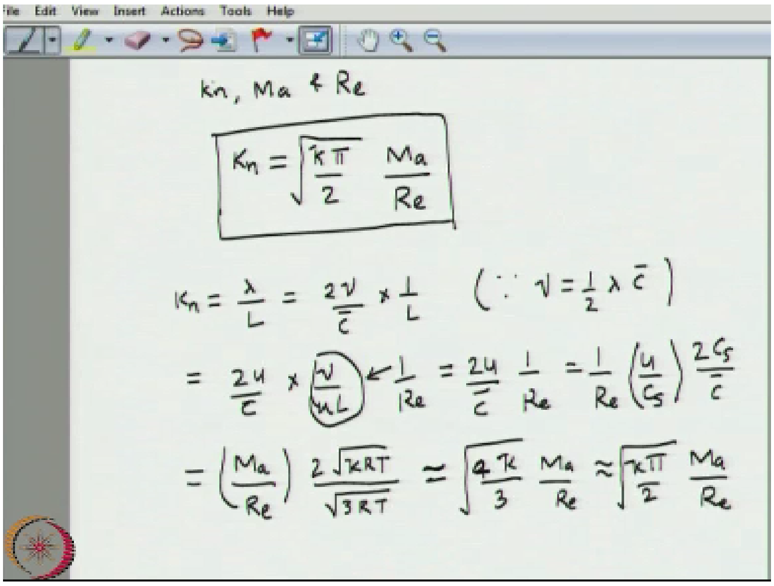
Microfluidics

Dr. Ashis Kumar Sen
Department of Mechanical Engineering
Indian Institute of Technology – Madras

Lecture - 04
Micro-Scale Fluid Mechanics (Continued)

The 3 dimensionless parameters the Knudsen number, mach number and Reynolds number can be related as follows.

(Refer Slide Time: 00:24)



Kn, Ma & Re

$$\boxed{K_n = \sqrt{\frac{k\pi}{2}}\,\frac{M_a}{R_e}}$$

$$K_n = \frac{\lambda}{L} = \frac{2\nu}{\bar{c}} \times \frac{1}{L} \qquad \left(\because \nu = \frac{1}{2}\lambda\bar{c}\right)$$

$$= \frac{2u}{\bar{c}} \times \boxed{\frac{\nu}{uL}} \leftarrow \frac{1}{R_e} = \frac{2u}{\bar{c}}\,\frac{1}{R_e} = \frac{1}{R_e}\left(\frac{u}{C_s}\right)\frac{2C_s}{\bar{c}}$$

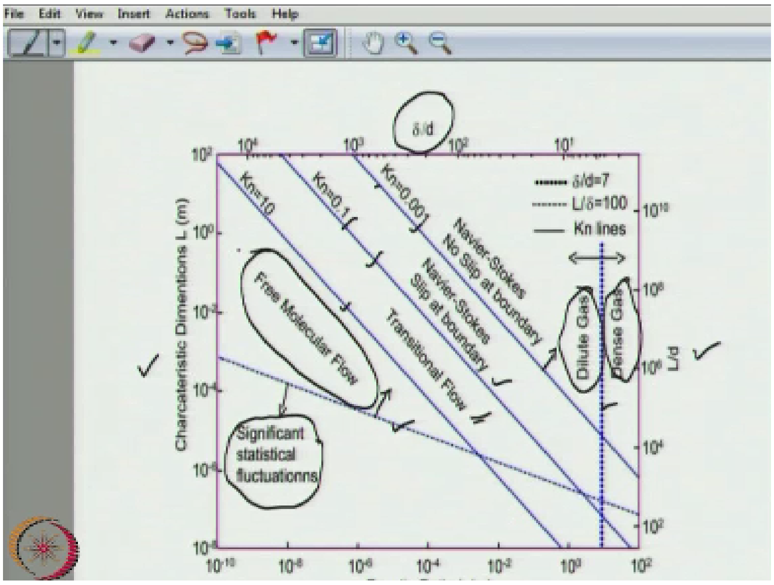
$$= \left(\frac{M_a}{R_e}\right)\frac{2\sqrt{kRT}}{\sqrt{3RT}} \simeq \sqrt{\frac{4k}{3}}\,\frac{M_a}{R_e} \approx \sqrt{\frac{k\pi}{2}}\,\frac{M_a}{R_e}$$

So here we would like to relate Knudsen number, mach number and Reynolds numbers they are related as follows. So Knudsen number is k which is the specific heat ratio pi/2 square root* mach number/Reynolds number. So this is how the 3 different dimensionless parameters are related. We can in fact mathematically show this we can start from the left hand side, we can write Knudsen number=lambda/L. Lambda is the mean free path and L is the length scale of the flow.

We can write lambda as 1/2, we can write 2 nu/c bar * 1/L because we know that nu is ½ lambda * c bar. This is what we know. So we can write 2 u/c * nu/uL. This is C bar. So this is nothing but this term is 1/Reynolds number. So we can write 2 u/c bar * 1/Reynolds number. Now we can write this as 1/Reynolds number *u/Cs * 2 Cs/c bar. And we can also write this as u/Cs is the mach number. So this is mach number/Reynolds number * 2 Cs is square root of k RT K is the specific heat ratio k RT/3 RT.

So this will be written as 4k/3 square root * mach number/Reynolds number. This can be approximated as k pi/2 * mach number/Reynolds number. So this is how we could prove the relationship between 3 different non dimensional parameter. The Knudsen number, mach number and Reynolds numbers.

(Refer Slide Time: 04:00)



So let us look at this plot that shows how this 3 numbers are related. It also gives you an overview of what happens to the flows at micro-scale. In the x axis we have the density ratio n/n0 where n0 is the density at standard conditions standard temperature and pressure. On the y axis here you have the characteristic dimensions L in meter. On the right hand side you have L/d ratio L is the characteristics length scale and d is the molecule diameter.

On the top you have delta/d delta is the spacing between molecules and d is the diameter of the molecular. Now below if you look at this dotted line here this dotted line on the left hand side the gas is considered as dilute gas where the delta/d ratio is >1. Delta being the separation (()) (05:13) giving the size of the molecule is>1 so it is called dilute gas and on the right hand side we have dense gas here is the dense gas.

If you look at this dotted line here above this line the continuum theory is valid. In the earlier we looked at the continuum theory we said that for liquid if the characteristics length scale is> 10 nanometer we can say that the flow is continuum. And if the gas if the characteristics size is> 1 micron we can consider flow to be continuum where do we the flow to be continuum first we consider that if we consider the point properties.

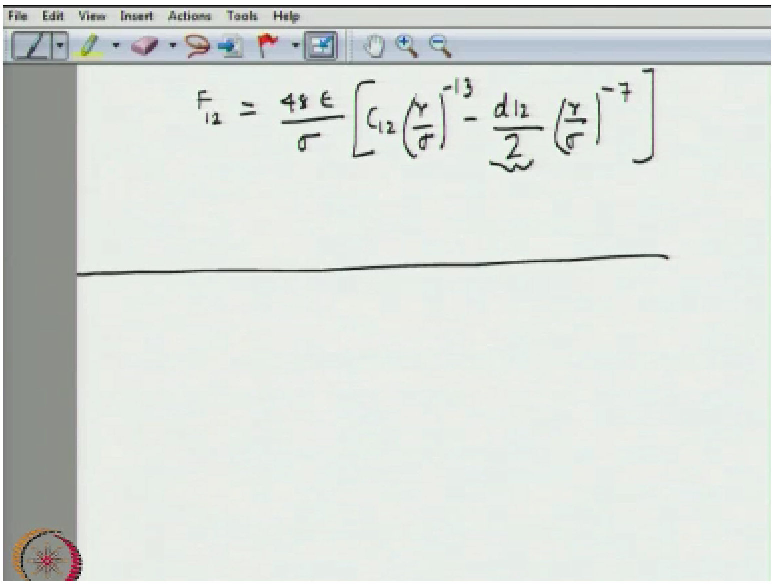
We told that let us assume they are 10 to the power 4 molecules in a cubic space, cubic control volume. There is a sampling volume that we talk about and for both liquid and gas we know the number density. So we can find out what should be the characteristics length scale of that point volume the cubic volume so that the fluid will be continuum and for the gas we found that the characteristics length scale is about 17 nanometer and for liquid we found that this should be around 10 nanometer.

Then we looked at the transport properties and looking at the transport properties we saw that for liquid the characteristics distance should be around 1 micron and for gas the characteristics length scale should be 1 micron above which it can be considered continuum. And for liquid the characteristics length scale should be above 10 nanometer. So above this dotted line it satisfies that the flow can be treated as continuum above this line and below this line we consider significant statistical fluctuations.

Now if you look at these lines these are called Knudsen number lines these 3 lines. Now above this line we have the Navier Stokes Equations applicable and the no slip at the boundary can be applied. So in which case we have Knudsen number<0.001 between Knudsen number 0.001 and Knudsen number 0.1 we have Navier Stokes Equation applicable, but with a slip at the boundary.

And between 0.1 and 10 we have transitional flow where we cannot use the Navier Stokes Equation that we normally use it needs to be modified. And if the Knudsen number is>10 we get into free molecular flow. So that is what we learnt from this plot. Now before we go ahead I would like to correct something earlier we were talking about the force between 2 molecules and there was a small corrections that needs to be made here.

(Refer Slide Time: 08:55)

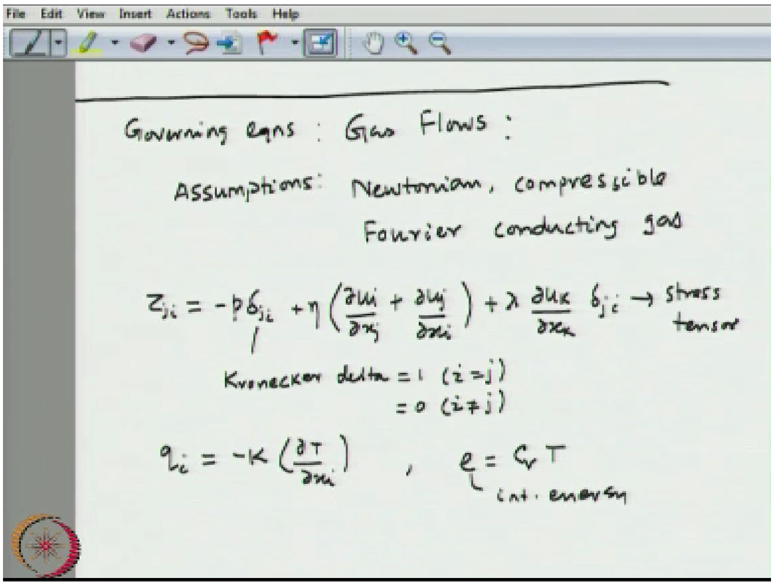


$$F_{12} = \frac{48\epsilon}{\sigma}\left[c_{12}\left(\frac{r}{\sigma}\right)^{-13} - \frac{d_{12}}{\underset{\sim}{2}}\left(\frac{r}{\sigma}\right)^{-7}\right]$$

So the force expression for the intermolecular force between 2 molecules 1 and 2 can be correctly written as this is 48*epsilon/sigma *c12* R/sigma to the power -13-d 12/2* r/sigma to the power -7. So this 2 was missing earlier when we are trying to write the expression for the force the factor of 2 was missing so that needs to be corrected. Now moving ahead let us talk about the governing equation for the gas flows. We have talked about the kinetic theory of gases where we can relate different parameters, different constant to understand how gas is behaved.

So let us look at the governing equations for the gas flows and we make some assumptions. We assume that the gas is compressible and we assume that the fluid to be Newtonian and we say that the Fourier law of heat conduction is valid. So under those assumptions we write down the governing equations for gas flows.

(Refer Slide Time: 10:30)



Governing eqns : Gas Flows :

Assumptions: Newtonian, compressible
Fourier conducting gas

$$\tau_{ji} = -p\delta_{ji} + \eta\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) + \lambda \frac{\partial u_k}{\partial x_k}\delta_{ji} \rightarrow \text{stress tensor}$$

Kronecker delta $= 1 \ (i = j)$
$= 0 \ (i \neq j)$

$$q_i = -K\left(\frac{\partial T}{\partial x_i}\right) \quad , \quad \underset{\text{int. energy}}{e} = C_v T$$

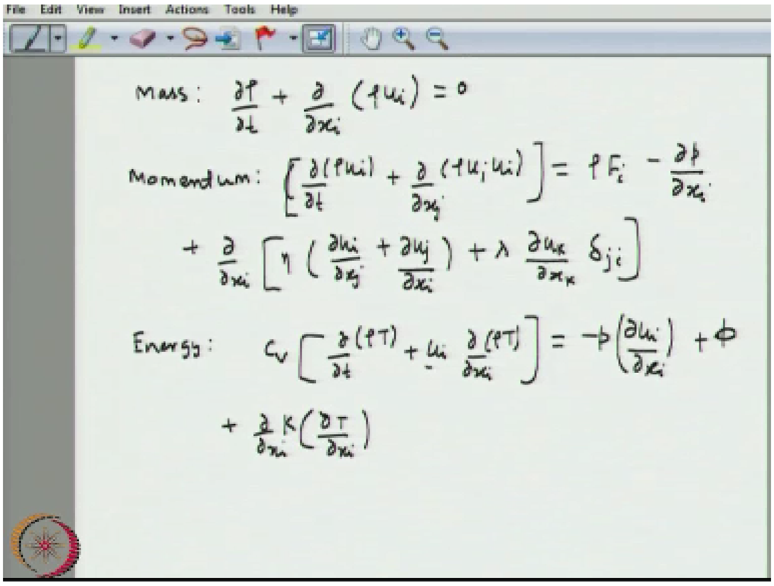
So we write down the governing equations for gas flows and we make some assumptions. So the assumptions are that the fluid is Newtonian and it could be compressible. And the Fourier conducting gas. So under those assumptions, we can write the conservation of mass, momentum and energy. Before we do that as we discussed yesterday we were talking about 5 differential equations 3 for momentum, 1 for mass, 1 for energy so it is 17 unknowns so it is not possible to solve those equation unless we come up with constitutive relationship.

And this constitutive relationship they can only be postulated they cannot be proved. So what are those constitutive relationship we relate the stress tensor to the velocity field the heat flux to the temperature field and we related the energy with temperature. So we write down the stress tensor tau ji. So that can be written as -p* delta ji+ eta * delta ui/delta xj+ delta uj/delta xi+ lambda delta uk/delta xk *delta ji. So this is the stress tensor.

We know what p is P is pressure this is called Kronecker delta function and this will be 1 only when i=j and will be 0 if I will be not=j. Eta is viscosity and we know all other parameters here. Similarly, we can write down the heat flux qi can be –k* delta T/del xi and the third constitutive relationship will be energy e will be Cv* T specific (()) (13:37) constant volume * temperature and this is the internal energy.

And if you use this constitutive relationship into the mass, momentum and energy equations.

(Refer Slide Time: 14:00)



Mass: $\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_i}(\rho u_i) = 0$

Momentum: $\left[\frac{\partial(\rho u_i)}{\partial t} + \frac{\partial}{\partial x_j}(\rho u_j u_i)\right] = \rho F_i - \frac{\partial p}{\partial x_i}$

$$+ \frac{\partial}{\partial x_i}\left[\eta\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) + \lambda \frac{\partial u_k}{\partial x_k}\delta_{ji}\right]$$

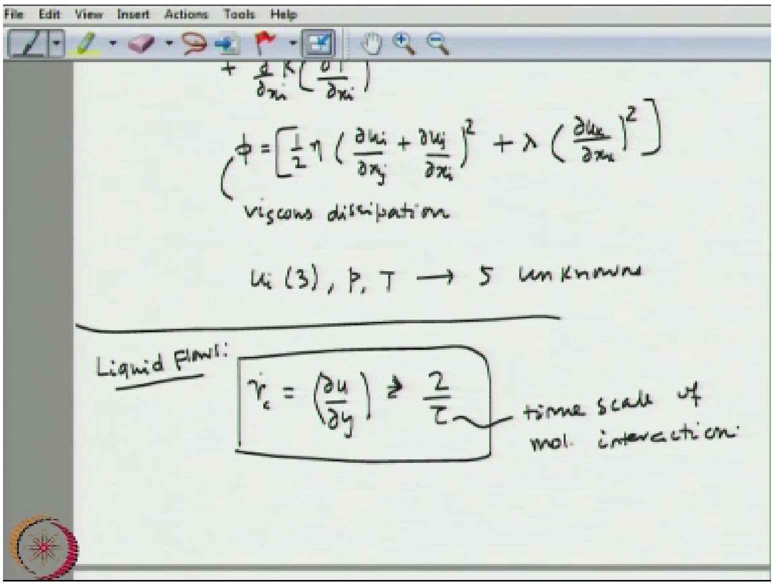
Energy: $c_v\left[\frac{\partial(\rho T)}{\partial t} + \underline{u_i}\frac{\partial(\rho T)}{\partial x_i}\right] = -p\left(\frac{\partial u_i}{\partial x_i}\right) + \phi$

$$+ \frac{\partial}{\partial x_i} k\left(\frac{\partial T}{\partial x_i}\right)$$

We can write the conservation of mass as follows. We can write it as del rho/del t+ del/del xi* rho ui to be 0 and we can write the momentum equation. Momentum equation will have rho we have rho will be inside so we will say del/del t* rho ui+ del/del xj* rho uj* ui that will be= rho * Fi-del p/del xi+ del/del xi* eta del ui/del xj+ del uj/del xi+ lambda *del uk/del xk* delta ji. So this is the momentum equation.

Similarly, the energy equation can be written the energy equation looks like this rho Cv again the rho has to go inside so del/del t* rho T+ ui* del rho T/del xi will be= -p* del ui/del xi+ phi is the viscous dissipation +del/del xi * k del T/del xi.

(Refer Slide Time: 16:26)



$$+ \frac{\partial}{\partial x_i} k\left(\frac{\partial T}{\partial x_i}\right)$$

$$\phi = \left[\frac{1}{2}\eta\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)^2 + \lambda\left(\frac{\partial u_k}{\partial x_k}\right)^2\right]$$

viscous dissipation

u_i (3), p, T → 5 unknowns

Liquid flows:

$$\boxed{\dot{\gamma}_c = \left(\frac{\partial u}{\partial y}\right) \geq \frac{2}{\tau}}$$

time scale of mol. interaction

Where the viscous dissipation phi can be written as follows where phi will be= ½ eta * del ui/ delxj+ del uj/del xi square+ lambda * del uk/del xk square and this is called the viscous dissipation. So now we have 5 equations and 5 unknowns what are those unknowns. Our unknowns are so we have 3 ui and we have pressure and temperature. So these are 5 unknowns so we have 5 equations so we can solve that.

So after that let us move on talk about liquid flows. Liquids flows in micros-scale is more complex is compared to the gas flows because the molecular theory for liquid flows is not very well developed. So there is no parameter equivalent to Knudsen number for gas flows in liquid flows. So the liquid flow is not very well understood and for example if you consider viscosity of a fluid when a fluid is passing through a micro channel.

It has been observed that in some cases the viscosity in the micro-channel a part in viscosity is same as the viscosity of the fluid for example in a micro-scale flow situation. In some cases, it has been found that the apparent viscosity in micro-scale flows is> the actual viscosity in some other cases it has been found that it may be less than the actual velocity so this is quite debatable.

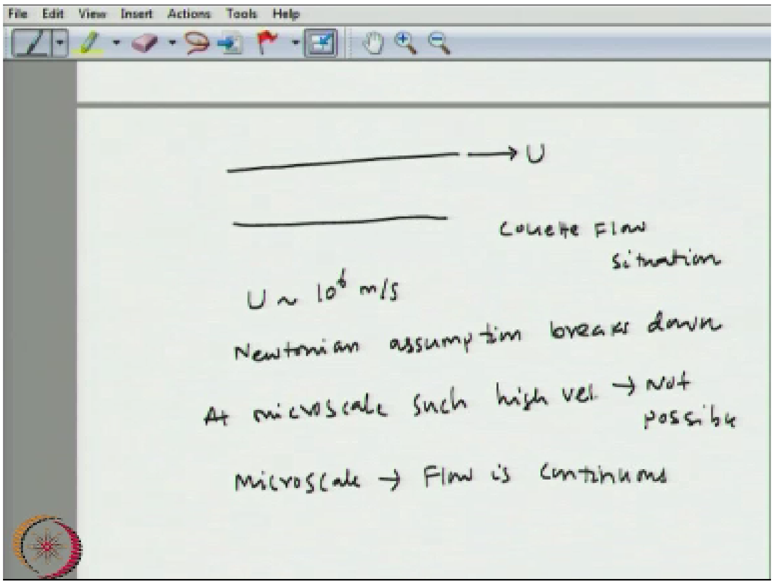
That is because the liquid flows in micro channels the molecular theory is not very well understood and the other important thing is that we know that if the length scale of the flow is larger than 10 nanometer we can consider the flows to be continuum, but whether we can consider the fluid to be continuum, but whether to consider the flow to be continuum is not very clear.

So and if you consider another example a fluid flowing through a relatively larger length scale can be considered as Newtonian the same fluid when it flows through a micro channel and the shear rate is too much above a critical value then the Newtonian assumption breakdown. When it is shear too much the Newtonian fluid assumption will break down. There is a critical value of strain which can be written as the strain rate gamma c dot which is= delta u/delta y.

So here we are talking about liquid flows so delta u/del y is 2/tau when it is> or = 2/tau and tau being the time scale of molecular interaction then the assumption that a fluid is Newtonian that breaks down. So if let us say between 2 parallel plates if you have a fluid flowing and we assume a Couette flow situation where one of the plates is stationery and the other plate is moving at a speed of 10 to the power 6 meter per second.

Then in that situation a Newtonian fluid will behave as a non Newtonian fluid. So the Newtonian assumption is going to break down. So that happens when the critical standard is twice the molecular interaction frequency.

(Refer Slide Time: 21:55)



→ U

Couette Flow situation

$U \sim 10^6$ m/s

Newtonian assumption breaks down

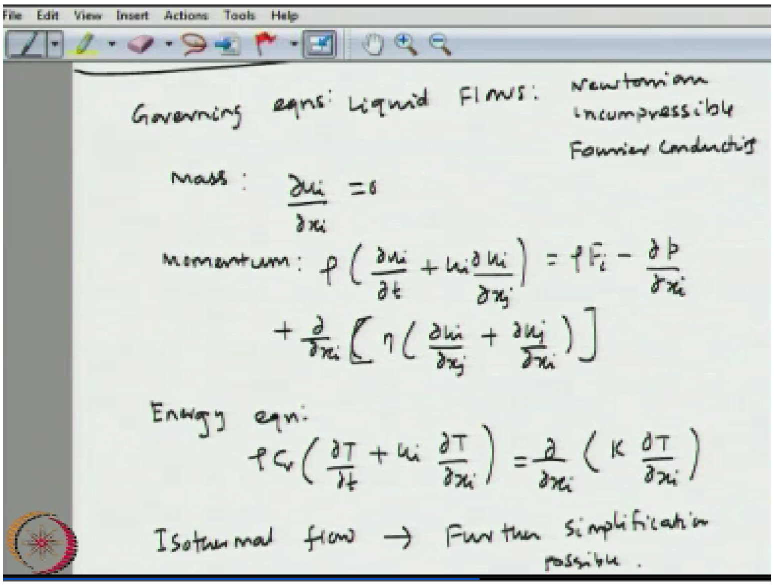
At microscale such high vel. → not possible

Microscale → Flow is continuous

So if we consider 2 parallel plates and 1 is moving at velocity u and say it is a Couette flow situation. When u is the order of 10 to the power 6 meter per second. Then the Newtonian assumption breaks down, but such speed is not possible at micro-scale to achieve a velocity of 10 to the power 6 meter per second is not possible at micro scale. So the conclusion is that for most micro scale we can assume the fluid to be continuous.

So at micro scale such high velocity not possible. So the conclusion is that micro-scale flow is continuous. Now having talked about you know the liquid flows let us try to write down the governing equations for liquid flows and in that case you can assume that the fluid is Newtonian. The density for the liquid does not vary that much we can assume it to be incompressible and we can also assume that the Fourier law of heat conduction is valid.

(Refer Slide Time: 23:57)



Governing eqns: Liquid Flows: Newtonian
Incompressible
Fourier conducting

Mass: $\frac{\partial u_i}{\partial x_i} = 0$

Momentum: $\rho\left(\frac{\partial u_i}{\partial t} + u_i\frac{\partial u_i}{\partial x_j}\right) = \rho F_i - \frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_i}\left[\eta\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)\right]$

Energy eqn:

$$\rho C_v\left(\frac{\partial T}{\partial t} + u_i\frac{\partial T}{\partial x_i}\right) = \frac{\partial}{\partial x_i}\left(K\frac{\partial T}{\partial x_i}\right)$$

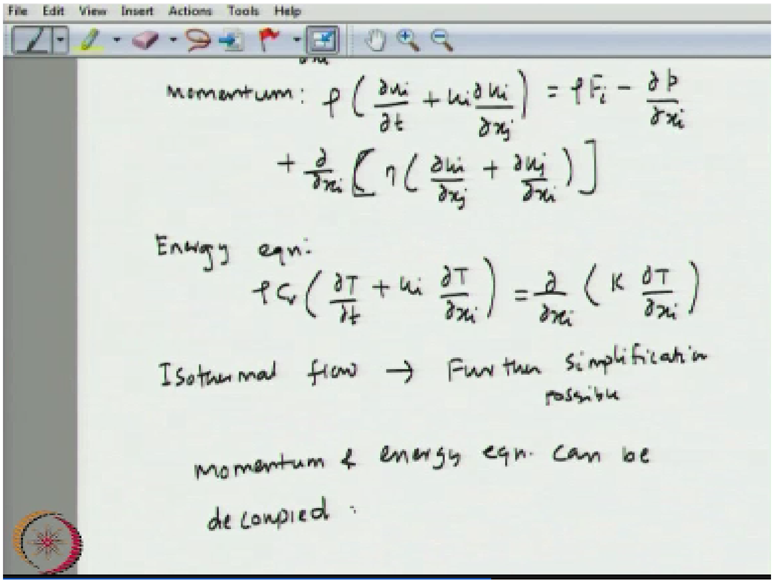
Isothermal flow → Further simplification possible.

So under those assumptions let us down the governing equation for liquid. So governing equations for liquid flows. So the first is the mass conservation equation our assumptions are Newtonian, fluid is Newtonian, fluid is incompressible and Fourier conducting. So we write down the mass conservation equation which will be del ui/del xi will be 0. The momentum equation will be rho* del ui/del t+ ui del ui/del xj wil be= rho* Fi-del p/del xi+ del/del xi* eta is the velocity del ui/del xj+ del uj/del xi.

So that is the momentum equation. Similarly, we can write the energy equation which is rho Cv * del T/del t + ui* del T/del xi= del/del xi *k del T/del xi. So this is the energy equation. Now if you assume that the fluid remains at constant temperature that means isothermal flow condition then the energy equation can be decoupled from the momentum equation. So if you want to solve the flow field where the isothermal flow condition is valid.

Then you can decouple the momentum equation from energy equation and you can just go ahead and solve the mass and momentum equation. So this equation can be further simplified when isothermal flow condition is valid. So isothermal flow further simplification possible.

Refer Slide Time: 27:00)



momentum: $\rho\left(\frac{\partial u_i}{\partial t} + u_i\frac{\partial u_i}{\partial x_j}\right) = \rho F_i - \frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_i}\left[\eta\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)\right]$

Energy eqn:

$$\rho c_v\left(\frac{\partial T}{\partial t} + u_i\frac{\partial T}{\partial x_i}\right) = \frac{\partial}{\partial x_i}\left(K\frac{\partial T}{\partial x_i}\right)$$

Isothermal flow → Further simplification possible

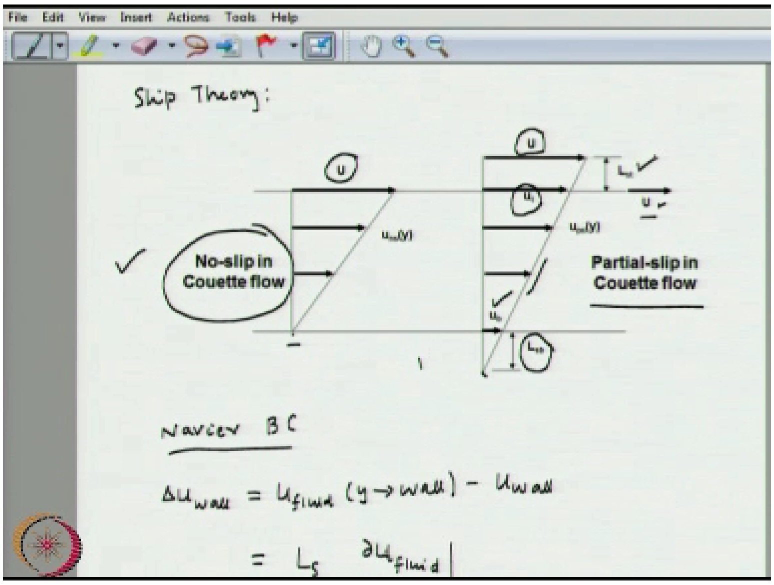
momentum & energy eqn. can be decoupled.

And the momentum and energy equation can be decoupled. So that is about the governing equations for the liquid flow. Next let us look at another important concept in the fluid flow which is the slip boundary condition. So while solving equations the mass momentum equation for micro scale flow situation many times when you try to solve it analytically we assume that no slip boundary condition is valid at the wall.

So that means at the wall the fluid takes the same velocity as that of the wall. Similarly, we have no temperature jump boundary condition the temperature of the fluid are the wall is same as the wall itself. So we have been using no slip, no temperature boundary conditions for analytical solutions because analytically we get the same results that we get using experiments so they have been widely accepted no slip boundary conditions have been widely accepted.

So if you want to generalize the no slip boundary condition or slip boundary conditions at the interface the fluids are already interface that can be done using Navier boundary conditions.

(Refer Slide Time: 28:52)



Slip Theory:

Navier BC

$$\Delta u_{wall} = u_{fluid}(y \to wall) - u_{wall}$$

$$= L_s \quad \partial u_{fluid} \Big|$$

So let us talk about Navier boundary conditions in slip theory. So the Navier boundary conditions tells that delta u are the walls will be u velocity of the fluid at y tending to wall-velocity of the wall. So the velocity of the fluid right at the wall-the velocity of the wall itself. So that we can write as Ls so for now for just write Ls and we will define it. So delta u velocity of the fluid/delta y at the wall.

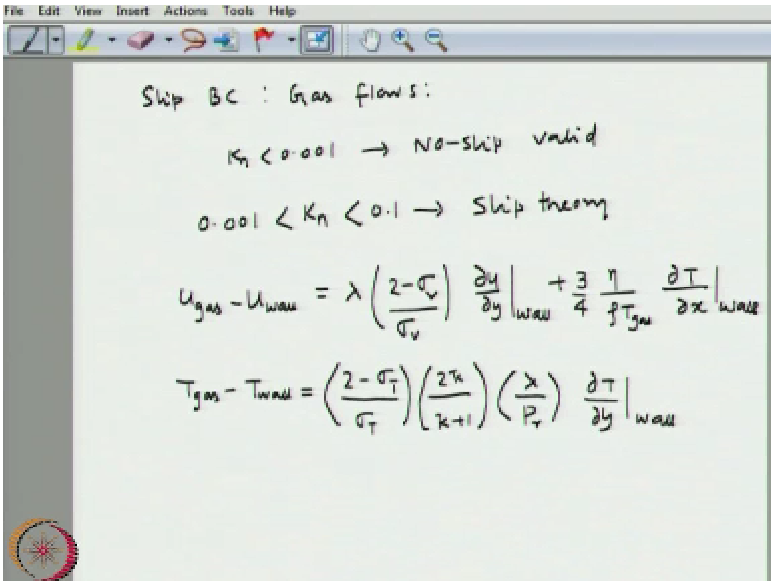
And Ls is known as slip length. As you can see here in this figure here we are considering a Couette flow situation and normally and we have no slip boundary conditions valid. So here we have the top plate moving at velocity u and the bottom plate is stationary. So the velocity of the fluid at the bottom wall is 0 and at the top wall it obtains the same velocity as the velocity of the upper plate.

So this is what we have seen in fluid mechanics. This is known as no slip in Couette flow. Now if you allow partial slip to occur what will happen is at the bottom plate instead of seeing 0 velocity we will see a finite velocity ub which is>0 and if you extend this profile into this bottom plate. At some point this velocity would vanish and the distance between the top surface of the bottom plate and the point at which the velocity vanishes is known as the slip length for the bottom plate.

Similarly, at the top plate if you allow slip we will get a velocity which is Ut which is smaller than U and if you extend the same profile into the top wall at some length this velocity is going to be= to the plate velocity and the length at which this would occur is known as the slip length for the top wall. So the slip length would depend on the interaction between the fluid and the surface.

Slip length depends on a particular surface. So with that let us move on and talk about the slip in gas flows.

(Refer Slide Time: 32:28)



Slip BC : Gas flows:

$K_n < 0.001 \rightarrow$ No-slip valid

$0.001 < K_n < 0.1 \rightarrow$ Slip theory

$$u_{gas} - u_{wall} = \lambda \left(\frac{2-\sigma_v}{\sigma_v}\right) \frac{\partial u}{\partial y}\bigg|_{wall} + \frac{3}{4} \frac{\eta}{\rho T_{gas}} \frac{\partial T}{\partial x}\bigg|_{wall}$$

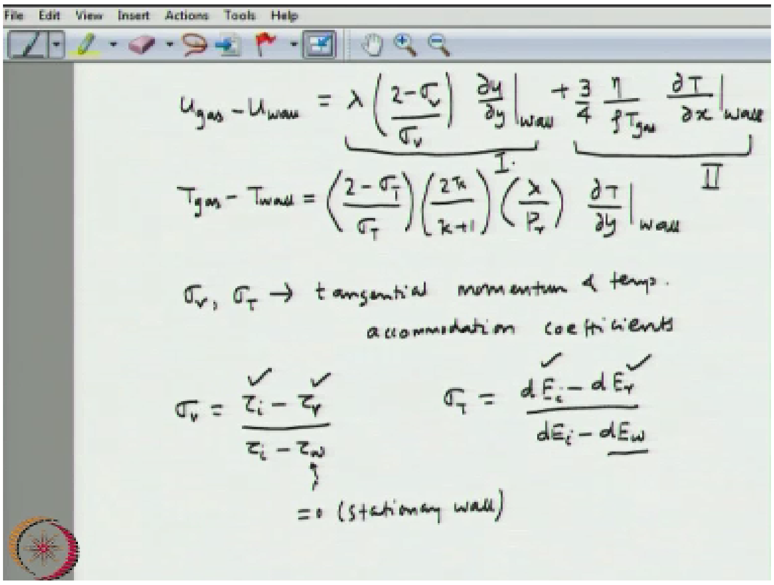
$$T_{gas} - T_{wall} = \left(\frac{2-\sigma_T}{\sigma_T}\right)\left(\frac{2k}{k+1}\right)\left(\frac{\lambda}{P_r}\right) \frac{\partial T}{\partial y}\bigg|_{wall}$$

So let us talk about the slip boundary condition in gas flows. We know that the no slip boundary conditions is valid when the Knudsen number is<0.001 and we can think of using Navier Stokes Equation where we may need to apply slip when Knudsen number is<0.01. So between 0.001 and 0.1 we need a theory to know what the value of slip is at the wall. So very well for gas flows we know if Knudsen number is<0.001 then no slip valid.

So between 0.001< Knudsen number<0.1 we need a slip theory and for gases we can write u gas-u wall to be lambda * 2-sigma v/sigma v* del u/del y at the wall + ¾ *eta/rho T gas * delta T/delta x at the wall. And we can write for temperature T gas-T wall will be= 2-sigma T/sigma T * 2k/k+1. K is the specific heat ratio * lambda/Prandtl number * delta T/ del y at the wall.

So here most of the constant we recognize eta is the viscosity of the gas and rho is the density of the gas. Here lambda is the mean free path.

(Refer Slide Time: 35:25)



$$u_{gas} - u_{wall} = \underbrace{\lambda \left(\frac{2-\sigma_v}{\sigma_v}\right) \frac{\partial u}{\partial y}\Big|_{wall}}_{I.} + \underbrace{\frac{3}{4} \frac{\eta}{\rho T_{gas}} \frac{\partial T}{\partial x}\Big|_{wall}}_{II}$$

$$T_{gas} - T_{wall} = \left(\frac{2-\sigma_T}{\sigma_T}\right)\left(\frac{2k}{k+1}\right)\left(\frac{\lambda}{P_r}\right) \frac{\partial T}{\partial y}\Big|_{wall}$$

$\sigma_v, \sigma_T \rightarrow$ tangential momentum & temp. accommodation coefficients

$$\sigma_v = \frac{\overset{\checkmark}{\tau_i} - \overset{\checkmark}{\tau_r}}{\tau_i - \underbrace{\tau_w}_{= 0 \text{ (stationary wall)}}} \qquad \sigma_T = \frac{\overset{\checkmark}{dE_i} - \overset{\checkmark}{dE_r}}{dE_i - \underline{dE_w}}$$

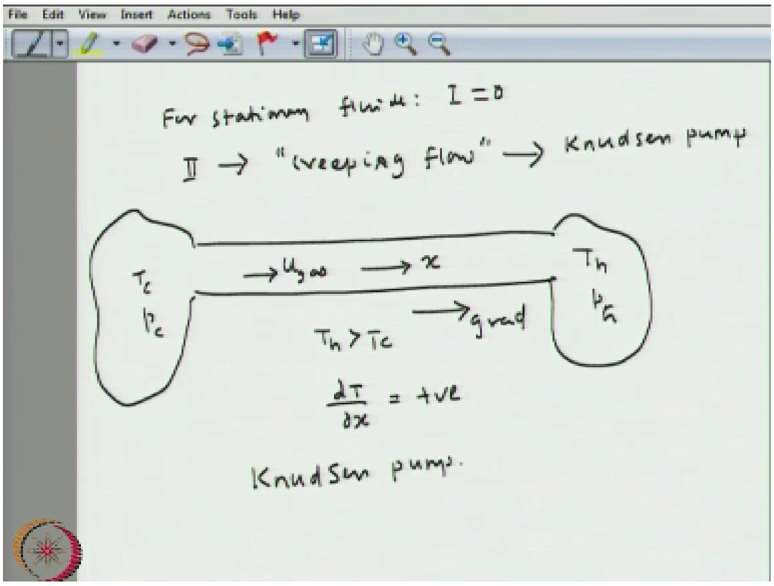
Here the new constant are sigma v and sigma T which are known as the tangential momentum and temperature accommodation coefficient. And they can be defined as follows. We can define the momentum accommodation tangential momentum accommodation coefficient sigma v as tau i- tau r/ tau i-tau w. Similarly, sigma t the temperature accommodation coefficient dEi- dEr/dEi- dEw.

So the Ti term here is the tangential momentum of the incoming molecules and Tr is the tangential component of the reflected molecules tau W is the tangential momentum of reemitted molecules which are absorbed to the surface and this term goes to 0 for a stationary wall. Similarly, in sigma T the dEi is the energy flux of the incoming molecules dEr is the energy flux of the reflected molecules and dEw is the energy flux of the reemitted molecules that have been absorbed to the surface.

So this is how we defined the tangential momentum and temperature accommodation coefficient. If you look at you know this equation here the second part of this equation what do we see is that if the flow is in stationary conditions the velocity is 0. So the first term is 0, then if we apply a temperature gradient along x direction then we would induce a slip along x direction and that is known as creeping flow.

So if you look at we call this is the second term.

(Refer Slide Time: 38:25)



For stationary fluids: $I = 0$

$II \rightarrow$ "creeping flow" $\rightarrow$ Knudsen pump

$T_h > T_c$

$\frac{\partial T}{\partial x} = +ve$

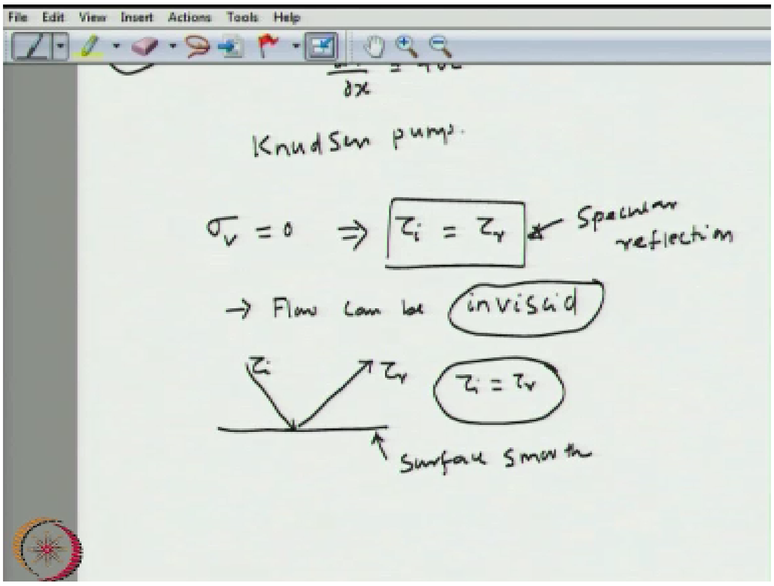
Knudsen pump.

And this is the first term and say that for stationary fluids the term one is 0 and term 2 in that case gives what is called as creeping flow. And it is possible to develop pumping action using creeping flow which would be known as Knudsen pump. For example, if you consider single channel here and here the temperature is T c here temperature is T h here pressure is Pc here pressure is T h and T h is> T c. So you maintain a gradient in the x direction. Let us call this x direction. So del t/del x is positive.

And if you look at this equation here so that would induce a flow so velocity of the gas with respect to the wall in the x direction. So you would have a flow occurring in this direction U of the gas and if you have let say 2 reservoirs and you maintain a temperature gradient in this direction. So we can induce a pumping action and that pump would be known as Knudsen pump.

Now if you talk about the tangential you know the momentum accommodation coefficient it takes care of the tangential momentum of incoming and deflected molecules tangential momentum of incoming and the reemitted molecules.

(Refer Slide Time: 40:45)



Knudsen pump.

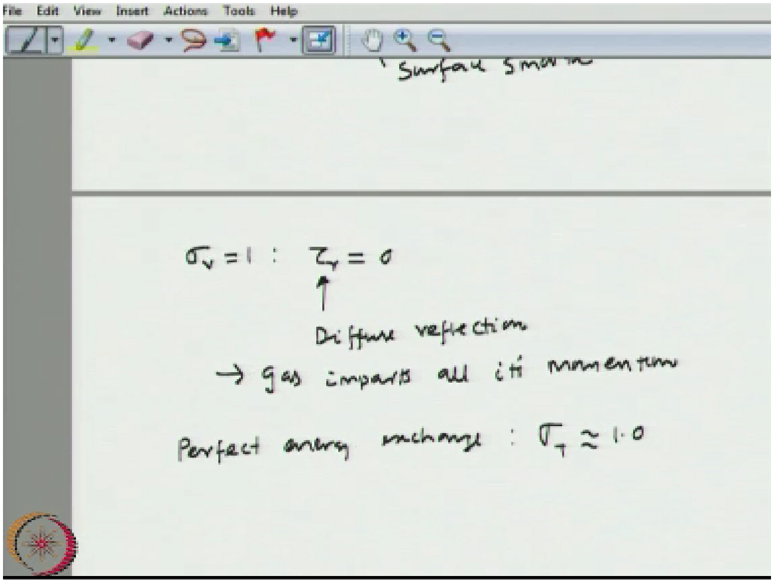
$\sigma_v = 0 \Rightarrow \boxed{\tau_i = \tau_r}$ ← Specular reflection

→ Flow can be inviscid

If the sigma v goes to 0 what that means is that the tangential momentum of the incoming molecules is going to be= to the tangential momentum of the reflected molecules. So in other words what that means is that the molecules do not impart any momentum to the wall. So the conclusion is that in that case the flow can be considered as inviscid flow. So there is no effect that is realized by the fluid in presence of the wall whereas if we and this situation can also be called as specular reflection.

So here if the molecules are coming at some tau i they will be going at tau r. So tau i=tau r. So this is the case of the specular reflection and this occurs when the surface is very smooth.

(Refer Slide Time: 42:15)



$\sigma_v = 1 : \tau_r = 0$

↑ Diffuse reflection

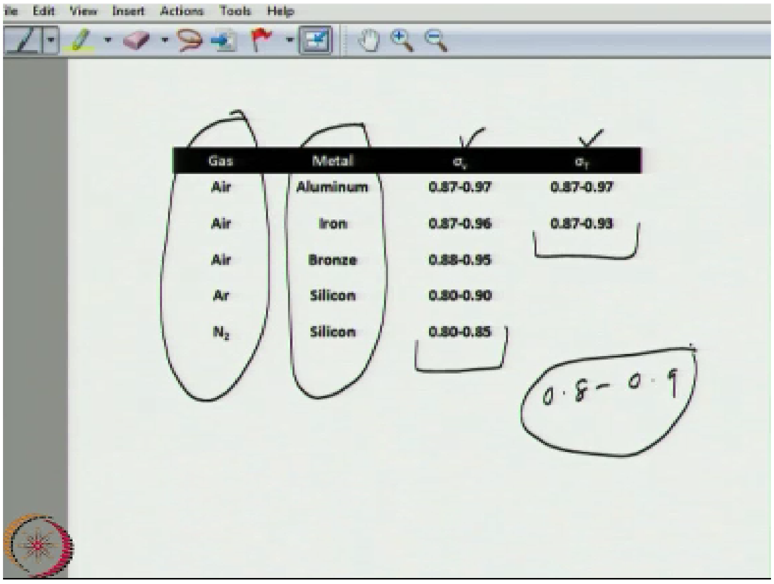
→ gas imparts all its momentum

Perfect energy exchange : $\sigma_T \approx 1.0$

Now if you look at the other extreme where the sigma v is going to be 1 then the reflected the tangential momentum of the reflected molecules will be 0 and what that will mean is that the incoming molecules impart all their momentum to the surface. So in that case you can say that this is the case of diffuse reflection. This is the case of diffuse reflection and where gas imparts all its momentum.

And if you say that perfect energy exchange taking place between the incoming molecules and the wall in that case the temperature accommodation coefficient sigma T will become 1. So for perfect energy exchange sigma T will be 1.

(Refer Slide Time: 43:38)



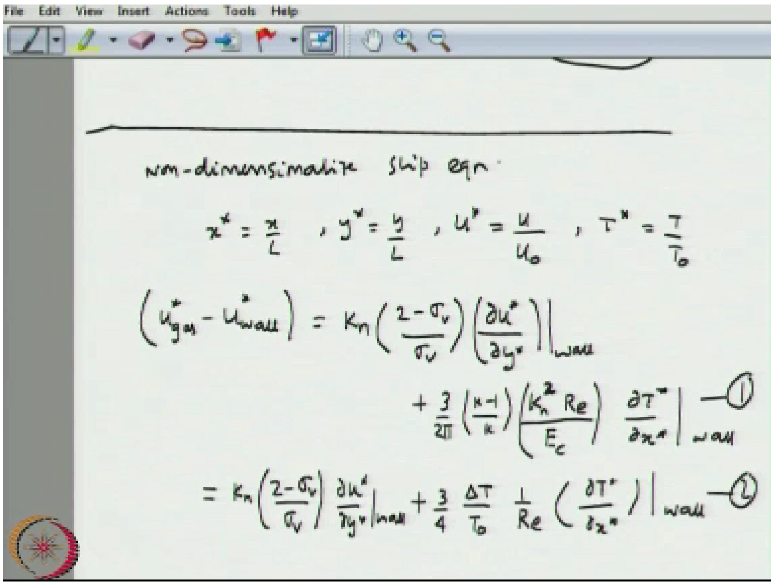
Gas	Metal	σ_v	σ_T
Air	Aluminum	0.87-0.97	0.87-0.97
Air	Iron	0.87-0.96	0.87-0.93
Air	Bronze	0.88-0.95	
Ar	Silicon	0.80-0.90	
N_2	Silicon	0.80-0.85	

0.8 – 0.9

Now let us some of the surfaces where we can know what are the values of momentum and the temperature accommodation coefficient. So these are different gases which are interacting with different surfaces and if you look at the values you can see that the values of the both momentum and temperature accommodation coefficients are in the range of 0.8 to 0.9 or little higher. So that are the values of the momentum and temperature accommodation coefficient.

So with that let us try to look at in the expression for the slip velocity what are the importance of different parameters. So in order to do that we need to non-dimensionalize the slip equation.

(Refer Slide Time: 44:43)



non-dimensionalize slip eqn·

$$x^* = \frac{x}{L}, \quad y^* = \frac{y}{L}, \quad u^* = \frac{u}{u_0}, \quad T^* = \frac{T}{T_0}$$

$$\left(u^*_{gas} - u^*_{wall}\right) = K_n\left(\frac{2-\sigma_v}{\sigma_v}\right)\left(\frac{\partial u^*}{\partial y^*}\right)\bigg|_{wall} + \frac{3}{2\pi}\left(\frac{k-1}{k}\right)\left(\frac{K_n^2 Re}{E_c}\right)\frac{\partial T^*}{\partial x^*}\bigg|_{wall} \quad \text{—①}$$

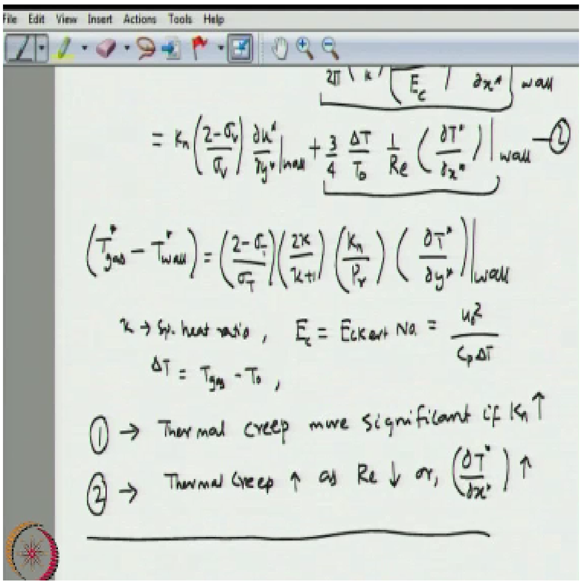
$$= K_n\left(\frac{2-\sigma_v}{\sigma_v}\right)\frac{\partial u^*}{\partial y^*}\bigg|_{wall} + \frac{3}{4}\frac{\Delta T}{T_0}\frac{1}{Re}\left(\frac{\partial T^*}{\partial x^*}\right)\bigg|_{wall} \quad \text{—②}$$

So let us try to non-dimensionalize slip equation. So we can choose a set of non dimensional parameters you can define x * as x/L. Y * as y/L. We can define u * as u/some characteristics velocity u0 and we can define T * as T/T0. If you do that then you can write the expression for the slip u gas-u wall all are non-dimensionalize now. So this is going to be Knudsen number * 2-sigma v/sigma v* del u/del y non-dimensionalize del u */del y *.

So this is at the wall+3/2 pi* k-1/k. So this is specific heat ratio * Knudsen number square Reynolds number this is square/ Eckert number which is the ratio between the kinetic energy to (()) (46:20) p of the flow * del T */del x * at the wall. We can also write this is one form of the equation. We can also write as Knudsen number * 2-sigma v/sigma v* del u*/del y * at the wall +3/4*delta T/T0.

This is the temperature difference between the fluid and wall * 1/Re* del t */del x * at the wall. So this is the second expression.

(Refer Slide Time: 47:18)



$$2\pi \quad k \quad E_c \quad \partial x^* \Big|_{wall}$$

$$= K_n\left(\frac{2-\sigma_v}{\sigma_v}\right)\frac{\partial u^*}{\partial y^*}\Big|_{wall} + \underbrace{\frac{3}{4}\frac{\Delta T}{T_0}\frac{1}{Re}\left(\frac{\partial T^*}{\partial x^*}\right)\Big|_{wall}} \quad — ②$$

$$\left(T^*_{gas}-T^*_{wall}\right) = \left(\frac{2-\sigma_T}{\sigma_T}\right)\left(\frac{2k}{k+1}\right)\left(\frac{K_n}{P_r}\right)\left(\frac{\partial T^*}{\partial y^*}\right)\Big|_{wall}$$

k → Sp. heat ratio, E_c = Eckert No. = $\frac{u_0^2}{C_p \Delta T}$

$\Delta T = T_{gas} - T_0$,

① → Thermal creep more significant if K_n ↑

② → Thermal Creep ↑ as Re ↓ or, $\left(\frac{\partial T^*}{\partial x^*}\right)$ ↑

Similarly, we can write for the temperature no temperature jump boundary condition or the temperature jump boundary conditions in this case. So T gas-T wall which will be= 2-sigma T/sigma T* 2 k+1* Knudsen number/Prandtl number * del T*/delta y * at the wall. So here K is the specific heat ratio and Ec is Eckert number which is the ratio of the kinetic energy to (()) (48:18) Cp delta T and delta T is T gas-T0.

So what we see here if you look at you know the term 1 and term 2. If you look at term 1 what we see that if you are increasing Knudsen number than the thermal creep this term which represents thermal creep become more significant. So thermal creep more significant if Knudsen number increases and we also observe from 2 is that the thermal creep increases as Reynolds number reduces.

From here we can see that thermal creep is going to increase as Reynolds number increase or the Reynolds number reduces or the delta T for temperature gradient is increasing. So those are 2 important conclusions that we make from the non-dimensionalization. So with that let us stop here and next we will be talking about slip flows for liquids.