Microfluidics
Dr. Ashis Kumar Sen
Department of Mechanical Engineering
Indian Institute of Technology - Madras

Lecture - 36
Micro Mixers (continued…)

Okay, so let us continue our discussion on micromixers, let us first take a very simple case where we tried to mix 2 parallel stream of liquids.

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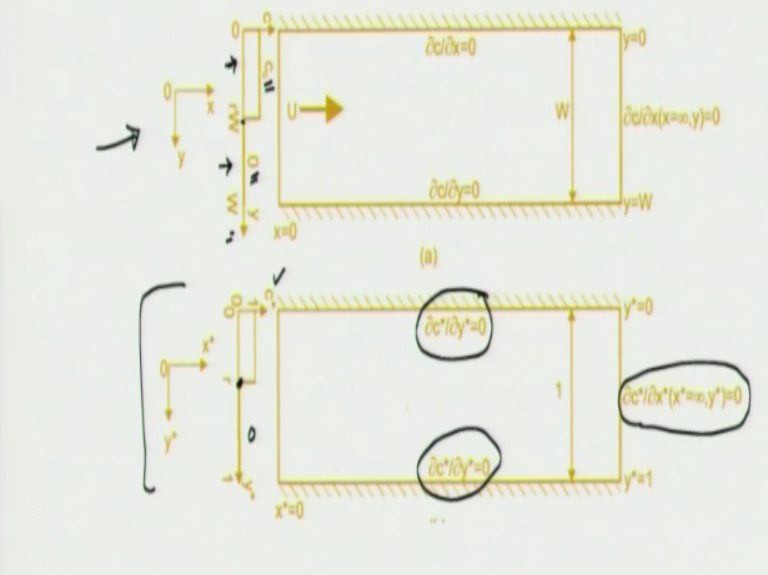

Look at the example here, here we are talking about mixing of 2 parallel streams, so parallel mixing with 2 streams okay. So as you can see we have 2 streams one stream here and one stream there, the concentration of stream one is the solute concentration is C0, in another one the concentration is 0. Then we can define a depending on the mass flow rate ratio r, we can define the interface here, so this will be r times W, and W is the total width of the channel okay.

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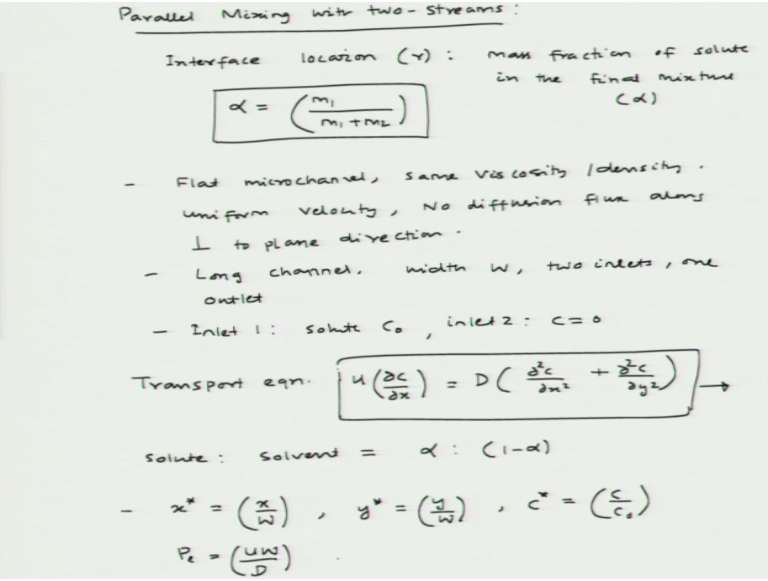

So we define interface location r, which will be proportional to the mass fraction of solute in the final mixer which is alpha okay, so alpha can be defined as m1/m1+m2 okay, so this is the mass flow rate of fluid 1 mass flow rate of fluid 2 okay. So we define you know we make some assumption we say that it is a flat microchannel okay, and we say the 2 fluids have same viscosity and density.

And we say that the velocity is uniform so uniform velocity condition, so velocity does not change in the axial direction, and we say there is no diffusion flux in the perpendicular to the plane, no diffusion flux along perpendicular to plane direction okay. So under that assumption we say that the channel is long and width is W. And we are talking about 2 inlets and one outlet okay, so inlet 1 the solute concentration is C0, inlet 2 solute concentration is C=0.

So you can write down the transport equation this is u del C/del x=D*del square C/del x square+ del square C/del y square right. Now we try to solve this equation for this geometry okay, so here we have 2 different fluids coming in with different boundary conditions we try to solve this equation okay. So the ratio between the solute to solvent ratio is=alpha is to 1- alpha okay, where alpha is given here the mass flow rate of solute/solute+ solvent okay.

So we can define non-dimensional parameters x star=x/W, y star=y/W, C star=C/C0 and we define peclet number as U W/D.

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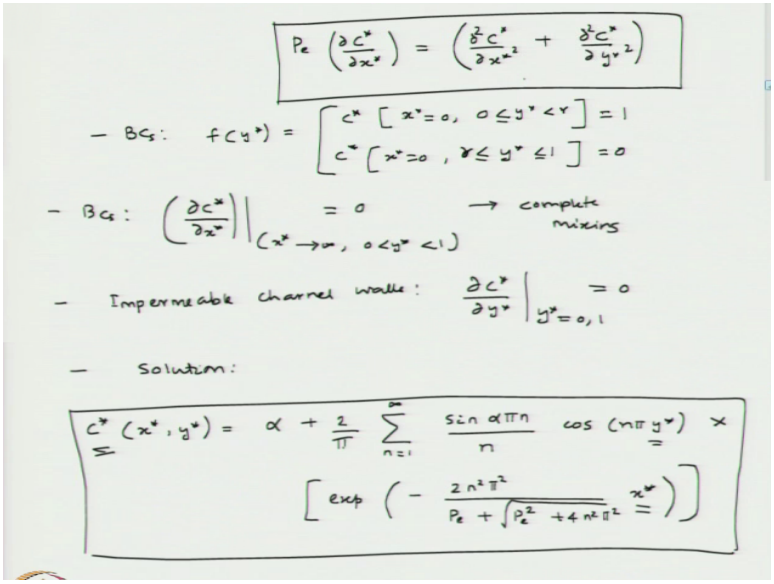

So under using these non-dimensional parameters that you can write the transport equation, so the transport equation becomes peclet number*del C star/del x star will be =del square C star/del x star square+ del square C star/del y star okay, so this is the non-dimensional equation that we are talking about okay which needs to be solved okay. So to solve this equation let us write down the boundary condition.

So the boundary condition f of y star is going to be C star and that is going to be if f star=0 and 0<y star<r, so between 0 to r the concentration C star will be =1 okay, so that is what we see here this is the non-dimensional representation of the original geometry so the interface is going to be at r okay, so y star between 0 to r the concentration is going to be C star, and if C star is between r to 1 then the concentration will be 0 okay, so it will be star if x star=0 and r<y star<1 that will be=0 okay.

So these are the concentration boundary conditions. Now the boundary conditions at the outlet, when complete mixing will happen then del C star/del x star at x star tending to infinity and 0<y star<1 this will be=0 so that says complete mixing okay, so there is no variation of concentration along x direction okay. So the other boundary condition is that we are assuming impermeable channel wall so nothing is crushing the channel wall okay.

So in that case the concentration variations del C star/del y star at y star=0, 1 is going to be 0 right, so that is the boundary condition that we see here del C star/del y star=0 on the bottom wall as well as in the top wall, and this is the boundary condition at x=infinity complete mixing boundary condition okay. So we have boundary condition on the all 4 boundaries to solve the transport equation.

So using these boundary conditions we can find out a solution for to the transport equation, so the solution can be written as so C star as a function of x star and y star can be written as alpha, alpha is the mass flow rate ratio of 2 fluids this is given there +2/pi okay summation n=1 to infinity sin alpha pi n/n*cos of n pi y star*we have exponential negative of 2 n square pi square/peclet number+ peclet number square+ 4 n square pi square, square root*x star.

So that is the expression for the concentration in x and y direction, now if you plot C star with x star and y star this is what we would get.

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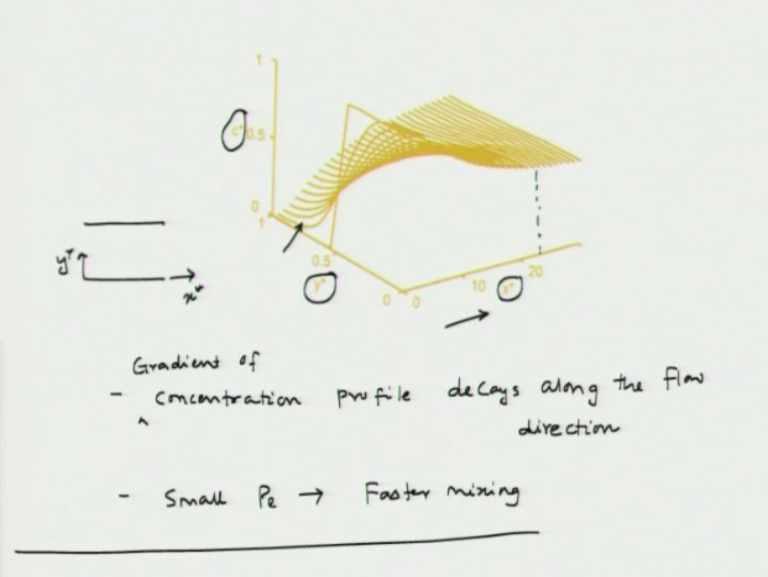

So if you look at here, here we are plotting C star with x star and y star, so y star is let us see if this is the channel y star is in this direction and x star is in this direction okay, so you can see that this is across the channel the concentration varies from 0 for the solvent to 1 for the solute okay, so this is at some x=0 okay. So there is a large gradient of the concentration across the middle line okay, now as we proceed along x direction this gradient is going to reduce.

And at some point so this is where the complete mixing is a occurring, so there is no gradient along the y direction okay. So what we observed from here is so the first observation is the concentration profile the gradient of the concentration profile decays along the flow direction okay, and other things that we observed from the equation if peclet number is going to be less, then the decay in the concentration gradient of the concentration is going to happen faster okay.

So what it means is that for small peclet number it leads to faster mixing okay, so you know we consider a case where we have 2 parallel fluids you know they are trying to mix with each other along channel, and we observed that the gradient of the concentration is going to be very high at the beginning of the channel, and along the direction of the flow along the x direction the gradient is going to reduce.

And at some point the gradient along the y direction is going to be 0 that is where the complete mixing would have occurred okay. So you make 2 important conclusions one is that the concentration gradient of the concentration profile along y direction decays along the flow direction, and the second conclusion is that the peclet number is less, then the mixing is going to be faster okay.

So with that let us move on and talk about mixing with you know multiple streams, how we can divide the mixing path into multiple smaller mixing parts and as you know that the mixing diffusion time scale varies as the square of the mixing path, and as mixing path will reduce the mixing time would also reduce, so that would lead to faster mixing. So what we see here is mixing with multiple streams.

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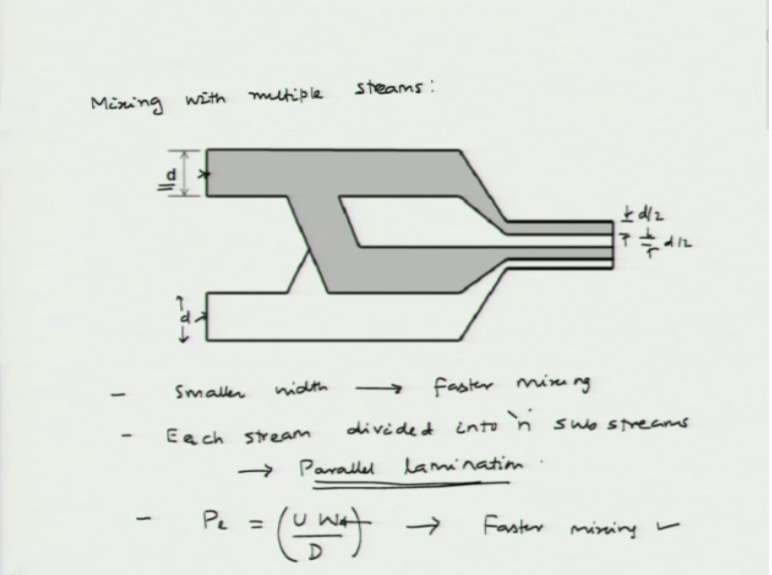

So look at mixing with multiple streams, so here we can look at this is you know one fluid so equal width is here, now instead of mixing these 2 fluids together what we do is we divide each stream into 2 parts as in this case, so d is divided into this is d/2, and this is the second fluid is divided into d/2 okay. So as the mixing path length is reduced by 1/2, the mixing performance will improve by 4 times okay.

So what we see here is that we know that smaller width leads to faster mixing okay, so what we do is each stream divided into n sub streams and that is known as parallel lamination okay. And in that case the peclet number which is U*W/D, so this W is going to reduce okay by going for parallel lamination so that would give us faster mixing okay.

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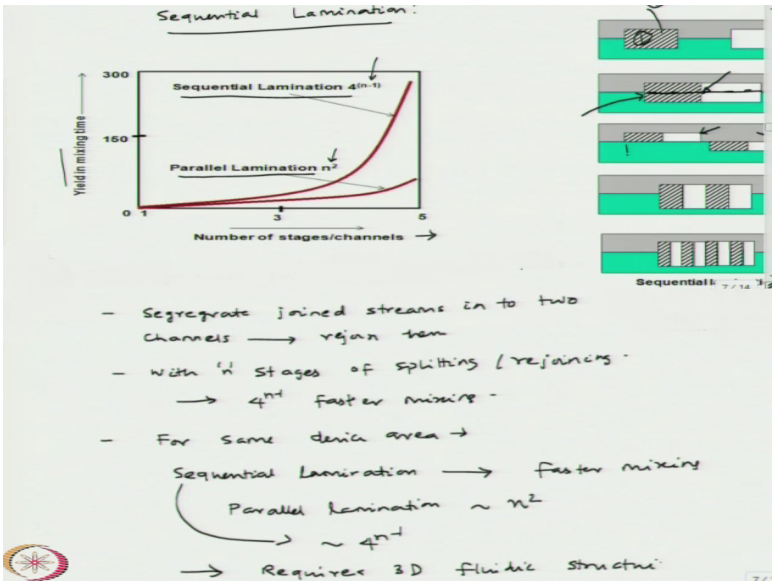

So now let us talk about sequential lamination, so in parallel lamination we talked about 2 you know streams which are trying to mix, so we divide each stream into n different parts, and allow this n different parts of fluid 1 to mix with n different parts in fluid 2 separately, and then bring them back together okay. In the sequential mixing the 2 fluids will come together and these 2 fluids are divided across a vertical plane okay across a horizontal plane and taken separately.

And the depth of each of these fluid layers are going to be increased to the original depth, so that the mixing interface area does not change, so they will mix separately and then brought back together. As you can see here, so this is fluid 1 and this is fluid 2, and so these 2 fluids are brought in contact here at this interface now mixing plane is divided across the middle plane there okay, so it is divided across the middle plane and then these 2 layers are taken to a separate to 2 different channels okay.

So they are divided across this plane and taken to 2 separate channels, then the channel width is channel depth becomes 1/2 here right, now the channel depth allowed to increase to the original depth as in here okay. So the channel depth is allowed to increase but the fluid width has become 1/2, and this process is continued n times okay, so it have n different sequential lamination layer, and if you do that then the mixing is going to improve okay.

So what we do here is we segregate joint streams into 2 channels and then rejoin them okay, so if you do n splitting and rejoining, with n stages of splitting and rejoining we would get 4 to the power n-1 faster mixing okay. So as you can see here in this plot so this is the mixing yield and this is the number of stages for channel, as you can see here by going for parallel lamination which we discussed earlier the mixing performance increases by n square time.

Whereas in sequential lamination it is increasing by 4 to the power n-1 times okay, so which is going to be higher as compared to parallel lamination okay. So what we conclude from here is for same device area the sequential lamination offers faster mixing okay, because the parallel lamination improves mixing n square, and sequential lamination improves 4 to the power n-1. But one downside of sequential lamination is that it requires 3-dimensional channel structure or fluidic structure and which are not quite easy to fabricate okay.

So even if the sequential lamination offers better performance as compared to parallel lamination the fabrication becomes the challenge okay. So with that let us look at one example okay.

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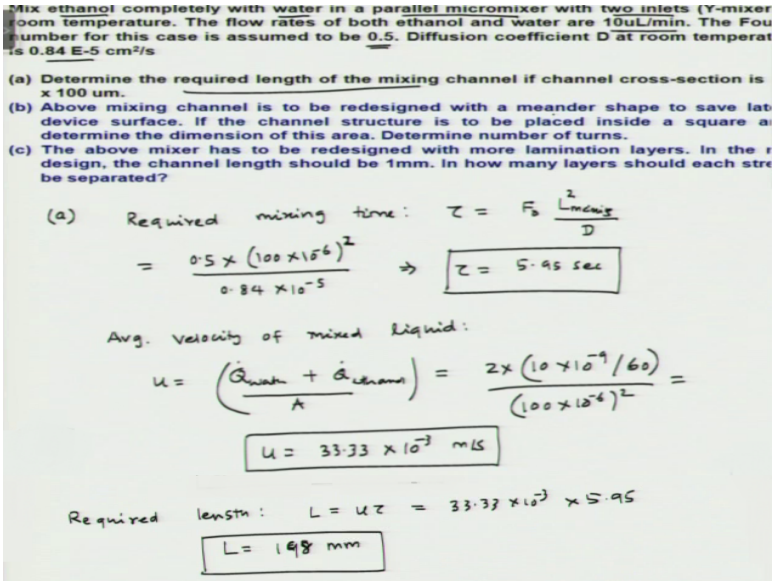

We take one example of a micromixer where we trying to mix ethanol completely with water in a parallel micromixer with 2 inlets okay using a Y-mixer at room temperature, the flow rates of both ethanol and water are 10 microliter per minute okay, each fluid has 10 microliter per minute

flow rate, the Fourier number for this case is assumed to be 0.5, and the diffusion co-efficient at room temperature is given as 0.84*10 to the power-5 centimeter square per second.

And we want to determine the required length of mixing channel, if the channel cross section is 100*100 micron okay, so we determine the required mixing length and given the channel cross section is 100 microns square, so that is the first we try to answer the first question. And the second is if the above mixing channel is to be redesigned with a meander shape or serpentine shape channel to save the lateral device surface.

If the channel structure is to be placed inside a squared areas determine the dimensions of this area, and also determine the number of turns okay. The third point will be if the above mixer has to be redesigned with more lamination layers, in the new design the channel lengths should be 1 millimeter in how many layers should each stream be separated okay. So let us solve the first question, so you try to find the required mixing time tau is given by the Fourier number*the mixing length scale square/D okay.

So this is going to be Fourier number is typically 0.5, and mixing length is 100 micron 100*10 to the power-6 square/diffusion co-efficient is 0.84*10 to the power-5, so tau timescale is going to be 5.95 second okay. We can find the average velocity of the mixed liquid, so u can be written as Q water+ Q dot ethanol/area, so this is 2*10*10 to the power-9/60 meter cube per second/area is 100*10 to the power-6 square.

So you find the velocity to be so u is found to be 33.33*10 to the power-3 meter per second okay. Now we find the required length of the mixing channel, so required length will be L will be u*tau so u is 33.33*10 power-3, tau is 5.95 second so we get the mixing length the length required for the mixing to occur is 198 millimeter okay. So that answers the first question, determine the required length of the mixing which is 198 millimeter.

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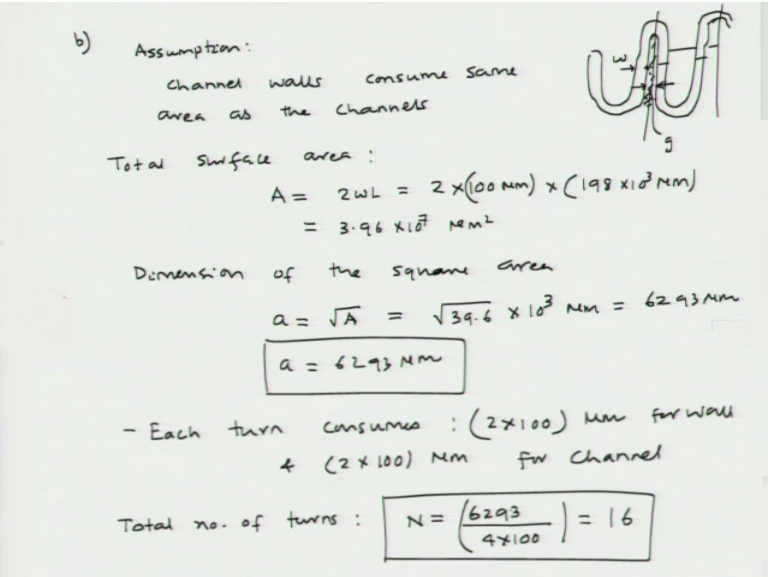

The second questions we are trying to use a serpentine shape to save the lateral device, and if channel structures to be placed inside a square area, determine the dimension of this area okay. So in that case you know here we would have a serpentine channel structure something like this, and we if you say so we make some assumption we say that the channel width is same as the gap, so this is where the material is and this is where the channel is.

So if this channel width is same as the gap between the channels okay, so you make assumption that the channel walls consume same area as the channels, we can find the total surface area A=2*W*L which is 2*100*10 to the power-6 we are calculating in micron so this is 100 micron*length is 198*10 to the power 3 micron, so the area is 3.96*10 to the power 7 micron square okay. So we can find out the dimension of the square area or the side of the square area.

The dimension of the square area a will be square root of a which will be square root of 39.6*10 to the power 3 micron so that will be 6293 micron okay, so a will be 6293 micron. So each side of the square area containing the serpentine channels would have 6293 micron size. Now if you say that each turn consumes twice the channel size, so 2*100 micron okay, so that is for the wall and another 2*100 micron for channel okay that is quite obvious.

So each turn will start from here to the middle of this okay, so you will have 2 channels and 2 channel widths okay, so this is one channel width here and there, so there will be 4 channel

widths, so we can find total number of turns N will be the total size 6293/4*100 micron that will be 16 turns. So it would require 16 turns to accommodate the serpentine future on the square area okay.

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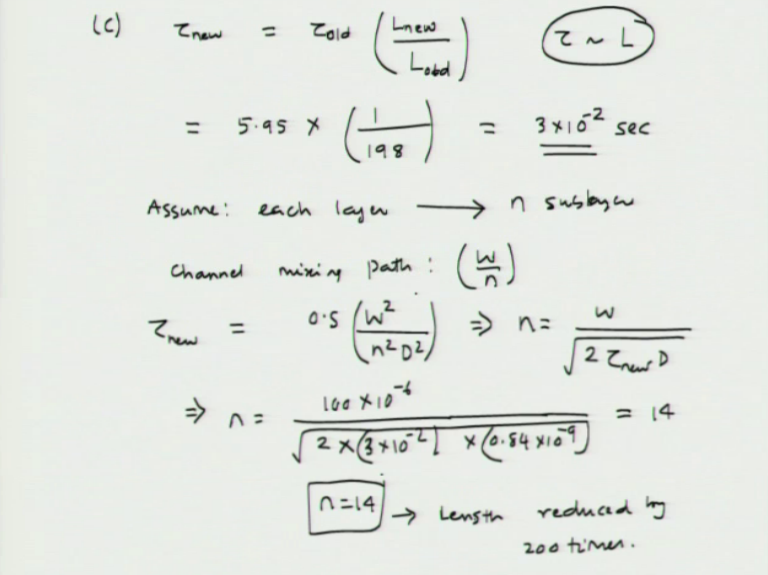

Next is the above mixer has to redesigned with more lamination layers, and so that the channel length should be limited to 1 millimeter okay, so we want to find how many layers should each stream be separated. So you know for lamination we can say that tau new is the time of mixing when you have more than one lamination let us say n lamination layers, the tau new will be tau old*L new/L old okay, so diffusion timescale is proportional to the mixing path okay.

So you can write tau old is 5.95 seconds, L new we want to limited to 1 millimeter but existing is 198 millimeter so the tau new is going to be 3*10 to the power-2 second okay, if you can make as fast as 3*10 to the power-2 seconds then the mixing required length would be 1 millimeter. Let us say we assume each layer is converted to n sub layers okay, so the channel mixing path will be W/n okay, so the new mixing time tau new will be=0.5 W square/n square D square okay.

So that is= so we can find n as W/2 tau new D square root, so n will be 100d*10 to the power-6 that is the width of the channel /2*tau new is 3*10 to the power-2 seconds so 3*10 to the power-2 seconds*D is 0.84*10 to the power-9 right square root of that, if you calculate n will be 14, so n=14. So what it says is that if you can divide each stream into 14 different parallel lamination

layers, we can limit the required length for the mixing to happen we can reduce it by 200 times okay. So in that case the length reduces by 200 times.

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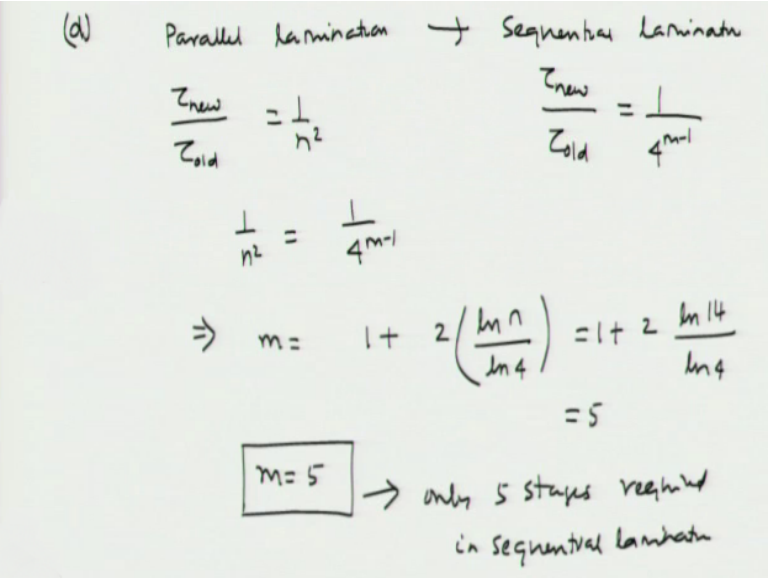

Now let us look at if instead of parallel mixing if you go for sequential lamination, so let us say instead of parallel lamination if you go for sequential lamination, then how many layers we would need okay. So in that case you can write for the parallel lamination tau new/tau old will be 1/n square, and sequential lamination tau new/tau old will be 1/4 to the power let us m different stages we are considering 4 to the power m-1.

So now for same improvement 1/n square will be=1/4 to the power m-1, so in that case we can find m=1 +2 of ln n/ln 4 which will be 1+2 ln 14 so 14 layers in parallel mixing/ln 4 so it will be =5 okay, so m=5. So what we saw that if instead of going for parallel lamination layers in which case we would require 14 different parallel lamination players, if you go for sequential lamination layer we can do it with 5 different stages okay, so only 5 stages required in sequential lamination okay.

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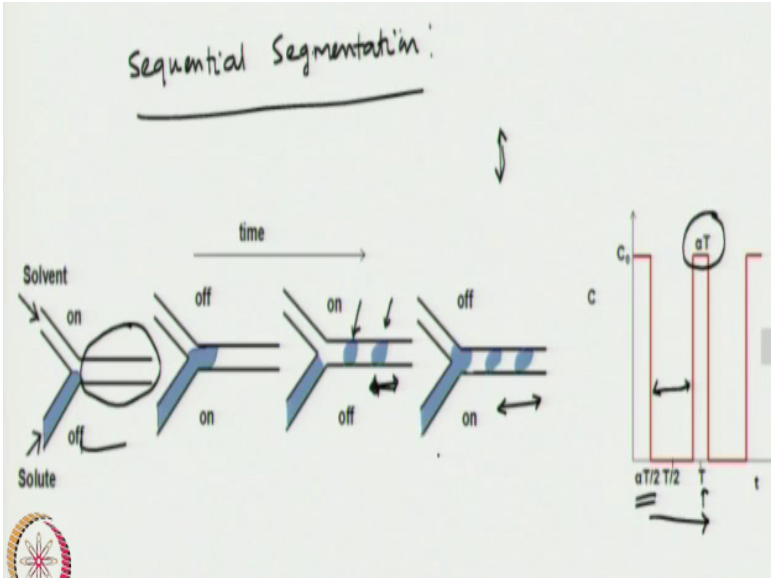

So with that let us talk about sequential segmentation, here we about sequential segmentation based mixing okay. So in sequential segmentation the mixing is going to happen along the direction of the flow, so we alternatively inject the solute and the solvent into a mixing channel alternatively by using valve, and we allowed this blocks of solute to mix with the solvent along the direction of the flow okay.

So that is what is shown here, if you look at here the solute and solvent they are injected into this mixing channel alternatively, so it creates blocks of the solute in the solvent, so the mixing is going to happen along the direction of the flow as supposed to across the channel has you have seen earlier okay. So now this control of the solute and solvent can be done by controlling the mixing ratio okay, so alpha is the mixing ratio which is the ratio between the solute to solvent.

And that can be done by controlling the valve mechanism okay. So let us say for alpha T/2, so the time period of the valve is T so this is the time period T okay. So for alpha T times the solute is switched on, and for T-alpha T/2 the solvent is turned on okay, so if you do that then we are interested to see how the mixing profile is going to change okay.

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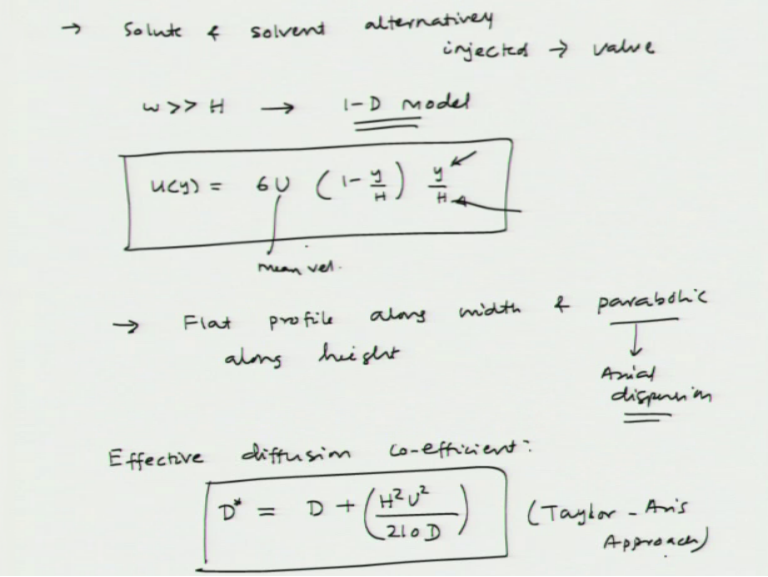

So you know we are doing solute and solvent alternatively injected okay and this is using valve, now let us consider a case where the width of the channel is>>height okay, this width we are talking about is >the height, so we use a 1-D model okay, similar to flow between 2 parallel plates. So in that case we can write down the expression for the velocity, so for flow between infinitely long parallel plates u y=6 U*1-y/H*y/H okay, and where U is the mean velocity.

And y is along the channel width, and H is the channel width okay. So now since we are talking about you know parallel plate where the width is large compared to height, then what you are saying is that profile is going to be parabolic across the height and it is going to be flat across the width, and this parabolic profile will lead to axial dispersion, so there is going to be axial mixing along the flow direction.

So what is going to happen is we will get a flat profile along width and parabolic velocity profile along height okay. So and this parabolic velocity profile will lead to axial dispersion okay, for which the mixing will happen along the flow direction. So we can define a parameter called effective diffusion co-efficient and which is given as D star=D+H square U square/2 10 d and this is using the Taylor-Aris approach okay.

Now if you assume that the concentration does not change across the height or in the width okay only it changes along the flow direction, then we can attempt to solve the transport equation analytically okay.

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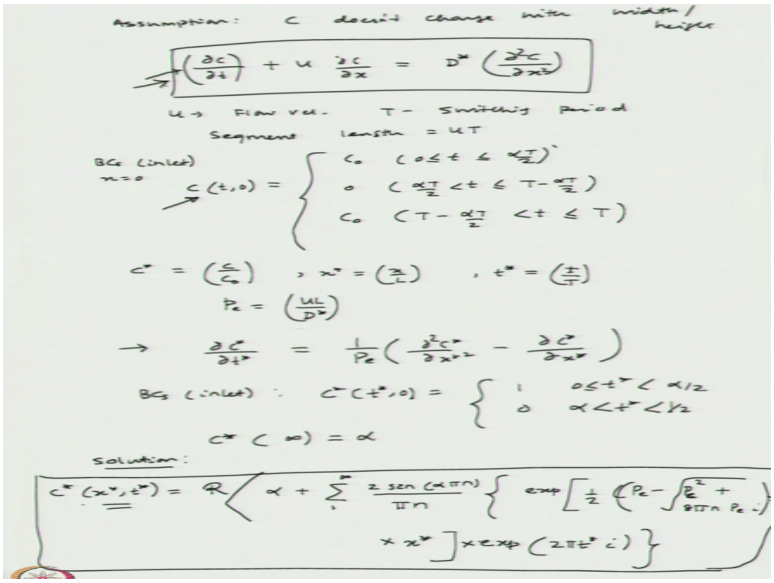

So we make assumption that concentration does not change with width or height, then you can solve the transport equation which is del C/del t+ u del C/del x=D star del square C/del x square okay, so this is the transport equation that we are trying to solve here, this is unsteady case because everything is time dependent, and so u is the flow velocity, and T is the switching period okay. So the segment length, length of 1 segment is going to be u*T.

So let us define the boundary conditions at inlet, so which is x=0 right, so you can define C anytime t at 0=it will be C0 when 0<t<alpha T/2, so because we are doing this solute injection for alpha T time this will be C0 at alpha T/2, so we start taking at the middle the first T=0 at the middle of the solute injection, so for alpha T/2 concentration will be C0, and then when this will be 0 when alpha T/2<t<=T-alpha T/2 okay, then it will be again C0 when T-alpha T/2<t<= capital T.

So that is how the concentration at the inlet is going to be control. Now we can non-dimensionalize this equation the transport equation using the following variables, we can use C star=C/C0 and x star=x/L and t star t/T and peclet number=u L/D star, so if you use these

parameters the transport equation will become del C star/del t star=1/peclet number*del square C star/del x star square-del C star/del x star.

And the boundary conditions that we just outlined these boundary conditions in the non-dimensional form will reduce to boundary conditions at inlet C star t star, 0=1 0<t star<alpha/2, 0 if alpha<t star<1/2, and for complete mixing star at infinity will going to be alpha right. So we can write down the solution for this case, the solution can be written as C star x star, t star is going to be the real part of alpha+1 to infinity 2 sin alpha pi n/pi n*exponential 1/2 peclet number-peclet number square+8 pi n*peclet number*i square root okay*x star then*exponential 2 pi t star*i okay.

So that is going to be the expression for the non-dimensional concentration that is how it is going to vary with x star and t star.

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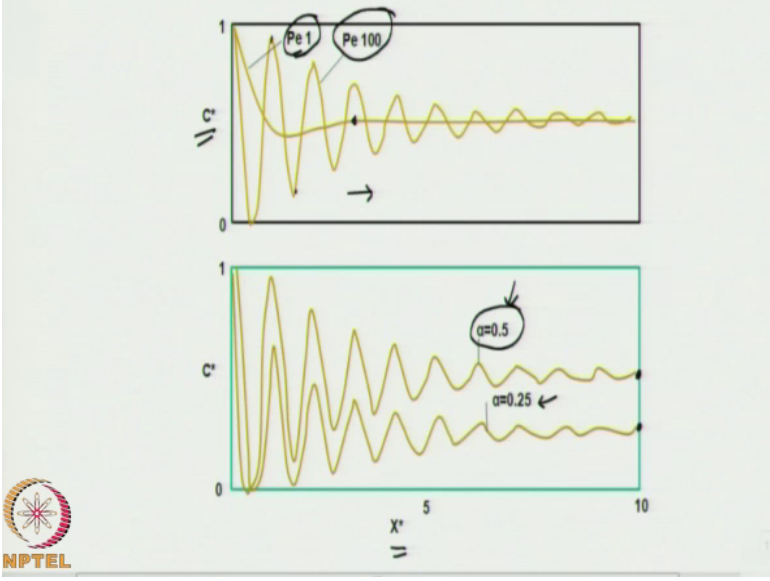

Now if you plot this with respect to the peclet number along the x star, so C star if you try to plot x star along the mixing length. You compare 2 cases in one case peclet number is 1, the other case peclet number is 100, and at low peclet number since the diffusion is more dominated okay, so diffusion is more dominant the final concentration is going to be attain pretty quickly okay, so this is where at this x it almost reaches a steady concentration very quickly.

If the peclet number is 100, so depending on the solute and solvent concentration, the concentration is going to fluctuate along the flow direction, and it takes much longer time to reach a mixed concentration state. Similarly, if you compare 2 solutes to solvent flow rate concentration ratios, one case alpha=0.5, and the other case alpha=0.25, and where we have more solute coming in as compared to solvent.

The final concentration is going to be higher as compared to a case where the solute concentration is lower, so those were 2 important conclusions that we make from here okay.

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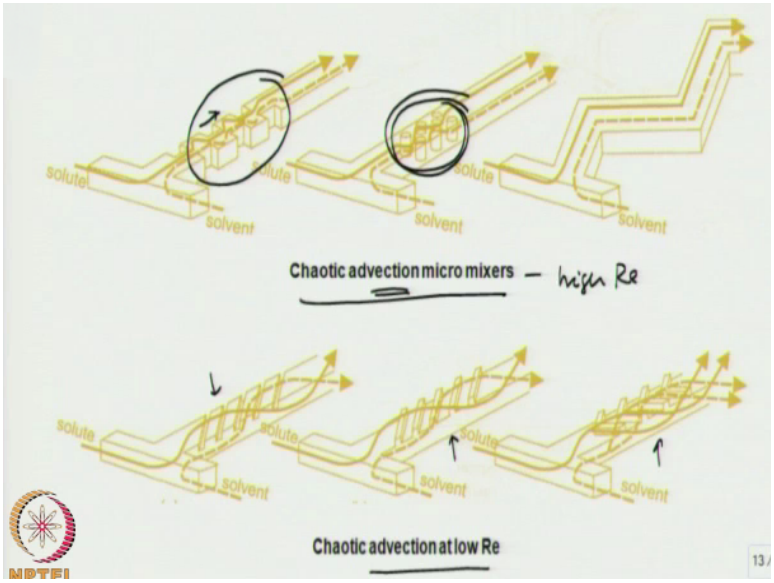

So let us quickly look at a few more types of passive and active mixers, so these are some of the chaotic advection micromixers, so we introduced structures in the flow channel itself which would create bulk moment of the fluid that would help the mixing of 2 fluids. So in this case you know we introduce the variations of the channel cross section along flow direction, also here we are introducing the post on the flow channel.

So this is at high Reynolds number how the advection can come into play to improve the mixing, at low Reynolds number you can use features like depression in the channel as well as ridges and this sabren structures in the flow channel to improve mixing.

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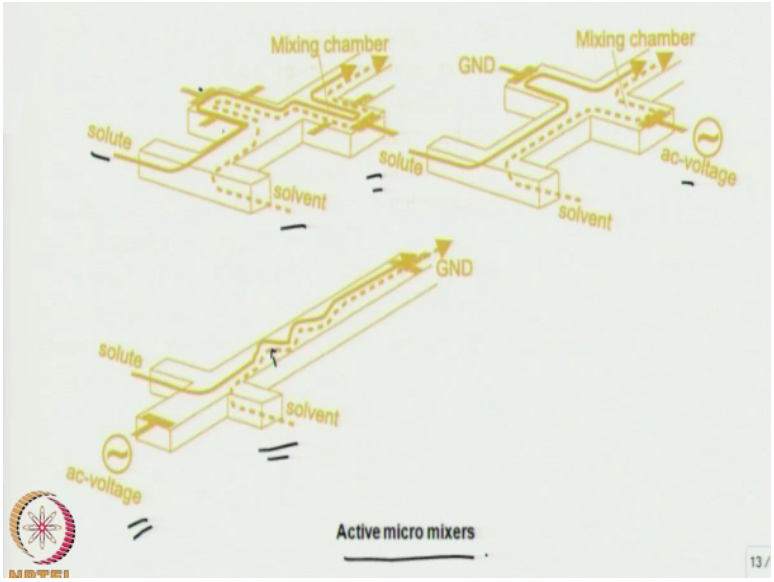

We can have active mixing where you can use external energy, as you can see that here solute and solvent are coming into a mixing channel, where we define you know electrode patterns are present along the channel, and by controlling the voltage in this case for example we can perturb the interface, so that the 2 fluids will talk to each other better to improve mixing, and this is what happens in active micromixers we can use electric field, we can use magnetic field, and we can use thermal gradient to improve mixing between 2 fluids okay.

So that completes our discussion on micromixer, so with that let us stop here.