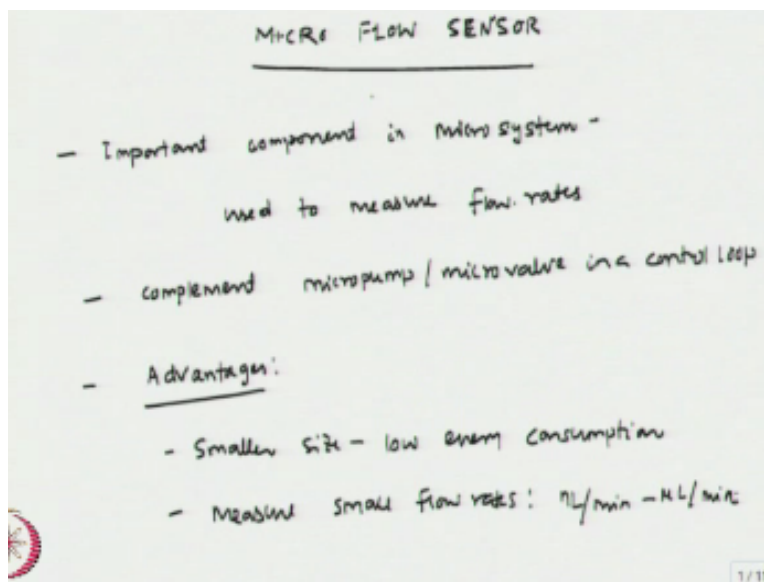


Microfluidics
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Lecture – 34
Micro Flow Sensor and Micro mixers

Okay, so let us talk about the micro flow sensors; micro flow sensors are the important component in a typical microfluidic system and they are used to measure flow rates in microfluidics, okay and these micro flow sensors can complement to a micro valves and micro pumps in a microfluidics system. In a micro pumps are used to generate flow rates and micro valves used for controlling flow rates.

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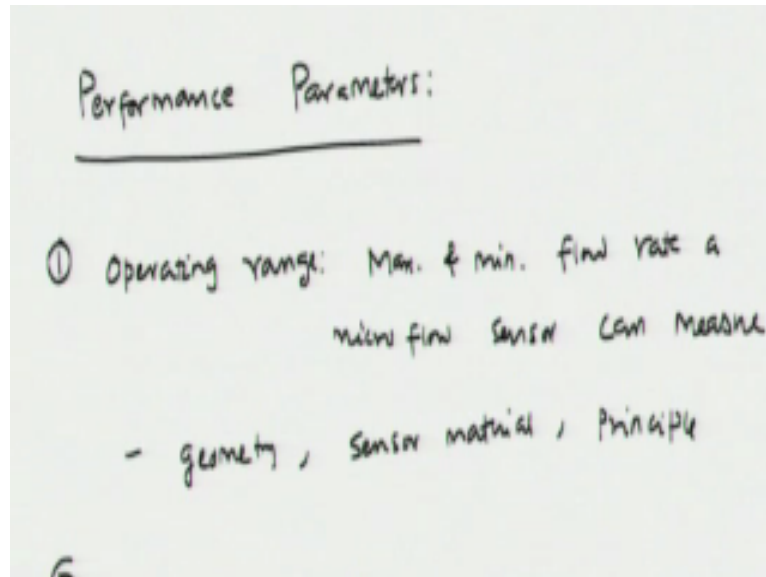


And micro flow sensors can measure the flow rate and check whether the micro pump or the micro valve is performing according to the requirement, okay. So, we talk about the micro flow sensor, okay. So, the micro flow sensors are important component in microfluidics system and their used for; used to measure flow rate and they can complement micro pump or micro valve in a control loop.

Now, what are the advantages of micro flow sensors; the advantages are that micro flow sensors are miniaturised in size, so they consume less power and the second advantages that the micro

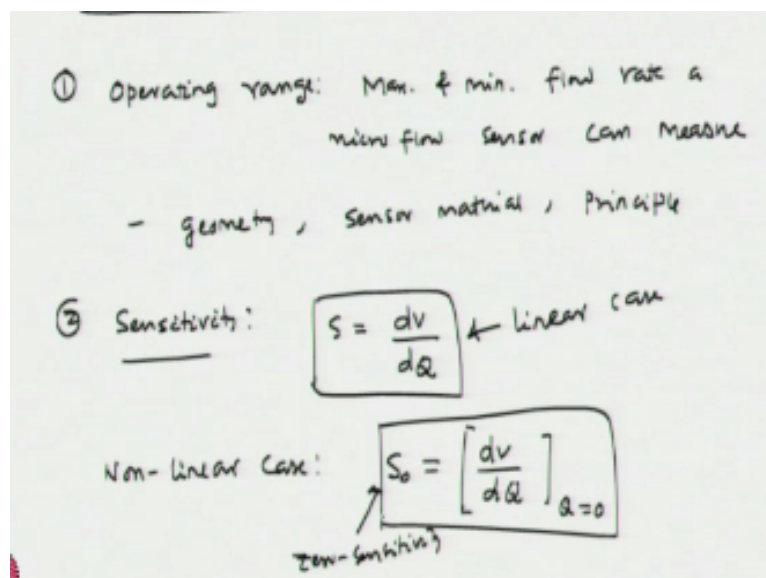
flow sensors enables measuring flow rates of the order of Nano litre per minute to microliter per minute, okay which may not be possible otherwise. So, the advantages are smaller size, so that gives low energy consumption.

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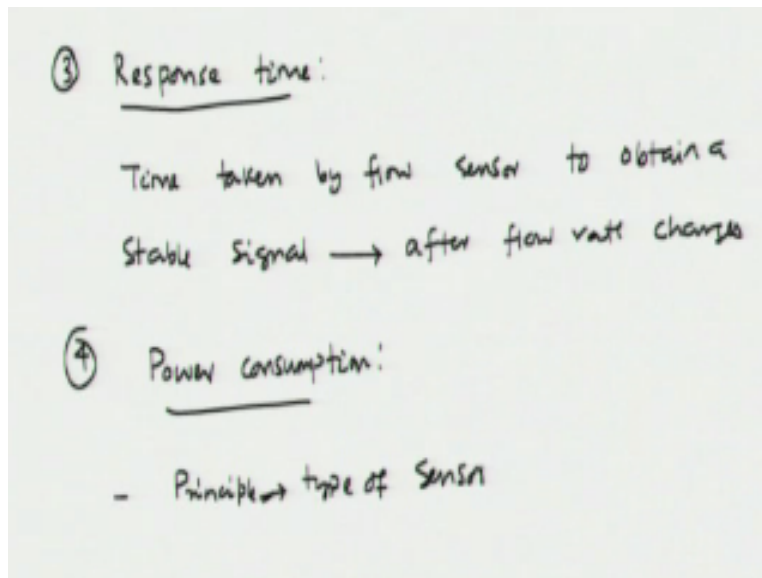
And the second is they can measure small flow rates of the nano litre per minute to microliter per minute, okay. Now, let us look at some of the performance parameters of a micro flow sensors; the first performance parameter that will look at is the operational range; in what range of flow rates is certain micro flow sensors can be used. So, we look at few performance parameters. The first parameter that we are going to look at is the operating range.

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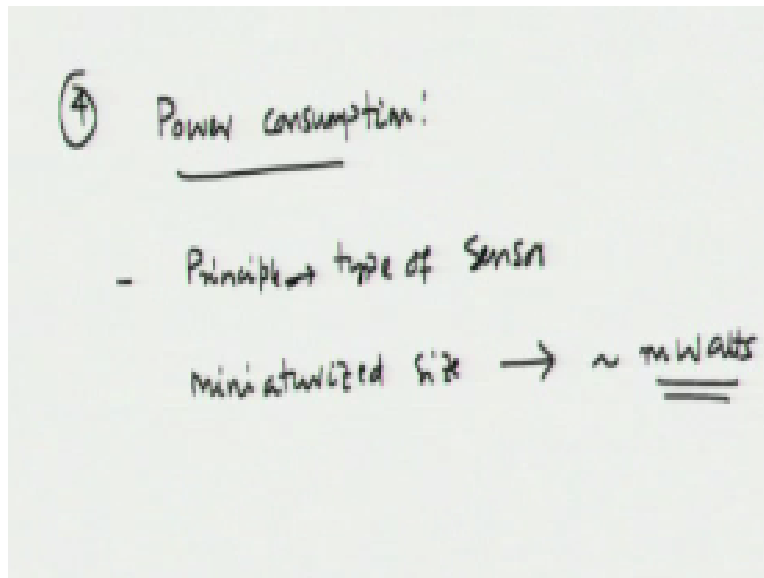
So, operating range is the maximum and minimum flow rates, a micro flow sensor can measure and this will depend on the geometry of the sensor, sensor material and as well as the principal, okay. The second parameter is sensitivity and a sensitivity is defined as the change in the output volt is for a given change in the flow rate. So, the sensitivity is defined as; is defined as dv over dQ , okay. Now, this is for a linear case.

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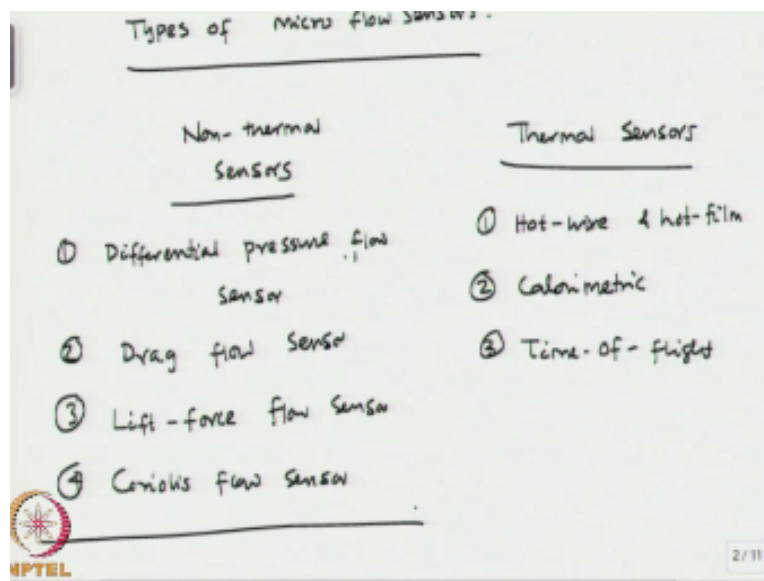
When output is linearly depending on the flow rate, for nonlinear case, we can define sensitivity are 0 flow rate; and this is called 0 sensitivity; as zero is defined as dv over dQ and that is at $Q = 0$, okay. The third parameter is the response time; response time is the time taken by the flow sensor of a signal to stabilise, once there is a change in the flow rate, okay. So, this is the time taken by flow sensor to obtain a stable signal after flow rate changes, okay.

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So, and the last parameter is the power consumption; power consumption can depend on the principle of the sensor and also the type of sensor, so the principle and type of sensor, so because of miniaturised size, the sensors have miniaturised size, so typically micro flow sensors will operate using the energy of the order of milli watts, okay. Now, let us look at what are the different types of micro flow sensors.

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Types of micro flow sensors; they can be broadly classified into long thermal sensors and thermal sensors, okay and in non-thermal sensors, there will be based on different principles; one is the differential pressure flow sensor, second one is the drag flow sensor, the third one is lift

force flow sensor and the fourth one is coriolis flow sensor, so these are 4 types of non-thermal sensors.

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Diagram illustrating the formula for pressure drop (ΔP) in laminar flow through a rectangular microchannel:

$$\Delta P = f Re \left(\frac{\eta L}{2 D_h^2} \right) u$$

Annotations:

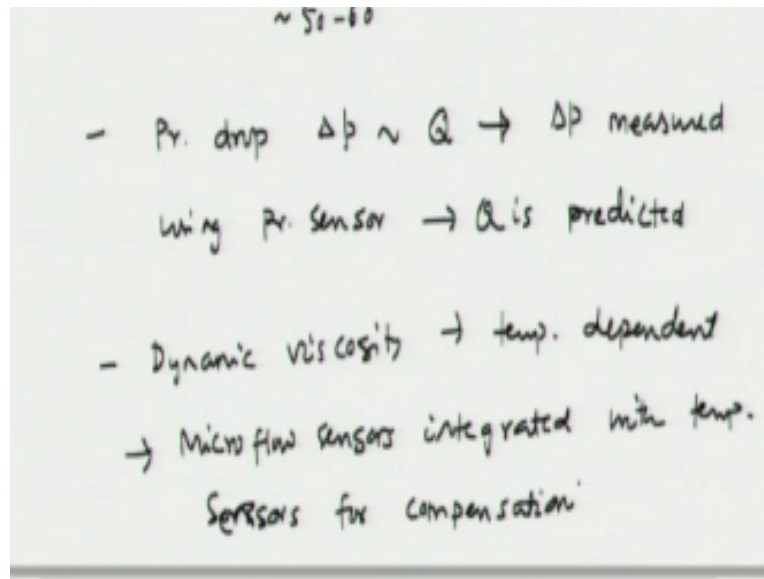
- $f Re$: rectangular $\sim 50-60$
- L : length
- D_h : hydraulic dia.
- u : velocity

In thermal sensors, we would have hot wire and hot film and we would also have colorimetric sensors and we would have time of flight, okay. So, look at each of these; different types of sensors; to start with let us look at the differential pressure flow sensors. So, let us look at the differential pressure, this is flow sensor, so in typically, in micro flow sensors, the channel sizes are small, so hydraulic diameter is small.

As a result, the Reynolds number are small which leads to laminar flow, okay. So, in micro flow sensors, we have laminar flow, so in laminar flow the pressure drop ΔP can be written as $f Re * \eta L \text{ over } 2 D_h \text{ square} * u$, so that is a expression for pressure drop, in case of laminar flow in a micro channel and $f Re$ as a; for a rectangular micro channel, which is the channel construction that we use quite often in micro fluidics.

The $f Re$ value is between 52, 60, okay and this is the hydraulic dia and this is viscosity, this is the length scale, the length of the channel, this is velocity okay. So, what we see here is that the pressure drop is linearly depends on flow velocity or flow rate, okay, so that is used to develop differential flow sensors. We actually measure the pressure drop and from measuring the pressure drop, we predict the flow rate, okay.

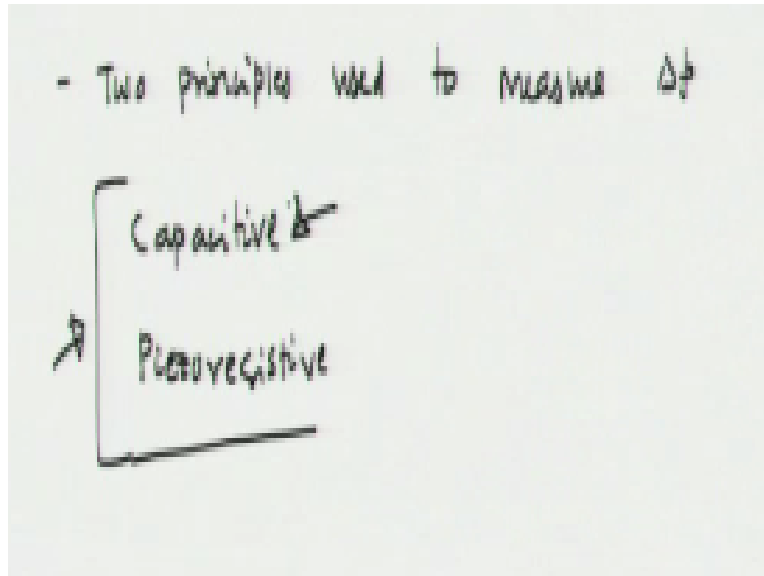
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So, the pressure drop of the Δp is proportional to Q and so Δp is measure using pressure sensor and from their Q is predicted, okay. Now, one important thing to notice here is that one parameter that depends on the temperature is η , okay; η is the dynamic viscosity which depends on temperature, okay and since the dynamic viscosity is depend on temperature, the flow rate will be affected by changing in the temperature.

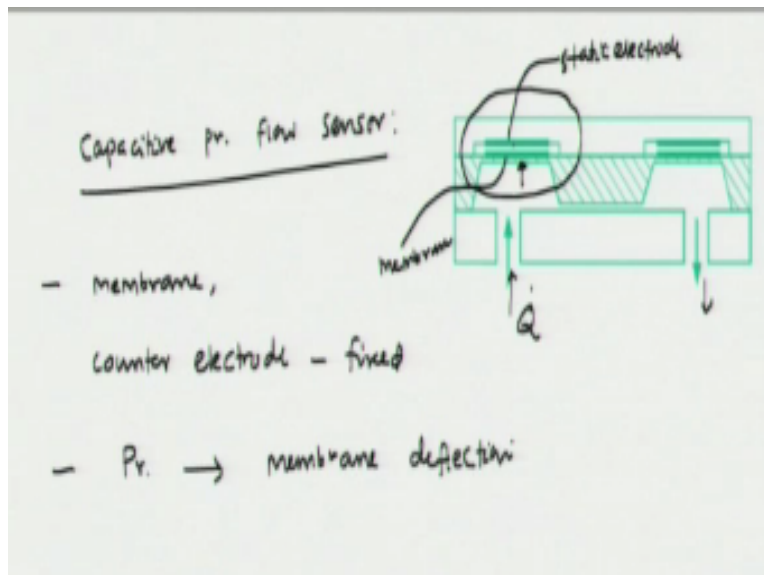
So many times this differential pressure and flow sensor are integrated with temperature sensors in order to compensate for the temperature change effect, okay. So, here the dynamic viscosity is temperature dependent, so the micro flow sensors are integrated with temperature sensors for compensation and based on; you know the principles; 2 principles that are used to measure the pressure drop.

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So, 2 principles used to measure Δp ; one is capacitive and the second one is piezo resistive, so these 2 principles are used to measure pressure drop and from their flow can be predicted, okay. Let us first look at capacitive type pressure flow sensor.

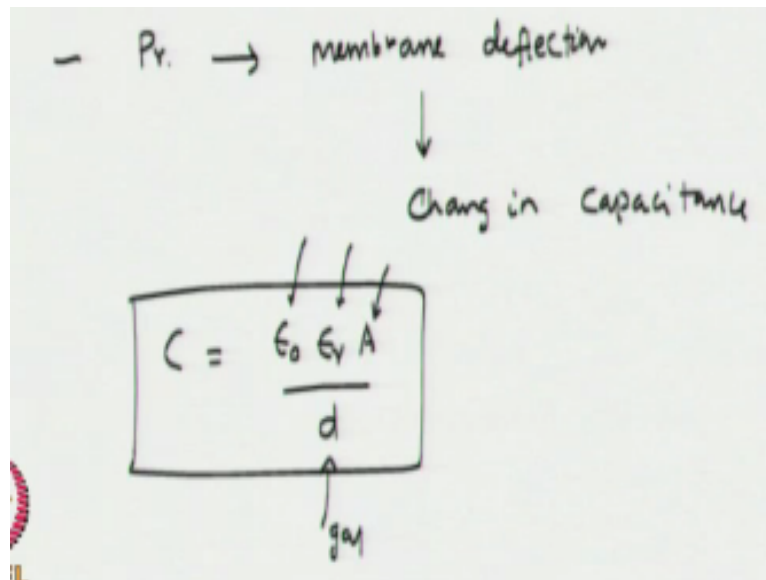
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So, we look at capacitive type pressure flow sensor. So, as I can see here, this is the inlet, okay and this is the outlet and here we have a capacitive type flow sensor. So, as the flow goes in through the microfluidic channel, here we have a membrane, okay and this is a membrane on which one electrode is position and there is the second electrode here, this is a static electrode and as the flow rate changes, so does the pressure drop.

So, and since the pressure drop is modify, the pressure inside this chamber is changing and when the pressure changes, there is this movable membrane moves with respect to the static electrode, so there is a change in the capacitance that happens, okay and that change in the capacitance is related to the flow rate, okay. So, here in the capacitive pressure flow sensor, we have different components, we have a membrane and we have a counter electrode.

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So, membrane itself as one electrode, the counter electrode which is typically fixed and what happens is the pressure causes the membrane deflection, okay; pressure causes the membrane to deflect and the membrane deflection will cause change in the capacitance, okay. So, we can write down the expression for capacitance which is $\epsilon_0 \epsilon_r A/d$, so this is permittivity of free space.

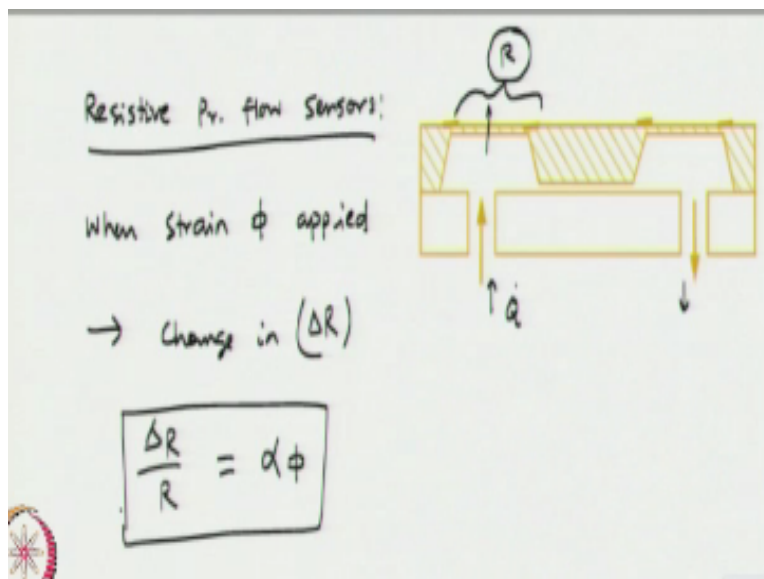
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$$\Delta C = \frac{\partial C}{\partial d} \times \Delta d \Rightarrow \Delta C = \left(-\frac{\epsilon_0 \epsilon_r A \Delta d}{d^2} \right)$$

$$\Delta C \rightarrow d \rightarrow Q$$

The dielectric constant of the medium, which is present between these 2 capacitor plates; A is the capacitor area and E is the gap, okay. So, this gap d actually changes, when there is change in the flow rate, okay so what is going to happen is there is going to be a change in the capacitance because of the change in the gap, so it will be $\frac{\partial C}{\partial d} \times \Delta d$, so $\frac{\partial C}{\partial d}$ is going to be $-\frac{\epsilon_0 \epsilon_r A}{d^2}$.

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So, that is how the capacitance is going to be related to the gap, which is a function of the flow rate, okay. So ΔC is related d , which is related to Q , so that is the principal based and which the capacitive pressure flow sensors work. Now, let us look at piezo resistive pressure flow

sensors. So, look at the resistive pressure flow sensor, okay. So, similarly here, we have flow rate Q coming in and is going out, this is a chamber.

So, as flow rate changes, the pressure drop is going to change, so the pressure inside the chamber is going to be modified, so the pressure modifies the membrane is going to expand; okay, the pressure increases, for example the membrane is going to expand and because the membrane is expanding, the resistance between these 2 electrode r is going to change, okay. So that, as flow change rate changes the pressure changes.

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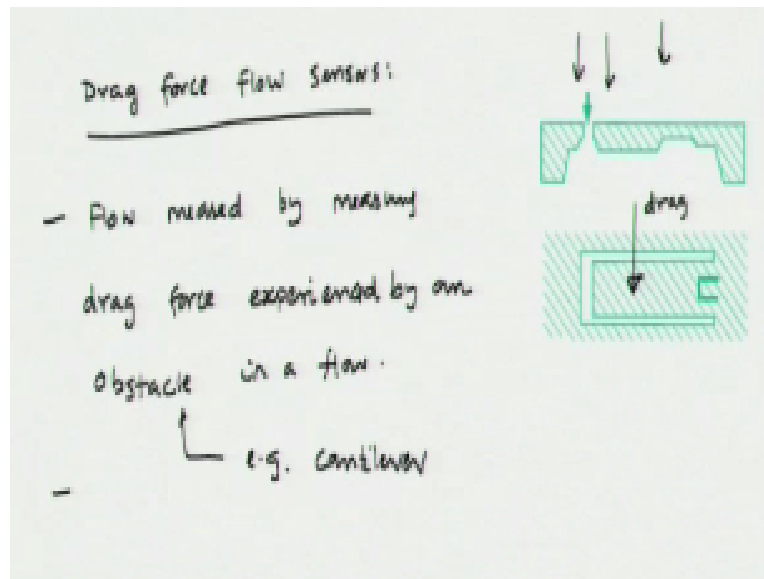
Handwritten notes on a slide:

- Change in (ΔR)
- $\boxed{\frac{\Delta R}{R} = \alpha \phi}$
- $\alpha = \text{Piezoresistive Co-eff. / gage factor}$
- $\phi = \text{strain}$
- Long. gage factor (metal) ~ 2.0
- Poly-Si : 10-40
- Crystalline Si : 50-150

And because the change in the pressure, there is a deflection of the member for which the resistance r between the 2 electrodes, which is a function of flow rate, okay. So, when the strain ϕ is applied, so we would have change in electrical resistance, okay. So, change in resistance, so ΔR ; so this ΔR over R is going to be a function of strain. So, let us $\alpha * \phi$ and this α is the piezo resistive coefficient or for the strain gage, it is the gage factor.

And ϕ is the strain and typically, if you have a strain gage, then the longitudinal gage factor; factor for metals is typically, 2 and for poly silicon, it is 10 to 40 and we have crystalline silicon; it is going to be between 50 to 150. So, by measuring the change in the resistance over original resistance will be related to strain, which is related to the flow rate. So, that is how the piezo resistive pressure flow sensors work, okay.

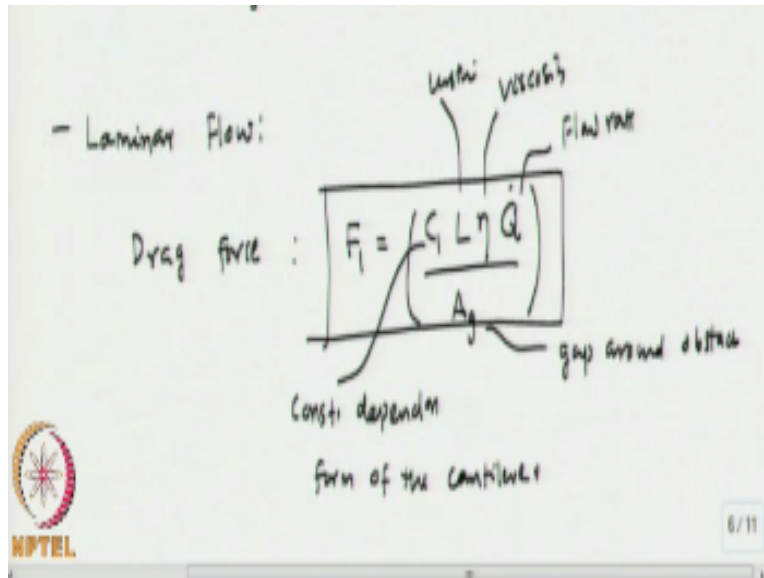
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Now, let us look at drag force flow sensors. So, let us look at drag force flow sensors; the drag force flow sensors work on the principle that we would introduce an obstruction like a cantilever structure into the flow field and when the flow rate would change, the drag experienced by the cantilever is going to change okay and by measuring the drag; change in the drag can be used to predict the flow rate.

So in a drag force flow sensor, the flow measure by measuring drag force experienced by an obstacle, okay; experienced by an obstacle in a flow. For example, a, cantilever as you can see here, if this cantilever structure is exposed to the flow that is coming in, so what is going to happen is this cantilever structure here is going to experience a drag, okay and that drag force can be measured by measuring the bending of this cantilever beam.

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And by measuring the drag, you can predict what is going to be the flow rate. Let us say, and that obstacle could be; for example, cantilever, okay. Now, if you assume laminar flow, let us say we are talking about laminar flow, which is often the case in micro channel, then the drag force will be experienced all in the direction of the flow, okay. So, the drag force is going to be F_d will be $C_1 * L \eta Q / A_g$.

So, here this is the flow rate, this is the viscosity; dynamic viscosity and this is a length scale and this is the gap around the obstacle, this is the gap around obstacles. So, in this case, this gap that you see here is going to be A_g , okay and c_1 is going to be a constant; this is a constant that depends on the form of the cantilever; form of the cantilever; so its length to; with ratio is going to determine this constant C_1 , okay.

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- Due to pr. loss in the gap
 → Additional drag

$$F_2 = \left(\frac{C_2 \rho}{A_0 A_g} \right) Q^2$$

↑
Surface area of the obstacle

This is viscosity and this is the length scale, okay. Now, if in addition to this drag force, there will be another drag force okay because of the pressure drop that occurs across the cantilever. So, due to pressure drop in the gap would have an additional drag force and that drag force yet to is going to be $C_2 \rho \text{ over } A_0 A_g * Q \text{ square}$ and where A_0 is going to be the surface area of the obstacle and C_2 is a constant, ρ is the density of the flow rate, okay.

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Total drag force: $F = F_1 + F_2$

$$F = \left[\underbrace{\frac{C_1 L \eta Q}{A_g}} + \underbrace{\frac{C_2 \rho}{A_0 A_g^2} Q^2} \right]$$

Low flow rate: $F \rightarrow \sim Q \Rightarrow$ linear character

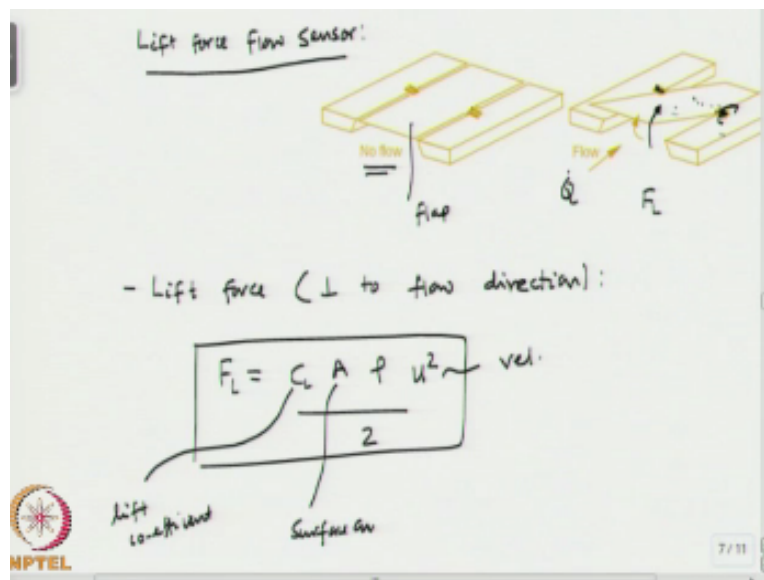
High flow rate: $F \rightarrow \sim Q^2 \Rightarrow$ quadratic character

So, you can find the total drag force; F will be $F_1 + F_2$ and so F is $C_1 L \eta Q / A_g + C_2 \rho \text{ over } A_0 A_g \text{ square}$; $A_g \text{ square}$ missing here; okay. So, this force; the drag total force has 2 components; one is a linear term, okay so this drag force depends on linearly on the flow rate and

the second one is a quadratic term, it varies as q^2 and when the flow rate is low, the linear term is going to be dominant (26:50).

And then in that case, the drag is going to be linearly dependent on the flow rate and when the flow rate is very high, the drag is going to be dependent on the flow rate following a quadratic function, okay. So, at low flow rate, F varies as Q , so this is linear characteristic, okay and at high flow rates; so at low flow rate, this term is going to be dominant and at high flow rate; this term is going to be dominant.

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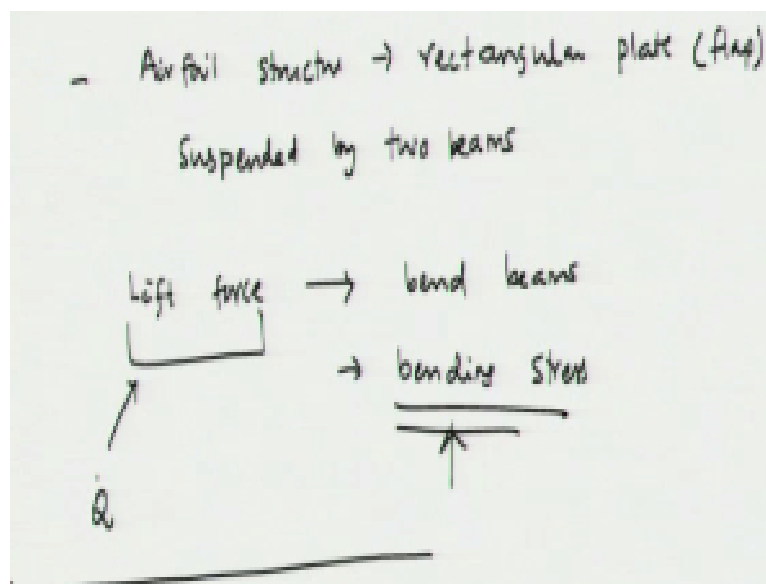
So, this will vary as Q^2 , the sensor will have quadratic characteristic. So, with that let us talk about lift force flow sensor; so, let us talk about lift force flow sensor. So, this lift force acts in a direction perpendicular to the flow direction; normal to the flow direction as opposite to the drag force, which occurs along the direction of the flow, okay and the lift flow sensor based on the principle that as the flow rate increases, that lift induced to an aerofoil kind of structure is increasing with the increase in the flow rate.

So, this is the kind of structure we have; there is a flap kind of structure and when there is no flow, the flap is going to be perfectly horizontal and when there is flow due to lift force, the flap is going to be lifted in one side, so this going to be twisted and so that lift forces is going to be

measured by measuring the bending of the beam. So, this beam is used around here, so there is going to be bending.

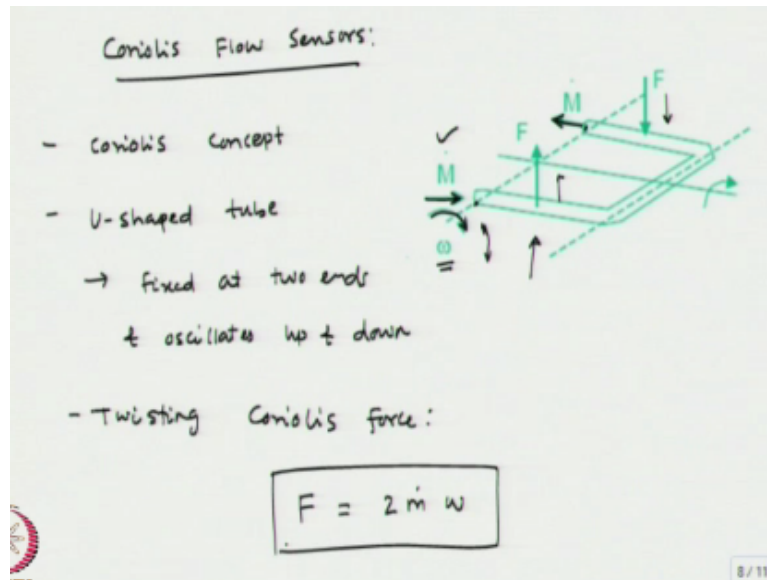
So, by measuring the torsional stress, we would be able to predict what is going to be the lift force, so that lift force will be related to the flow rate, okay. So, what is going to happen here is that lift force which acts perpendicular to flow direction is FL is going to be $CL A \cdot \rho \cdot u^2$ over 2, okay. So, u is the flow velocity, which is related to the flow rate and CL is the lift coefficient, A is the surface area and rho is the density, okay.

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So, what we have here is like an Air foil structure, you can have a rectangular plate or flap and it will be suspended by 2 beams, so as you can see here. So, the lift force, which is proportional to the flow rate would tend to bend the beams, so what we can measure is the bending stress or torsional stress and by measuring the stress, we will be able to predict the lift force, which will be related to the Q.

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So, that is the principle of operation of the lift force flow sensor. Now, let us look at Coriolis flow sensors. So, let us look at the Coriolis flow sensors; so, the Coriolis flow sensors are based on the principle of Coriolis concept, okay. So, as you can see here, this is the U shaped bent tube, okay and the flow is getting in there, so this is the must flow rate sensor; some mass flow rate \dot{M} is going inside the tube and coming out there, right.

And this tube is going to oscillate up and down, okay, so this tube is going to; so there is a; you know, U shaped bent tube and this tube is going to oscillate up and down as you can see in the images, okay. As the tube oscillates up and down, it is going to get a bending in the other direction, okay that is known as the Coriolis force, okay. As you can see here, the tube is hinged around here, there is mass flow rate \dot{m} going in, \dot{m} coming out.

And this tube is going to move; you know up and down at some frequency and when it does so, there is going to be Coriolis force develop in this direction, okay, so the beam is an experience Coriolis force in this direction, okay and that Coriolis force is related to the mass flow rate, so this is based on the Coriolis concept and we have a U shaped tube and the U shaped tube is fixed at 2 ends, okay and it oscillates up and down, okay it oscillators up and down.

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- Twisting Coriolis force: mass flow rate

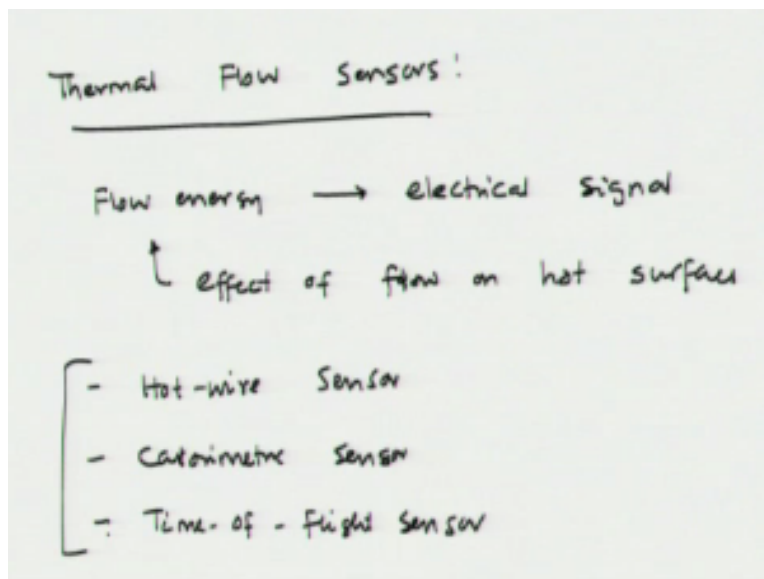
$$F = 2 \dot{m} \omega$$

force transducer

Angular frequency of oscillation

So, in that case we would have a twisting Coriolis force that acts on the tube and that Coriolis force F is given by $2 * \dot{m} * \omega$, okay. So, \dot{m} is the mass flow rate and the ω is the angular frequency of oscillation, okay. So, by measuring this twisting Coriolis force which can be measured, using a force transducer. We will be able to predict \dot{m} knowing the angular frequency of oscillation.

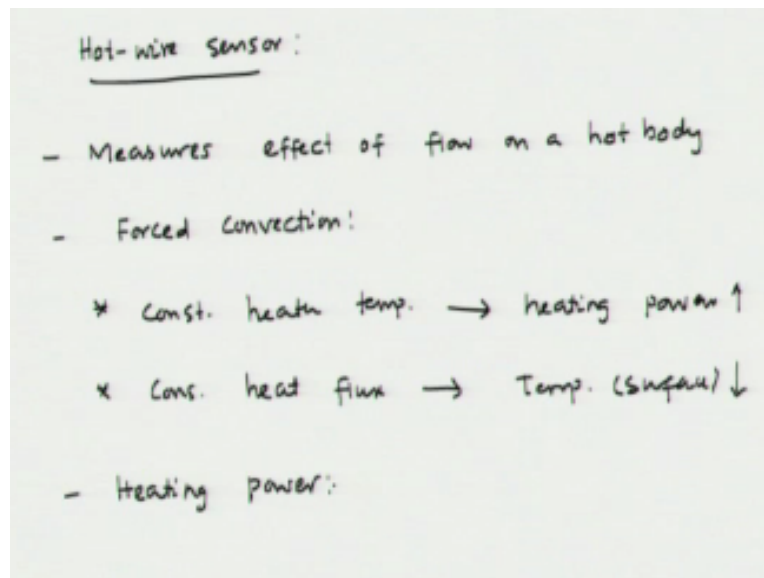
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So, that is how the Coriolis mass flow sensors work, now let us move on and talk about thermal flow sensors. So, the thermal flow sensors basically, utilise the effect of flow rate on a hot body okay. So, the different concept; different principles that are available, so let us look at that. Let us

look at thermal flow sensors; so here, the basically the flow energy is converted into electrical signal and it considers effect of flow on hot surfaces, okay.

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So, there are different principles available; one is the hot wire sensor, the second one is the calorimetric sensor. The third one is the time of flight sensor. So, first look at the hot wire sensor. So, let us look at the hot wire sensor, so the hot wire sensor basically, measures effect of flow on the hot body and we are talking about forced convection and when there is fluid movement. So, what is going to happen is you know; if you have a constant heat flux coming on to (()) (37:37).

And if the flow rate is going to increase, then the temperature is going to go down at a constant heat flux, okay and if you are trying to maintain the surface of the constant temperature, then the heat flux has to go up with increase in the heat flux. So, there are 2 things that can happen; one is the; at constant heater temperature, the heating power or heat flux will go up or at a constant heat flux, I hit the power the temperature of the surface is going to go down, okay.

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* Cons. heat flux \rightarrow Temp. (Surface) \downarrow

- Heating power:

$$P = \frac{N_u K A_H \Delta T}{L}$$

Annotations for the heating power equation:

- N_u : Nusselt number
- K : thermal conductivity
- A_H : heat area
- L : ch. length scale

- Laminar Flow:

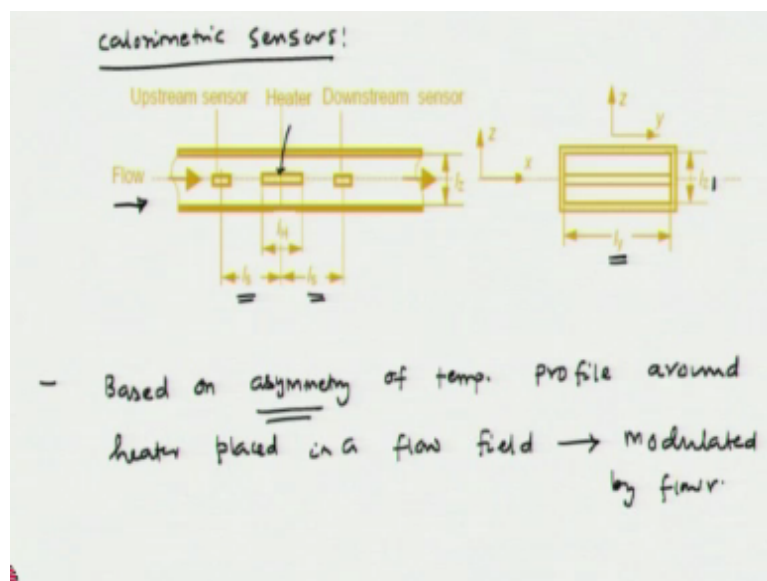
$$Nu = 0.664 \sqrt{Re} \sqrt{Pr}$$

Annotations for the laminar flow equation:

- Re : Reynolds number
- Pr : Prandtl number

So, the heating power p Nusselt number \cdot conductivity \cdot area of the heater $\Delta T / L$, okay. So, this is the Nusselt number, this is thermal conductivity and this is the characteristic length scale, okay, this is the heater area, ΔT is the temperature difference and for laminar flow, we have an expression for Nusselt number, which is square root of $Re \cdot Pr$ number to the power $1/3$, okay.

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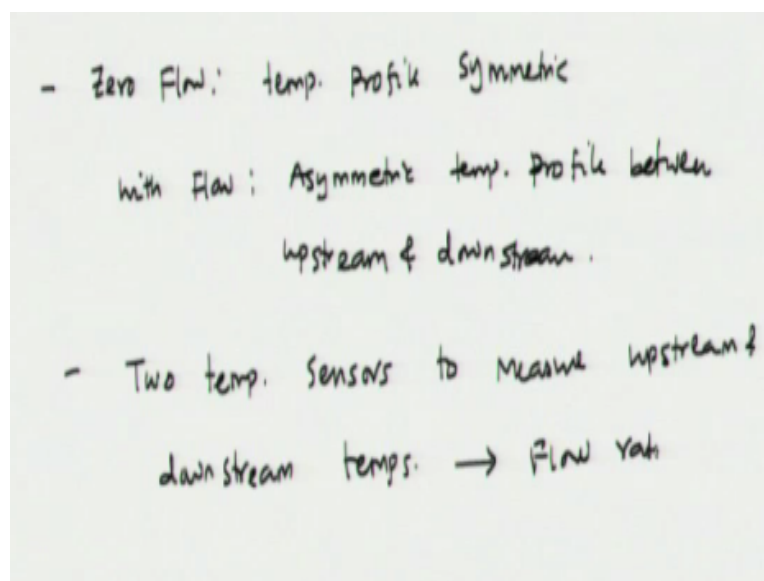
So, by knowing the heating power, we can calculate Nusselt number and from the Nusselt number will be able to calculate the flow velocity, so you can predict what is going to be the flow rate, okay. So, that is how the hot wire sensors work. Now, let us look at calorimetric sensors. So, the calorimetric sensors work on the principle that if you have a hot surface and the hot body and

flow is coming from one direction, the temperature distribution up stream is going to be different from that in the downstream.

If there is no flow, the temperature profile is going to be symmetric about the hot body but if there is a flow, the temperature profile is going to be asymmetric and this is asymmetric temperature distribution is going to be exploited to predict the flow rate, okay. So, as you can see here in this image; here we have a heater of length L_H and the upstream sensors is located at L_s from the centre of the heater and the downstream is located at a distance L_s , okay.

So, on both sides, we have 2 sensors; temperature sensors and flow is coming in. When there is no flow, the temperature profile is going to be symmetric and when there is going to be flow, the temperature profile is going to be asymmetric, okay. So, this is here the channel transduction is given; L_y and L_0 , the lengths of the channel and the heater is located at the centre of the channel height, okay.

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So, you know, it is basically based on the asymmetry of the temperature profile around heater placed in a flow field, okay and this is going to be the; asymmetry is going to be modulated by flow rate, okay. So, when you have 0 flows, when nothing is flowing, then we have symmetric temperature profile; temperature profile symmetric and when you have flow; with flow, we have asymmetric temperature profile, okay between upstream and downstream, okay.

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Handwritten equation for temperature difference $\Delta T(u)$ as a function of velocity u :

$$\Delta T(u) = T_0 \left\{ \exp \left[\gamma_2 \left(l_s - \frac{l_H}{2} \right) \right] - \exp \left[\gamma_1 \left(-l_s + \frac{l_H}{2} \right) \right] \right\}$$

temp. difference between upstream & downstream

temp. defined below

1 → upstream, 2 → downstream

And we would have 2 temperature sensors to measure upstream and downstream temperatures, so that would give us flow rate, okay. So, the difference in the temperature between upstream and downstream; ΔT as a function of the velocity has been estimated as $T_0 * \exp(\gamma_2 * l_s - l_H/2) - \exp(\gamma_1 * -l_s + l_H/2)$, okay. So, this is the expression for the temperature difference between the upstream and downstream.

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Handwritten equation for $\gamma_{1,2}$:

$$\gamma_{1,2} = \frac{u \pm \sqrt{u^2 + 8a^2/l_s^2}}{2a}$$

1 → upstream, 2 → downstream

thermal diffusivity

$$= \left(\frac{K}{\rho} \right)$$

This is the temperature difference between upstream and downstream, we have an expression for this T_0 , this is the temperature defined below; 1 is for upstream and 2 is for downstream as you can see here, so this is going to be 1, this is going to be 2, so in that case, expression for gamma

l_2 is estimated as $u + \sqrt{u^2 + 8a^2} / L_2$, square root/2a, okay, where a is the thermal diffusivity, this is k over l .

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Handwritten equation for T_0 :

$$T_0 = \left[\frac{P_H}{\frac{8k l_y l_H}{l_z} + A k (\gamma_1 - \gamma_2)} \right]$$

Below the equation, an arrow points from the term $A k (\gamma_1 - \gamma_2)$ to the expression $(l_3 \times l_z)$.

Below the equation, the following definitions are written:

$\Delta T(u) \rightarrow \text{measured}, \quad R_{HS} \rightarrow \text{calculated}$

$(u) \rightarrow \underline{\underline{\dot{Q}}}$

So, we can define T_0 as follows; so, the heater power/ $8k l_y \cdot l_H / l_z + A \cdot k \cdot \gamma_1 - \gamma_2$, so this is expression for the temperature T_0 , okay. In this area of cross section is going to be $l_y \cdot l_H$, okay everything else is defined here, so what is going to happen is we are going to measure the temperature difference and all the values in the right hand side will be either the fluidic property or a flow property, okay or depends on the geometry, locations of the heater with; the sensors with respect to the heater and the geometrical parameters.

So, by calculating left hand side and right hand side, we should be able to predict what is the value of u ? okay. So, ΔT is measured and right hand side calculated except u , so we can measure; we can predict u , so predicting u , we can predict the flow rate, so that is how the calorimetric sensors are going to work and next let us talk about the time of flight sensors. So, let us talk about time of flight sensors.

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Time-of-flight sensor

- Based on time taken by a heat pulse to travel through a fluid

$$T(x,t) = \frac{q_0}{4\pi k t} \exp \left[-\frac{(x-ut)^2}{4at} \right]$$

So, this is based on time taken by a heat pulse to travel through a flight, okay. So, we have an analytical solution for this, so $T; x, t$ is a function of time is going to be q_0 dash $4\pi K * t *$ exponential - $x - ut$ square / 480 , okay. So, the temperature is going to be function of x and t , this is the heat flux that is generated by the heater, k is the thermal connectivity, t is time and u is the velocity, x is the direction in the x direction, A is the thermal diffusivity, okay.

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$z \rightarrow$ time at which signal would pass through the temp. sensor

\rightarrow Time-of-flight

$4at \ll \text{heat sensor distance}$

diffusion length scale

\rightarrow Forced convection dominates over diffusion

So, we can define a parameter called time constant, so this is the time at which the signal would pass through the temperature sensor. So, it basically measures the time of flight of the heat signal; the time of light is measured, okay. Now, for this to happen, we have to make sure that the

diffusive flux; so we have to ensure that $4a \cdot t$ will be \ll the heater sensor distance, okay. So, $4at$ is the diffusion length scale.

So, in this case what we mean by this is that forced convection dominates over diffusion, okay. So, we have a point; you know heater, which is generating some heat and we are locating a temperature sensor at some distance and we are saying that before the heat goes by diffusion, the fluid flow actually carries the heat, okay that is what we mean by the diffusion length scale is \ll the heater sensor distance.

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- When diffusive effects considered:

$$\tau = \frac{-2a + (4a^2 + u^2 L^2)^{1/2}}{u^2} \quad \text{for } u \neq 0$$

$$= \left(\frac{L^2}{4a} \right) \quad u=0$$

And when diffusive effect is considered, so here the diffusive effect is negligible; when diffusive effects considered, then the time scale τ will be $-2a + 4a \text{ square} + u \text{ square } L \text{ square}$ over $1/2 / u \text{ square}$ and that is for $u \neq 0$ and when $u = 0$, this is going to be $L \text{ square over } 4a$, when $u = 0$, okay. So, by measuring the time that it takes for the heat pulse to reach from heater to the sensor, we will be able to predict what is going to be the flow rate that is how the time of flight flow rate sensors work, okay. So, with that let us stop here.