay with respect to control mass it's only a function of time therefore it is one derivative single derivative and with respect to the control volume you have property which is varying with respect to time and space therefore you have partial derivatives so as simple as that okay so this can also be written in a tensor notation okay so this is a general notation also in tensor form.

I think many of you have learned tensors in your incompressible flows so I am just going to use some notation here so the left hand side term tastes the same α DV should be = to $D\alpha$ by $DT + D$ by DX so I am going to use the subscript K to denote the divergence here okay so this should be again αu_K okay in fact I can remove the vector so this itself means DV okay so this is in a tensor representation okay.

So this here right here means that you are basically going K is = to 1 2 3 and you are summing it up I hope all of you can remember your tensor notations okay $k =$ to 1 becomes DX okay similarly here $D u$ by DX $k =$ to 2 becomes this DV by DY so you can keep changing this particular thing with respect to whatever suppose you write your X momentum so you can write your momentum in the X direction and keep this as 1 and we will see what is this α there similarly in terms of energy so you have to define α and then the index also will keep changing accordingly okay.

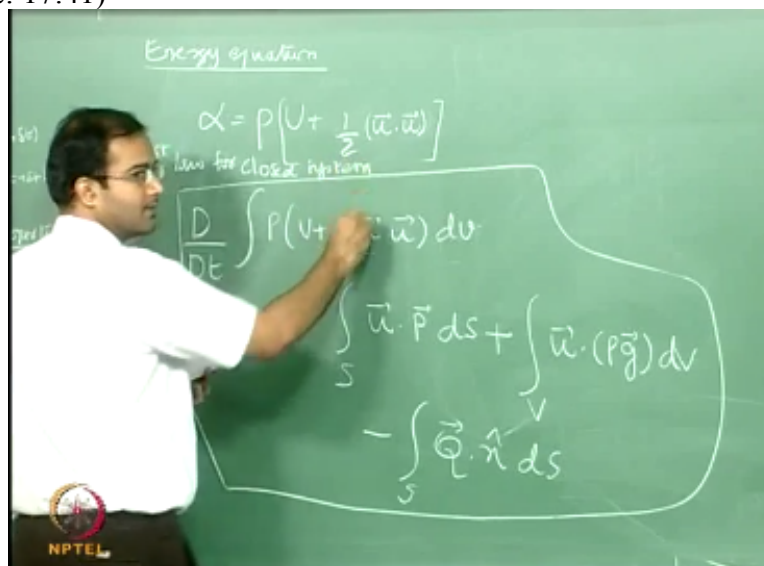
So this is your theorem we will apply this to derive the conservation principle number one conservation of mass ok so when you talk about conservation of mass so what should be your property α be α we are defining property per unit one the Q conservation of mass so this is mass per unit volume right so your α should be P so simply replace your α by P so what it essentially states what your conservation of mass says if you have a closed system there is no

mass entering and mass leaving because this is a closed system so that is the advantage of using Reynolds transport theorem you do not have to consider any flux of mass okay.

So all you are bothered about is this total derivative so what it says you are D by DT integral over V your $P DV$ should be this is the mass the total mass 0 right because as I said this is the control mass so the mass cannot change with time okay so this is zero so now what you know is this is the fundamental starting point for a closed system so now you also know from the Reynolds transport theorem the link between the total derivative and the partial derivative okay so now you can write from a control mass to a control volume the same expression.

Okay so that that will say that this is $=$ to V of T okay $D P$ by $DT + \text{del dot } P u DV =$ to what 0 from the Reynolds transport theorem okay so now for this two condition to be valid the integrand has to be 0 correct. So therefore this gives you your continuity equation okay so in a coordinate free representation so momentum I think you can do it yourself you have to just apply the Newtonian law okay for start with the closed system apply the Newton's second law for the closed system and then you can link the total derivative with respect to the partial derivatives okay so I am going to do that for energy equation so you can probably apply the same principle for the momentum equation

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Also okay now what I am going to do is for this particular case I am going to consider the corresponding transport or maybe the different forms of energy which are acting on this control wall right so one component which I will combine together the body force and the viscous forces together I will say that they are acting in some direction okay let me call this as some P

vector okay this does not mean pressure here it has all the components of viscous and pressure and body viscous body on pressure forces.

Okay and one single component, I am showing all these sub components are included under that \mathbf{P} vector okay so naturally what will be the work if you have a \mathbf{P} vector acting on this differential volume and you have a Lassa \mathbf{T} vector in this direction so the work will be $\mathbf{P} \cdot \mathbf{u}$ vector okay so the corresponding displacement that is along this the particular velocity vector direction should give me the work okay so apart from that I can also have heat transfer okay and I can just say I have some heat transfer like this \mathbf{Q} vector okay so now I cannot do this in any coordinate system it is arbitrary in any direction.

So I am just using the vector notation and I am going to substitute there okay so this is your heat transfer rate okay in fact I can use the heat flux here right here okay \mathbf{Q} double prime vector heat flux okay so you have some internal energy for this particular system you have this is a controlled mass so essentially what is happening you are not considering any flux of energy okay it is close to mass but it allows work and energy transfer to happen therefore you are considering a component of work that is speed ordered with you vector and heat transfer so now we can write the first law of thermodynamics for a closed system okay now first we'll define what is the A here you have to tell me property α for when you consider energy.

So you have components of internal energy and kinetic energy right per unit volume okay so you have $u + u^2$ okay in fact I can write this as some magnitude of \mathbf{U} vector square okay so I am just expanding it in the conventional Cartesian coordinate system okay just for your convenience and so what else is this correct this is per unit mass okay but I need property per unit volume divided by density multiplied by density right that's it okay so this is my α now I will write the conservation for energy to this particular closed system and that should give me what D by DT I can integrate this α that is \mathbf{P} into so in fact.

I am going to use a coordinate free representation here so write this as half of you vector dotted with you vector right that is the same $U^2 + v^2 + W^2$ if you write it in a Cartesian coordinate system so I am just going to write this as $+\mathbf{U} + \frac{1}{2} \mathbf{u} \cdot \mathbf{u} \, DV$ this is my left hand side term for a control mass I am writing the first law of thermodynamics so that should be $=$ to the work okay and the contribution of work and heat okay only this can contribute to the change of energy of this particular system I am now going to split into two components.

So I am saying that this \mathbf{P} has components of viscous and pressure I can also have components of gravity which I will say it is acting in some direction like this okay so therefore this is the gravity contribution I am separating that from the viscous and pressure contributions so I will write those contribution separately so as far as the work done by the pressure for the pressure and the viscous forces are concerned so it should be $\mathbf{u} \cdot \mathbf{P}$ and they are acting on the

surfaces so it should be integrated over the surface so this is one contribution the other is coming from the gravity so that is $\mathbf{u} \cdot \mathbf{P} \mathbf{G}$ and this is a body force right this is acting on the entire volume so this should be a volumetric integral so what is the final contribution so you have the work and energy heat okay.

So the heat will be $\mathbf{q} \cdot \mathbf{n}$ dotted with the normal and once again it is a surface integral right now what should be the magnitude of the should it be what should what should be the sign of this + or - okay so in fact I should give it correct representation here so in my thermodynamic definition the heat transfer to a system is positive therefore I will not denote that my heat flux is actually entering its particular so therefore $\mathbf{Q} \cdot \mathbf{n}$ will be negative so I have to put a negative sign here to make sure that this convention is the positive heat transfer all right so according to this is opposite to the normal okay so this is the positive definition of heat that is added to a system and this has to be negative so that's it so once you have written this for this particular control volume control mass so okay so this is for a control mass okay.

This is the first law for closed system all of you agree there are no flux terms so this is very straightforward to write then for an open system now you know directly the Reynolds transport theorem which links the closed system to a control volume so just replace the left-hand side with the partial derivative terms okay and therefore you will finally have your conservation of energy for an open system okay so if you do that so this will on the left-hand side.

I'm going to replace that with the Reynolds transport theorem formulation I have $\frac{D}{Dt} \int_V \rho \mathbf{u} \cdot \mathbf{u} \, dV$ okay the entire thing okay anyway + I have I can use the tensor notation rather than using the divergence notation here $\frac{D}{Dt} \int_V \rho \mathbf{u} \cdot \mathbf{u} \, dV$ I am going to use this particular notation into $\rho \mathbf{u} \cdot \mathbf{u} + \frac{1}{2} \rho \mathbf{u} \cdot \mathbf{u}$ okay multiplied by what ρ okay the entire term dV okay so I am replacing this control mass derivative with a control volume derivative that should be $= \frac{D}{Dt} \int_V \rho \mathbf{u} \cdot \mathbf{u} \, dV$ okay so this should be just as it is integral is $\rho \mathbf{u} \cdot \mathbf{u} + \frac{1}{2} \rho \mathbf{u} \cdot \mathbf{u} \, dV$ okay where my \mathbf{p} vector here is nothing but the viscous stresses okay so that is $\sum_{i,j} \tau_{ij} \mathbf{e}_i \cdot \mathbf{e}_j$ into \mathbf{n} vector okay.

So this is my compact notation for these viscous forces which are acting on this particular control is it clear so only the left hand side I am writing in terms of the Reynolds transport theorem control volume derivatives. So now you can apply the Gauss divergence theorem you can also express this and this and write this in terms of volume derivatives so therefore I can say my integral $\mathbf{u} \cdot \mathbf{P} \mathbf{D} \mathbf{s}$ will become integral $\mathbf{D} \cdot \mathbf{X} \mathbf{U} \mathbf{J} \sum_{i,j} \tau_{ij} dV$ and integral $\mathbf{Q} \cdot \mathbf{n} dS$ will be integral over volume $\mathbf{del} \mathbf{Q} \cdot \mathbf{J} \mathbf{by} \mathbf{Del} \mathbf{X} \mathbf{J} \mathbf{into} dV$ okay so you please understand the tensor notation here so in this case you simply say $\mathbf{DQ} \cdot \mathbf{X} \mathbf{by} \mathbf{DX} + \mathbf{DQ} \cdot \mathbf{y} \mathbf{by} \mathbf{DY} + \mathbf{DQ} \cdot \mathbf{Z} \mathbf{by} \mathbf{DZ}$ okay here this is the this is a tensor okay, \sum this is a stress tensor okay you have all the nine components so you have to go one by one so you start with say $i = 1$ and $j = 1$ and then $i = 2$ and $j = 2$ so and you keep expanding this okay so this is a very compact way of

writing all those terms okay you can go back and check with your Cartesian coordinate system okay whether this makes sense okay so I am just going to write this in terms of volume derivatives

and therefore if I put them together I can say the integrand should be 0 because I can collect all the terms which have volume integral okay on the left hand side and that will be = to zero and therefore the integrand has to be zero therefore the integral becomes d by DT of my $P u + \frac{1}{2} UJ u J$ so this is the compact notation again + D by DX $K P u + \frac{1}{2} UJ u j U K$ okay so now you see if you are going from $k = 1$ to 3 so you are adding all the special derivative terms with respect to X Y and z in Cartesian coordinate system okay and for each of this you are saying this is $u^2 + v^2 + W^2$ so you have two notice notations here 1 for J and 1 for K they are independent ok so this J will go within the k loop alright so then you have to expand that for different coordinate systems $k = 1, 2 \& 3$ so on the right hand side you have UJ ok so this should be D by DX $I \rightarrow UJ \Sigma IJ + UJ \rightarrow P G J$ this is the body force term - you have DQ_j by DX J all right so this is your energy equation .

Now this has to be closed and one more thing we can do is this has all these complex terms which we can probably simplify once again we can write the mechanical energy equation in a tensorial form and we can subtract that from this ok so the mechanical energy equation if you multiply your momentum equation by the velocity terms okay and sum all the three momentum equation .

So your mechanical energy equation in the tensorial form can be written as d by DT $P UJ UJ$ by 2 okay this is like $P u^2 + v^2 + W^2$ by 2 okay that is the + you have D by DX $K P UJ u J$ by 2 $u K$ so what you are doing is you are multiplying with UJ so that is for X momentum you have multiplication with u Y momentum V and that momentum W and this K is basically your spatial derivative the divergence operator essentially so you have $UD u$ by DX + $VD u$ by DY so that that is taken care by this K so that K is different.

From this J ok that that is that is = to $UJ D \Sigma IJ$ by DX $I + P UJ GJ$ so for a particular momentum X momentum you will have gravity in the X Direction y direction and so on okay so this also is the stress tensor J here indicates the direction along which that stress is acting so in the X direction you will have Σ_{xx} and no tau YX and so on and this is the since it is a tensor it has to be the gradient in that particular direction okay so for x direction you have $D \Sigma_{xx}$ by DX the Y direction you have $D \tau_{YX}$ by DY and so on okay so that is the compact way of representing that all those terms in the Cartesian system so now if you subtract let us call this number we all we all ready use number one no okay somewhere we use before I think but let me redefine as number one and this is two and subtracting two from one so I will be left with an equation for only the internal energy

So subtracting two from one I can say that I will get an equation like $\frac{D}{Dt} \rho u + \frac{D}{Dx} K$ so these two terms get cancelled off I will get another term here $\rho u u_K$ on the right hand side I will have $\sum IJ$ because if you expand this is $\sum IJ$ into $\frac{d}{dx} UJ$ by $\frac{D}{Dt} I + UJ$ into $\frac{D}{Dt} \sum IJ$ by $\frac{D}{Dt} X$ so $u J$ into $\frac{D}{Dt} \sum$ is n this when you subtract they are gone okay so you will have $\sum IJ$ into $\frac{d}{dx} UJ$ by what $\frac{D}{Dt} I$ and the body force term gets cancelled off directly then you will have $-\frac{d}{dx} QJ$ by $\frac{D}{Dt} J$ all right so what I am going to do. I am writing this term.

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Subtracting ① from ②

$$\Rightarrow \frac{D}{Dt}(\rho u) + \frac{D}{Dx}(\rho u u_K) = \sigma_{ij} \frac{\partial u_i}{\partial x_j} - \frac{\partial p}{\partial x_j}$$

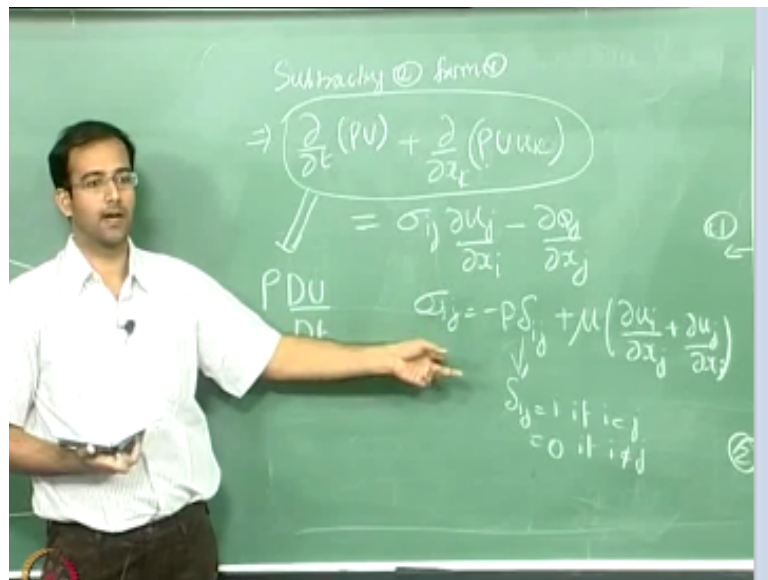
In terms of total derivative again which you are which you know is $\frac{d}{dt} u$ by $\frac{D}{Dt}$ and you know for incompressible fluid now I am going to bring in the relationship between the stress and strain rate so I can say $\sum IJ = \tau - P$.

So I am going to write this in terms of tensorial notation and you will see that this is a very compact you do not have to write this for each and every component by $\frac{D}{Dt} u_j + \frac{D}{Dx} u_j$ by $\frac{D}{Dt} X$ does it make sense where Δ is your Kronecker Δ so Δ_{IJ} will be = to 1 if $i = j$ = to 0 if i is not = to j right so what it means only for your normal stresses you have $-P$ the pressure forces come to balance the normal stresses whereas the tangential stresses do not have the pressure force to balance that so there they will disappear because this is your Δ function okay so wherever you have normal for normal stresses only there the pressure stresses will be there otherwise it's zero and this is a common representation okay so this will be said $\frac{D}{Dt} u$ by $\frac{D}{Dt} Y$ if you suppose you have $i = 1$ $j = 1$ then this will be $\frac{D}{Dt} u$ by $\frac{D}{Dt} X + \frac{D}{Dt} u$ by $\frac{D}{Dt} X$ that is twice $\frac{d}{dt} u$ by $\frac{D}{Dt} X$ okay if you are writing your tangential stresses so $i = 1$ $j = 2$ so this will be $\frac{d}{dt} u$ by $\frac{D}{Dt} Y + \frac{D}{Dt} v$ by $\frac{D}{Dt} X$ which are the components under shear stresses okay so depending on the

particular index I and J this can be written for all the components normal component or tangential components.

It is a very compact notation ok for those of you who have probably not known any of you not heard of tensorial notation okay so, I think you may have to brush through any basic fluid mechanics book maybe usually in the appendix there is something on tensorial notation or you can just quickly read through tensors it is I mean it is very common since you can see that from yourself you do not have to expand that you can substitute.

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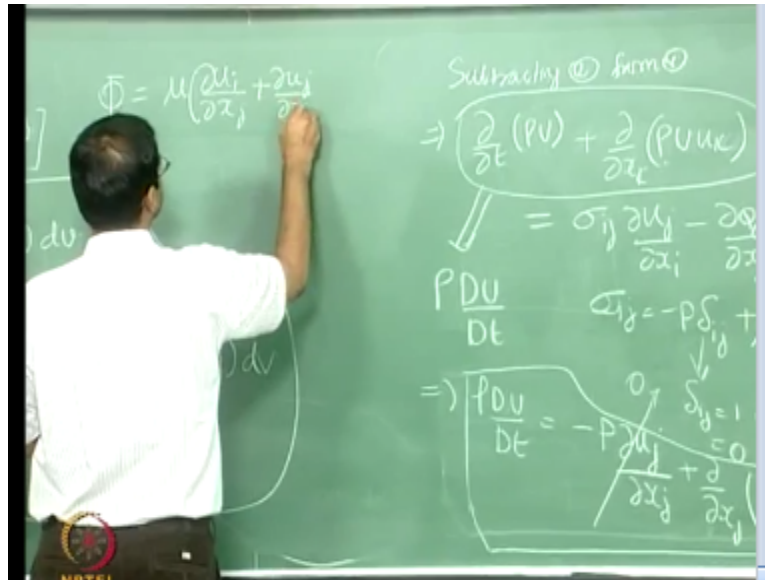


If it is 2 dimension I and J can maximum go to 2 if it is 3 dimensions so you can just expand it in that manner and if you represent something like this that means this is a summation operation you do not just stop like this you have to expand it ok for different values of J and I that is what it means ok so therefore now if I substitute this you see how easy it reduces to the final form in the tensorial notation.

So this gives me $\rho \frac{Du}{Dt}$ okay so this will be I am substituting for $\sum_{i,j} \sigma_{ij}$ in terms of this so $-P$ into $\frac{Du}{Dt}$ or $\rho \frac{Du}{Dt} = \mu \nabla^2 u + \rho \frac{Du}{Dt}$ so I am also using the Fourier's heat conduction law for Q and I am writing this in terms of the temperature gradient $\frac{Du}{Dt}$ okay + you will have some other terms which form the viscous dissipation right here in fact this will be 0 by continuity right so what I am doing is I am clubbing all the pressure term so if you write this is $P \Delta u$ and this will be 1 only if $i = j$ therefore if you are $i = j$ only that term will be there and when they are \neq that is nothing but the continuity equation for incompressible flow

okay so that will automatically be satisfied therefore you will have your total derivative that is = to your laplacian operator right here + your viscous dissipation term and for incompressible flows. So this Φ what I have written here.

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You can see that for yourself will be new DUI by DX j + d UJ by DX I times what fill in the blanks what plot will be the multiplication factor. Here, so I am just substituting this into this particular term ok The pressure term I have cancelled it DUJ by DXI right okay now you can go back and check for your Cartesian system whether you will get all the terms you know you can start with say you I = to 1 J = to 1 and you will get 2 D U by DX into D u by D X so that will be 2 D u by D X the whole Square to new D u by DX the whole Square and if you have + then you have to expand in Y Z Direction. So you have other D DV by DY + DV by DY the whole square + DW by DZ 2 whole square +

Now if you go to I = to 1 J = to 2 then you will get your other terms D u by DY + DV by DX now this again has to be expanded so for different values of J so again if you if you if you do that you will get DV by D DX + D u by DY the whole square okay which you have derived in the Cartesian coordinate system. So this is a very compact notation alright so now therefore that that is your final energy equation which you have you can write down in a coordinate free representation okay for a control volume so this is finally for a control volume .

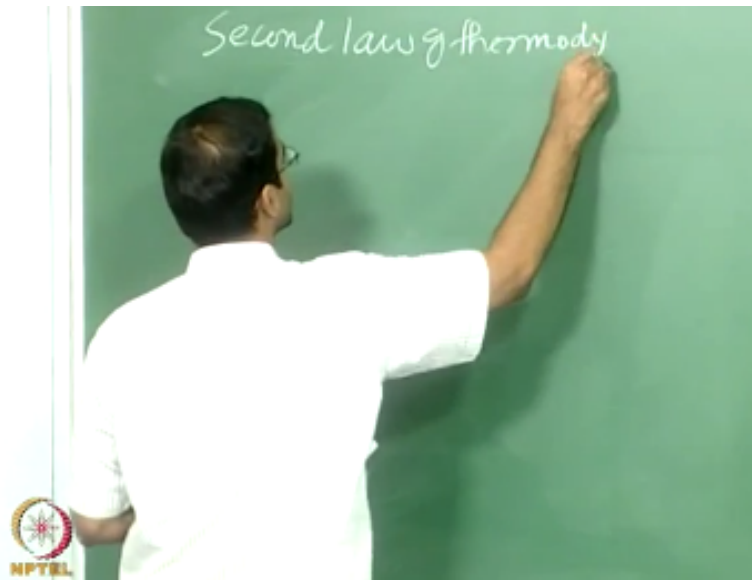
Okay so therefore I think this gives you a pretty good idea how the Reynolds analogy can be applied to derive the conservation equation so all the Reynolds analogy says is you can transform your variation with respect to a control mass to a control volume okay and you have to write down the conservation laws for a control mass which is much simpler and then you

apply that relationship and then you write the final equation for a control volume okay and this is a completely ordinate free representation so any questions.

So what you have to do is go home and then check once whether all these tensor notations make sense to you and if you have any questions you can ask me okay so it takes some time for you to you know understand this completely but it is not that difficult either okay so working with tensors you don't make any mistake so that is another advantage and finally it's up to you to expand that in what are coordinate system that you are working.

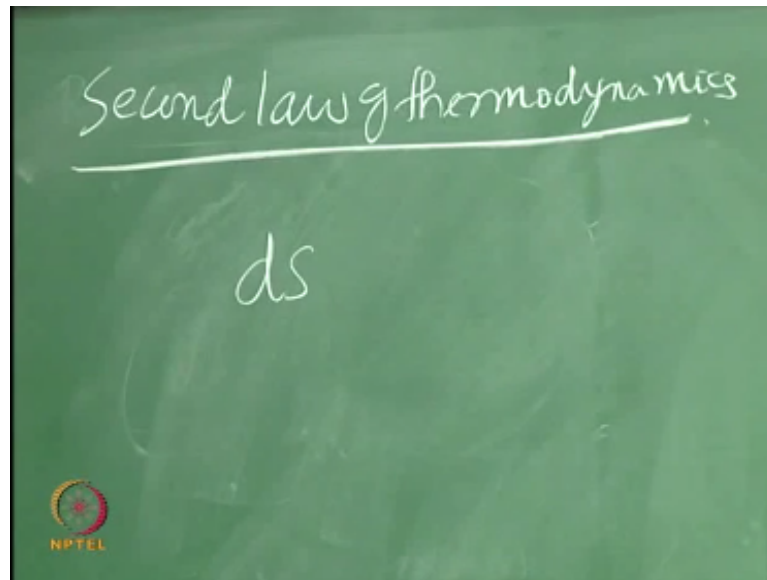
Okay so very quickly what I am going to do is another five minutes I am going to go over to the second law of thermodynamics because we have derived all our required conservation equations applying the first law for energy but we have to see something about more important fact which is entropy generation okay when you have heat transfer you have some kind of entropy generation inside the system as well so the heat transfer results in some internal irreversibility and we will see how this internal irreversibility can be characterized okay very briefly okay I am to go into too much of detail okay. So I am just talking about the second law aspects now.

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Okay so I am talking about irreversibility is and entropy generation and you all know we can start with the clashes in= ITY you know which relates your change in entropy to the amount of heat transferred.

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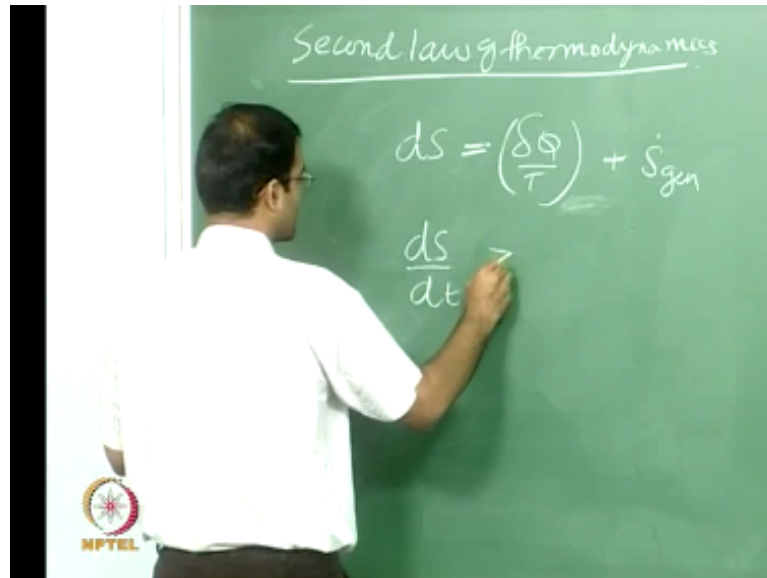


what it says if you have a perfectly reversible process so your Ds is $=$ to ΔP this is your perfectly reversible process okay if you had differential amount of heat so you can relate your change in entropy directly with this particular relationship you can calculate the change in entropy if you integrate this particular equation okay.

Now if you have some internal irreversibility what it says is your Ds is greater than or $=$ to this so usually for internal if it should be greater than this so we can also say or otherwise Ds is $=$ to this $+$ some entropy which is generated inside the system due to irreversibility is you know these irreversibility is could be one of them could be from the viscous dissipation okay the energy equation you have a contribution of viscous dissipation right so what it says even if you don't transfer heat that viscous dissipation is sufficient to increase the energy of the system that means part of the kinetic energy is converted into energy into internal energy of the system.

Okay so that can cause additional irreversibility is apart from that your conduction or heat diffusion itself will generate some internal irreversibility okay so we will now try to estimate okay what is the contribution of the heat transfer and your viscous dissipation to the generation of heat entropy okay that is a very important thing because it is not just enough to understand the conservation law we should also understand and characterize what is the contribution of these components to your entropy generation in a system okay therefore we will just quickly expand this for a closed system. We can write this.

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in the rate form as DS by DT is greater than or $=$ to 1 by dQ by DT okay this is nothing but the heat transfer rate okay so you can call this as \dot{Q} or whatever heat transfer rate so for an open system okay. So what I am going to do is I will express this total derivative this is for a closed system. I can use the Reynolds transport theorem which I have expanded the total derivative in terms of the partial derivatives I can say that this is my total entropy now if I define an entropy per unit volume as a property.

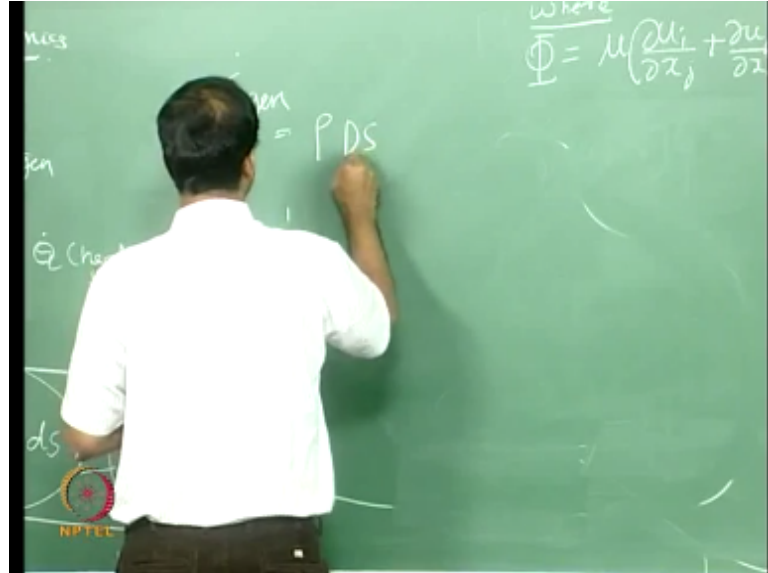
So I can write this as ρ into s so this is the entropy per unit mass Here I am sorry okay this is me this is your entropy specific entropy therefore they α which is entropy per unit volume will be ρ times s and integrated over the different entire volume + what is the second term. Okay so this is your s into so this is ρ is times so this will be the velocity component okay which will make your derivative here $V \cdot n DS$ this is integrated over the control surface this should be greater than or $=$ to if you integrate this again over the control surface because your heat transferred is transferred across the control volume boundaries okay \dot{Q} by DT okay DS all right so this is your equation for conservation of entropy.

So this is the entropy conservation equation you can say for an open system alright so it is just like your energy conservation you can write something in terms of entropy flux okay and your rate of change of entropy with respect to time and that should be greater than or $=$ to your heat transfer which is happening divided by time the temperature sorry so what we will do now is that we will expand this particular term on the right hand side okay.

So we will apply Gauss divergence theorem for this term as well as this right hand side term and we can expand that term a little bit okay so if you write in terms of the Gauss divergence theorem so what I am going to do is that so this difference ideally if you don't have any internal irreversibility is this left hand side - the right hand side term should be zero right if that is

greater than zero that means you have some internal irreversibility so what I am going to say you are internally irreversibility is dot gen.

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Is = to this -. Okay so which is nothing but your rho ds by DT. Okay so I am just writing this in terms of the total derivative + I am going to expand this particular term I will write that using Gauss divergence theorem as del dot one by T this can be written as del dot 1 by T del Q by DT for the entire control volume all right so this can be expressed as 1 by T I can write this as del dot Q double prime now the del Q by Del T is nothing but the heat flux primarily so this can be written as del dot Q I can take 1 by T out - I can take 1 by T Square Q double prime dot del.

T okay I'm just splitting this derivative I mean just expanding I'm just saying this is 1 by T if I take out this is del dot Q double prime - if I take Q double prime out so this is 1 by T Square - double prime dot del T I'm just expanding and rewriting this so this is my final expression which says that my generation term has something like a total derivative + this + this so if you ideally balance all these terms together you can calculate your irreversibility internally Okay so we will stop here and tomorrow we will continue and show what are the final expressions for the contribution to internal irreversibility .

Reynolds Transport Theorem

End of Lecture 6

**Next: Entropy Generation and streamfunction-
vorticity formulation**

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