Microfluidics
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Lecture - 05
Micro-scale fluid mechanics (continued.)

Okay so we will continue our discussion on slip flow or liquid and we know that for liquids the no slip boundary conditions and no temperature jump boundary conditions are valid.

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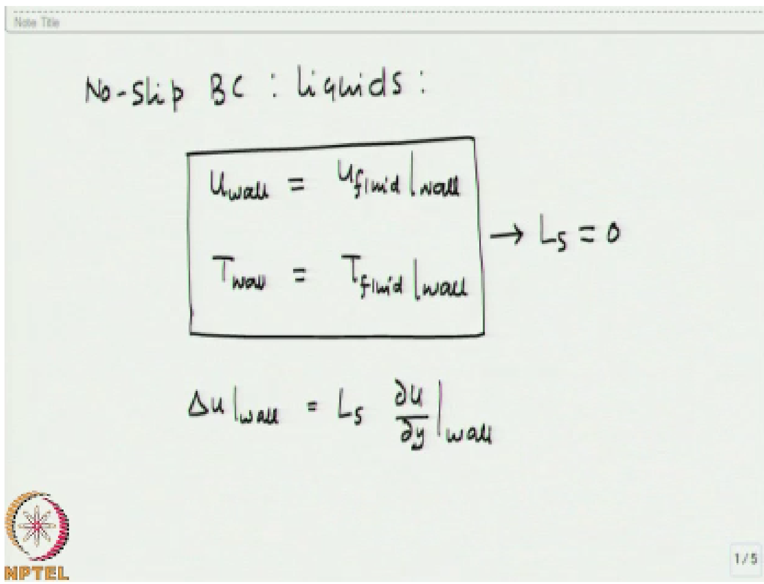

So, no slip boundary condition or liquid you can write that u at the wall will be=u fluid at the wall. So, velocity of the wall is velocity of the fluid at the wall. Similarly, T wall is temperature of the fluid at the wall. So, this is the no slip boundary condition for liquids so what does it mean it means that our slip length in this case is 0 right we can generalize we can write delta u at the wall is Ls*delta u/delta y at the wall.

This is the general equation for slip. Now the slip length is very difficult to determine slip length by experiment okay. So, there are no experimental data available however people have tried to do molecular dynamics simulation to estimate slip length okay. So, the slip length has been determined using molecular dynamic simulation by Thompson and Troian and you know they have considered a case where the fluid flows through 2 parallel plates.

Okay and one plate is moving so there is a quake flow situation and you know they have found that the slip length is going to be a property of the fluid as well as property of the solid material. Okay if you consider a case where the inter atomic distance in the wall is of the same order as the distance between the molecules then the slip length is going to be 0. Okay whereas if the inter atomic distance is about 4 times the distance between the molecules.

Then in that case the slip length could be as high as 16.8 times the intermolecular distance in the fluid.

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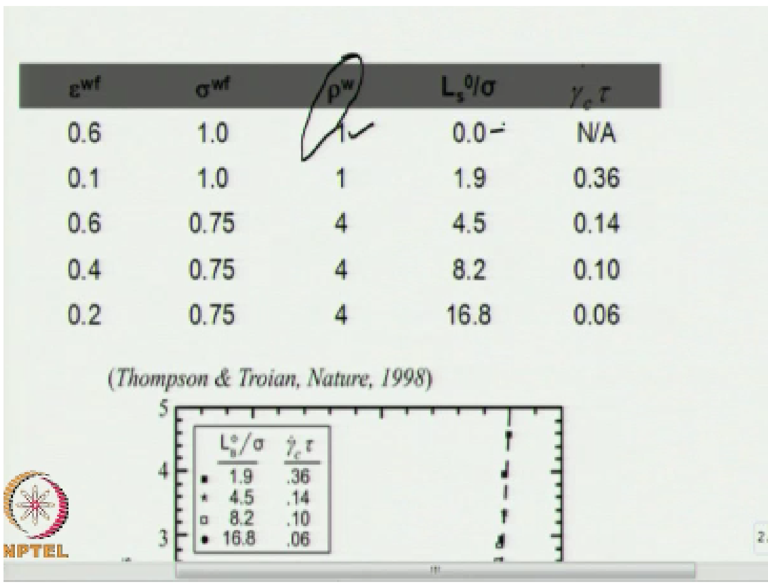

If you look at this table here this term is representing the ratio between the inter atomic distance to intermolecular distance when this ratio is 1 the slip length is 0 okay and when this ratio is becoming 4 the slip length could be as high as 16.8 times than the intermolecular distance in the fluid. Okay so that is what we learn from this molecular dynamics simulation data now we know that the slip length is not a function of the shear rate or the strain rate.

When the value of strain rate is very low ok so the slip length remains constant for a pair of solid and liquid. Okay however the change this if only if the solid or the liquid changes okay but this assumption that slip length is not a function of the strain rate or the shear rate breaks down at very large value of strain rate. So, in that case what will happen is the slip length

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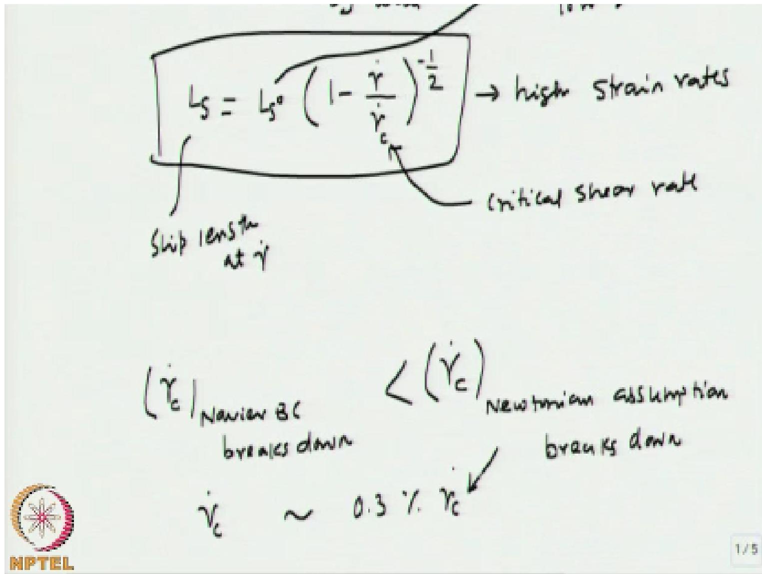

Ls can be written as Ls0*1- this is the strain rate gamma or gamma. Critical *-1/2 ok so this is how the slip in the is going to vary at high strain rates. Okay when this is going to happen when the shear rate or the strain rate is higher than the gamma critical okay. So, this is the critical shear rate this is this is going to be this is going to be the slip length at certain gamma certain strain rate and this is the slip length at low shear rates which is constant okay.

Now here we talk about a critical shear rate or a critical strain rate okay it has been found that this critical shear rate at which the you know Linear Navier boundary condition breaks down is low as compared to the shear rate at which Newtonian assumption break down. Okay so earlier we had discussed that if the shear rate exceeds twice the molecular interaction frequency then the Newtonian assumption breaks down.

Okay now here we talk about another critical shear rate at which the Linear Navier boundary condition breaks down so what we say here is that the critical shear rate at which Navier boundary conditions breaks down is less than critical strain rate at which the Newtonian assumption break down. So, typically this gamma c.is about 0.3% of the strain rate at which Newtonian assumptions would break down okay.

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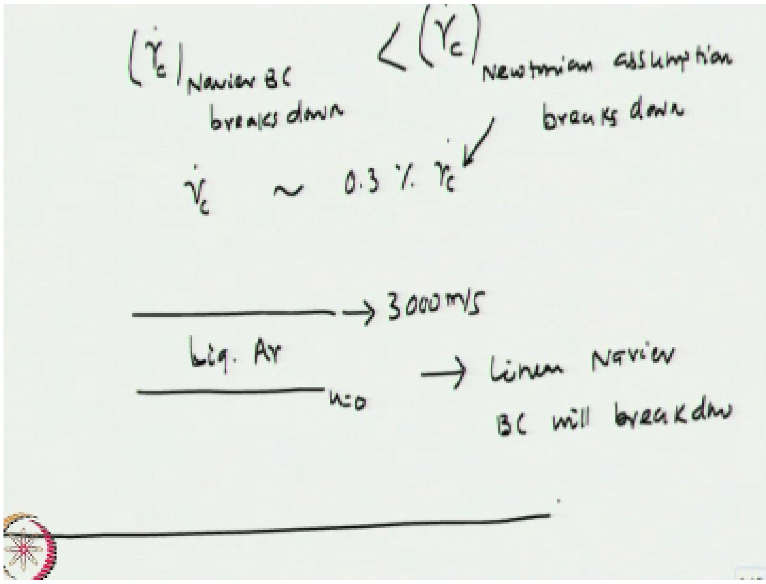

So, for example if you have you know if you consider the same example where you know we have liquid argon between 2 parallel plates and 1 is fixed. The other if it is moving at about 3000 meter per second then we would you know encounter a situation where the Linear Navier boundary condition will break downs, okay and this velocity although it is sounds high quite possible to achieve in micro scale flows okay.

So, if you have you know 2 parallel plates and 1 is stationery and the other is moving at 3000 meter per second then here we would encounter a situation where the Linear Navier boundary condition will break down. Okay so here we are considering liquid argon as the fluid right.

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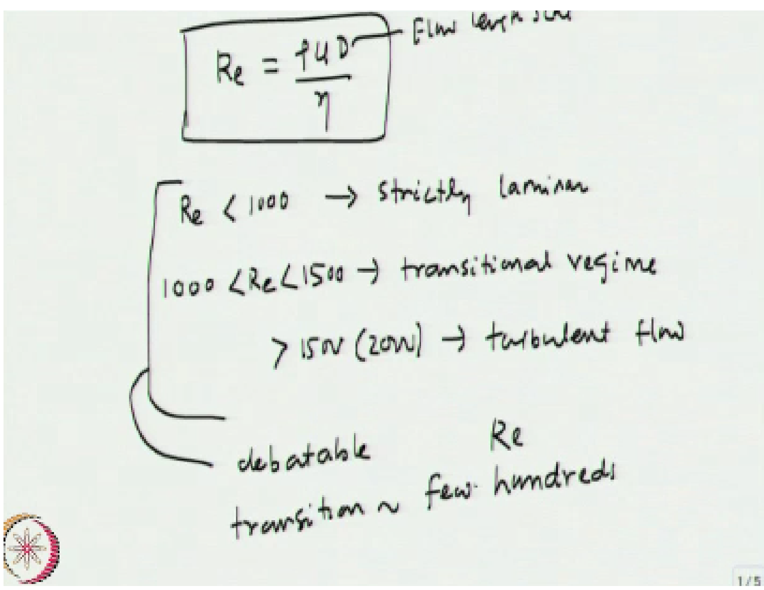

Next we move on to talk about Reynolds number okay how the transition occur from Laminar flow to Turbulent flow. So, we talk about transition to turbulence can be defined using Reynolds number which is rho ut/eta so rho is the density u is the velocity d is the flow length scale okay and eta is the viscosity. So, you know we know that for a pipe flow situation the Reynolds number is between 1000 to 1500 then the transitional flow starts to occur.

Okay so about 1000 the flow will be strictly Laminar between 1000 to 1500 the flow would try to exhibit a turbulent behavior okay and if it exceeds 1500 or in some cases 2000 then the flow will turbulent. Okay so we know that if Reynolds number is < 1500 the flow is strictly Laminar and if it is between this is 1000 between 1000 to 1500 it starts to exhibit the transitional regime and if it is > 1500 hundred in some cases 2000 and then it exhibits turbulent.

However, so this is what happens in micro scale tube flows or channel flows okay but in micro scale this critical Reynolds number or transitional Reynolds number is debatable there have been some of divisions. Where the transition occurs as low as you know a few 100s Reynolds number of a few 100 okay so this is quite debatable. Okay so transition of the order of a few 100s re of the few 100s have been observed.

So, you know more work needs to be done okay so the Reynolds number at which transition would occur in micro scale is not very clear okay and more work needs to be done. Next we talk about low Reynolds number flows as you know in microchannel flows the Reynolds number is very less okay so that has an effect on how the behavior the flow of the flow is okay the nature of the flow.

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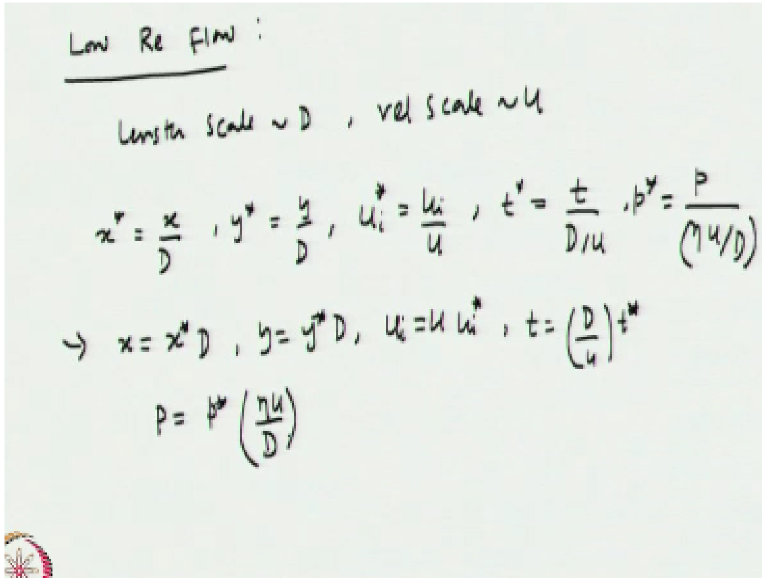

So, we talk about flow Reynolds number flow here we consider you know the length scale to be D and velocity scale to be u okay and so we consider some scaling parameters to non-dimensionalise the governing equations so we consider x star as x/D y star y/D ui star is ui/u and define time star t star which is the non-dimensional time as t/D/u and in non dimensionalise pressure p/eta u/D.

Okay now if you do inverse scaling we get x=x star d, y=y star and you get ui as ui star. And t you get D/u*t star and we get p as p star*eta u/d.

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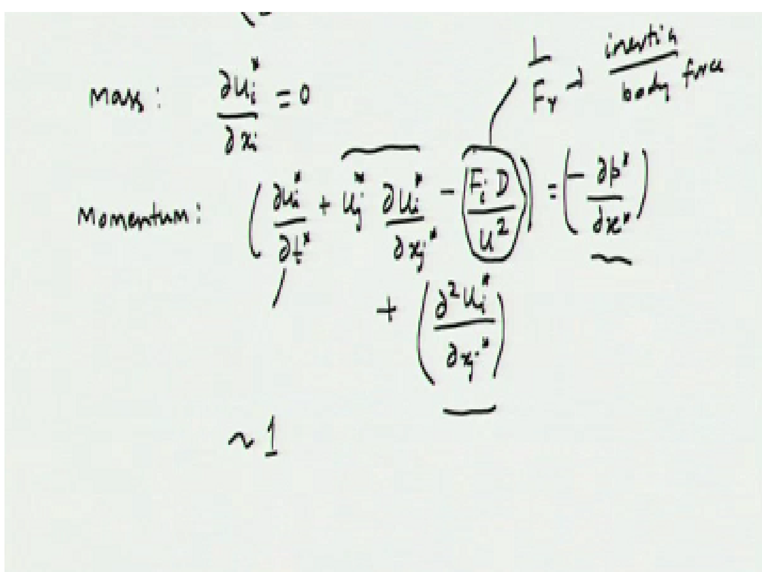

And now if you use non-dimensional parameters in the conservation of mass and the momentum we can bring this the momentum equation to a form where you know Reynolds number and Froudes number emerged as 2 important non-dimensional numbers. So, if you put in the mass conservation equation the mass conservation equation will appear like this del ui star/del xi=0. And the momentum equation will appear to us as.

Rho*del ui star/del t star+uj star del ui star/del xj star-Fi the body force * characteristic length scale/u square=-del p star/del x star+del square ui star /del xj star. So, this is you know this term is 1 over Froude number okay where Froude number is defined as inertia to body force or gravitational force. Okay so you know if you look at this equation since we are non dimensionalizing this each of these terms.

Okay this term this term this term and that term all these terms okay are of the order of 1 okay. However, you know we have so we have you sorry we know this and Reynolds number appearing here, the Reynolds number and the Froude number are you know emerging as 2 important non-dimensional parameters that would characterize the microscale flows. Now you know in microchannel flows or they said that the Reynolds number is very small. Okay so the order of 0 then so in that case the entire left inside will drop off okay.

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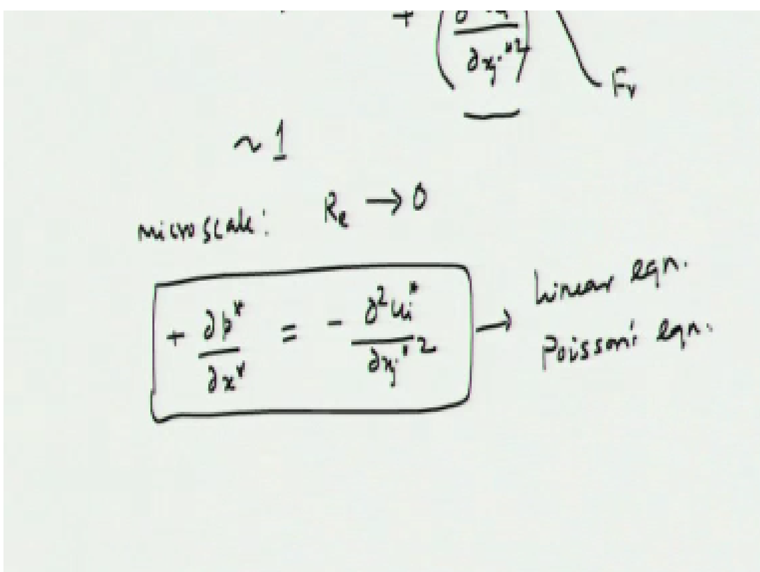

So, micro-scale the Reynolds number would turn to 0 okay. So, we will have an equation so the entire left hand side will drop off. So, what we would have here is you know –delta p star /delta x star okay=or we can make it positive del square ui star/del xj star square there is a square here okay so this is the equation that we would arrive at. So, now if the you know the pressure gradient increases by a factor a.

Then velocity at each point in the flow field will increase by a factor a. Okay so you know this is a linear equation this is a linear equation easy to solve and this is nothing but the Poisson equation okay. So, if you solve this you would see that the as the pressure gradient increases by a factor a velocity at each point in the flow field will be increasing by a factor a. So, what it means is that if you do an experiment at low Reynolds number and you obtain a flow field.

You can increase the Reynolds number of course we are talking about smaller relatively smaller Reynolds number which are you know close to 1. Then you can you know Reynolds the experiment performed at 1 Reynolds number and you have obtained the flow field is applicable to predict the behavior of the flow at different Reynolds numbers. Okay so this linear equation enables you to write that you know.

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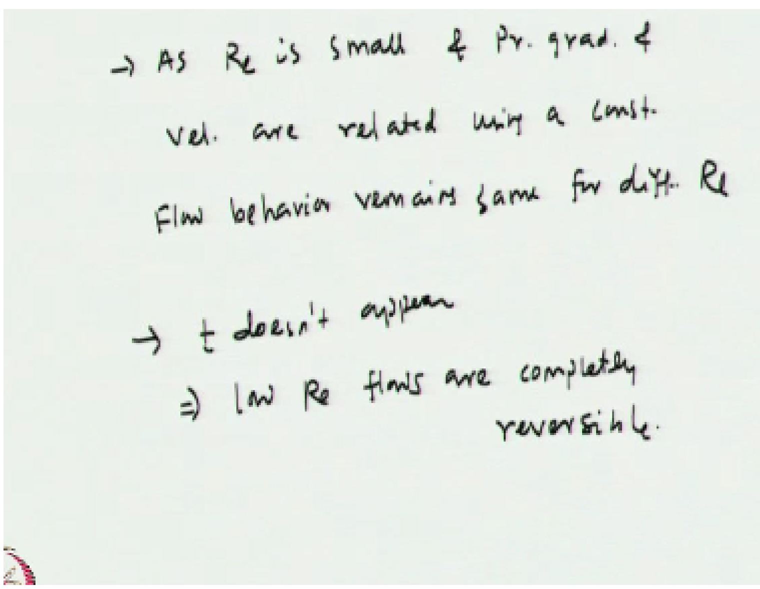

As Re it is small and the pressure gradient and velocity are related using a constant. So, you know Reynolds number in a flow behavior remains same for different Reynolds number also you can say that here in this equation you see that time does not appear as a term if okay we do not have appear as a term here. We do not have steady term here so that tells us that these low

Reynolds number flows are reversible in nature.

For example, if you are considering you know flow occurring due to oscillation of a Diaphragm the nature of the flow that you see in the forward stroke of the diaphragm is going to be exactly reverse when the diaphragm goes in the backwards stroke okay. So, this is what we see for example in case of PJ electric micro pump so the flows are exactly reversible okay so t does not this time does not appear so low re flows are completely reversible.

Okay so those are 2 important conclusions of low Reynolds number flows. Next we talk about the entrance effect at micro scale okay we talk about how the you know behavior of the fluid at a very entrance to the channel you know is appearing. So, at micro-scale you know you know that the flow is more or less uniform in both horizontal and vertical direction before into it enters a flow situation for example flow 3 tube or a pipe.

But in micro scale typically you have an inlet plenum okay and in micro fluid macro channels the height of the plenum and the height of the channel are almost the same. So, what happens is that the flow gets developed in the vertical direction whereas in the other direction it is still developing. So because the flow gets developed in the vertical direction in microchannel as compared to microscale flows.

Actual entrance length that we get in microchannel is less than what model would predict okay.
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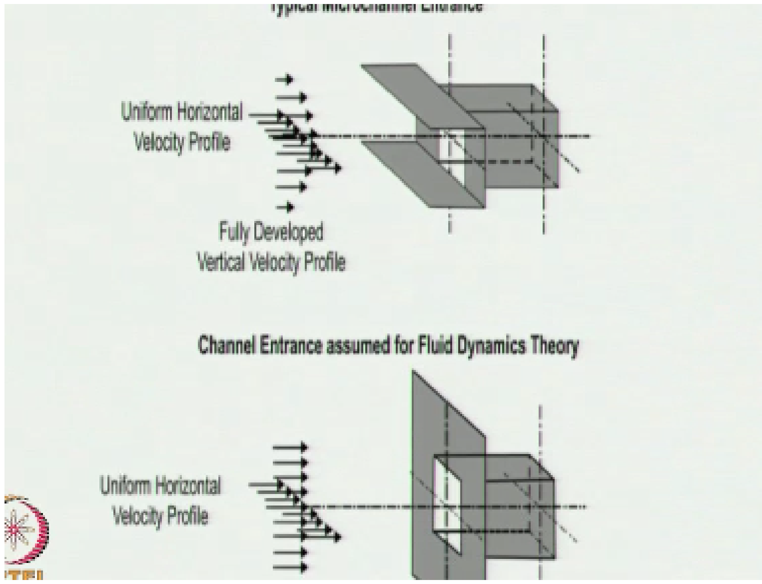

So, let us see here what is happening if you look at this plot here okay if you look at this plot this is what happens in a typical you know microchannel flow situation conventional fluid dynamics theory you have you know a bigger entrance volume before it enters this micro-scale flow channel and where you know the horizontal velocity and the vertical velocity profiles they are uniform okay if you compare that with a micro channel entrance.

Here we are talking about a plenum okay where this dimension is long as compared to the other dimension okay so the vertical dimension is very small and the height of this plenum=the height of this channel here. So, that is the reason why the flow gets developed in the vertical direction where in the horizontal direction it is still developing okay so you know in typically in a microchannel.

We will see that when the flow comes from the inlet flow and gets into you know the inlet plenum.

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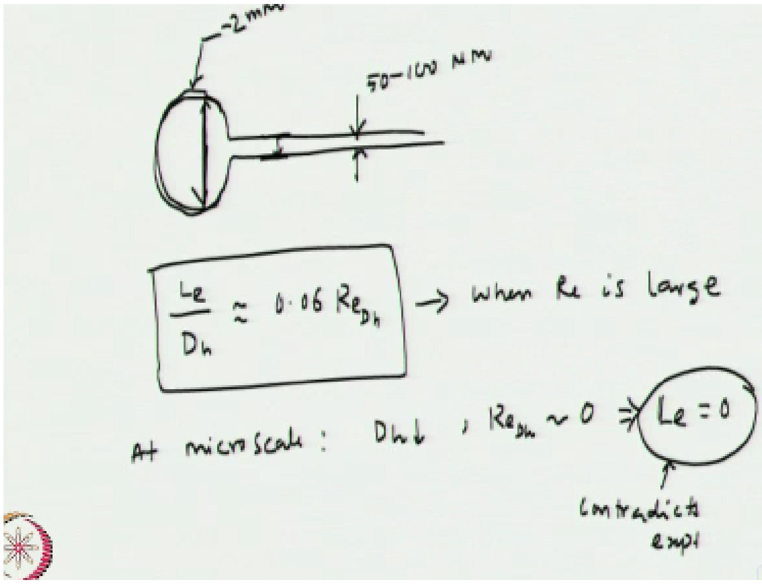

Which is typically in a rounded shape and then the flow gets into the microchannel and these are typically of the order of you know a few millimeter and these are of the order of the say 50 to 100 microns right. So, the height here in this plenum and here in the channel area they are the same but if you look at the other direction this is large as compared to here. So, in this direction in the horizontal plane the flow is uniform.

Whereas in the vertical plane the flow is developed okay so that is the reason why the entrance length predicted by model is actually higher compared to what we actually measure experimentally. So, you know if you want to maybe you know theoretically predict what the entrance length would be there are different correlations that are available so one such correlation is entrance length Le.

Le/Dh is going to be 0.6*Reynolds number based on the hydraulic diameter okay. So, here this is you know valid for when Reynolds number is large which is a case in micro scale flow situations. Now at micro scale at micro scale our hydraulic diameter will be very small and the Reynolds number based on the hydraulic diameter will be small. So, what this equation would give is entrance length will be 0 but this is not what is observed.

So, this contradicts our experiment that have been performed contradicts experiment. So, the conclusion is that we cannot use this relation and there have been you know different other correlations developed for predicting entrance length in microchannel.

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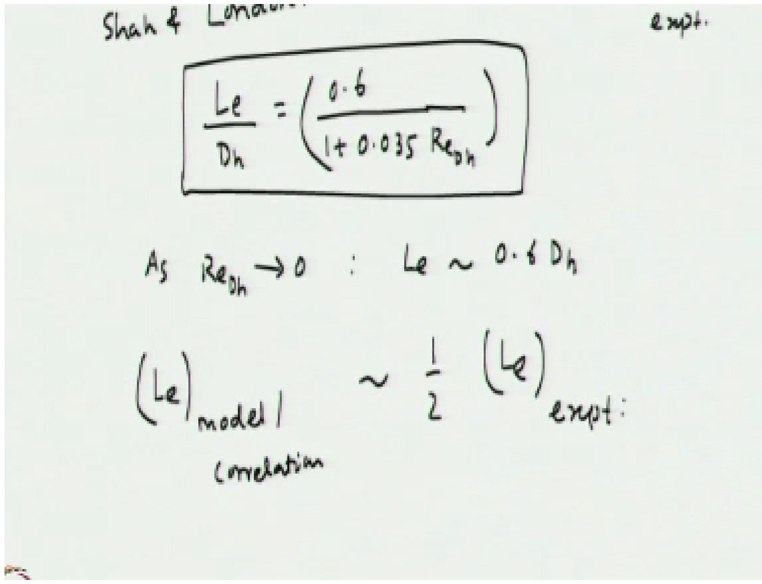

One such correlation has been developed by Shah and London which tells that the entrance length Le/Dh is 0.6/1+0.035*ReDh. Now here you could see if the Reynolds number goes to 0 as ReDh turns to 0. Your entrance length will scale as 0.6* Dh. So, this correlation can be used to predict the entrance length in microscale flow situation. However, it has been observed that the entrance length predicted by model.

Okay or this correlation is about 1/2 entrance length predicted from this experiment.

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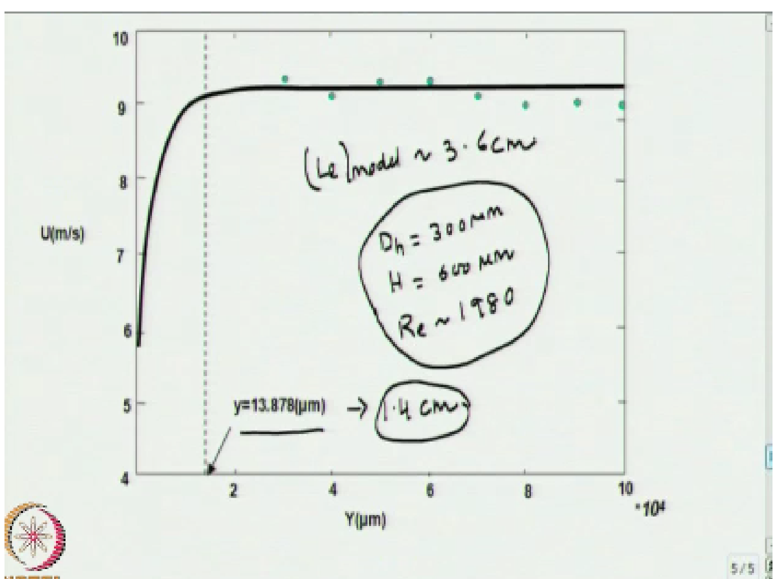

So, if you look at this plot here this is an experiment which was performed at Purdue University for a channel of Hydraulic diameter you know dh of the channel was about 300 micron. And the height of the channel was 600 micron and the Reynolds number was typically 1980.So, in that condition this was this experiment was performed and here we see that the entrance length that was predicted by the experiment is about you know 1.4 centimeter.

Okay these many microns this is about you know 1.4 centimeter whereas you know the model that we see here okay this model predicts an entrance length you know Le from the model is about 3.6 centimeter. Okay so there is a difference between the entrance length predicted by the model and that is measured from the experiment. So, entrance length predicted by experiment is about 1/2 of what is predicted from the model.

This has important consequence in you know micro heat exchangers and as well as in biomedical devices for example in micro heat exchangers we are talking about channel length typically you know in millimeter to centimeter. So, the entrance length that is found experimentally or calculated from the model is not very different compared to the overall length of the channel okay.

And as you know that the heat transfer characteristics in the entrance region is different from that or that of the fully developed region. So, it will have some consequence in case of micro heat exchanger design. If we are talking about biomedical devices you know as we know the shear stress that is found in entrance region is different as compared to that is found in the fully developed region and for example in 1 biomedical application.

For example, you are trying to you know bind 1 bio molecule to some antibody in the wall in that case shear stress that is acting on the molecule becomes very important and as you know the shear stress will be different in entrance to fully developed knowing understanding what exactly the entrance length is will affect the design of such a biomedical device. Okay so this is where we move on to our next topic.

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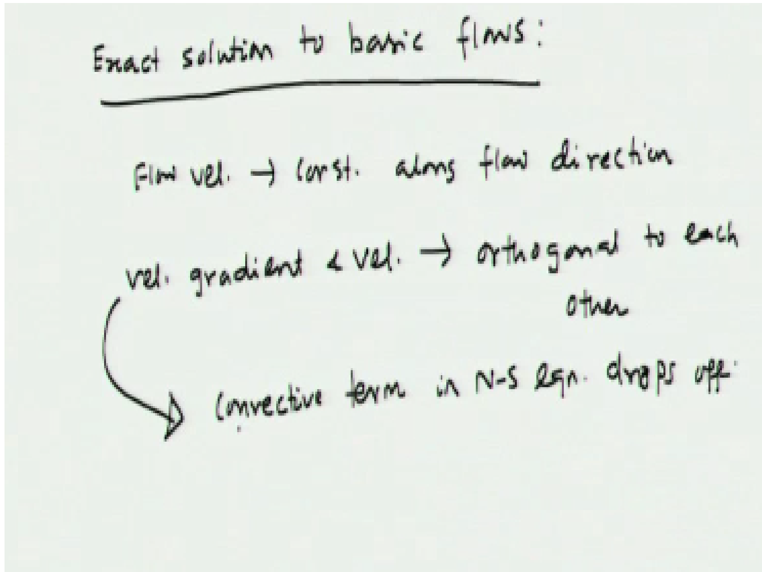

Where we talk about you know some solutions exact solutions to some basic flow situations. So, we talk about exact solution to basic flows you know in microfluidics in many situations we have long straight micro channels where you know the perception does not change over the length. So, here we will talk about a few simple you know flow solutions. We will talk about weight flow, partially flow.

You know both steady as well as we will take one case where the transient flow situation you know will be solved and well see how the solution evolves over time. But in such cases will say that the flow is not accelerating in the direction okay so the velocity remains constant along the flow direction. Okay so what happens is the gradient of the velocity and the velocity component itself become orthogonal to each other.

So, the convective term in the Navier Stokes situation drops out okay. So, as a in a way to simplify the equations we could say that the flow velocity remains constant along flow direction. So, the velocity gradient and velocity become orthogonal to each other. So, what to do least to is the convective term okay in Navier Stokes equation drops. Okay and you would assume here that the fluid is Newtonian would assume that it is isothermal.

And you know in most of the flow situations we assume that the flow is steady but we will also look at 1 or 2 transient flow situations and we would say that you know the flow is fully developed in a range that we will be considering.

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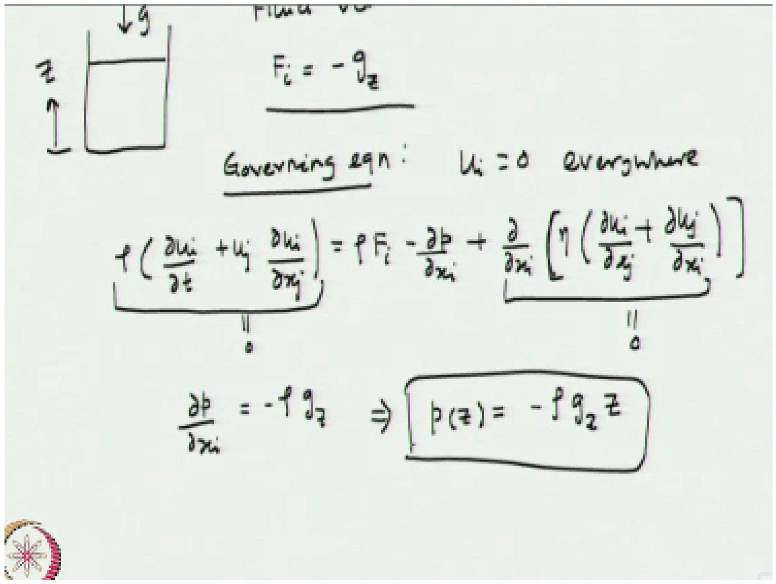

So, the first example we will take is the hydro static pressure this will find for a fluid in mechanical equilibrium. So, this is the simplest case possible where we will try to derive ore we can find out the hydrostatic pressure for a static liquid in a container okay from Navier Stokes equation here the fluid is stationery so the velocity will be 0 and the only terms that will exist is the pressure term and the body force term okay.

So, let us consider you know fluid present in a container and we will say that g is acting in this direction and z is in this direction and here the fluid velocity is 0 velocity is 0 and so the body force Fi is going to be –g acting in the z direction. So, if we write down the governing equation okay the first is the velocity will be 0 everywhere we said because we first you know the case that we consider is we are considering that the fluid is in mechanical equilibrium.

That means the flow velocity at every point is going to vanish if it does not then the fluid will move and it will dissipate energy continuously because of the viscous interactions and since it is in mechanical equilibrium and it is dissipating energy that is not possible. So, the velocity has to vanish okay and next we talk about the momentum equation which is rho*del ui/del t+uj del ui/del xj=rho Fi-del p/del xi+del/del xi*eta del ui/del xj+del uj/del xi .

So, that is the momentum equation the velocity is 0 also all this terms will be 0 is terms okay these terms will be 0. So we are left with only the pressure and the body force term. So we can write delta p/del xi-will be=rho gz- because the Fi is –gz. So, if you integrate you get p z will be-rho*gz*Z. So, this is nothing but our hydro static pressure equation where you know pressure varies linearly with the hydro static height.

Now here we have made one important assumption we have assumed that the density and the pressure and density does not change with the pressure but actually pressure and then density they are related.

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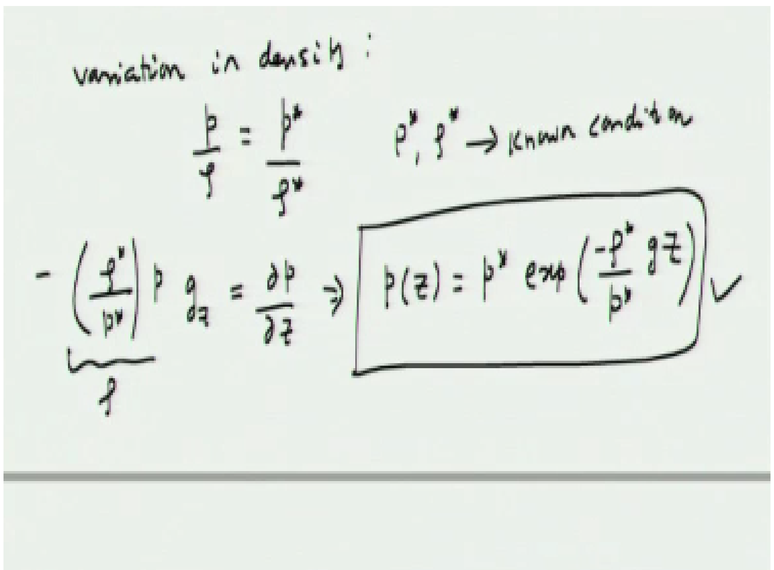

Okay so if you consider variation in density and in density we can write that p/rho=p star /rho star, so p star and rho star are pressure in density at some known conditions. Certain known conditions let us say p star and rho star are pressure and density. So, you know here this equation we can write as rho star/p star*p is the rho, this is the term, this is nothing but rho*gz-of that=delta p/delta z.

So, this is delta z, so we can solve pz=p star* exponential-rho star/p star *g*z. So, that is when you are considering variation in the density with the pressure okay. So, this is what you would arrive at now you can use this hydrostatic pressure equation to calculate what the height of the atmosphere is okay. Because you know the pressure condition at the decibel and from there you can find what would be the height of the atmosphere that would lead to that much pressure okay.

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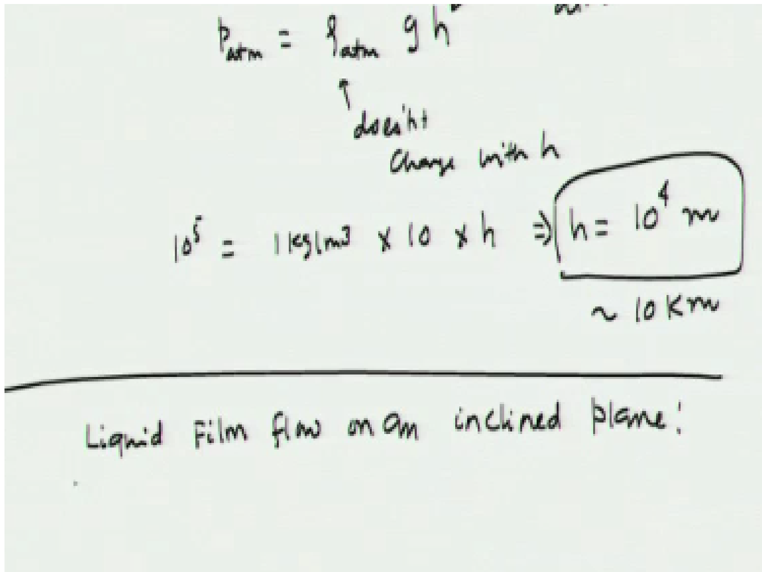

For example, here you can say that you know p atmospheric you know that is about 10 to the power 5 pascal right and so p atmospheric is rho atmospheric*g*h, h is the height of the atmosphere and we say that let us say for example this does not with h. Let us just try to scale it to see how the height of the atmospheric looks like, so 10 to the power 5 is going to be this is about 1 kg per meter cube*10*h, so h is about 10 to the power 4 meter okay.

So, this is about 10 kilometer okay so we can find out that the height of the atmosphere is about 10 kilometers and next we consider a case where a thin layer of liquid is falling along an inclined plane. So, we talk about a case of liquid will flow on an inclined plane.

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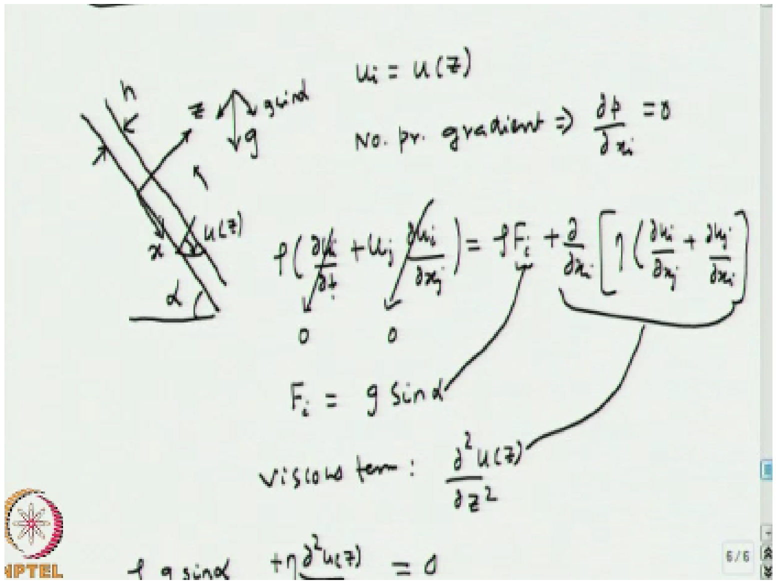

So, here let us consider an inclined plane something like this. Let us say this angle is alpha and we say this direction here is x and this direction is z. So, we have a thin layer of fluid okay and the velocity will vary something like this. At some point velocities uz okay and let us call this thickness to be h okay and we say that gravity is in this direction okay. So, we can write that ui is going to be a function of z.

Since we are talking about you know uni directional flow here and also there is pressure gradient the fluid flow is occurring because of gravity. So, no pressure variant so that means the delta p/del xi is going to be 0. So if you talk about the Navier Stokes equation rho*del ui/del t+uj del ui/del xj it will be=rho Fi+del/del xi*eta del ui/del xj+del uj/del xi. So, the convective term will be 0.

Because that is the assumption that we have made in the beginning a steady flow so this is going to be 0 and the body force Fi is going to be g sin alpha because it is going to be the in this direction so it is going to be g sin alpha. So, here your viscous term will have the gradient only in the z direction. So, this will be del square uz/del z square. Okay, so this is what this term would be and Fi is going to be this.

So, you can write rho*g sin alpha+del square uz/del z square will be=o right sorry you would have eta here missing. So, basically you would have viscous force coming in this direction and the body force coming in this direction. So, they are balancing each other so the boundary conditions you can write the boundary conditions one is u at o, z=0 is 0 that is the no slip boundary condition.

And the other boundary condition is going to be since we are talking about a free surface this is a free surface okay.

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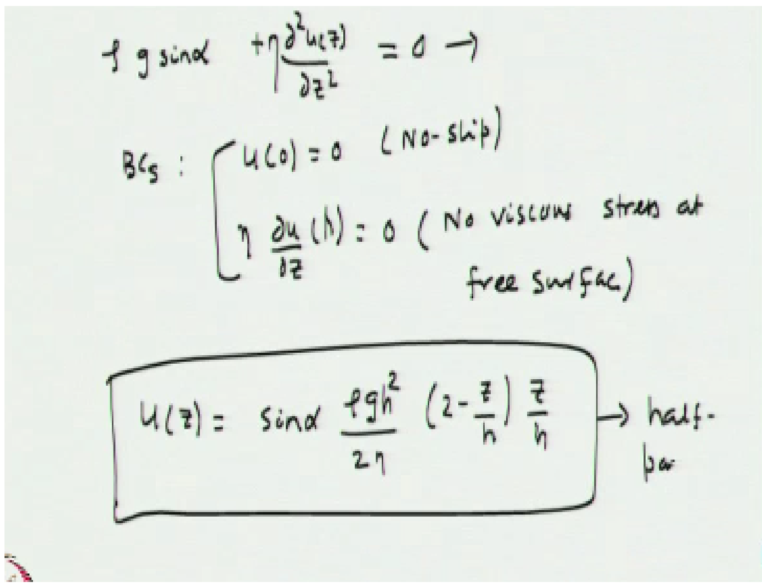

The free surface the shear stress is going to vanish so eta*delu/delz at height h=o. So, this is no viscous stress at free surface. Okay no viscous stress at free surface now if you apply these 2 boundary condition to this equation you get this solution uz=sin alpha*rho gh square/2 eta*2-z/h*z/h. So, this is nothing but an equation of half parabola and this satisfies the boundary conditions the no slip boundary condition is satisfied.

If you put z=0 then the velocity is going to be 0 and when z=h the gradient of the velocity is going to vanish okay. So, the shear stress at the free service is going to be 0 if you draw the velocity profile.

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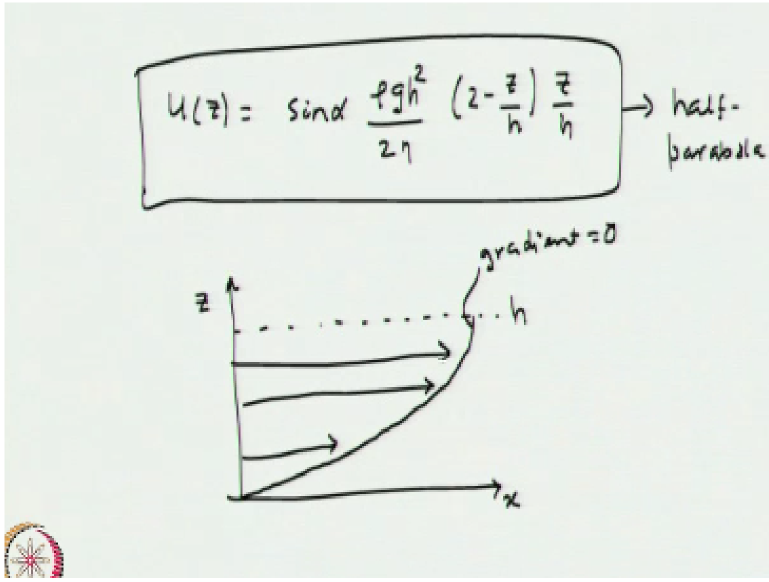

So, this is z this is x along the flow direction in the inclined plane and this is perpendicular to the inclined plane. And let us say we are talking about this as the film height okay this is the film height. So, the velocity is going to look like this like a half parabola and right at the surface it is going to be okay here the gradient is going to vanish the gradient=0 and this is the general velocity how it is going to vary along h.

And it typical you know film of 100 micron thickness will have a velocity of the order of one centimeter per second. So, that is the type of velocity you can expect from a thin liquid film. So, we stop here.