Microfluidics
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Lecture - 35
Micro Mixers

Okay, so in the previous lecture, we are talking about Microflow sensors, this lecture we will take 2 examples, and then move on and talk about Micromixers okay.

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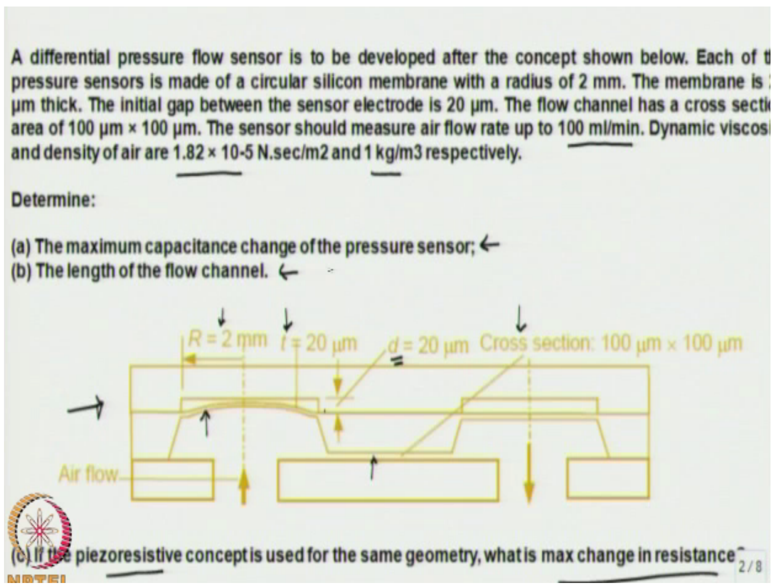

So first let us consider this example on micro flow sensor, so we are talking about a differential pressure flow sensor that would like to develop, and that is as shown in the figure here, and each of the pressure sensors is made up of a circular silicon membrane, so this is the circular silicon membrane with the radius of 2 millimeter as shown here. The membrane is 20 micron thick, so the thickness is 20 micron, and the initial gap between the sensor electrode is 20 micron okay.

So the initial gap here this gap we are talking about is 20 micron okay. The flow channel has a cross section of area of 100 micron 100 micron, so that is the flow channel area of cross section as you can see here 100 micron/100 micron. The sensor should measure air flow rate up to 100 milliliter per minute okay, and the dynamic viscosity and density of air is given, this is the dynamic viscosity and density is 1 kg per meter cube.

Now we would like to find the maximum capacitance change of the pressure sensor okay, the length of the flow channel this flow channel that we are talking about, and if the piezoresistive concept instead of capacitive that we have shown here, if you use piezoresistive concept for the same geometry what is the maximum change in resistance okay. So first let us try to find, what is the maximum capacitors change of the pressure sensor? Okay.

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Assumption: Max. deflection < membrane thickness

$$y(r) = \frac{2(1-\nu^2)}{16 E t^3} (R^2 - r^2)^2 p$$

Max. deflection at centre $(r=0)$:

$$y(0) = y_0 = \frac{2(1-\nu^2)}{16 E t^3} R^4 p$$

$$p_{max} = \frac{y_0 \times 16 E t^3}{2(1-\nu^2) R^4} = \frac{(20\times 10^{-6}) \times 16 \times (170\times 10^{9}) \times (20\times 10^{-6})^3}{}$$

$$p_{max} = \frac{y_0 \times 16 E t^3}{2(1-\nu^2) R^4} = \frac{(20\times 10^{-6}) \times 16 \times (170\times 10^{9}) \times (20\times 10^{-6})^3}{2 \times (1-0.25^2)(2\times 10^{-3})^4}$$

$$= \underline{9671 \text{ Pa}}$$

So we make an assumption okay, we assume that the maximum deflection that is going to occur is <the thickness of the membrane itself okay, so the maximum deflection is going to be the < the membrane thickness. So in that case we can write the expression for deflection, which is the function of r which is 2*1-nu square/16 E t cube*R square-r square whole square*the pressure okay, so that is the expression for the deflection along r.

So you can find the maximum deflection at the center, so that is r=0, so you can find y 0 is going to be let us call it y subscript 0=2*1-nu square/16 E t cube*R 4*pressure okay. So for this case, so from there we can find the maximum pressure that the sensor can work with you can find p max is going to be y0*16 E t cube/2*1-nu square*R 4, so we can find 20*10 to the power-6 is the maximum deflection.

So that is the maximum gap*16*170*10 to the power 9 is the Young's modulus of silicon*thickness is 20 micron*10 to the power-6 cube/2*1-0.25 square is the Poisson’s ratio

*2*10 to the power-3 is the radius to the power 4, so you can find the pressure to be 9671 Pascal okay. So that is the maximum pressure that the sensor will work with.

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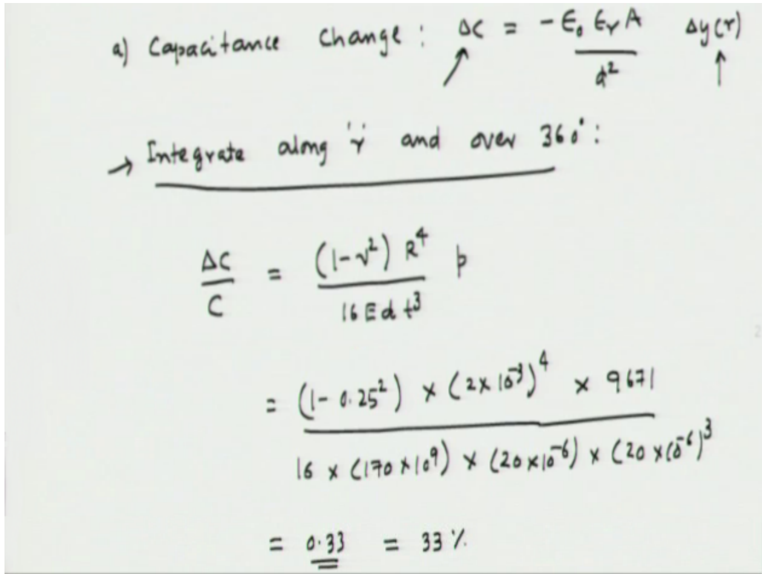

So you can find the capacitance change, so the capacitance change delta c as per definition is going to be -epsilon 0 epsilon r A/d square*delta y r okay, this is the change in the deflection. Now if we integrate this equation, so we integrate along radial direction r and over 360 degrees the azimuthal direction, then we can find the total change in the capacitance over original capacitance will be 1-nu square*R 4/16 E d t cube*pressure.

So if you substitute for delta y r and do the integration this is what we will end up with okay, so from here we can calculate the maximum capacitance change delta c max/c=1-nu square*R 4/16 E d t cube*p max, so that will be=1-0.25 square*2*10 to the power-3 4*p max is 9671 which we just found out so/16*170*10 to the power 9 *20*10 to the power -6 *20*10 to the power -6 cube that is the thickness q.

So we can find an expression for the maximum capacitance change it will be 0.33 okay, so the change in the capacitance is about 33% of the original capacitance okay. Now if you I want to find the length of the channel okay, so you want to find the length of the flow channel, what is the length of the flow channel to do that first let us try to find the maximum velocity.

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b) $u_{max} = \left(\frac{Q_{max}}{A}\right) = \frac{(100\times10^{-6}/60)}{(100\times10^{-6})^2} = 16.7\ m/s$

Reynolds no. at this velocity :

$$Re_{max} = \left(\frac{u_{max} D_h \rho}{\eta}\right) = \left(\frac{16.7\times10^{-4}\times1}{1.82\times10^{-5}}\right)$$

$$= \underline{\underline{918}}$$

↳ Flow is laminar

$$\Delta p = f\,Re\left(\frac{\eta u L}{2 D_h^2}\right)$$

$$\Rightarrow L = \left(\frac{1}{fRe}\right)\left(\frac{2\,\Delta p_{max}\,D_h^2}{\eta\,u_{max}}\right)$$

$$= \frac{1}{55}\times\frac{2\times9671\times(10^{-4})^2}{(1.82\times10^{-5})\times16.70}$$

$$= \underline{\underline{1.16\ mm}} \qquad \boxed{L = 1.16\ mm}$$

So u max=Q max/the area of cross section, q max is 100 microliter per minute so 100*10 to the power-6/60 meter cube per second/area will be 100*10 to the power-6 square okay, so 100 micron channel size, so we can find the velocity to be 16.7 meter per second okay. So the Reynolds number at this velocity, so Reynolds number Re max is going to be u max hydraulic diameter density/eta viscosity.

So that will be 16.7*hydraulic dia is 10 to the power-4 that is 100 micron right*1 is the density of air viscosity is 1.82*10 to the power-5 okay, so we can find the Reynolds number to be 918, so at this Reynolds number what it means that the flow is laminar. So the flow is laminar and so in that case we can write delta p the pressure drop will be f Re*eta u L/2 Dh square okay, this is the Hagen-Poiseuille formula.

So from there we can find L the channel length is 1/f Re *2 delta p max*Dh square/eta u max okay. So if we plug in the values, so 1/ as you know the f Re value for laminar flow in a rectangular microchannel has a value between 50 to 60 okay, let us consider f Re is 55, so f Re is 55*2 delta p is 9671 Dh square is 10 to the power-4 square/eta is 1.82*10 to the power-5 and the velocity is 16.7 meter per second okay. So we can calculate L to be 1.16 millimeter, L=1.16 millimeter right.

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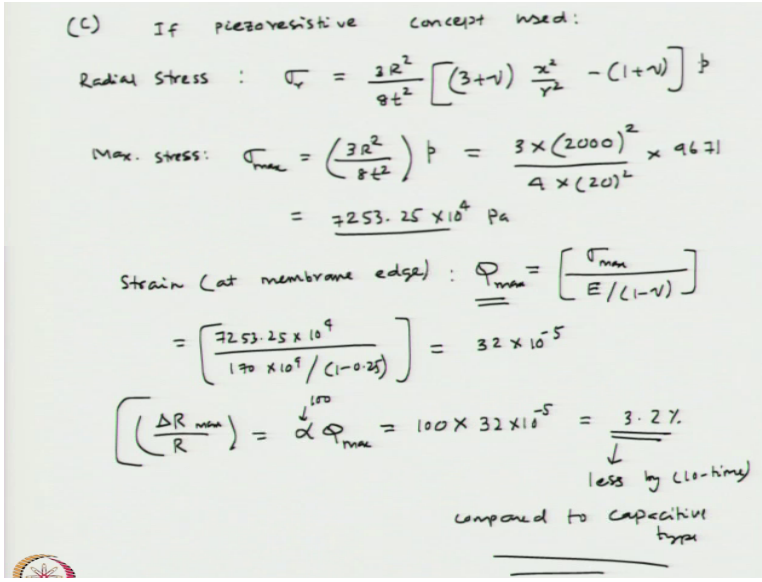

Now we are interested to see, if piezoresistive concept is used for the same geometry instead of capacitive type concept, we want to see what is the maximum change in resistance? Okay. So we would like to see if piezoresistive concept is used, then the stress along radial axis the stress radial stress sigma r is going to be 3 R square/8 t square*3+nu x square/r square-1+nu*pressure, so that is the formula for the radial stress in the membrane.

So you can find the maximum stress sigma max which will be 3 R square/8 t square*p okay, so here we can say it is 3*2000 square/4*20 square so this is 2000 micron which gets cancelled so 10 to the power-6 can gets cancelled with the 20 micron in thickness okay, so 2000 square/20 square*the pressure is 9671, so we can say this is 7253.25*10 to the power 4 Pascal okay. So we can find the strain which is going to be maximum at the edge of the membrane.

So at membrane edge, so let us called phi max=sigma max/E/1-nu okay, so that will be 7253.25*10 to the power 4/170*10 to the power 9 is E/1-0.25 okay Poisson's ratio nu is 0.25, so we found that 32*10 to the power -5 right. So knowing the strength we can find delta R max/R=alpha*maximum strength, and for alpha has typical value about 100 for silicon so 100*32*10 to the power-5, so that gives a value which is 3.2% okay.

So if you compare the change in the resistance over the original resistance, which is about 3.2%, with the change in the capacitance over original capacitance which is 33%, so this is less by 10 times compared to the capacitive type okay. So now let us take on the second example problem which is on a drag flow sensor okay.

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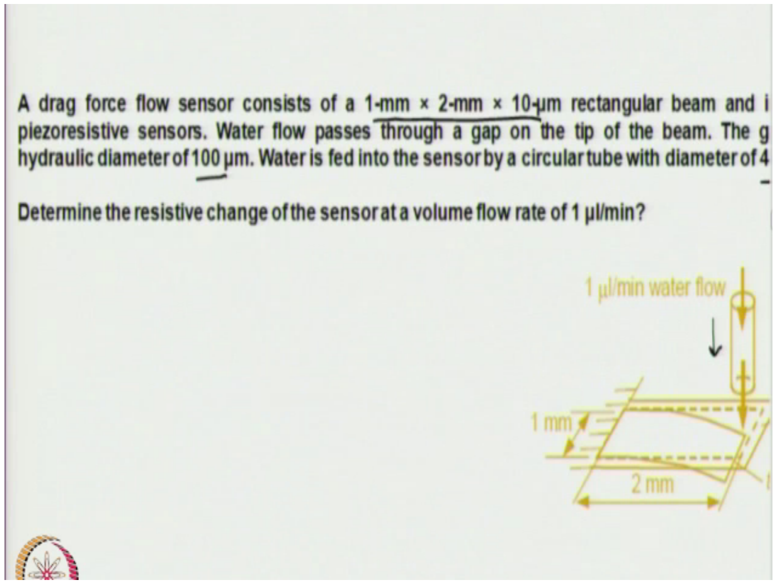

So we are talking about a drag flow sensor that consists of a 1 millimeter, 2 millimeter, 10 micron rectangular beam, and that is integrated with piezoresistive sensors, water flow process through a gap on the tip of the main beam as you can see here, and the gap has a hydraulic diameter of 100 micron, so there is gap around this the beam and water is fed into the sensor by a circular tube of diameter 400 micron okay. And we are interested to determine the resistive change of the sensor at a volume flow rate of 1 microliter per minute okay.

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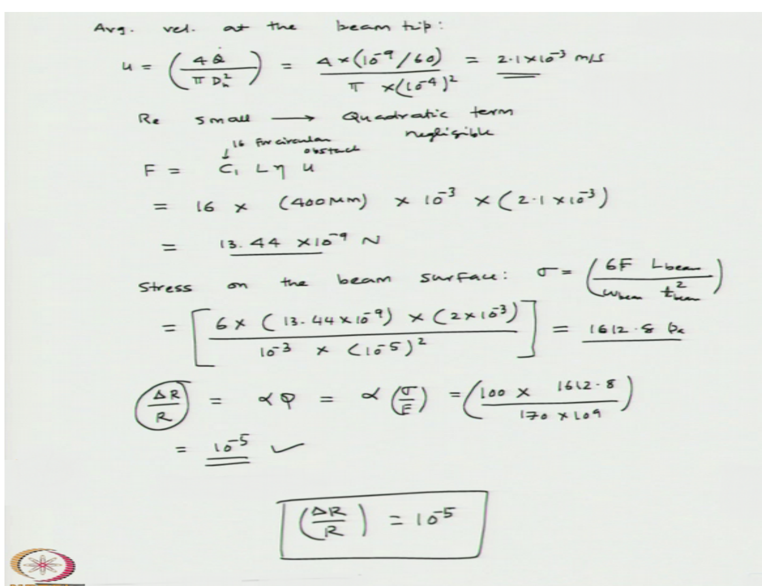

So first we find out the average velocity of the beam tip, average velocity at the beam tip okay, average velocity of the water at the beam tip, so u=4 Q/pi Dh square okay this flow rate hydraulic diameter, so 4*10 to the power-9/60/ so this is in meter cube per minute and the -9 meter cube/60 meter cube per second pi*10 to the power-4 square, so this is going to be 2.1*10 to the power-3 meter per second.

So at such low velocity the Reynolds number is going to be small okay, so Reynolds number is going to be small so as a result of that the quadratic term in the drag force equation will be negligible, so the quadratic term is negligible okay. So we discussed that the total drag force will be a combination of the linear term and the quadratic term, the linear term is dominant at low Reynolds number, and the quadratic them is dominant at high Reynolds number.

so since here the Reynolds number is low the quadratic term is negligible okay, so we take the linear term so the expression for the force is C1 L eta*u okay, now so we can write C1 is 16 for a circular obstacle, so this is 16 for circular obstacle, so the characteristic length L is 400 micron the obstacle size*eta is 2.1*10 to the power-3 sorry the viscosity of the water, eta is viscosity of water which is 10 to the power -3 and the velocity is 2.1*10 to the power-3.

So the force is 13.44*10 to the power-9 Newton okay. So we can find the stress on the beam surface sigma is typically 6F*L of the beam length of the beam/width of the beam, thickness of the beam square okay, so you can find that 6*13.449 10 to the power-9*2*10 to the power-3/10 to the power-3 10 to the power-5 square, so that is about 1612.8 Pascal okay. So with that we can find the change in the resistance over original resistance which is alpha*the strength.

So alpha*sigma/E is the strength, so alpha is the value of 100 for silicon, so 100*1612.8/170*10 to the power 9 which will be =10 to the power-5 okay. So the change in the resistance over original resistance is 10 to the power-5, so that is what we want to find out, determine the resistance change of the sensor, so delta R/R=10 to the power -5 okay, so with that we complete our discussion on micro flow sensor. Now let us move on and talk about micromixers okay.

So as we discussed initially micromixers are an important component in microfluidic devices microfluidics systems, where you know they have significance in biomedical applications, they have significance in chemical analysis, where we are interested to mix 2 different chemicals, mix a buffer with sample okay. So the mixing has prime importance in microfluidics okay, at the same time the mixing is very challenging in microchannels okay.

At microscale we exploit the velocity of the fluid so that the flow can be easily turn into turbulent mode by increasing the flow velocity, so this turbulence in the flow helps in mixing at microscale. Now at microscale the turbulence effects are not present flows are inherently laminar because of the small length scale of the system, so since flows are laminar mixing becomes very challenging, and mixing and laminar flow is primarily based on diffusion okay.

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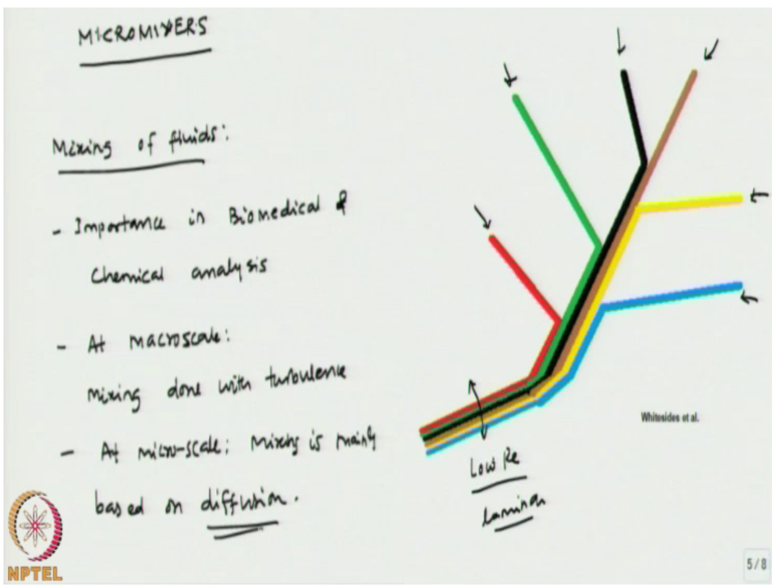

So let us look at micromixers okay, so the mixing of fluids have importance in biomedical and chemical analysis okay, in at macroscale mixing is done with turbulence. Now at microscale the mixing is based on diffusion okay this is mainly based on diffusion, in some cases advection also plays the role. Now if you look at here why mix mixing becomes challenging at microscale, here we are injecting in a different colored eyes at different inlets.

And in the microchannel these colored eyes remains as it is they do not interact with each other better, because of the low Reynolds number okay because of the low Re, and since flow is lamina they do not interact much and the only mode they can mix is through diffusion okay, so diffusion is the primary mode of mixing at microscale okay. So with that let us look at the physics of micromixing okay.

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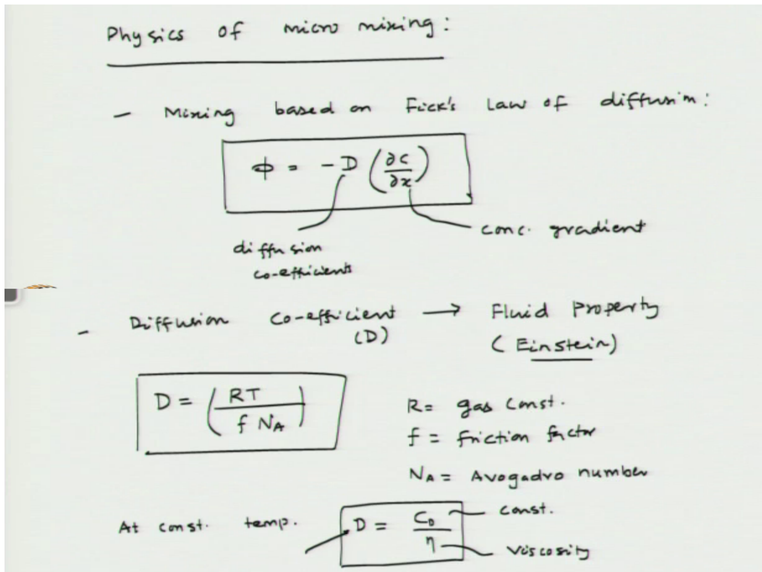

Now the mixing of 2 fluids in a micro channel is purely based on the Fick's law of diffusion okay, so the mixing based on Fick's law diffusion, which states that phi which is the mixing rate=-D*the concentration gradient okay right, where D is the diffusion co-efficient, we will see you know what are the typical values of diffusion co-efficient for different materials, del C/del x is the concentration gradient okay.

Now this diffusion co-efficient is a fluid property, and an expression for the diffusion co-efficient has been derived by Einstein okay, so the diffusion co-efficient D is the fluid property okay, and it has given by Einstein so D=RT/f*NA, where R is the gas constant and F is the friction factor and NA is Avogadro number okay. So now at constant temperature the diffusion co-efficient can be written as CD/eta, where CD is a constant and eta is the viscosity dynamic viscosity okay.

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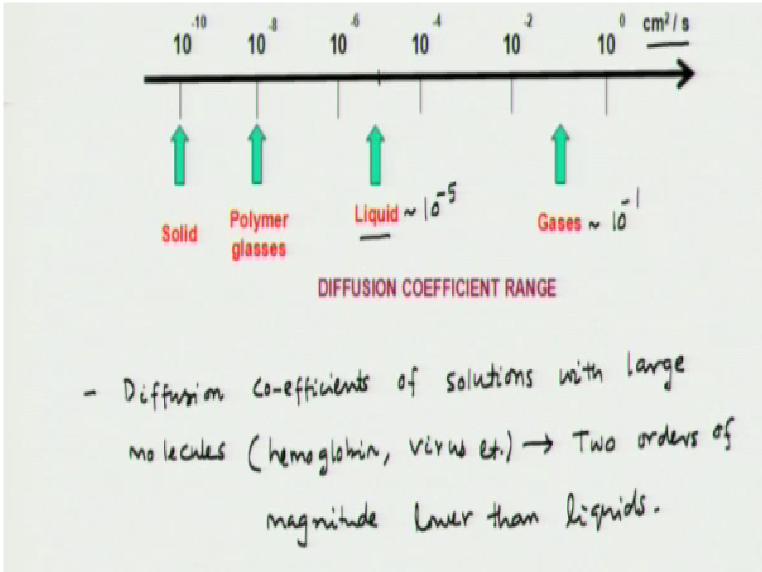

So you know you can see what is the diffusion co-efficient for different materials, if you look at here we have the diffusion co-efficient in the unit centimeter per second, liquids have a diffusion co-efficient of the order of 10 to the power-5 okay, and gases have larger diffusion so gases have about 10 to the power-1, so 4 order of magnitude higher than that of liquid. Polymers and glasses have of the order of 10 to the power-8, and the solids have of the order of 10 to the power-10.

Now the diffusion co-efficient of a liquid will reduce by at least 2 orders of magnitude, when it will contain some biomolecules okay. For example, if it contains hemoglobin, it contains viruses or larger molecules the diffusion co-efficient will reduce okay. So the diffusion co-efficient of solutions with large molecules such as hemoglobin, viruses etc. will have 2 orders of magnitude lower, so 2 orders of magnitude lower than liquids okay.

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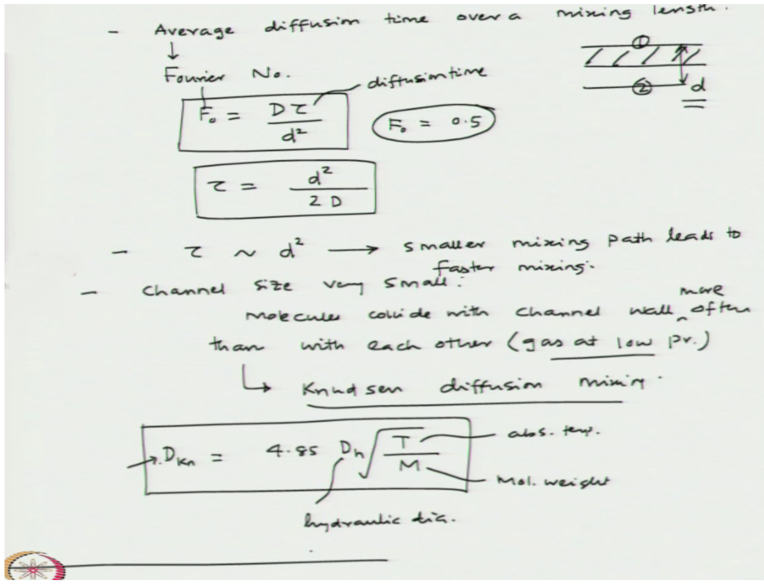

Now the average diffusion time over a mixing length let us say the 2 fluids, and so this is fluid 1, this is fluid 2, they are trying to mix and this the channel width is W. Then we would see that the average diffusion time over a mixing length would depend on what is called a Fourier number okay. And the Fourier number is defined as let us call this d Fourier number is defined as diffusion co-efficient*timescale of diffusion/square of the mixing length scale okay.

Now Fourier number has a value typically 0.5 okay, so we can write down the expression for the diffusion timescale as the d square/2*diffusion co-efficient okay, so tau is the diffusion time, F0 is Fourier number and D is the mixing length scale. So what we have observed here is that the diffusion timescale varies as the square of the mixing length scale okay, so the diffusion time scale tau varies as square of the mixing paths okay.

So what we can infer from here is smaller mixing length or smaller mixing path leads to faster mixing okay. And if channel size so this is what we can conclude right, now if the channel size is very small let us say of the order of nanometer and we are talking about gas at very low pressure, the gas molecules will collide with channel wall quite often as compared to the intermolecular collision okay. So in that case the mixing length will be called as Knudsen mixing length okay.

So channel size is very small, so that the molecules collide with channel wall more often than with each other okay, so in that case so this will be called as Knudsen diffusion mixing, so the

Knudsen diffusion co-efficient so this is the case for gas at low pressure. So the Knudsen diffusion co-efficient Dkn is given by 4.85*the hydraulic diameter*temperature/the molecular weight square okay.

So this is the Knudsen diffusion co-efficient, and this is molecular weight, and this is the absolute temperature, and this is the hydraulic dia okay. So with that let us proceed and look at the peclet number low Reynolds number diagram.

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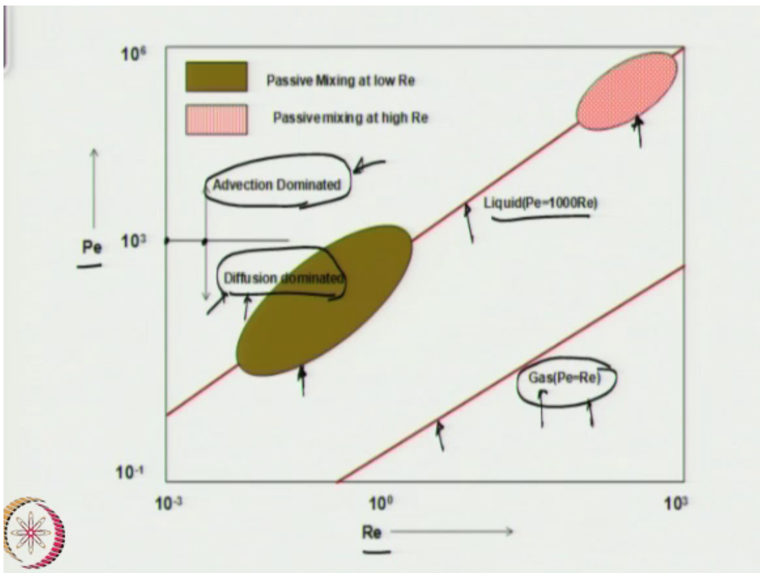

You know for mixing at microscale Reynolds number is not the only parameter that would govern mixing okay, because we are talking about mixing based on diffusion, then peclet number will come into play okay. So here we look at the peclet number, Reynolds number diagram which is an important concept in the mixing in microchannels, so here we look at the peclet number Reynolds number diagram.

So on x-axis we have Reynolds number and y-axis we have peclet number okay, now if you look at the definition of peclet number and Reynolds number.

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$$\left(\frac{P_e}{R_e}\right) = \frac{(uL/D)}{(uD_h/\nu)} = \left(\frac{L}{D_h}\right)\left(\frac{\nu}{D}\right)$$

Micro mixers: $L \sim D_h$

$$\boxed{\left(\frac{P_e}{R_e}\right) = \left(\frac{\nu}{D}\right)}$$

$10^{-5}\ cm^2/s$

Liquids: $\nu = 10^{-6}\ m^2/s$, $D = 10^{-9}\ m^2/s$

$$\frac{P_e}{R_e} \sim 1000 \Rightarrow \boxed{P_e \sim 1000\ R_e}$$

Gases: $\nu = 10^{-5}\ m^2/s$, $D = 10^{-5}\ m^2/s$

$$\frac{P_e}{R_e} \sim 1 \Rightarrow \boxed{P_e \sim R_e}$$

So the peclet number/Reynolds number is given by u L/D that is peclet number, and Reynolds number is u Dh/nu kinematic viscosity, so this will be L/Dh*nu/D. Now in case of micromixers the hydraulic diameter and the mixing length they are of the same order okay, so for micromixers L and of the order of Dh okay. So what it means is that the peclet number/Reynolds number is varying as nu/D okay.

Now for liquids the kinematic viscosity is of the order of 10 to the power-6-meter square per second okay, and the diffusion co-efficient as we just saw it is of the order of 10 to the power-9 meter square per second, so you know we saw this as 10 to the power-5 centimeter square per second which translates to 10 to the power-9 meter per second. So nu/D so peclet number/Reynolds number which is nu/D will scale as 1000 times the so this will be 1000.

So the peclet number will be 1000 times the Reynolds number and this is for liquid. Now for gases, the kinematic viscosity for gases is about 10 to the power-5 meter per second, and the diffusion co-efficient is 10 to the power-1 centimeter square per second so that is 10 to the power-5 meter square per second. So we can see that the peclet number/Reynolds number will be the order of 1, so peclet number will be off the order of Reynolds number for gases.

And peclet number will be 1000 times Reynolds number for liquids, so with that understanding if you look at the Pe, Re diagram, then we can see that for gases Pe is about Re, so this is the line which governs the mixing of gases in microchannels okay. Now this line governs mixing of liquids in microchannels, for liquids the peclet number is 1000 times Reynolds number. And there are 2 regions for mixing of liquids in the Pe, Re diagram.

There is one region where the Reynolds number is high, and 1 region the Reynolds number is low okay. Now for a region based on the peclet number we can differentiate 2 regions, one region is diffusion dominated if the peclet number is <1000 then the mixing is diffusion dominated okay, and peclet number is >1000 the mixing is advection dominated okay. And this is based on the Reynolds number, this is low Reynolds number, this is high Reynolds number okay.

So here we can see that for the gas since the peclet number is below 10 to the power 3 below 1000, then you know mixing in gas is mainly because of diffusion okay, so there is no advection present when you are talking about mixing of cases. However, for liquids when the peclet number is <1000 are we are talking about low Reynolds number, then mixing will be diffusion dominated, and if the Reynolds number is high or the peclet number is >1000 the advection mixing will come into picture okay.

So that is an important understanding when you talk about mixing of liquids and gases at microchannels, for gases the mixing is always based on diffusion, and for liquids if the Reynolds number is low or the peclet number is <1000 we can say that mixing is diffusion dominated, if the Reynolds number is number is high and peclet number is >1000 we can say that mixing is advection dominated okay.

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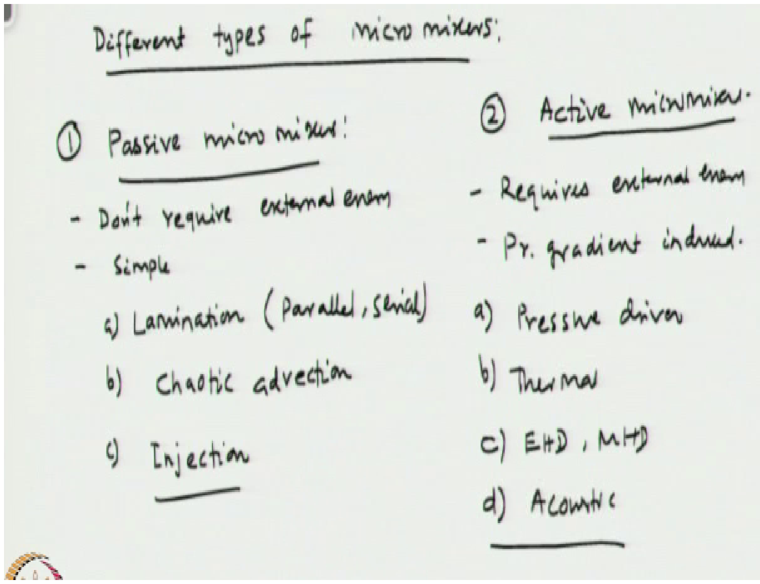

So with understand that let us move on and talk about different types of micromixers okay, so we discuss different types of micromixers. So micromixers can be categorized as passive micromixers and second active micromixers, the passive micromixers do not require external energy, and they are simple. Active micromixers requires external energy such as electric field, magnetic field, pressure fluctuations to enhance mixing.

And mainly they will induce pressure gradient which helps in mixing okay. Different types of passive micromixers are laminar, so lamination type micromixers, which would have parallel lamination, also serial lamination, and it would have chaotic advection, it would also have injection type where one sample will be injected into second sample for mixing. In active micromixers we would have pressure driven, thermal.

Then electrohydrodynamic, magneto hydrodynamic and acoustic okay. So these are different types of active micromixers. So with that let us stop here, we will continue our discussion on different types of micromixers, and look at the physics of different micromixers in the subsequent lecture.