Microfluidics
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Lecture - 29
Micropump

Okay, so let us continue a discussion on micropump, we have talked about the acoustic micropump, let us talk about electrokinetic micropump. We have already discussed that electrokinetic micropump operate on the principle of electro-osmosis and we have already discussed electro-osmosis in the previous lectures, so here will be briefly introducing to electro-osmosis and then talk about a design problem, okay.

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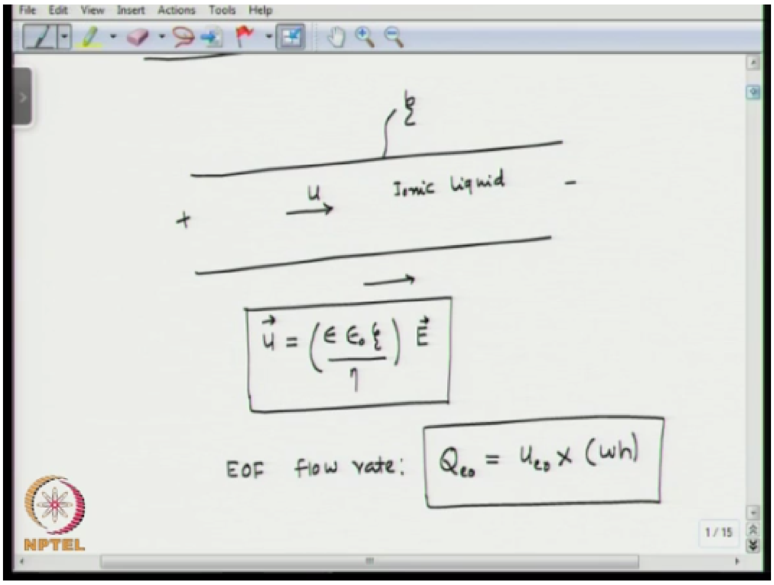

So, we talk about electrokinetic micropump, so here we have a channel which has some zeta potential, okay and here inside we have an ionic liquid, liquid with non0 conductivity and if we establish an electric field, okay then we induce a electro-osmotic flow, okay and this electro-osmotic flow velocity is given by epsilon epsilon 0, so that is the permittivity of free space, this is a dielectric constant of the liquid*zeta potential over the viscosity of the liquid*the electric field, okay.

So that is the expression for the electro-osmotic velocity. Now, knowing the area of cross-section of the channel, we can find the electro-osmotic flow rate. So the electro-osmotic flow rate for a rectangular channel, we have seen that the electro-osmotic flow rate is given by*the area of cross-section per width*height, okay. So, let us consider a design problem, so you consider this situation here.

We are talking about electrokinetic pumping ne2rk for capillary electrophoresis. So this is a typical microchannel ne2rk use for capillary electrophoresis. Normally, you would have you know sample present in one of reservoirs, so here would be applying vacuum. So that the sample plug comes to junction and then by applying a voltage between this port, port 2 and port 4, the sample will get separated along this channel, okay.

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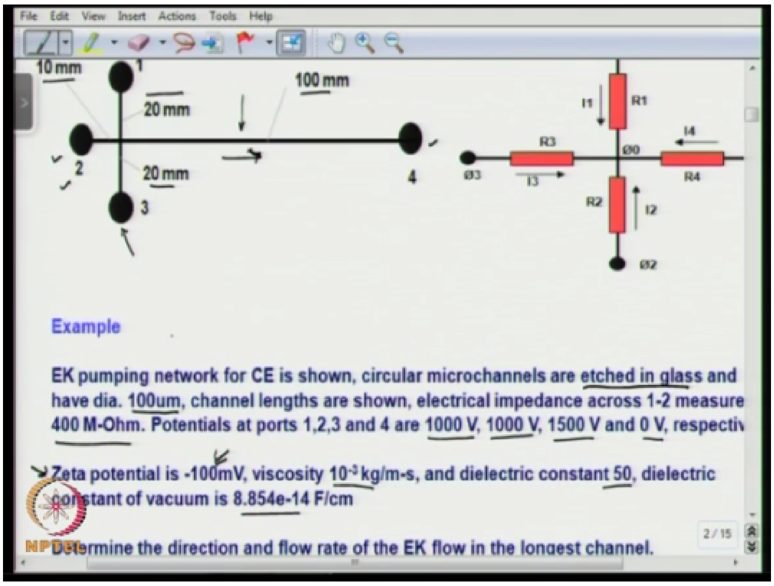

So that is the principle of capillary electrophoresis. Here one such ne2rk is shown and we have circular microchannel cross-section that are etched in glass and have diameter of 100 micron and how do you make circular microchannels? We can have bulk etching of silicon to have hemispherical channel and we can bond 2 such microchannel to get a circular microchannel cross-section and here all the channel lengths are shown.

You can see this is 10 millimeter, this is 20 millimeter, this is 20 millimeter and the long channel is 100 millimeter. The electrical impedance across 1 to 2 is major, so you can major the electrical impedance between port 1 and port 2. So that is measured to be 400 mega-Ohm, okay. Now, the potentials at the port 1, 2, 3 and 4 are given. So port 1, which is under 1000 volts, port 2 is 1000 volts, port 3 is 1500 volts and port 4 is 0 volts, okay.

So, it is also given the zeta potential of the channel (()) (04:42) is given is -100 millivolt, okay and the viscosity is 0.001 kilogram per meter second. The dielectric constant is 50 and the dielectric constant of vacuum is 8.854*10 to the -14 Faraday per centimeter. Now, we are interested to know the direction and flow rate of electrokinetic flow in the longest channel, okay.

So, we are interested to know in which direction, the electrokinetic flow is going to occur in the long channel and what is going to be the flow rate, okay. Now, how do we solve this. So, here we can assume that the fluid in the microchannel is homogenous, okay. So, we make an assumption is that the fluid in the microchannel is homogenous meaning the electrical properties are same throughout, okay.

So in that condition, the electrical impedance will be proportional to the channel length, okay and we can analyze the system as a ne2rk of resistors. So in that case, we can represent these ne2rk as an equivalent electrical ne2rk, okay. Now since we now the impedance between 1 to 2 that is 400 mega-Ohm, we can predict the resistances assuming that the fluid is uniform throughout, okay that is the assumption we make.

So, under that assumption we get R1 as 200 mega-Ohm because from the length of this channel 1 junction and junction to 2, so 1 junction is having twice the length from junction to 2. Sorry, we are majoring 1 to 2, so this is actually 2, this is 3, okay. So, 1 junction and junction to 2 have same length 20 millimeter. So in that case, etch will have resistance 200 mega-Ohm.

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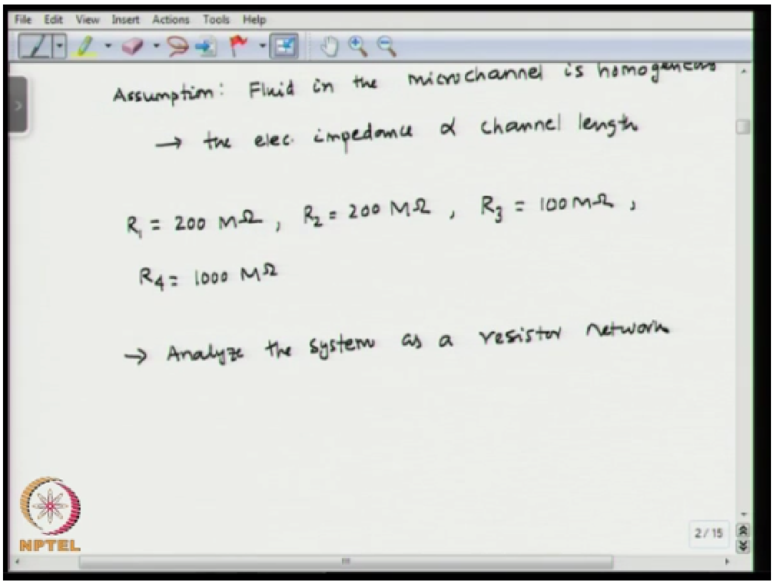

Because the total is 400 mega-Ohm, so R2 will be 200 mega-Ohm and R3 will be 100 mega-Ohm and R4 will be 1000 mega-Ohm. So R3 has half the channel length, okay. So R3 is only 10 millimeter, so it will be half of R12 junction, okay so that is 100 mega-Ohm and R4 is 10 times R3 so that will be 1000 mega-Ohm. So, we can analyze the system as a resistor ne2rk.

Now, if we consider the direction of the potential and current, we arbitrarily define the direction of the current and the potential, so this is at junction 1, it is phi1, so this is at 2, it is phi2, this is phi3, and that is phi4 and we define the resistances R1, R2, R3 and R4 and the current I1, I2, I3, I4. So for convenience, we defined all the currents are coming towards the junction of course that will not happen due to Kirchhoff's law.

So, we will get some current in fact in the reverse direction. Now if we assume Kirchhoff's law for this electrical ne2rk, then we can arrive at a solution, okay. So, we use potentials and current directions, so using that we can establish the relation between the 4 currents and we can do that using the Kirchhoff's law, okay. So, we can write I1+I2+I3+I4, so all the currents coming towards the junction, the summation of that is going to be 0.

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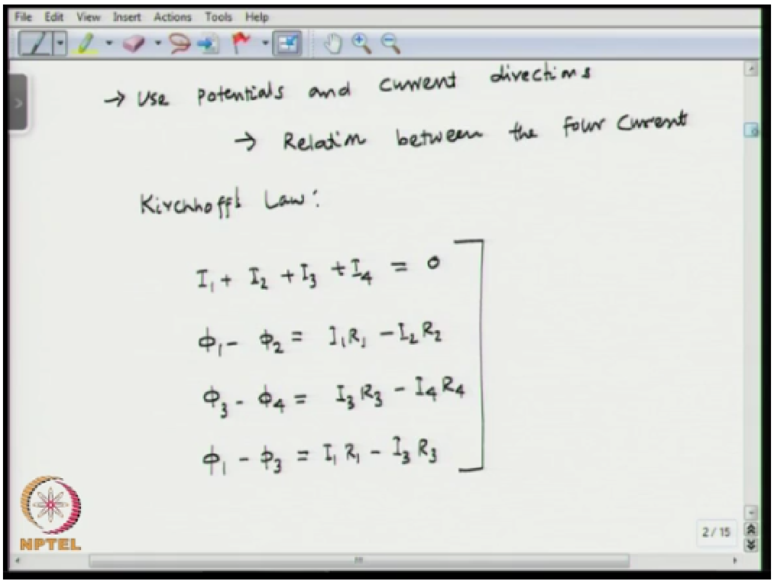

Now, phi1 - phi2, so here phi1 - phi2 is going to be I1 R1-I2 R2, because the direction of I2 is in this direction, so it will be –I2 R2, so I1 R1-I2 R2. Similarly, we would have phi3 - phi4 will be I3 R3-I4 R4 and we have phi1 - phi3 as I1 R1-I3 R3. So, here we have 4 equations and we have 4 unknowns. The unknowns are I1, I2, I3 and I4, everything else is known. So, if you solve the 4 equations simultaneously, we can get an expression for I4, okay.

So, we can solve for I4, I4 will be phi4 - phi1*R2 R3 + phi4 - phi2*R1R3 + phi4 - phi3*R1 R2 divided by R1 R2 R3+R2 R3 R4+R3 R4 R1+R4 R1 R2, okay. So, this is going to be the expression for I4. So if we plug in the values, we know all the values here. So if we plug in the values, we can calculate I4 is -1.2*10 to the power ampere. So, this is negative because here if you see here, we have taken I4 to be in this direction, but in reality the current is going to be in this direction, okay.

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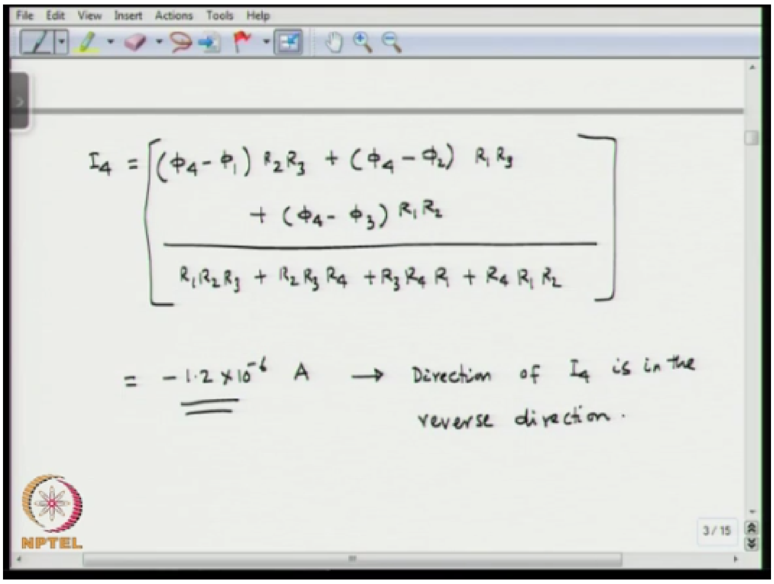

So that will be same as the direction of the flow, okay. So, what that means is direction of I4 is in the reverse direction. So, it is going from the junction towards the port 4, is in the reverse direction. Now, you can find the electric field strength in the longest channel, which is going to be phi0, phi0 is the potential at the center - phi4 divided by L. So, this is going to be I4R4 divided by L.

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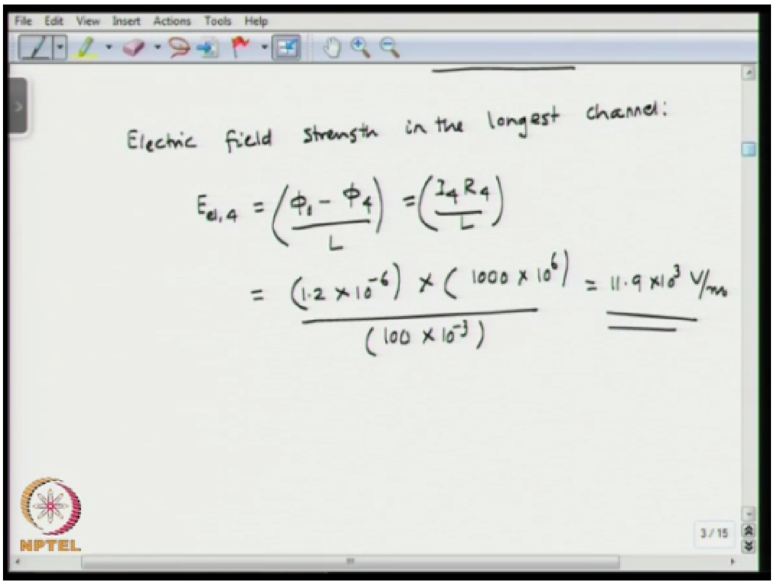

So you have all the values, we can substitute I4 is going to be 1.2*10 to the power -6, so that is from the junction towards port 4*R4, R4 is 1000 mega-Ohm, so 10 to the power 6 Ohm divided by the length of the channel is 100 millimeter, so 10 to the power -3, so that would be 11.9*10 to the power 3 volt per meter, okay. So that will be from the junction towards the port 4 that will be the direction of the electric field.

Now, the electro-osmotic mobility can be determined as, so new electro-osmolality is going to be epsilon epsilon 0 xi over eta. So this is 50 that is the dielectric constant of the liquid that is given and of the vacuum is 8.854*10 to the power -14, so*xi is given which is 100*10 to the power -3, 100 millivolt thus given and viscosity is 10 to the power -3. So that is going to be 4.43*10 to the power -8 meter square per volt second, right.

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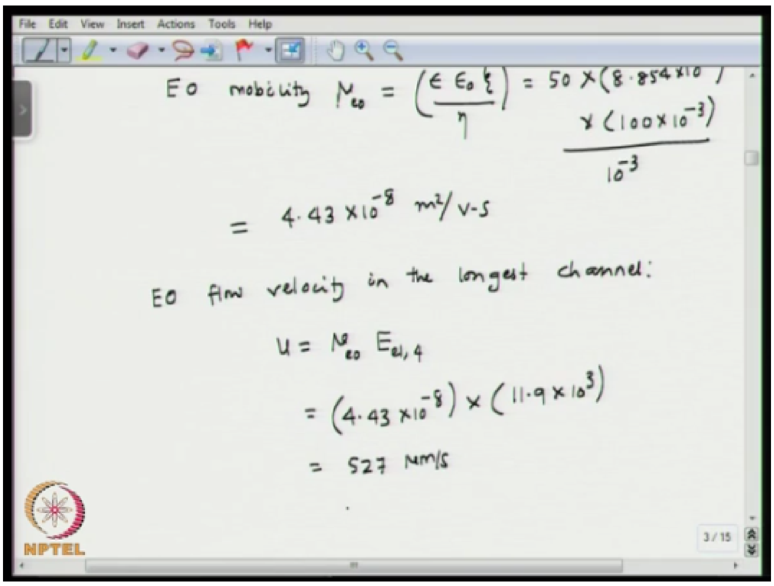

So, you can find the electro-osmotic flow velocity in the longest channel, so u is going to be mu electro-osmotic*the electric field which is going to be 4.43*10 to the power -8*11.9*10 to the power 3, so that will be 527 micron per second. So, knowing the electro-osmotic flow velocity, we can find the flow rate, okay. So, the electro-osmotic flow rate is going to be, so that is in channel 4 is going to be u4.

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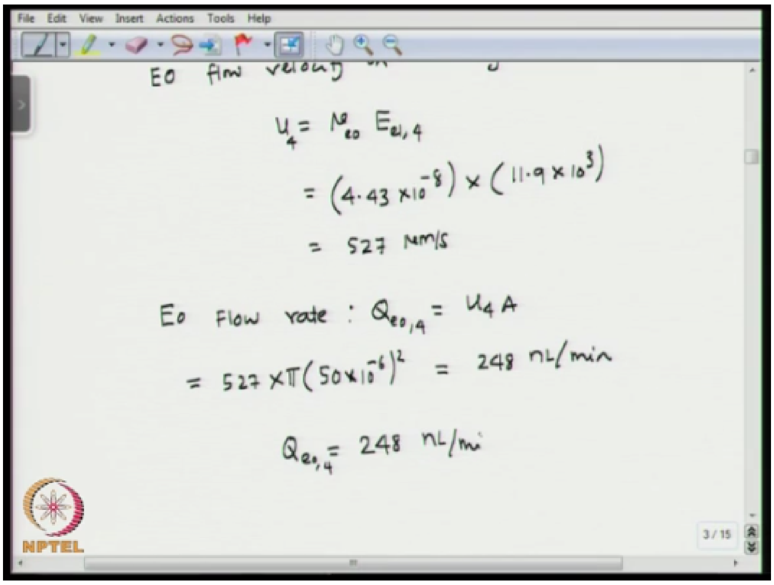

So this is in velocity in channel 4*the area of cross-section is 527*the area of cross-section, this is 50 micron radius, pi*50*10 to the power -6 square, so that is going to be 248 nanoliter per minute. So, Q electro-osmotic is 248 nanoliter per minute, okay. So, that gives you an example how to solve for the electro-osmotic flow rate in a typical microfluidic ne2rk when you are electro-osmosis for capillary electrophoresis applications, okay.

Now, let us talk about magneto-hydrodynamic micropump. So, we talk about magneto-hydrodynamic micropump, MHD micropump, okay. So, magneto-hydrodynamic micropumps work on the principle of Lorentz force. So, typically if you have a channel and you are applying a magnetic field, you have an electro through which you are applying the current through the fluid, okay.

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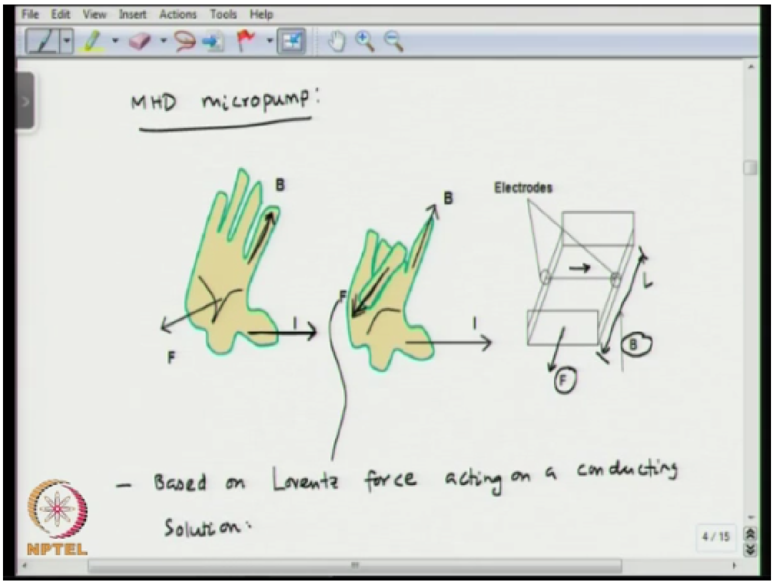

So, if you have a channel such that you are applying a current in the thumb direction and you are applying a magnetic field along the index finger, the liquid will be subjected to a force along the middle finger, okay. So that is the principle based on which the magneto-hydrodynamic pumping works. So as we can see here, so this is the direction of the current and this is the direction of the magnetic field.

So, you would have a force induced in this direction, okay. So, you can see this is the length of the channel, okay and this is the direction of the magnetic field and you have a current which is going in this direction between the 2 electrodes, so induced the force F in this direction. So, magneto-hydrodynamic pumping is based on the Lorentz force, okay. So, this force is called Lorentz force that is acting on a conducting solution.

So, you need a conducting solution because you need to pass current through it, okay. So the force, F is going to be current*cross magnetic field, so that means force is going to be on the third direction, okay. So, current is along one direction and magnetic field along the direction 2, so this will be along the third direction, okay*w, right. So here, I is the current and B is the magnetic flux intensity, okay and w is the distance between the electrodes, okay.

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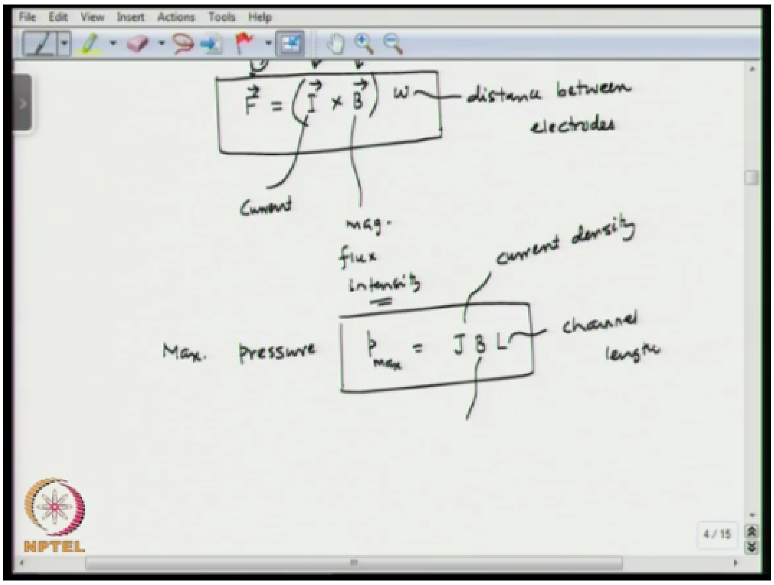

So, you can find out the expression for the pressure and flow, so the maximum pressure P max has been estimated as J*B*L, okay. So, J is the current density, okay and B is the magnetic flux intensity which is defined here and L is the channel length, okay and the maximum flow rate Q can be defined as J*B*pi Dh4 divided by 128, okay. So that is the expression for the flow rate.

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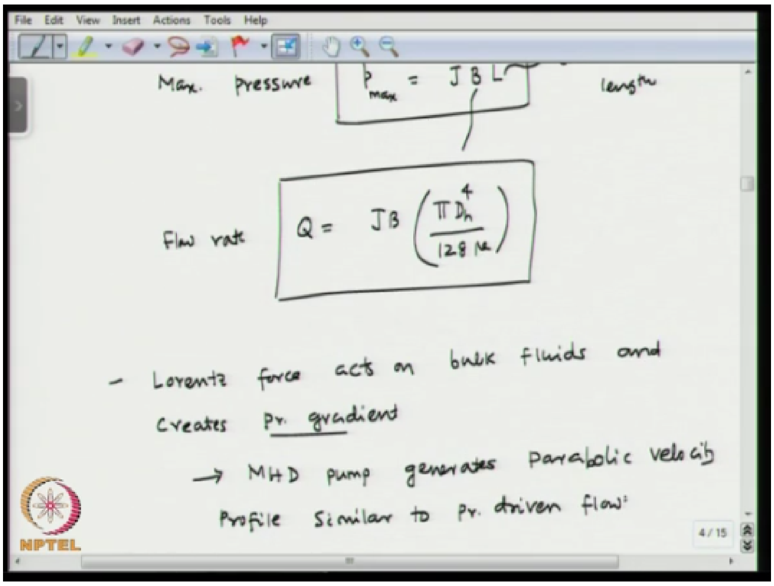

So, the Lorentz force what it does is, it induces a pressure gradient in the bulk of the fluid, okay. So, since it induces a pressure gradient, the flow induced because in the magneto-hydrodynamic effects is parabolic in nature, okay similar to that is encountered in pressure driven flow, okay. So, the Lorentz force acts on bulk fluids and creates a pressure gradient, so the velocity profile similar to that obtained for pressure driven flow.

So the magneto-hydrodynamic pump generates parabolic velocity profile similar to pressure driven flow, okay. So that finishes our discussion on magneto-hydrodynamic pump, now let us talk about the micropump based on surface tension driven effects, okay. So, we first talk about passive capillary effect, okay. So typically, when we have a liquid droplet present on a solid surface, this is the contact line that we encounter.

So here, we have the surface tension between solid gas and surface tension between solid liquid and surface tension between liquid gas, okay and this angle is known as the contact angle. So, we can write the force balance at the interface. So if you do force balance along the interface, we can write sigma solid gas - sigma solid liquid is going to be sigma liquid gas cosine theta.

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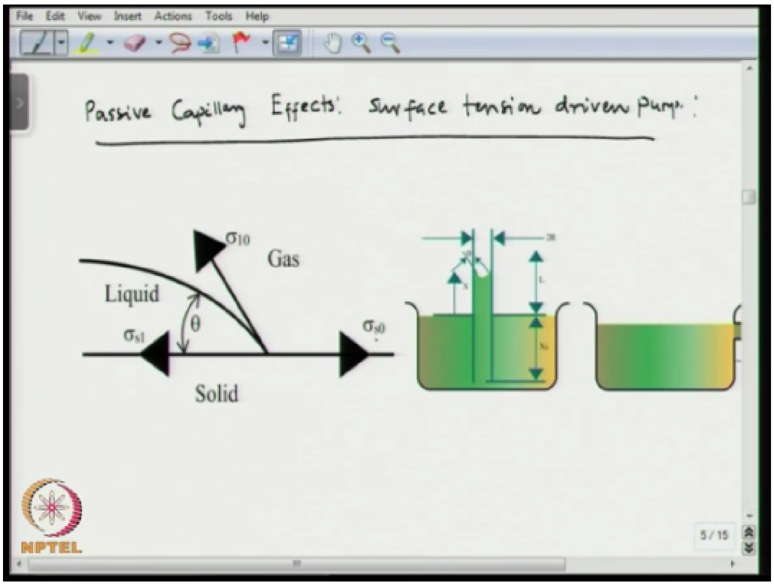

So, you make a component of sigma lg in this direction and that is what the force balance is at the contact line, okay. So, you know if you have a hydrophobic surface what that means is that the surface energy between a solid gas interface is < that between a solid liquid interface, okay. So, the surface does not like liquid to waited, okay. If you have a hydrophilic surface, then the surface energy between a solid liquid interface is < that between the solid gas interface, okay.

So, if we have a hydrophobic surface, then the solid gas surface energy is < the solid liquid surface energy, okay and if you have a hydrophilic surface, then the solid liquid surface energy is < the solid gas surface energy, okay. So, you know the surface tension between the liquid gas interface can be obtained using a capillary tube experiment that we have seen in previous lecture, so here we just briefly revisit that.

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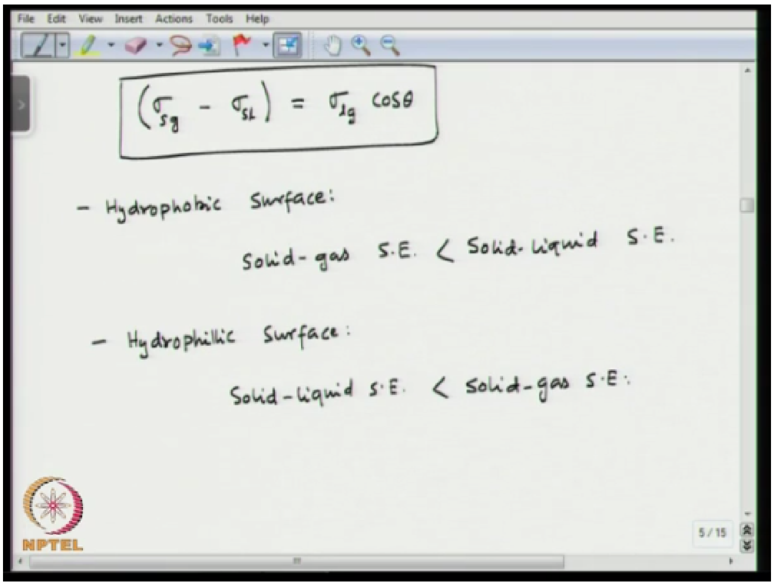

So as you can see, in a vertical tube put in a liquid bath, if we do force balance at the interface, so say that the liquid gas surface tension can be determined by capillary rise experiment and if you do force balance at the interface, then it will be 2 pi R*sigma solid gas - sigma solid liquid will be delta P*pi R square, okay. So, you can find sigma lg as rho, so this you can express in terms of sigma lg cosine theta.

So this will be rho g*l*R divided by 2 cosine theta, okay. So, this is the expression for the liquid gas surface tension, okay. So here, you see we have 2 configuration, here the tube is connected vertically, so this is the vertical configuration, here the capillary tube it is inserted horizontally to the liquid container, okay. Now, this vertical capillary rise and horizontal capillary advancement have different applications.

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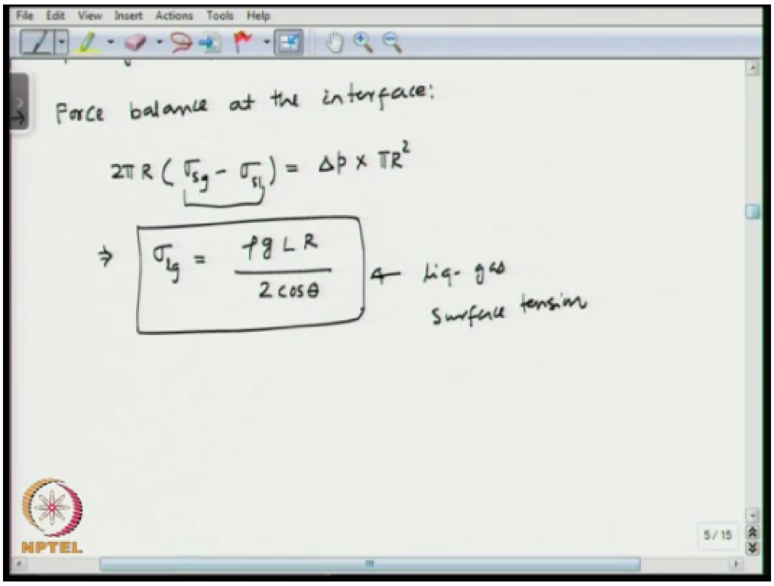

Vertical capillary rise is used when you want to collect some sample from a container using a capillary and horizontal capillary advancement is used in case of a filling of microchannel for lab-on-a-chip applications, okay. So, you have 2 cases here, the vertical tube configuration is used for sample collection and the horizontal tube configuration is used for the self filling of microchannels in lab-on-a-chip application, okay.

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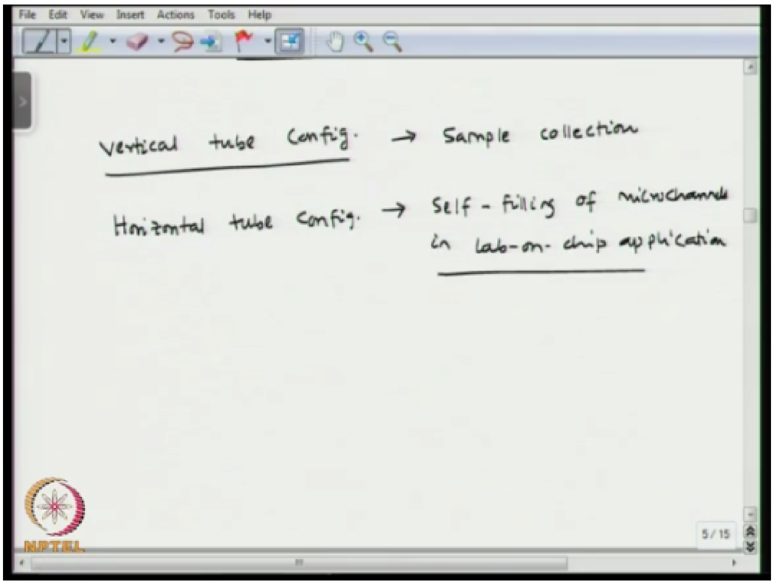

So, let us look at first case where the liquid is trying to raise in the vertical tube configuration, okay. So, we consider passive capillary in vertical configuration, here we do balance of different forces, okay. So, when this liquid column is trying to move off, because of the capillary action we can do a force balance on this liquid column, okay. So, what is the different forces that are acting?

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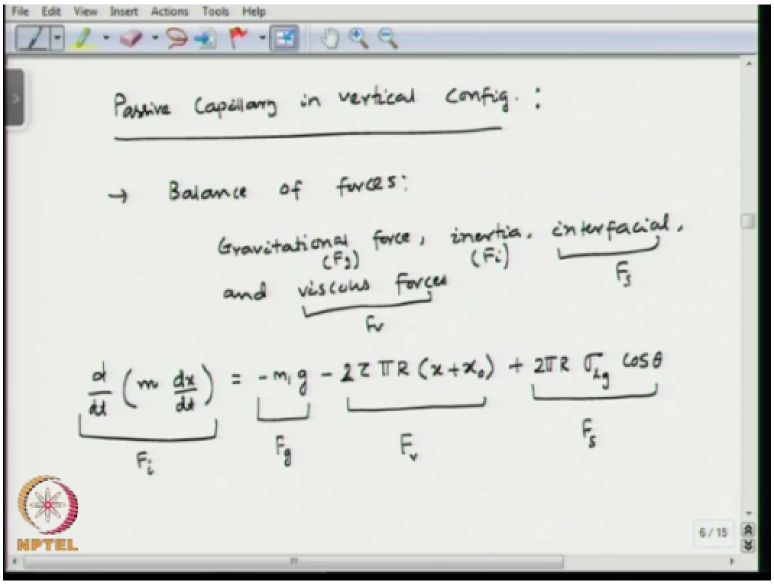

We have gravitational force, then we have inertia force, we have interfacial force and viscous forces, okay. So, we can write done the expression for these forces, so we can write d over dt*m dx over dt - m1g - 2 tau pi R*x + x0 + 2 pi R sigma lg*cosine theta. So, this is the inertia force, so this is nothing but mass*acceleration as mass is changing with time, so we have d/dt outside the bracket.

This one is the gravitational force, okay, m1*g, where m1 is the mass of the liquid column ever the outside liquid surface. So, anything ever so this mass will be ever this line, this mass will be m1, okay and this is you know the viscous force and this is the surface tension force, okay. So this is the inertia, this is the gravitational force and this is the viscous force, okay and this is the surface tension force, okay.

Here, the x is the position variable and x0 is going to be the immersed length, tau is the shear stress, m is the mass of the liquid column, so total liquid column from the bottom to the top is m and m1 is the mass of the liquid column above reservoir surface, okay. So now, we can simplify this equation, we can write d/dt*rho pi R square, so we write the expression for m pi R square*x + x0*dx over dt will be = - rho pi R square*x*g – pi R square*dP/dx, this is pressure gradient*x + x0, okay.

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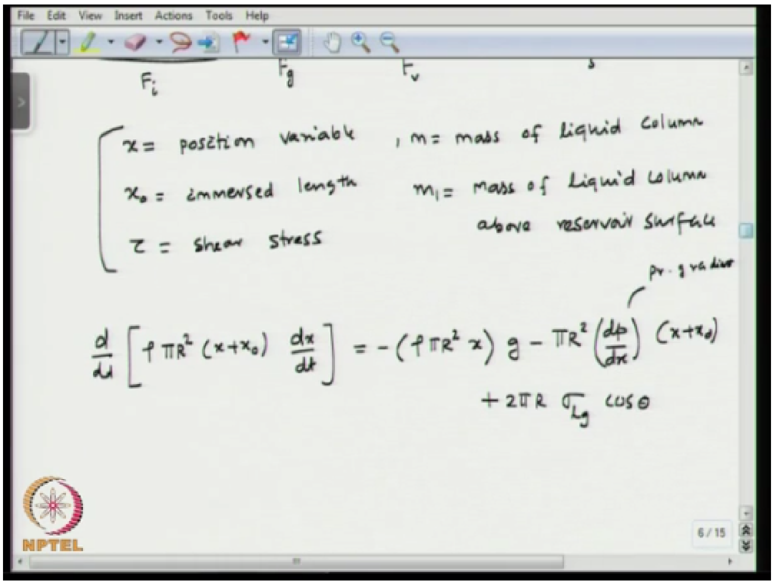

So, this is the pressure gradient, this is the viscous force and + 2 pi R*sigma lg*cosine theta, okay. So here, what we have used? We have used the expression for the pressure gradient in this expression. So now, if we use Hagen-Poiseuille model for pressure drop, then we can write dp over dx, the pressure gradient can be written as 8 mu over R square*dx over dt, right so this is the Hagen-Poiseuille model.

That we can substitute in this equation now, so we get this equation, d square x over dt square + 1 over x + x0*dx over dt whole square + 8 mu divided by R square rho*dx over dt, right. So this is the inertia term and this is the viscous term + g*x divided by x + x0 and this is the gravitational force term, which will be = 2 sigma lg cosine theta divided by rho R*1 over x + x0, okay.

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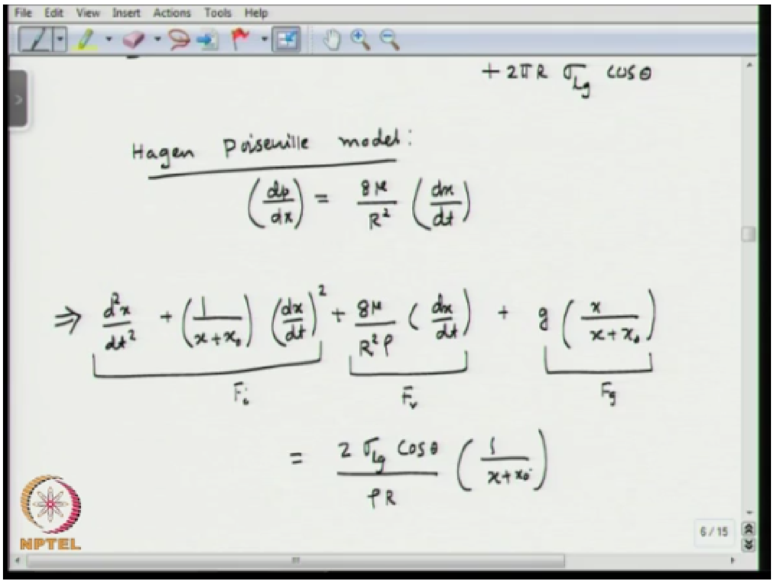

So that is the equation we have, this is the surface tension force, okay. Now, we can be obtained for the solution for the 2 extents, one when the time scale is you know very less, so it is basically the beginning of the capillary filling process and the second time step is towards the end of the capillary filling process, so solution can be obtained for 2 extremes, okay. T tends to 0, x0 << x infinity, so that is the beginning of the filling process.

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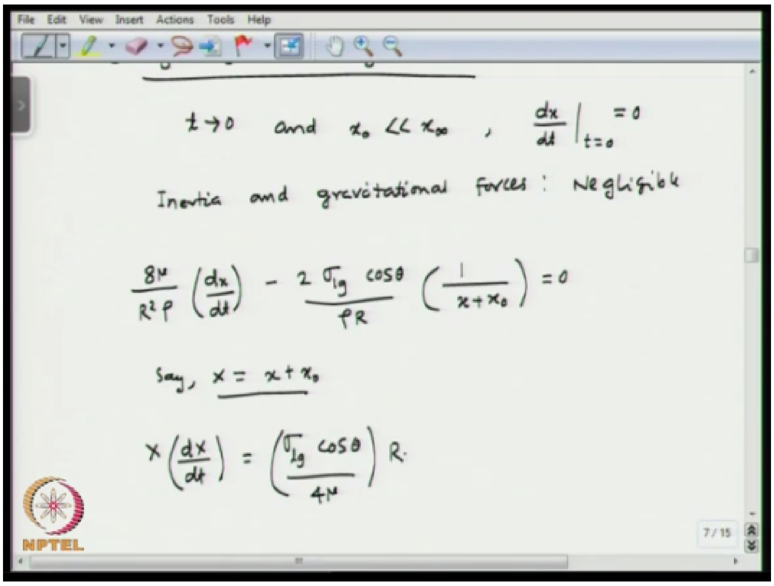

The second process is t tends to infinity and x infinity tends to L, okay. So first let us talk about the beginning of the filling process, so beginning of the filling process t tends to 0 and x0 << x infinity and also dx over dt at t = 0 is going to be 0. So in this case, the inertia and the gravitational force going to be negligible, okay. Since we are talking about the beginning of the filling process, the fluid had just started to move.

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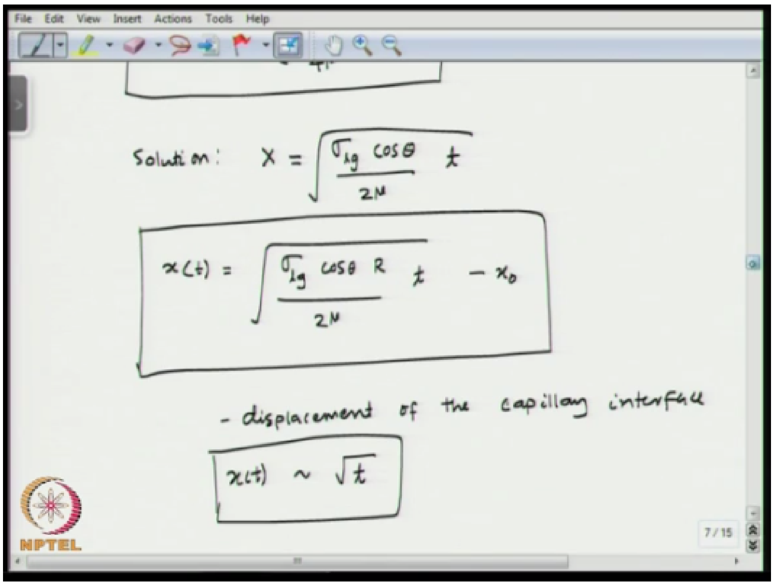

So, the inertia and the gravitational force are going to be negligible, so this is the general equation we work on and for the beginning process, we drop the inertia and the gravitational force from this equation and if we do that we get this simple equation, we get 8 mu over R square rho*dx over dt - 2 sigma lg cosine theta over rho R*1 over x + x0 is going to be 0. So, we can do a conversion of a variable, we can say X is x + x0, okay.

So, if we say that then we can write X dx over dt will be sigma lg*cosine theta divided by 4 mu*R, okay. So this is the equation we get which is easy to solve, so we can get the solution as this. The solution would be X is going to be sigma lg*cosine theta divided by 2 mu*t, okay or we can convert back to the original variable, in that case we can say xt is going to be sigma lg*cosine theta R divided by 2 mu - x0, okay.

So, what we see here is at the beginning of the filling process, the displacement is going to vary as square root of t, okay. So, the displacement of the capillary interface varies as square root of t, right.

So next, we will look at what happens at the end of the filling process. So at the end of the filling process, since the liquid column is going to slow down and the driving force which is the surface tension force is going to die down, the inertia and the surface tension forces are going to be negligible. So in the general equation, we drop the inertia and the surface tension force from the equation.

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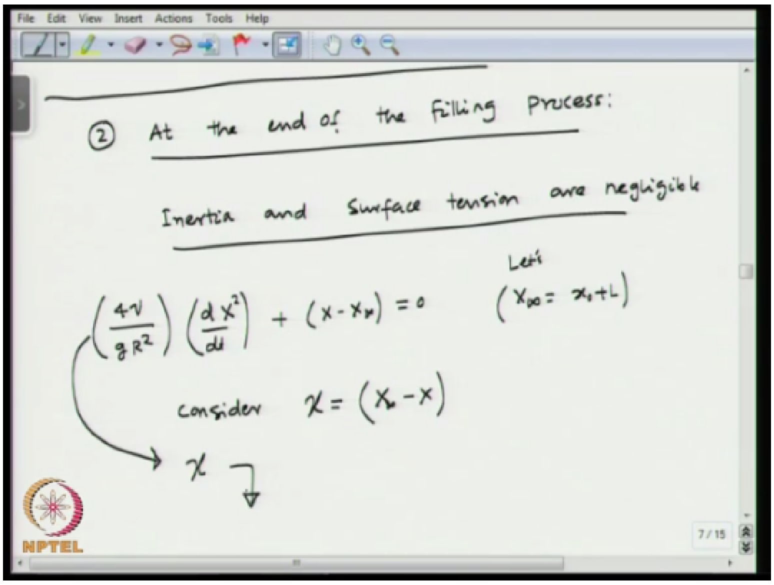

So at the end of the filling process, the inertia and the surface tension are negligible, okay. So in that case, we can get 4 nu over g R square*d x square over dt + x - x infinity is going to be 0 and here, we define x infinity as x0 + L, okay. So, now if we consider a parameter x as x infinity - x and in that case, we can convert this into an equation in terms of the new variable and try to solve that. We will continue with this in the subsequent lecture.