Microfluidics
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Lecture - 30
Micropump

Okay, so we have been looking at the capillary filling process. So, if we looked at how the interface of the liquid changes at the beginning of the filling process, just look at what happens at the end of the filling process. So, if we look at capillary filling and we look at what happens at the end of the filling process, okay. So, we said that at the end of the filling process, the inertia and the surface tension forces are negligible.

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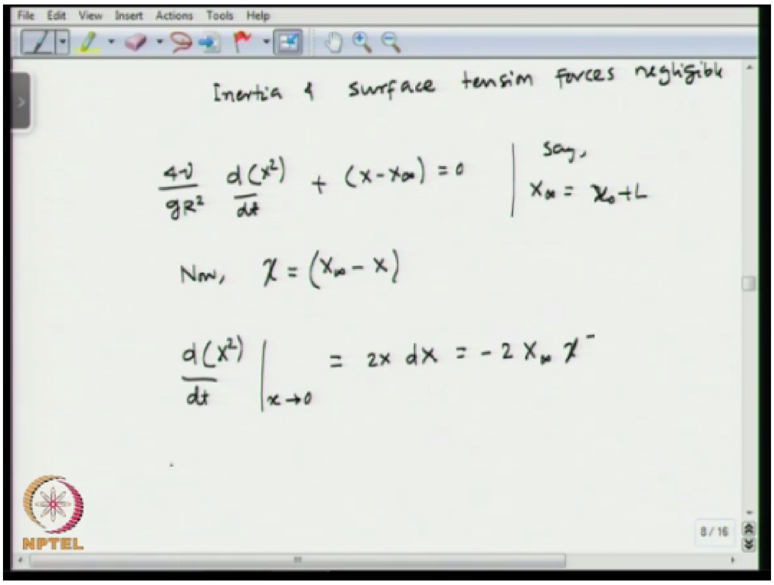

So in the general equation, if you drop of the inertia and surface tension forces, then we get 4 nu over g R square*d of x square over dt + x - x infinity are 0, where we say that x infinity is x0 + L, okay. So, now if you consider a parameter x as x infinity - X, we can say that d x square over dt at x tends to 0 will be 2x*dx this will be = - 2 x infinity*x. So, this is what we can say.

So now in this equation, we can write this as 8 nu over g R square*x + x is going to be 0, okay. So, this equation is very easy to solve, right. So, we can find a solution x t is going to be x infinity exponential - R square g over 8 nu x infinity*t. Now, if you convert it back to the original variable X t can be written as x infinity*1 - exponential - R square g over 8 nu x infinity*t, okay.

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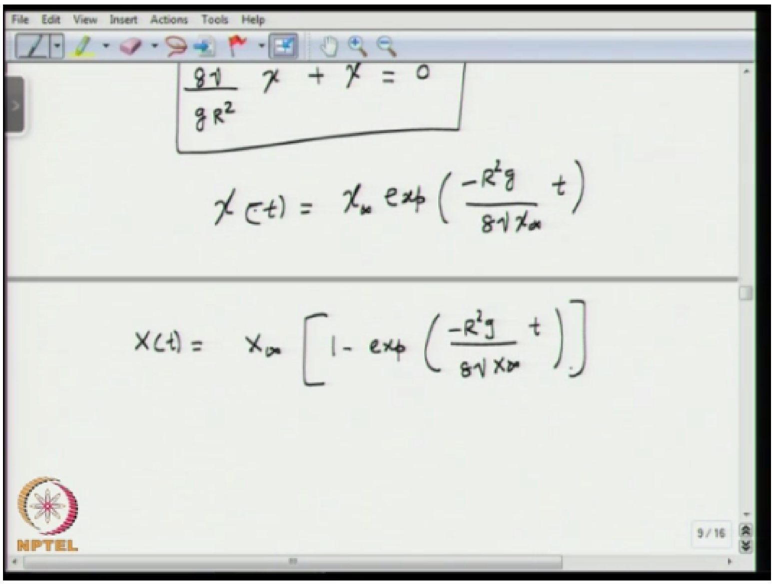

Now, we can convert it back to the small x t which is our original variable, x t is going to be L - L + x0 exponential - R square g divided by 8 nu*L + x0*t, okay. So that is the solution we would have at the end of the filling process when t tends to infinity, okay. So, what we see here is that the profile is going to reach a steady state exponential, okay. So, x t varies as exponential t, okay.

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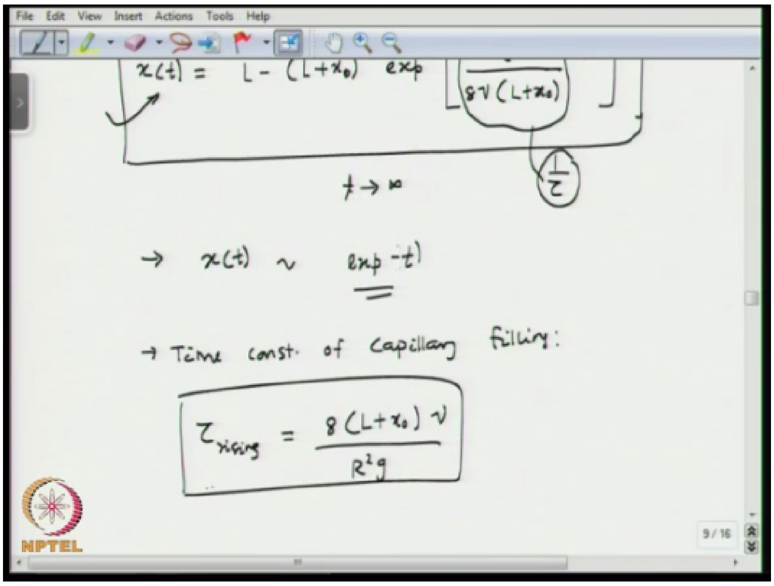

So, we can define a time constant for the filling process which will be given by this expression. So this is the time constant, this actually inverse of t, 1 over tau, we can define a time constant of capillary filling, okay. So, tau rising 8*L + x0 nu divided by R square g, okay. So, the time constant is an indicator of how much time is going to take for the capillary rise to occur.

So that is the observation that we make for a capillary filling in case of a vertical tube. Now, let us talk about for a horizontal tube case. So for the dynamics of passive capillary in horizontal configuration, so for the horizontal configuration of course the gravitational force will be negligible, gravitational force is neglected, okay and in that x0 is going to be 0, okay.

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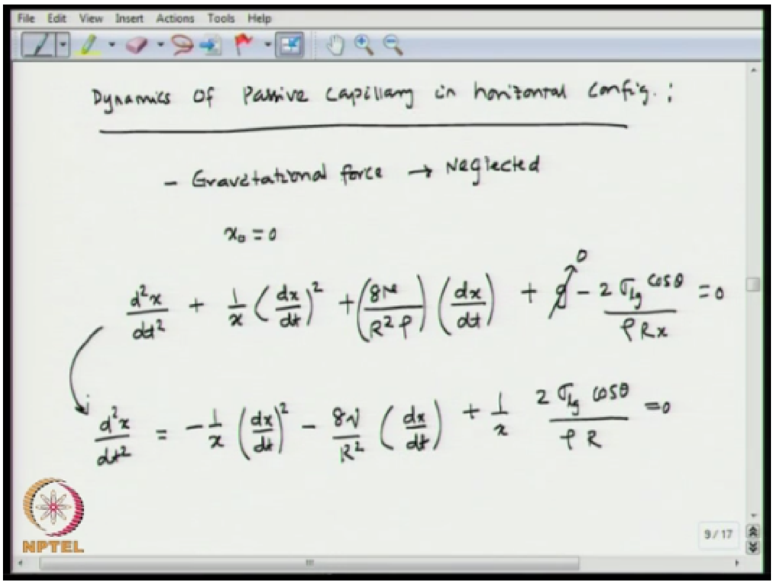

So what we defined x0 as, so if we look at here, we defined x0 as you know the portion of the capillary which is dipped inside the liquid container since here we are talking about a horizontal tube, the x0 is going to be 0. So here, x0 is 0. So from the general equation, we can simplify by dropping up the gravitational force and putting x0 = 0. So, the equation that we would have is d square x over d t square + 1 over x*dx over d t square + 8 mu divided by R square rho*dx over dt + g - 2 sigma lg*cosine theta divided by rho Rx is going to be 0.

Now, we can drop of the gravitational force, so we can write d square x over d t square as - 1 over x dx over d t square - 8 nu over R square*dx over dt + 1 over x 2 sigma lg cosine theta divided by rho r is going to be 0, okay. So, the term 1 over x*dx over d t square because we are talking about you know horizontal tube and we are talking about capillary flow where the velocity is very small and this term is squared here, okay.

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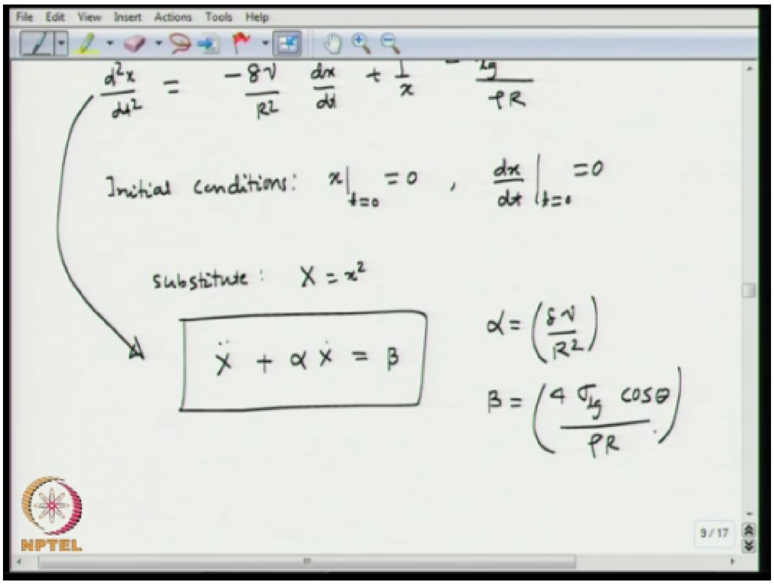

This term is going to be negligible, okay. So this term is going to be negligible, so we can simplify the equation for this term vanishes, so d square x over d t square = - 8 nu over R square dx over dt + 1 over x 2 sigma lg cosine theta over rho R, okay. Now, we can write the initial conditions, the initial conditions are x at t = 0 is going to be 0 and dx over dt at t = 0 is going to be 0.

So that is at the very beginning of the capillary rise advancing process. So, if you substitute x = x square in the above equation, so this equation would reduce to d square X over d t square + alpha*d X over dt will be = beta, okay and here, alpha would be 8 nu over R square and beta would be 4 sigma lg cosine theta over rho R, okay. Now, this equation is easy to solve, so we can find a solution for this.

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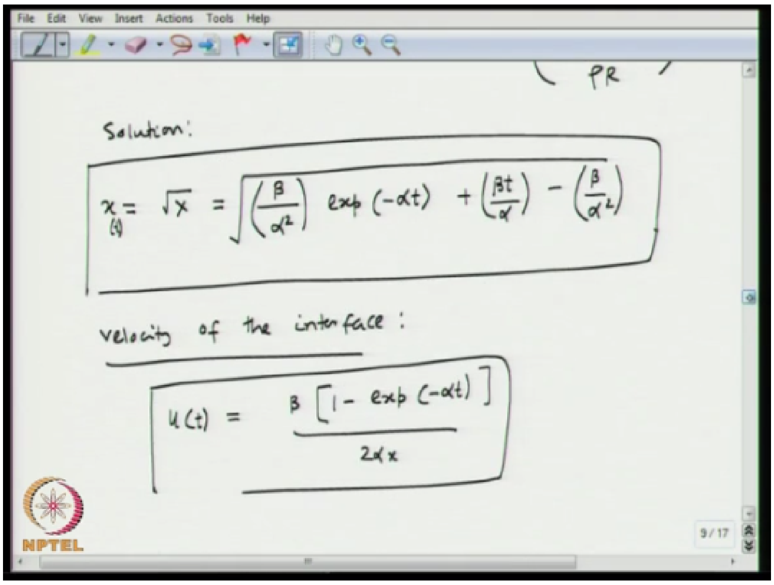

So, the solution would be x will be square root of X which is beta over alpha square*exponential - alpha t + beta t over alpha - beta over alpha square, okay square root. So that is how x is going to vary with time, okay. So, x is function of time, from there, we can obtain an expression for the velocity, okay. So, the velocity of the interface u is a function of t is going to be beta.

So, you can find dx over dt there, so it will be beta*1 - exponential - alpha t divided by 2 alpha x, okay. So that is the expression for the velocity. So, we have you know looked at 2 cases, one vertical tube case where at the beginning of the filling process, the interface is going to follow a square root of t and at the end of the filling process is going to vary as the exponential of time.

And we have also looked at that for the horizontal case, where that is the profile is going to vary as exponential of the time, okay. Now, let us move on and talk about the transpiration cooling. So, transpiration cooling is what we see on the leaves based on that the water moved from the root of the tree to the leaf because the water moves out of the leaf because of the transpiration cooling effect and it gives a negative pressure on the surface of the left, which basically draws liquid from the ground to the leaf, okay.

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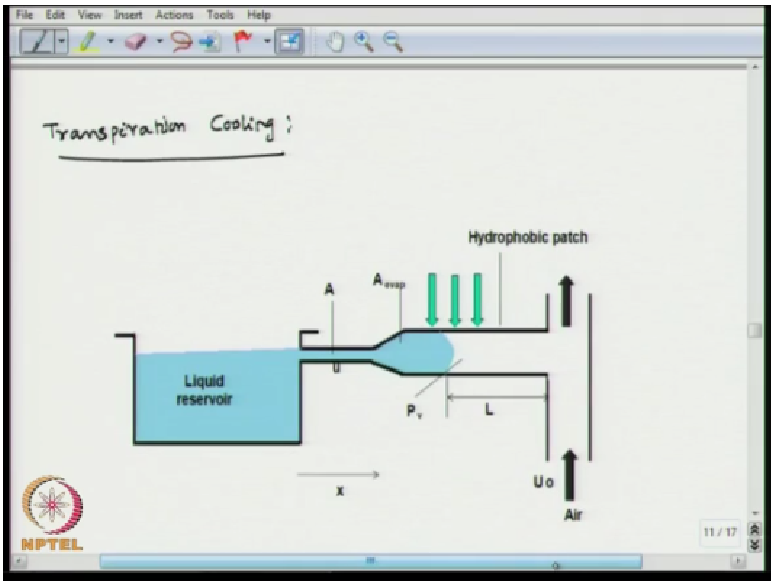

So, similar effect that is discussed here, so we have a liquid reservoir in which liquid is present and so by capillary action, the liquid is allowed to go through this capillary until a point where we have a hydrophobic patch, so this area of capillary has got a hydrophobic patch. So the liquid forms a meniscus there, okay after the capillary liquid filling, liquid form a meniscus.

And here, we apply the heat, okay. So this is the heat that we got at the interface and because of which the evaporation occurs and the air coming in here fix of the vapor and exits and this process repeats as long as we have an interface there, okay and as the air carries this vapor, it creates a negative pressure and the liquid from the reservoir continues to draw into the interface, okay.

So, what we have here is you know the liquid in a reservoir which is drawn into a capillary, so liquid from a reservoir fills horizontal channel and this is due to capillary effect and the meniscus is stopped using a hydrophobic patch, okay and we are hitting the meniscus, so the liquid heating is done at the meniscus and this increases the vapor pressure at the interface, okay.

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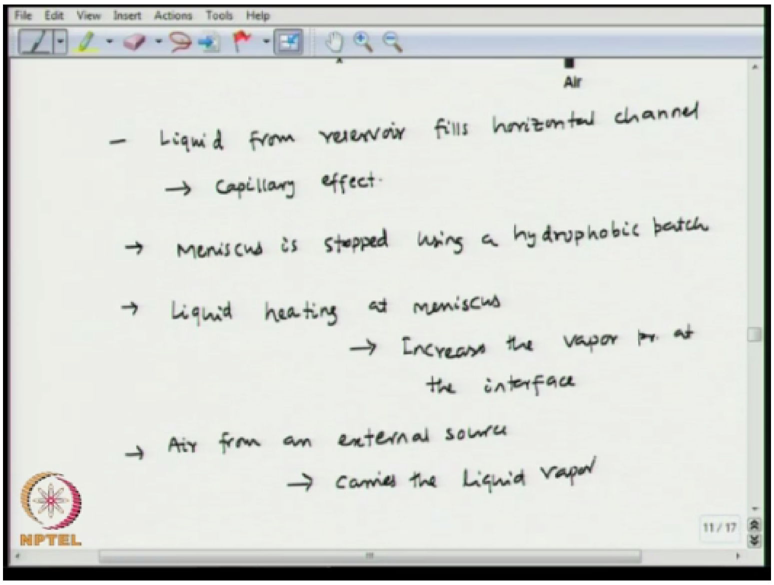

So, we would have air from an external source, the air will carry the liquid vapor and that would create a vapor pressure gradient, that would create a gradient of pressure, okay. So that will enable the liquid to come in continuously to the interface. So, liquid comes in from reservoir through interface continuously, okay and that velocity is given by this expression, M*D over rho R T and*A*evaporator over A Pv - Pv0 divided by L*1 + U0 Dh over D.

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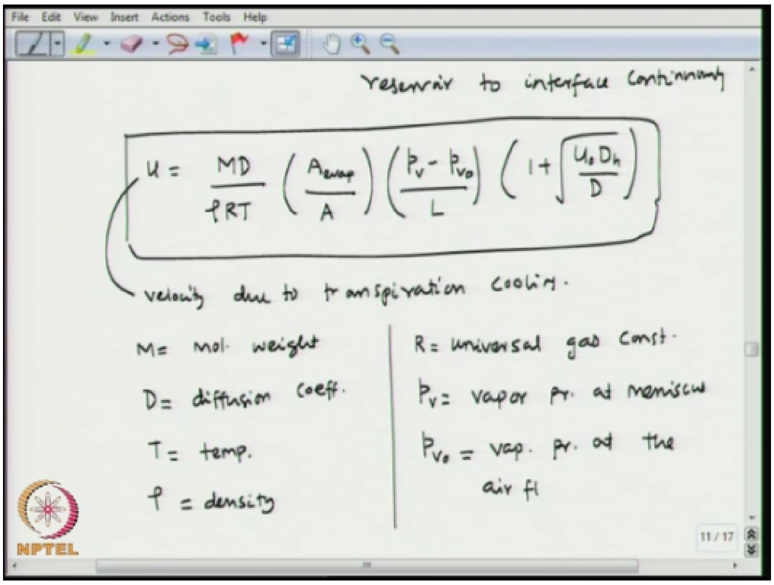

So this is the expression for the velocity due to transpiration cooling, okay. So, U is the velocity due to transpiration cooling, M is the molecular weight and D is the diffusion coefficient, T is the temperature and rho is the density, R is the universal gas constant and Pv is the vapor pressure at the meniscus and Pv0 is the vapor pressure at the air flow side, okay. The area of the evaporator and the area of the cross-section of the channel are given.

The length of the channel is given for you, okay. So that finishes our discussion on the transpiration cooling based micropump. Now, let us consider few examples, okay. So, this is the first example we would consider, we have a thermopneumatic check-valve pump that delivers a maximum flow rate of 340 microliter per minute and a maximum back pressure of 50 kilopascal, okay and the heater resistance is 150 Ohm.

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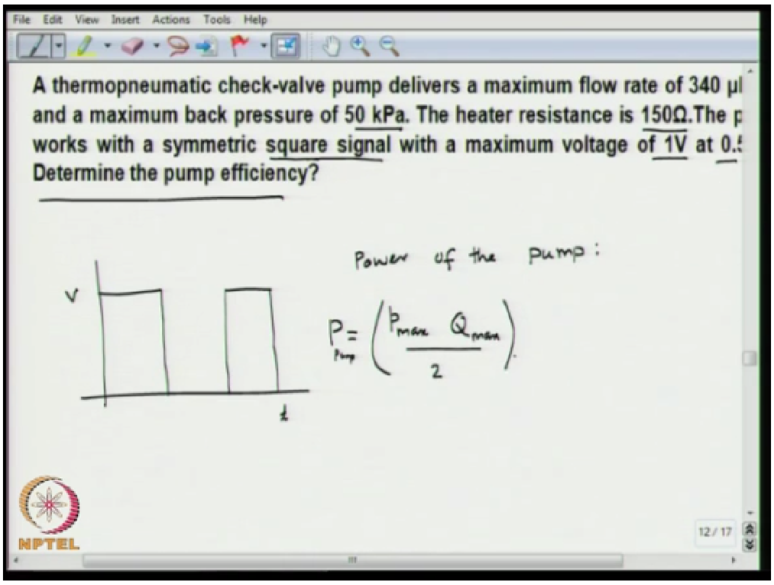

The pump works with a symmetric square signal with a maximum voltage of 1 volt at 0.5 hertz and we want to determine the pump efficiency, okay. So, how do we do that? So first, you know we would have this is time and this is voltage, so we have a power signal that is being applied. So this is applied for only one half of the cycle, okay. So, we can find out the power of the pump.

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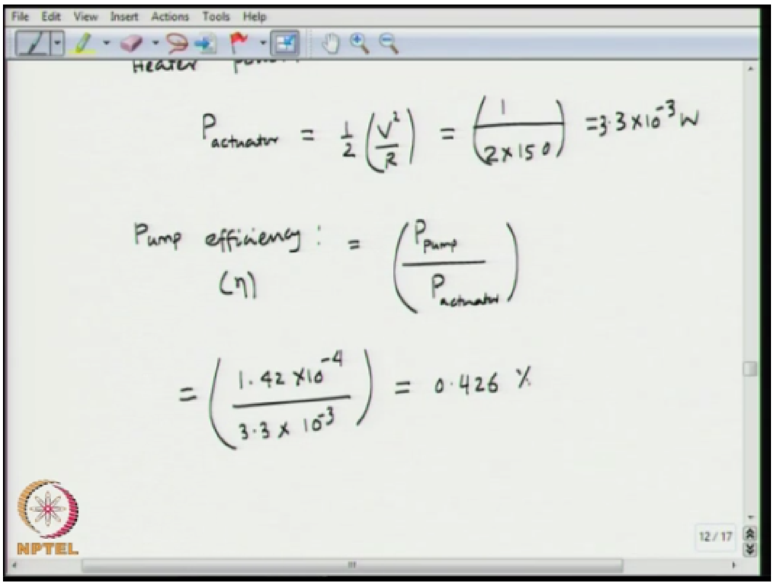

We can find the power of the pump as P max, so this is pump of P max Q max divided by 2, so that will be P max is 50 kilopascal, 50*10 to the power 3 and Q max is 340 microliter per minute, so 340*10 to the power -9 divided by 60 meter cube per second divided by 2, so that would be 1.42*10 to the power -4 watt, okay. Similarly, you can find heater power, okay.

Now, the heater is actuated only for the half of the cycle, okay. So the other half, the voltage is not present, okay. So, we would have P actuator will be = 1/2*v square over R, which is going to be 1/2*150, so this 1 volt here, right and R is 150 ohm, so this is going to be 3*10 to the power -3 watt. So, you can find the pump efficiency. Pump efficiency is going to be the power of the pump divided by power of the actuator, this is the detail power.

It is going to be 1.42*10 to the power -4 divided by 3.3*10 to the power -3, so that is going to be about 0.426%. So, the efficiency is about 0.426%. Now, let us consider another example, so we have a check valve pump that has circular orifices as inlet and outlet and the orifices have diameters of 400 micron and we want to determine the minimum compression ratio for self-priming, okay.

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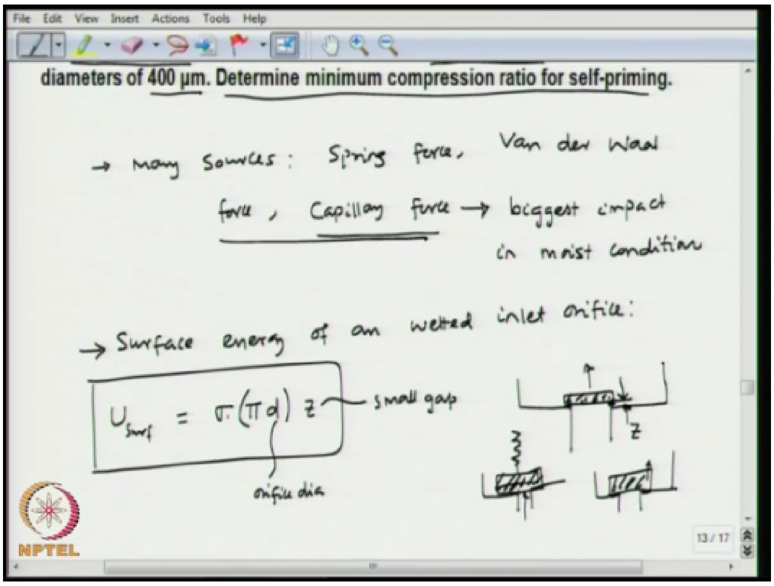

Now, for a check valve pump, there are many sources to hold the valve in place, we have the spring force or we can have thermopneumatic force which can hold it in place, okay. So, we will have many sources like spring force or Van der Waals force or we can have capillary force, okay. So, these are different sources that can be used to close a valve. Now, for a valve that is handling liquid since the condition is moist, the capillary force based valve closing is more significant, okay.

So, this becomes most significant in moist condition, so this has the biggest impact in moist condition, okay. So, these are the sources which can close the valves. So the surface since we are telling that in the moist condition, capillary force is going to the strongest force, you can find the surface energy. The surface energy of a wetted inlet orifice, so what is going to happen is this?

So we have a valve force something like this and here this is the lesser the valve seat, so we have this tiny gap between the valve seat and the valve actuator, okay. So, this gapless is z. So in moist condition, this is handling liquid, we would have a liquid film seating here, right. So, there is a liquid film seating here. Now, if we want to open this valve, the applied energy has to overcome the surface energy that is already created because of the presence of this liquid here, okay.

So, we can find what is going to be the surface energy of a wetted inlet orifice and applied force has to overcome this force, okay. Many times, when we are talking about 2 wetting surface, if let us say you know this is dry and this surface is wetting there, in some cases, we have some spring force that is trying to close the valve.

So the applied force has to overcome the spring force or in some cases, if there is no spring present, but we have just you know the valve actuator is physically in contact with the valve sit, there is Van der Waals force, so to separate that we have to overcome this Van der Waals force, which are normally very weak forces, okay. But in this case, since we are dealing with liquids in microchannels, we would have this liquid you know interfaces.

And the applied force has to overcome the liquid interfaces, so first we calculate the surface energy of a wetted inlet orifice. So, the U surface is going to be sigma*pi d*z, okay. So that is going to be the surface energy, sigma is the surface tension coefficient and d is the orifice dia and z is the small gap, okay. So here, we assume that this you know there is physical contact throughout.

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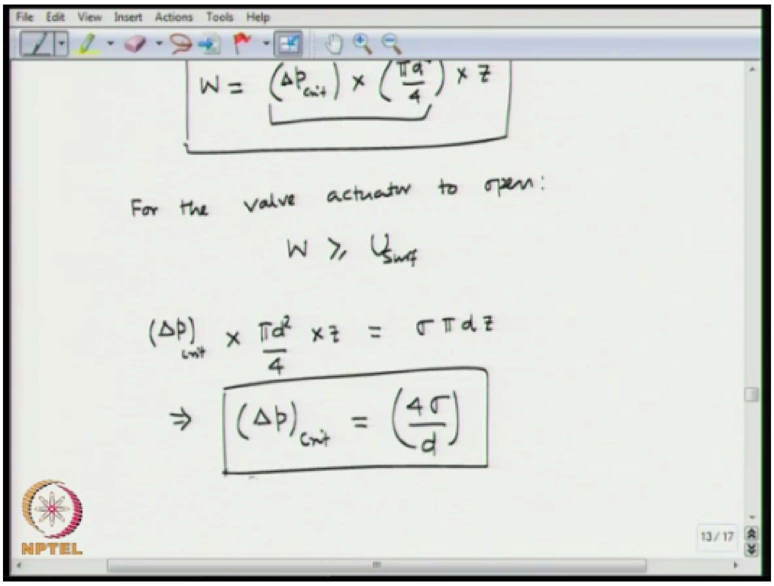

So, we get the surface energy as sigma*pi d*z, right. Now, the external force will be applied, okay so this would cause mechanical valve. So, the external force applied has to overcome the critical pressure that is required to open the valve, okay. So, the external force, so total worked on would be delta P critical that is the critical pressure to open the valve*the area which is pi d square over 4*so that force is applied over a distance z, okay.

Since it is tries to open that, so this is the energy that is applied, right. So, this work has to be greater than the surface energy that is trying to prevent this actuator to open up, okay. So for the valve actuator to open, W has to be > the U surface, okay. So if that is the case, what you would get, in the limiting case, this will be = the surface energy. So, delta P critical*pi d square over 4*z has to be = sigma pi d z, okay.

So we can find the delta P critical, the delta P critical is going to be 4*sigma over d. Now, knowing the critical pressure, we can find the compression ratio. So, the surface tension of water considering water as the liquid, considering sigma as 72*10 to the power -3 Newton per meter for water, we can find delta P critical that will be 4*72*10 to the power -3 divided by d, d will be 400 micron it is given.

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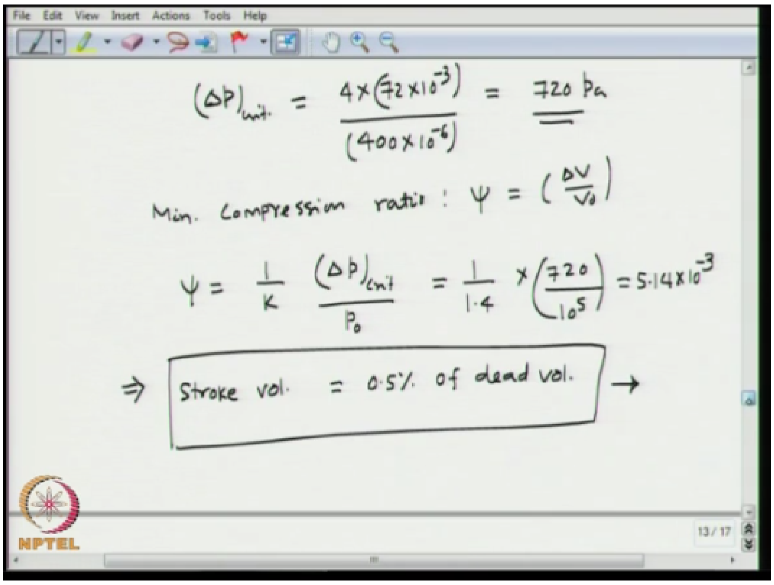

So, 400*10 to the power -6, so that is going to be 720 Pascal. So, we need minimum 720 Pascal to open the valve, okay. So, we can find the minimum compression ratio, the minimum compression ratio which is psi which is nothing but delta v over v0, right. So, we have an expression for psi for liquid, so that is going to be 1 over K*delta P critical divided by P0. So, this is something that we have seen in the previous lecture.

So, this will be 1 over 1.4*delta P critical is 720 and P0 is the atmospheric pressure which is 10 to the power 5, so this will be above 5.14*10 to the power -3, okay. So, what it tells is that the stroke volume delta v has to be about 0.5% of the dead volume, okay. So, then only the valve seat will open and the pump will get self prime, okay. So, next we consider another example.

We consider an example of a peristaltic pump that has got 3 pump chambers and 3 circular piezodisc actuators and the pump membrane has a diameter of 4 mm, okay and the pump works with a frequency of 100 hertz and we are going to determine the volume flow rate at 0 back pressure, okay given that the maximum membrane deflection is 40 micron. So, first we can assume, make an assumption that the deflection of the membrane follows that affecting circular plate.

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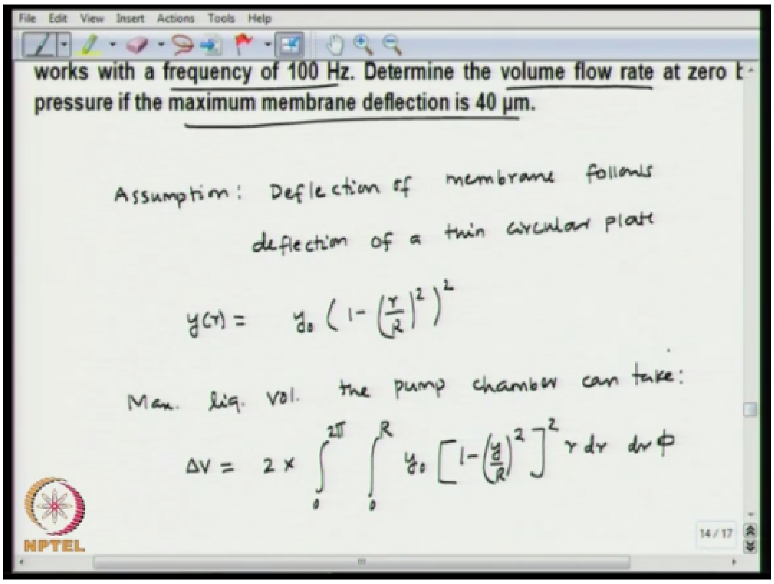

So, the deflection of membrane follows deflection of a thin circular plate, so in that case we can write the deflection y r as y0 1- r over R square whole square. So, we can find the maximum volume that the pump chamber can take, the maximum liquid volume the pump chamber can take which is given by delta v will be twice 0 to 2 pi*0 to R y0*1 - y over R square whole square r dr*dr*phi.

So, basically what we are trying to do here is let us say this is the undeformed position of the membrane and this is the deformed position, we are trying to find what is going to be this area, okay. So that will be the stroke volume, right. So actually, this is the volume that the membrane is storing. So, we take a symmetry about this, so that is why the factor 2 is coming.

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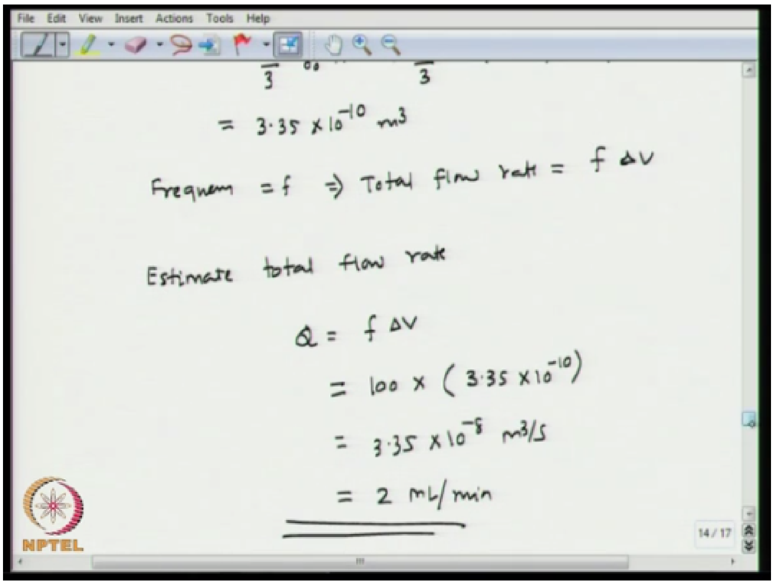

So, we get 2 pi over 3*y0 R square that will be 2 pi over 3, the maximum deflection is given as 40 micron, so this will be 4*10 to the power -5*the radius of the membrane is given, so it is 4 millimeter diameter, so the radius is 2 millimeter, 2*10 to the power -3 square, so that will be 3.35*10 to the power -10 meter cube. So this is going to be the liquid volume, the pump chamber can take in a single stroke.

As it is expanding, it can only take this much volume delta v, okay. So, if you know a frequency and so for known frequency, so frequency = f, then total flow rate will be f*delta v, okay. So, total flow rate we can calculate, so we can estimate total flow rate u going to be f*delta v, the frequency is given, so the frequency is 100 hertz*delta v is going to be 3.35*10 to the power -10.

So, this is will be 3.35*10 to the power -8 meter cube per second which will be about 2 milliliter per minute, okay. So that is the flow rate that the pump can deliver. Next, we consider another example, so we consider a diffuser/nozzle pump and that has a pump membrane of diameter 10 millimeter and the maximum deflection is about 10 micron and we like to determine the volume flow rate with a pump frequency is 100 hertz.

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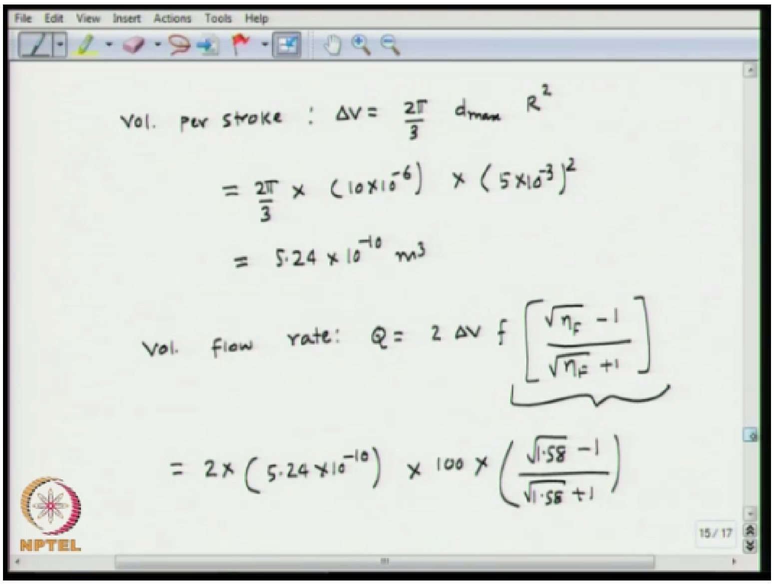

And the fluid diodicity of the diffuser/nozzle structure is known. So, what we first do is find the volume for stroke so that we calculated in the previous example delta v is 2 pi over 3*d max*R square through the maximum deflection, so this is going to be 2 pi over 3*10*10 to the power -6*the radius of the membrane is 5 millimeter, so 5*10 to the power -3 square, so that is going to be 5.24*10 to the power -10 meter cube.

So, we can find the volume flow rate Q would be 2*delta v*f square root of diodicity - 1 divided by square of eta f + 1, okay. So that is known as the rectification efficiency, so that will be 2*delta v, we just calculated 5.24*10 to the power -10*f, the frequency is 100 hertz and the diodicity is given 1.58 - 1 divided by 1.58 + 1, so we can calculate this to be 1.19*10 to the power -8 meter cube per second, which is 715 microliter per minute, okay.

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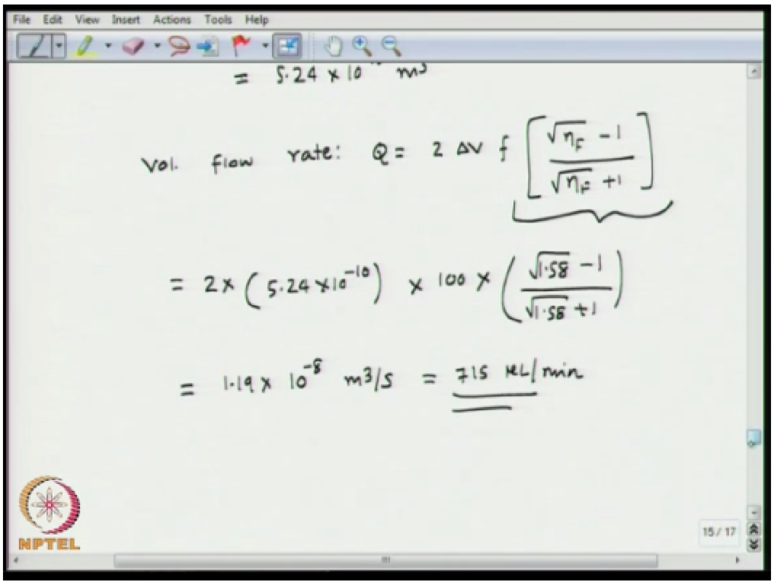

So, let us consider another example, so this is the rotary gear pump that we have discussed and it has the following parameter, it has 596 micron of gear diameter and it has 12 number of teeth and the pressure angle is 20 degrees, okay. So, 20 degrees pressure angle and 515 micron center to center distance, okay and the thickness is 500 micron which is equal to the channel height and we are going to determine the volume flow rate a speed of 300 micron.

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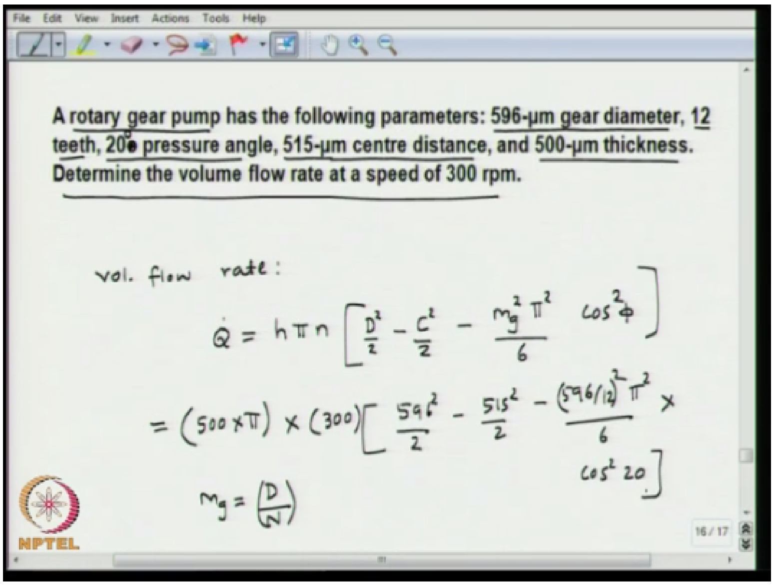

So this is the very simple question, if you remember the formula. The volume flow rate for a gear pump is given by h*pi n d square/2 - c square/2 – mg square pi square over 6*cosine phi square, okay. So this is the expression for the flow rate for the gear pump, so h is the channel height, so it is given which is 500 micron*pi*n is the rpm which is 300*d square by 2 is 596 square by 2 – the center to center distance is 515 square by 2.

Mg can be calculated by the diameter divided by the number of teeth, okay. So, Mg will be diameter of 596 divided by 12*pi square over 6*cosine square 20, okay. So, if we do a calculation, we get this as 19.5*10 to the power 9 micron cube over minute, okay. So this is another example, the last example will consider in this lecture. So, we want to determine the burst frequency of a water column.

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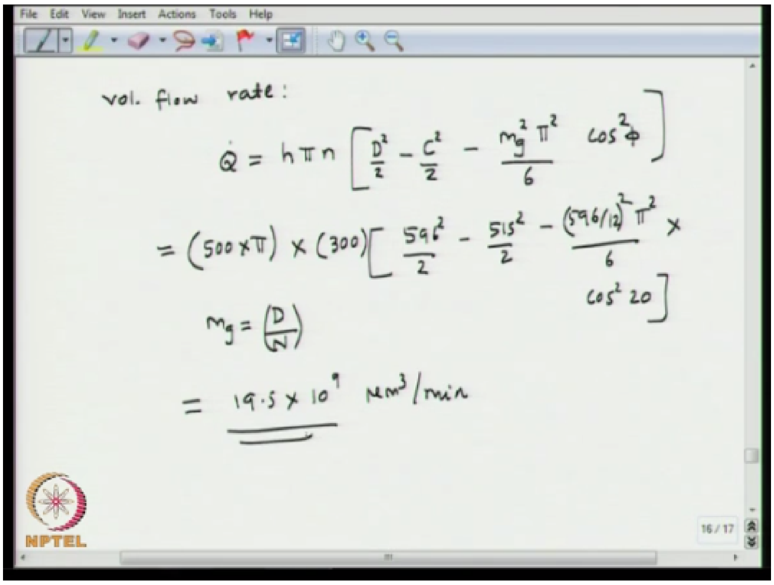

So I talking about a centrifugal pump which is 2-millimeter-long and an average distance to disc center is 4 millimeter. The channel cross-section is 100 micron by 50 micron. The contact angle is 30 degrees and the surface tension and density of water 72*10 to the power -3 Newton per meter and 100 kg per meter cube. What is the frequency to burst of the same water column at an average distance of 16 millimeter, okay.

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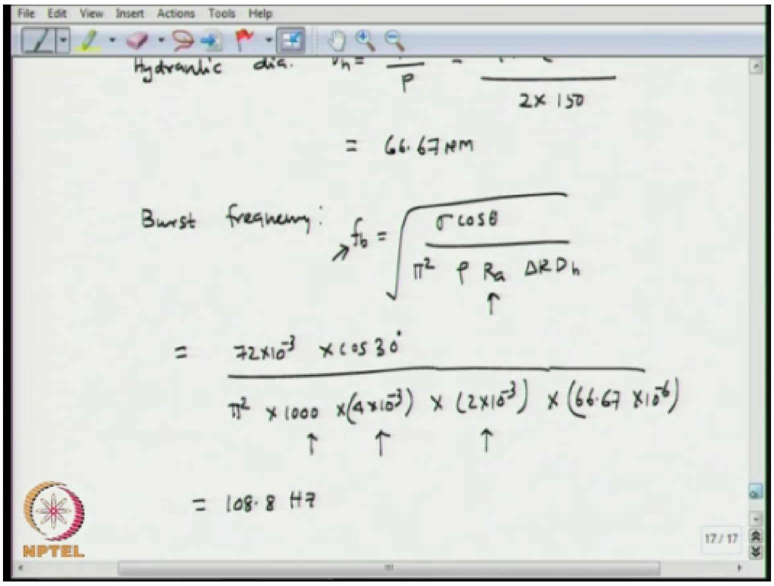

So first, we have to calculate what is the burst frequency when at the distance where the liquid column is located 4 millimeter, then we can calculate what will be burst frequency when it will be located at 16 millimeter, okay. So, the first thing to do is calculate the hydraulic dia, the hydraulic diameter can be calculated as Dh is 4*area of cross-section over perimeter.

So, 4*the channel is 100*50 divided by 2*150, so this is about 66.67 micron and so, we can find the burst frequency, we have the formula, so the burst frequency fb is sigma cosine theta divided by pi square rho Ra delta R*Dh square root. So, we can substitute the values 72*10 to the power -3*cosine 30 degrees is the contact angle, pi square*1000*4*10 to the power -3*2*10 to the power -3.

So this is the density, this is the average location, this is the length of the column*66.67 is the hydraulic diameter*10 to the power -6, okay. So that will be about 108.8 hertz. So, we can notice that the burst frequency is inversely proportional to the square of the average location, okay. So, what we can say that f2 over f1 is going to be R1 over R2 square, okay. So now, this is the burst frequency at a locating of 4 millimeter, okay.

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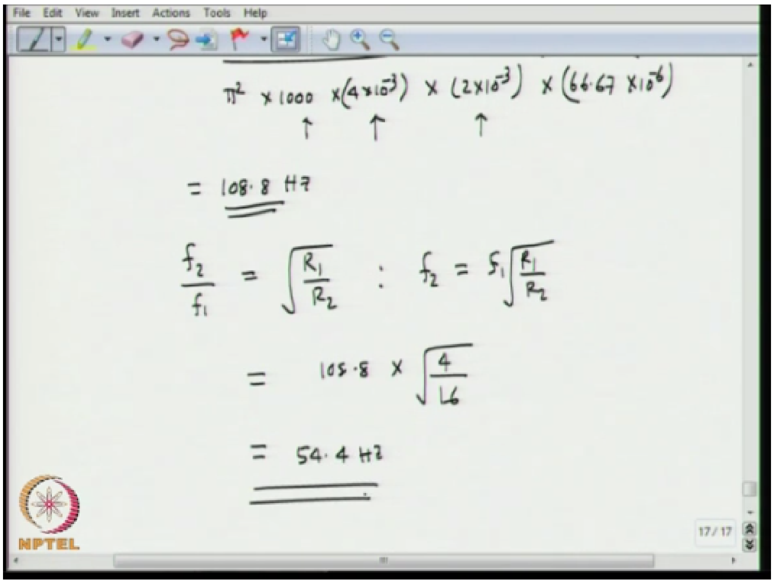

So what is going to be at 16 millimeter, we can find f2 or 16 millimeter is going to be f1*R1 over R2 square root, so this will be equal to 108.8*4 over 16 square, so that will be 54.4 hertz, okay, right; So with that, let us stop here.