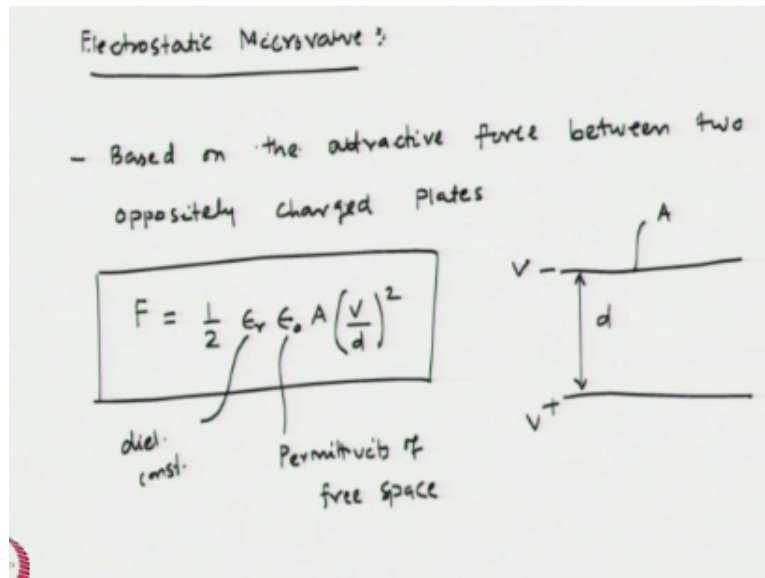


Microfluidics
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Lecture – 33
Microvalve (continued.)

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Okay, so let us continue our discussion on micro valve, let us first talk about electrostatic micro valve. Electrostatic micro valves are based on the attractive force between 2 oppositely charged plates, so if you have 2 oppositely charged plates facing each other, we can write down the expression for the electrostatic force between them. So, let us look at the electrostatic micro valve, okay; let us look at electrostatic micro valve.

So, these are based on the attractive force between 2 oppositely charged plates; 2 oppositely charged plates okay, so we have 2 charged plates, let us say the overlapping area is A and the distance between them is d and the voltage is let us say, V okay. So, if that is the case you can write down the expression for the electrostatic force F which is given by $\frac{1}{2} \epsilon_r \epsilon_0 A \left(\frac{V}{d}\right)^2$ okay, where epsilon r is the dielectric constant of the medium; dielectric constant or relative permittivity.

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- Based on the attractive force between oppositely charged plates

$$F = \frac{1}{2} \epsilon_r \epsilon_0 A \left(\frac{V}{d} \right)^2$$

ϵ_r : dielectric const.
 ϵ_0 : Permittivity of free space

- Insulator of thickness d_i and dielectric const ϵ_i :

$$F = \frac{1}{2} \epsilon_0 A \left(\frac{V}{d} \right)^2 \left(\frac{\epsilon_i d}{\epsilon d_i + \epsilon_i d} \right)$$

And this is permittivity of free space okay, this permittivity of free space. Now, if you have an insulating material of the dielectric constant ϵ_i between the 2, let us we have a layer whose dielectric constant is ϵ_i , then what is going to be the expression for the electrostatic force? So, if you have an insulator of thickness d_i , okay this thickness is going to be d_i and the dielectric constant, ϵ_i .

Then, we can write down the expression for the electrostatic force, which will be $\frac{1}{2} \epsilon_0 A \left(\frac{V}{d} \right)^2 \left(\frac{\epsilon_i d}{\epsilon d_i + \epsilon_i d} \right)$, okay. So, ϵ_0 is the dielectric constant of this medium here and ϵ_i is the dielectric constant of the insulating medium okay, so this is the permittivity of the medium here; this is the permittivity of the insulating medium.

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- Advantages: Fast response

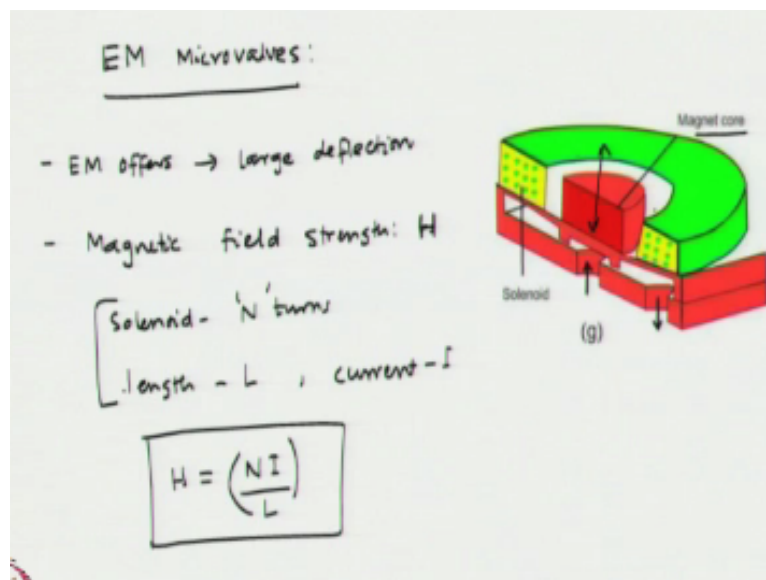
→ requires large actuating voltage → difficult to integrate with microfluidic system

→ Small displacement

So, this is the expression for the electrostatic force between the plates. Now, what are the advantages of using electrostatic actuators, so the biggest advantage is that the response time of the electrostatic actuators are very good okay, so it has got fast response okay and however the electrostatic actuators require a large actuating voltage okay, so that is one downside of electrostatic actuators.

So, requires large activating voltage and so that is why difficult to integrate with microfluidic system okay. Now, the other; so it has large actuating voltage also small displacement okay. So, the downsides are large actuating voltage in small displacement and the advantage is that it offers fast response okay, so with that let us move on and talk about electromagnetic based micro valve okay.

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We talk about electromagnetic micro valve, okay, so in an electromagnetic micro valve, the actuation is based on the magnetic field okay and one advantage is that these electromagnetic micro valves offer large displacement okay; large force okay, so the electromagnetic micro valve offers large deflection, okay. As you can see here there is the magnetic core and with a known magnetic field, it introduces a force in the vertical direction okay.

So, let us say if you have a magnetic field strength; so the magnetic field strength, let us say equal to H and we can create this magnetic field by using a solenoid okay. So, let us say we have a solenoid with N turns and the length is L , so and we are passing a current I , so the magnetic field strength in that case can be written as $N \cdot I$ over L , okay. So, it is possible to create such magnetic field using solenoid okay.

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- Magnetic flux density:

$$B = \mu H = \mu_0 \mu_r H = \mu (1 + \chi_m) H$$

↑
↑
↑

Permeability

Magnetic Susceptibility

Now, the magnetic field strength and the flux density are related as follows. The flux density; the magnetic flux density B will be $= \mu H$, where μ is the permeability okay, so this is magnetic permeability of the medium and H is the magnetic field strength will be $\mu_0 \mu_r H$, so this is relative permeability and this is permeability of free space, so this can be written as $\mu_0 (1 + \chi_m) H$, so this is called magnetic susceptibility.

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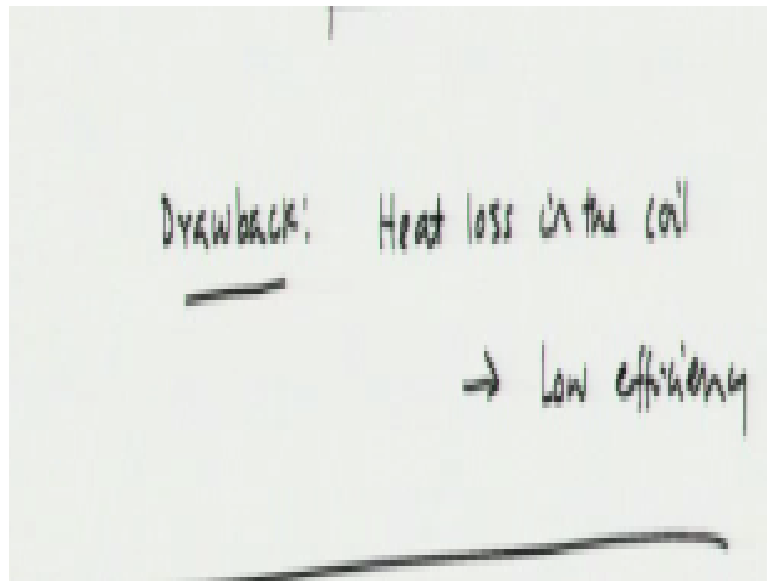
- B , magnet $\rightarrow$ magnetization M_m

Vol. $\rightarrow V$

Force $F = M_m \int \left(\frac{dB}{dz} \right) dz = \int M_m dB$ $\rightarrow z$ -direction

So, this we have seen; when you are talking about the magnet of forces okay, so this is the expression for magnetic flux density. So, you know with that magnetic flux density; so with magnetic flux density B and if you have a magnet with the magnetization as M_m and the volume of the magnet is V , then you can find the force that is acting on the magnet okay. So, the force F can be written as the magnetization $\times$ integration of dB over $dz \times dV$, okay.

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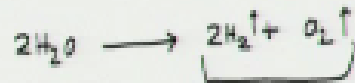
So, that is the force that is acting on; acting in the z direction okay. So, this will be the gradient of the magnetic flux density in the z direction into the integration over the entire volume of the magnet into the magnetization that will give you the force due to the magnetic field okay and so the drawback of the magnetic actuator, the drawback is that there is going to be heat loss in the coil okay.

So, the heat loss in the coil and that gives rise to low efficiency okay. So, with that let us talk about electrochemical valve. Now, electrochemical valves are based on the principle of electrolysis that is occurring at the electrode because of the electrochemical process and due to the electrolysis process, you would have you know water bubbles generated and these bubbles will expand and they will act as the electrochemical valves, okay.

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Electrochemical micro valves :

→ Electrolysis :



Electrochemical valves are actuated using
gas bubbles generated in electrolysis

So, let us talk about electro chemical micro valves, so electrochemical micro valves are based on the principle of electrolysis and in electrolysis, we would have water breaking into hydrogen and oxygen, okay. So, this is; 2 gases are generated and the electrochemical valves are actuated using these gas bubbles okay; using gas bubbles generated in electrolysis. So, we can write the expression for the pressure inside a bubble.

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- Pressure inside a bubble

$$\Delta p = \frac{2\sigma}{R}$$

Surface tension
Radius of curvature

- Pressure drop across a liquid-gas interface

$$\Delta p = \frac{2\sigma_{lg} \cos\theta}{r_0}$$

contact angle
liq-gas surface tension
channel radius

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Pressure inside the bubble will be proportional to the surface tension inside a bubble will be proportional to Δp will be proportional to the surface tension and inversely proportional to the radius of the curvature okay. So, this is surface tension and R is radius of curvature okay. So, we can find out the pressure drop across a liquid gas interface; the pressure drop across a liquid gas interface.

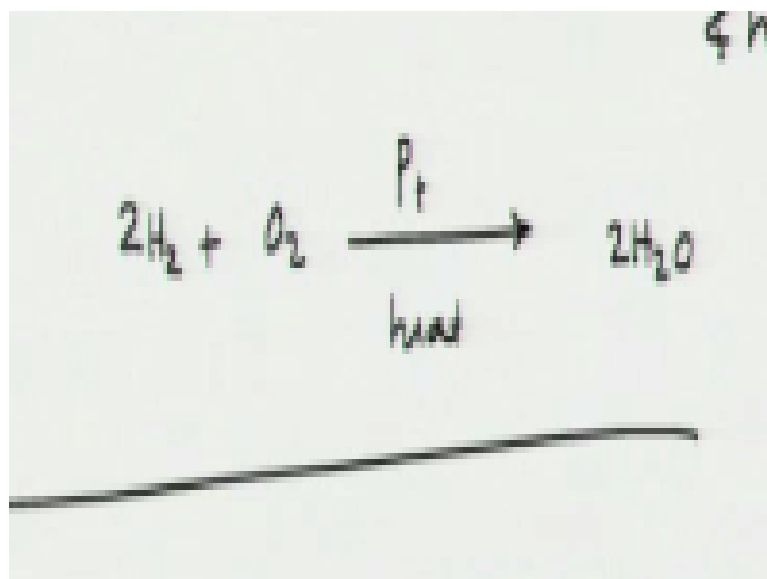
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P_c that actuates the microvalve.
 - Reverse reaction possible
 $\rightarrow$ Reaction needs catalyst & heat.

And that can be written as Δp will be $= 2 \times \text{liquid gas surface tension} \cos \theta$; θ is the contact angle/ r_0 , okay and r_0 is the channel radius, this is contact angle and this is liquid gas surface tension okay. So, this is the pressure that is at the liquid gas interface, which controls the valve mechanism okay. So, this pressure that actuate the micro valve, okay. Now, you know once you want to start let us switching off the valve.

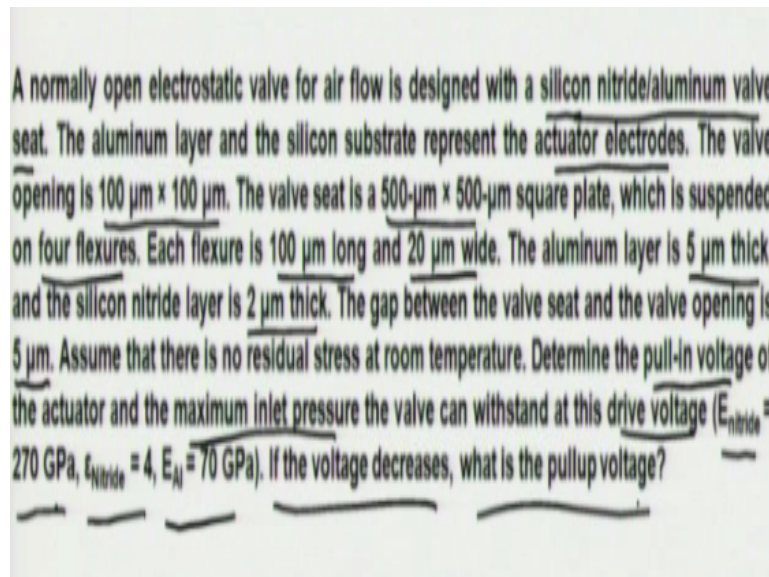
That means, going to stop the flow. you can have the electrolysis process. so that these bubbles will grow and stop the inlet, so that the flow cannot come inside. Now, if you want to take these bubbles away, the reverse reaction is also possible okay. So, that the reverse electrolysis process the hydrogen oxygen will again combine together to produce water, so the valve will diffuse okay.

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So, the reverse reaction is possible but that would require a catalyst okay, so the reaction needs catalyst and also heat okay. So, what would happen is in the reverse reaction, hydrogen and oxygen would combine in presence of platinum and heat to produce water okay. So, that is how the electro chemical micro valves will work, so with that let us take a one example of an electrostatic actuator based micro valve.

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So, let us talk and take this example on electrostatic based micro valve. So, here we have a normally open electrostatic valve for air flow and it is designed with a silicon nitride aluminium valve okay, so silicon nitrate aluminium valve seat. The aluminium layer and the silicon substrate represent the actuator electrodes, the valve opening is 100×100 micron and the valve seat is 500×500 micron square plate which is suspended on 4 flexures, okay.

And each flexure is 100 micron long and 20 micron wide, the aluminium layer is 5 micron thick and the silicon nitride layer are 2 micron thick, so there are 2 layers on the valve, so these 2 layers on the electrodes have different thicknesses, they have different materials and the gap between the valve seat and the valve opening is 5 micron, okay and we assume that there is no residual stress at room temperature.

We want to determine the pull in voltage of the actuator and the maximum inlet pressure, the valve can withstand but this drive voltage okay. So, the two things you want to determine; one is the maximum pull in voltage and the second one is the maximum inlet pressure, the valve can withstand and so the properties are given the Young's modulus of nitride is 270 giga pascal and the permittivity dielectric constant of nitride is 4 .

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Active area of the electro-static actuator :

$$A = (A_{\text{seat}} - A_{\text{orifice}})$$
$$= (500 \times 10^{-6})^2 - (100 \times 10^{-6})^2$$
$$= \underline{24 \times 10^{-8} \text{ m}^2}$$


And the Young's modulus of aluminium is given 70 giga pascal. If the voltage decreases, what is the pull of voltage and how the valve is going to open? So, if you consider this problem, the first thing we would like to calculate is the active area okay. So, the active area of the electrostatic actuator; so we find active area as the area of the valve seat - the area of the orifice okay.

So, basically we would have an orifice and the valve seat will have something like this, so you want to subtract the total area from the orifice area, so we find the active area. So, the active area is $A_{\text{seat}} - A_{\text{orifice}}$, which is given by the seat has 500 microns square, so 500×10 to the power -6 square - the orifice is 100 micron square; 100 micron square, so this is going to be 24×10 to the power -8 meter square okay, so that is the active area of the actuator.

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- Spring const. for the composite beam:

$$K_0 = \frac{12 (EI)_{\text{beam}}}{L^3}$$

$$= \frac{(b_1 E_1 t_1^3)^2 + (b_2 E_2 t_2^3)^2 + 2 b_1 b_2 E_1 E_2 t_1 t_2 \times (2 t_1^2 + 3 t_1 t_2 + 2 t_2^2)}{L^3 (t_1 b_1 E_1 + t_2 b_2 E_2)}$$

$$= \underline{\underline{468.78 \text{ N/m}}}$$

Now, if you do the model for the composite beam, so here in this particular case, we have silicon nitride and aluminium; 2 layers forming a composite beam. So, for the composite beam, we can find out the spring constant okay. So, the spring constant for the composite beam; the composite beam is going to be; so k_0 is going to be 12, the flexural rigidity of the composite beam / L cube, okay.

So, that can be written as $b_1 E_1 t_1$ square whole square + $b_2 E_2 t_2$ square whole square + $2 b_1 b_2 E_1 E_2 t_1 t_2 * 2 t_1$ square + $t_1 t_2 + 2 t_2$ square / L cube * $t_1 b_1 E_1 + t_2 b_2 E_1$. So, if we use this formula to calculate the spring constant for the composite beam, we can plug in the values for the silicon nitride as well as aluminium, so let us say this is silicon nitride and this is aluminium, then we can find out this to be 468.78 newton per meter okay.

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The spring const. of valve seat:

$$K = 4 K_0 = 1875 \text{ N/m}$$

Dielectric const. of air = 1 (Assumed)

Force acting on the valve seat:

$$F = \frac{1}{2} \epsilon_0 A \left(\frac{V}{d} \right)^2 \left(\frac{\epsilon_r d}{\epsilon_r d + \epsilon_0 d} \right)^2$$

So, that is going to be the spring constant of the composite beam, so from there we can find out the spring constant of the valve seat. The spring constant of the valve seat, so the valve seat has 4 of these flexures, okay, so we can find out the total spring constant by multiplying the spring constant for each flexure with 4, so we can find the total spring constant will be $4 \cdot k_0$, so that will be 1875 newton per meter okay.

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Handwritten notes on a slide:

$\epsilon = 1$ (say)

$$F = \frac{1}{2} \frac{\epsilon_0 A V^2}{\left(d + \frac{d_i}{\epsilon_i}\right)^2}$$

Initial gap = $d_0 = 5 \mu m$

Net force acting on the valve seat:

Now, if you say that the dielectric constant of air is 1; the dielectric constant of air is 1, so this is assumed here, so you can find out the expression for force acting on the valve seat. So, the force acting on the valve seat F is going to be $\frac{1}{2} \cdot \epsilon_0 A V^2$ by $d^2 \cdot \epsilon_i$ d over $\epsilon_i d + \epsilon_i d$ whole square. So, here you say that $\epsilon = 1$, if you do that you can get an expression for force F .

So, that is going to be $\frac{1}{2} \epsilon_0 A V^2 / d + d_i$ over ϵ_i whole square okay, so that is the expression for the force if you say air dielectric constant is 1 okay, that under that assumption. Now, the initial gap is given; the initial gap between the electrodes is given to be 5 microns, so say d_0 is 5 micron. So, the net force acting on the valve seat, so this is the electrostatic force.

(Refer Slide Time: 26:22)

Initial gap: $d_0 = 5 \mu m$

Net force acting on the valve seat:

$$F_{net} = (-F_{el} + F_{spring})$$

$$= \left[-\frac{\epsilon_0 A V^2}{2 \left(d + \frac{d_i}{\epsilon_i}\right)^2} + K(d_0 - d) \right]$$

So, the force that acts on the valve seat will be the electrostatic force as well as the spring force okay, so we have to combine them together to find the net force. So, the net force can be found out; F_{net} is going to be $F_{electric} + F_{spring}$ and since they will be opposing each other one of them has to be negative, okay. So, you can write this as $-\epsilon_0 A V^2 / 2(d + d_i/\epsilon_i)^2 + K(d_0 - d)$, okay, so that is the expression for the net force.

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$$\Delta F_{net}(d) = F'_{net}(d) \Delta d$$

$$= \left[\left(\frac{\epsilon_0 A V^2}{4} \right) \times \frac{1}{\left(d + \frac{d_i}{\epsilon_i}\right)^3} - K \right] \Delta d$$

At pull-in voltage the actuator reaches unstable equilibrium

$$\begin{cases} F_{net} = 0 \\ \Delta F_{net} = 0 \end{cases}$$

Now, since this net force it is going to be a function of the separation distance okay and this separation distance would change during the actuation okay. So, we can find the change in the force, so ΔF_{net} , which is going to be a function of the separation distance between the plates is going to be $F'_{net} \Delta d$, okay. So, you can find this out, so that will be $\epsilon_0 A V^2 / 4 \times 1 / (d + d_i/\epsilon_i)^3 - K \Delta d$, okay.

So, that is what will get for expression for the change in the net force as a function of the separation distance okay. So, are the pull in voltage okay, so when; during the pull in voltage when the valve tries to close it, the membrane will reach some kind of an unstable equilibrium okay, so during an unstable equilibrium, the ΔF_d is going to be 0, okay. The change in the force as a function of change in the gap will be 0.

Because it has to reach the maximum deflection okay and after which it can only go back, so at the peak; the change in the force as a function of d will be 0. So, at pull in voltage, the actuator reaches unstable equilibrium and there, the net force is going to be 0 because that is going to be stopping and as this is going to pass through the maximum, this ΔF_{net} will also be 0. Now, if you apply these 2 conditions on the equations here 1 and 2, what we get is this.

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Handwritten derivation on a whiteboard:

$$\left[\begin{array}{l} F_{net} = 0 \\ \Delta F_{net} = 0 \end{array} \right] \rightarrow K = \left[\frac{\epsilon_0 A V^2}{4} \times \frac{1}{\left(d + \frac{d_i}{\epsilon_i}\right)^3} \right] \quad \text{--- (a)}$$

$$\frac{\epsilon_0 A V^2}{2 \left(d + \frac{d_i}{\epsilon_i}\right)^2} = K (d_0 - d) \quad \text{--- (b)}$$

Substitute (a) in (b):

→ a quadratic eqn. in 'd'

Solve above eqn. → take positive solution

We get $k = \frac{\epsilon_0 A V^2}{4 \cdot \left(d + \frac{d_i}{\epsilon_i}\right)^3}$, so this is one equation that we get and the other equation would get is $\frac{\epsilon_0 A V^2}{2 \cdot \left(d + \frac{d_i}{\epsilon_i}\right)^2} = k \cdot (d_0 - d)$ okay. Let us call this equation a, let us call this equation b. Now, if you substitute a in b, so we substitute for K in here okay, what we get is; so if you do that we will get a quadratic equation in d , okay.

(Refer Slide Time: 32:11)

Critical gap:

$$d_{cr} = \left(\frac{d_0}{3} - \frac{2d_i}{\epsilon_i} \right)$$

thickness of nitride layer

$$d_{cr} = \left[\frac{5 \times 10^{-6}}{3} - 2 \times \frac{(2 \times 10^{-6})}{3} \right]$$

$$d_{cr} = 1.33 \times 10^{-6} \text{ m}$$

So, this will result in; so substitute a in b, what we get is a quadratic equation in d okay and if you solve this quadratic equation and take the positive solution, if you do that then, you get an expression for the critical gap. So, the critical gap, which is the d that you obtain here; the d critical is going to be d_0 by 3 – $2d_i$ over ϵ_i . So, we can find out d critical for this case; d_0 is given, so it is 5×10 to the power -6/ 3 – $2 \times 2 \times 10$ to the power -6 by 3, okay.

So, this will be = 1.33×10 to the power -6 meter okay, so that is the value for d critical. So, this is the total gap d_0 and this is the thickness of the nitride layer, which is 2 micron thick okay, so this is thickness of nitride and this is the gap okay. Now, if you substitute the expression for the critical gap in this equation; in a, then you can find the expression for voltage okay, that will be the pull in voltage.

(Refer Slide Time: 34:09)

Substitute expression for d_{cr} in eqn. (a):

→ Expression for V_{cr}

Pull-in voltage:

$$V_{cr} = 2 \sqrt{\frac{\kappa \left(d_{cr} + \frac{d_i}{\epsilon_i} \right)}{\epsilon_0 A}}$$

Now, substitute expression for critical separation distance in equation a, then what you would get is the expression for the pull in voltage okay. So, the pull in voltage V_{critical} will be $2 \cdot K \cdot d_{\text{critical}} + d_i$ over $\epsilon_i / \epsilon_0 A$ square root okay, so that would be the expression for the pull in voltage that you would get from the above expression.

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→ Expression for V_{cr}

Pull-in voltage:

$$V_{\text{cr}} = 2 \sqrt{\frac{K \left(d_{\text{cr}} + \frac{d_i}{\epsilon_i} \right)^3}{\epsilon_0 A}}$$

$$V_{\text{cr}} = 2 \times \sqrt{\frac{1.875 \left(1.33 \times 10^{-6} + \frac{2 \times 10^{-6}}{4} \right)^3}{(8.854 \times 10^{-12}) \times (24 \times 10^{-8})}} = 147 \text{ V}$$

Now, if you substitute the values, V_{critical} can be calculated as $2 \cdot K$ is $1.875 \cdot d_{\text{critical}}$ is $1.33 \cdot 10^{-6}$, d_i is $2 \cdot 10^{-6} / 4$ and here we have missed a cube, so cube under the square root / $8.854 \cdot 10^{-12} \cdot \text{area}$; A is $24 \cdot 10^{-8}$, okay, so that is the effective area that we have calculated first, so square root of that that will give us 147 volts.

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$$\sqrt{(8.854 \times 10^{-12}) \times (24 \times 10^{-8})}$$

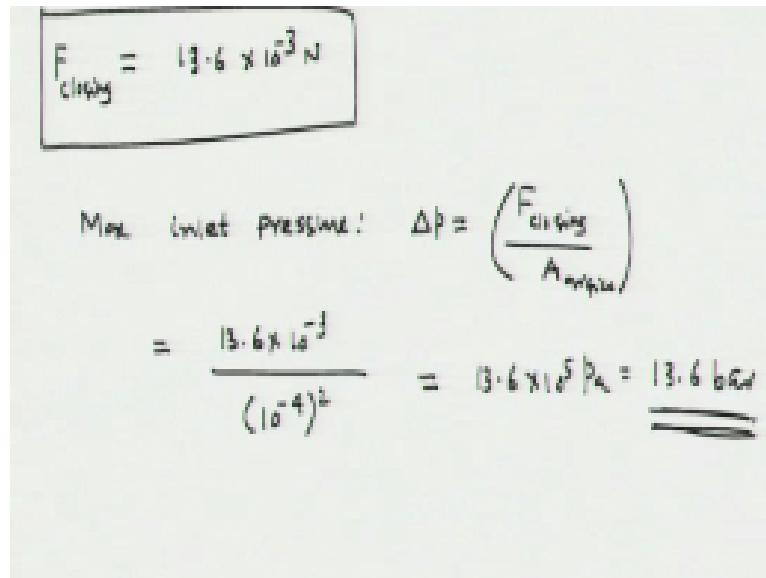
→ $V_{\text{cr}} = 147 \text{ V}$

Closing force: $F_{\text{closing}} = \left(\frac{\epsilon_i \epsilon_0 A V^2}{2 d_i^2} - K d_o \right)$

$$= \left[\frac{4 \times (8.854 \times 10^{-12}) \times (24 \times 10^{-8}) \times (147)^2}{2 \times (2 \times 10^{-6})^2} \right] - 1.875 \times (5 \times 10^{-6})$$

So, the critical voltage in this case is going to be 147 volts okay. Now, at this critical voltage, what is going to be the closing force? So, the closing force F_{closing} is found as $\epsilon_0 \frac{V^2}{2 d_i^2} - \frac{K d_0^5}{E_i}$, okay. So, the closing force will be the electrostatic force minus the spring force, so that can be calculated as so, E_i is $4 \times 8.854 \times 10^{-12}$ area is $24 \times 10^{-8} \text{ V}^2$; V is 147 volts square / $2 \times$; d_i is $2 \text{ micron} \times 10^{-6}$ square - K is $1875 \times d_0$ is 5×10^{-6} , okay.

(Refer Slide Time: 38:00)



The image shows handwritten calculations. At the top, a box contains the result for the closing force: $F_{\text{closing}} = 13.6 \times 10^{-3} \text{ N}$. Below this, the formula for maximum inlet pressure is given as $\Delta p = \frac{F_{\text{closing}}}{A_{\text{orifice}}}$. The calculation then proceeds to $\Delta p = \frac{13.6 \times 10^{-3}}{(10^{-4})^2} = 13.6 \times 10^5 \text{ Pa} = \underline{\underline{13.6 \text{ bar}}}$.

So, that is going to be 13.6×10^{-3} Newton okay, so that is going to be the closing force okay. So, you can find the maximum inlet pressure Δp can be found as F_{closing} ; the closing force / the orifice; area of the orifice, so that is going to be $13.6 \times 10^{-3} /$ the orifice area is 10^{-4} square, that is going to be 13.6×10^5 pascal or 13.6 bar.

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The pull-up voltage! → To open the valve

$$F_{el} = F_{spring}$$

$$\frac{\epsilon_i \epsilon_0 A V^2}{2 d_i^2} = K d_0 \Rightarrow V = \sqrt{\frac{2 K d_0 d_i^2}{\epsilon_i \epsilon_0 A}}$$

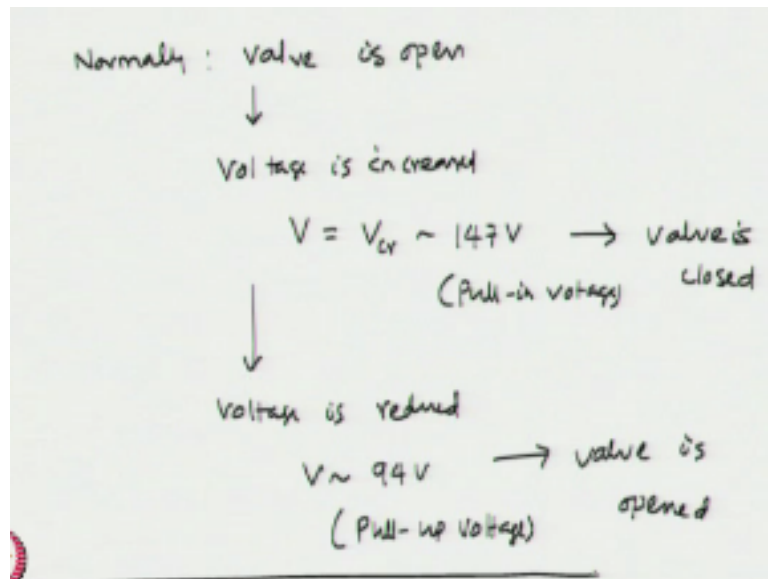
$$= \sqrt{\frac{2 \times 1875 \times (5 \times 10^{-6}) \times (2 \times 10^{-6})^2}{4 \times (8.854 \times 10^{-12}) \times (24 \times 10^8)}} = \underline{\underline{94V}}$$

So, you know in order for the valve open, we would need 13.6 bar of pressure to open the valve, now the pull up voltage; the pull up voltage, so the pull up voltage is defined as the voltage at which the valve will again start to open okay. So, to determine the pull voltage, we would say that to open the valve and to open the valve, the electrostatic force, if it is equal to the spring force that is trying to take the membrane to its original position.

And meaning; the electrostatic force if it is equal to the spring force, then the valve will start to open that is the pull up voltage. So, in that case we would have the electrostatic force will be equal to spring force okay. So, if that is the case, $\epsilon_i \epsilon_0 A V^2 / 2 d_i^2$ will be $= K d_0$ okay. So, if that is the case then you can find the V , which is $2 K d_0 d_i^2 / \epsilon_i \epsilon_0 A$ square root and you can substitute the values.

It is going to be 2×1875 is the spring constant $\times 5 \times 10$ to the power $-6 \times 2 \times 10$ to the power -6 square / $4 \times 8.854 \times 10$ to the power $-12 \times \text{area} \times 24 \times 10$ to the power -8 square root, so that will be $= 94$ volts, so that is the pull up voltage, okay. So, you know then normally, the valve will be open so then when you increase the voltage and if the voltage exceeds the pull in voltage, the valve is going to close and as the voltage reduces.

(Refer Slide Time: 42:33)



And the voltage reduces value, so that the spring force becomes equal to the pull up voltage then the valve again tries to open. So, what we have the situation is this; so, normally the valve is open and now voltage is increased, let us say V becomes $= V$ critical, let us say in this case it is 147 volts. So, if that is the case, this is the pull in voltage and valve is closed and now the voltage is reduced.

And if V is of the order of 94 volts, this is the pull up voltage for this case and in this case, the valve is open okay. So, this is a case that we saw for electrostatic micro valve, so with that we complete our discussion on micro valves.