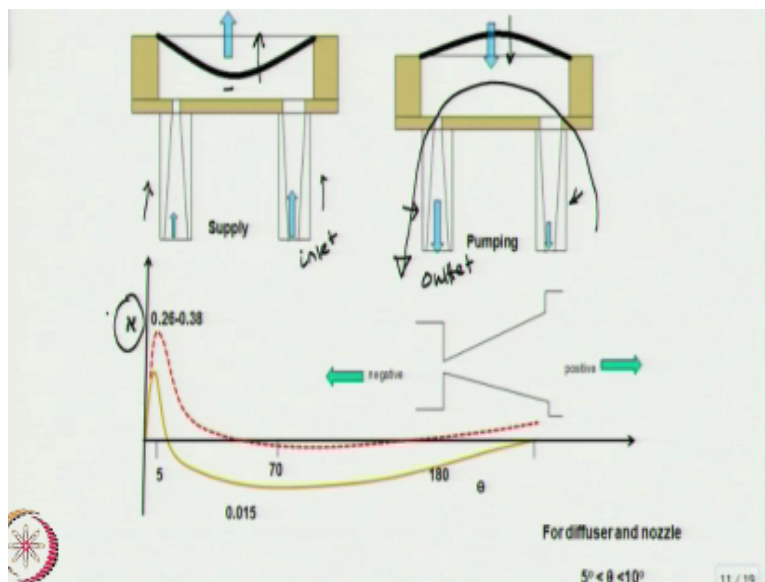


**Microfluidics**  
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**Lecture - 28**  
**Microfabrication Techniques (Continued...)**

Let us continue our discussion on micro pump so we start talking about valve less micro pump ok valve less micro pump will not use any valves. They will require nozzle diffuser arrangements to selectively you know transport fluid from the inlet to outlet by utilizing the difference between the pressure drop that happens across the nozzle to that across a diffusion.

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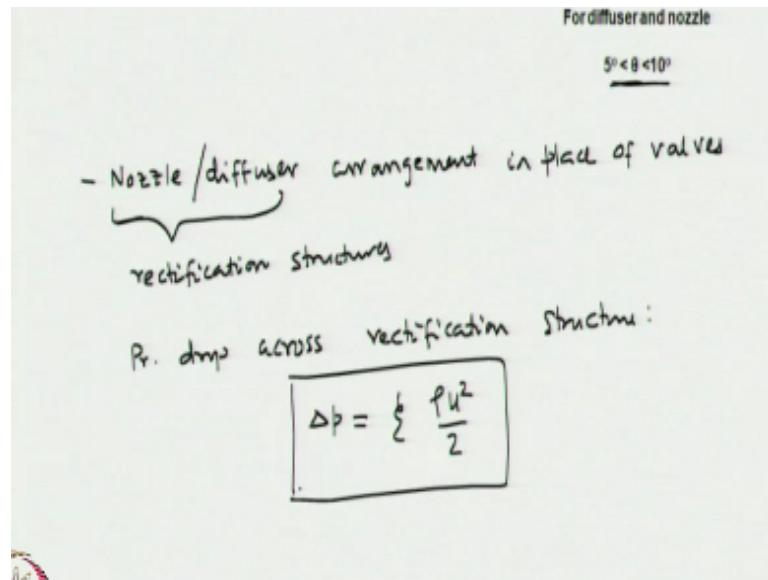
So this is a configuration of a valve less micro pump that you see here and what happens is when the membrane is trying to move up as you can see here then it creates a negative pressure inside the chamber and so during that time. So, this is the inlet it okay so this is the inlet and that is the outlet so when the negative pressure is created here the liquids move from the inlet into the chamber also some liquid move from the outlet into the chamber.

However, at the inlet we have a diffuser arrangement and as we know that and at the outlet we have a nozzle arrangement and we know that the pressure drop across the diffuser is less as compared to that across a nozzle. So, far that more fluid goes from the inlet to chamber as compared to what goes from the outlet into the chamber, so we have more fluid coming from the

inlet to the chamber.

So, in the downward stroke when the membrane goes down this act as a nozzle and this acts as a diffuser, so more fluid will go to the outlet then that will go to the inlet. So, we have essentially net fluid going from the inlet to the outlet. So, typically you know this is known as rectification efficiency which we will discuss and this is found to be maximum for a nozzle diffuser angle which is about 5 to 10 degrees okay.

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So, here we use nozzle diffuser arrangement in valve less micro pump we use nozzle diffuser arrangement and instead of in place of valves okay. So, the pressure loss across nozzle diffuser that is called rectification structure. So, these are called rectification structures okay so that means the flow has a directionality depending on the pressure drop nozzle diffuser okay. So, the pressure drop across the rectification structure.

Pressure drop across the rectification structure  $\Delta p$  is given by  $\xi \cdot \rho u^2 / 2$ . So,  $u$  is the flow velocity  $\rho$  is the density and  $\xi$  is the pressure loss coefficient and this pressure loss coefficient is going to be different for the nozzle and the diffuser.

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Fluid diodicity: Ratio between pr. loss coefficient at nozzle to that at diffuser

$$\eta_F = \left( \frac{\xi_-}{\xi_+} \right) \quad \eta_F = \underline{\underline{(1-5)}}$$

So, we can define the parameters which is called the fluid diodicity which is the ratio between pressure loss coefficients at nozzle to that at diffuser okay. So, we defined fluid diodicity we defined where the parameter eta f which is given by xi- pressure less coefficient for the nozzle/pressure loss coefficient for the diffuser okay. So, eta f has a value typically between 1 to 5 okay.

So, definitely the pressure loss coefficient for nozzle is more so that results in larger pressure drop across the nozzle okay.

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$$\eta_F = \left( \frac{\xi_-}{\xi_+} \right) \quad \eta_F = \underline{\underline{(1-5)}}$$

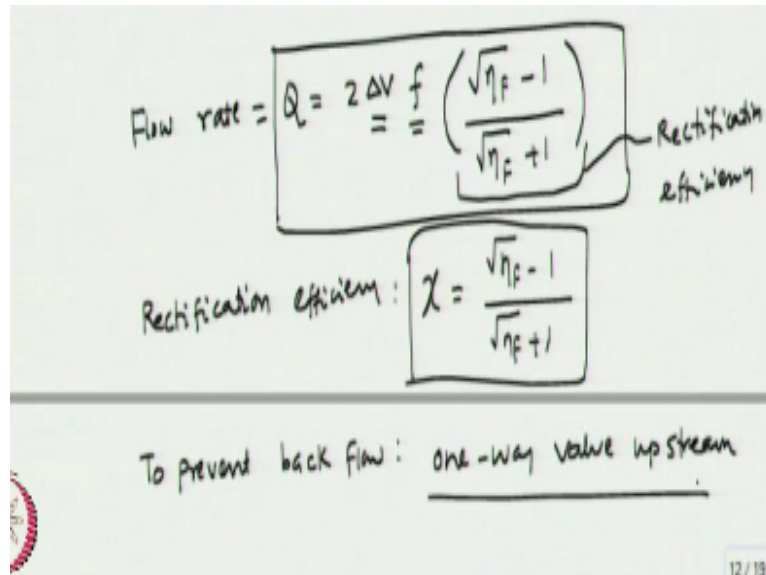
$\xi_-$  = pr. loss co-eff. across the nozzle  
(-ve flow direction)

$\xi_+$  = pr. loss co-eff. across the diffuser  
(+ve flow direction)

So, xi- is the pressure loss coefficient pressure loss coefficient across the nozzle so that is the

negative flow direction okay  $\eta_f$  is the pressure loss coefficient across the diffuser which is the positive flow direction.

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Flow rate =  $Q = 2 \Delta V f \left( \frac{\sqrt{\eta_f} - 1}{\sqrt{\eta_f} + 1} \right)$  Rectification efficiency

Rectification efficiency:  $\chi = \frac{\sqrt{\eta_f} - 1}{\sqrt{\eta_f} + 1}$

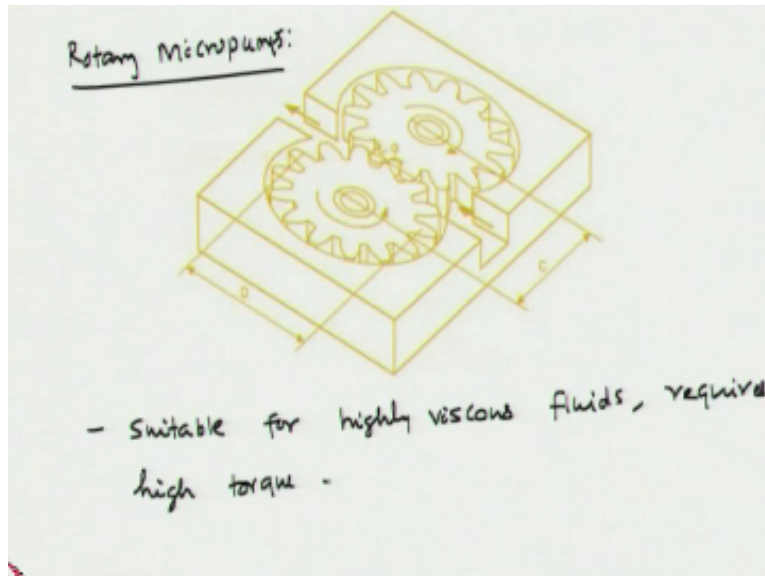
To prevent back flow: one-way valve upstream

So, we can write an expression for the flow rate can be written as  $Q$  is  $2 \cdot \Delta V \cdot f$   $f$  is the frequency of the oscillation of the membrane  $\cdot \frac{\sqrt{\eta_f} - 1}{\sqrt{\eta_f} + 1}$ ,  $\eta_f$  is the fluidic diodicity So, that is how we can express the flow rate given by a valve less micro pump in terms of the stroke volume and the frequency and the diodicity. Now this parameter here is known as the rectification efficiency okay.

So, for a pair of nozzle and diffuser the rectification efficiency is higher meaning the difference between the pressure drop that happens for a nozzle and diffuser is more then we will result in a larger flow rate okay. So, the rectification efficiency is square root of  $\eta_f - 1$  / square root of  $\eta_f + 1$ , Now in case of valve less micro pump when it is not in operation since were not using any valve there maybe possibility of backflow okay.

So, to prevent backflow we use one-way valve upstream so with that let us move on and talk about rotary micro pumps.

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So, we talk about rotary micro pumps so rotary micro pumps have 2 gears which are driven in opposite to each other to drive very viscous fluids okay. So, rotary micro pumps are used for driving viscous fluids and so it requires high torque is necessary. Okay the rotary micro pumps are suitable for highly viscous fluid and so it requires high torque so we need external actuator and the flow rate of such rotary pumps have been estimated.

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- Flow rate:

$$Q = h \pi n \left( \frac{D^2}{2} - \frac{c^2}{2} - \frac{m_g^2 \pi^2}{6} \cos^2 \phi \right)$$

$c$  = dist. between the centres of two gears  
 $h$  = channel height  
 $n$  = RPM  
 $m_g = \left( \frac{D}{N} \right)$  No. of teeth  
 $\phi$  = Pr. angle

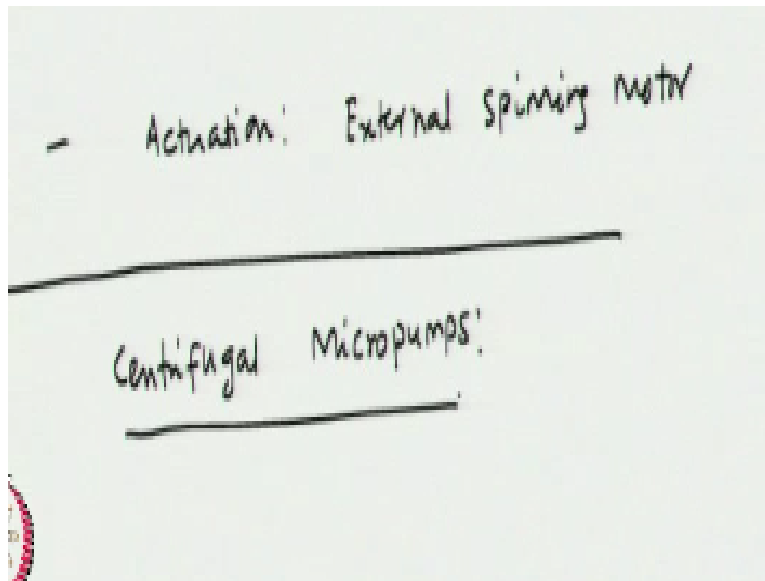
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So, flow rate has been estimated we can express in terms of a formula  $h \pi n \frac{D^2}{2} - \frac{c^2}{2} - \frac{m_g^2 \pi^2}{6} \cos^2 \phi$ . So, this is the expression for the flow rate here you can see  $D$  is the diameter of the gear okay and so this is the gear dia and  $c$  is the distance between the centers of the 2 gears. So,  $c$  is the distance between the centers of 2 gears and  $h$  is the channel

height  $h$  is the channel height and  $n$  is the RPM okay.

So,  $m_g$  is the diameter of the gear/number of teeth and  $N$  is the number of teeth and  $\phi$  is the pressure angle that is defined according to the geometry the profile of the teeth pressure angle. So, knowing these parameters which are defined for a set of gears and if you know the channel height and the RPM we can calculate the flow rate of delivered by a rotary micro pump.

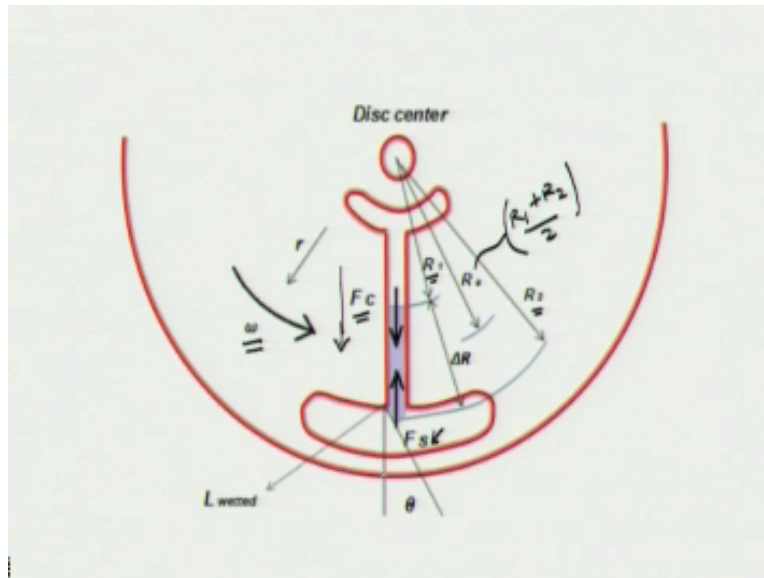
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Now because we are talking about viscous fluid so the actuation is due to the external spinning motor okay so that is about rotary micro pump next we talk about centrifugal micro pumps. So, the centrifugal micro pumps work on the principle of the centrifugal force. Okay so normally the channels are carved and they typically CD configuration and by using the centrifugal force this CD platform is rotated.

And when it rotates because of the centrifugal force the liquid moved from the center of the CD to the outside perimeter of the CD okay so.

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So this is the configuration of a typical centrifugal micro pump so this is the disc center about which the disc is rotated at under velocity of omega and this is the liquid column that we are trying to analyze here. So, this liquid column the bottom end is at a radius of  $R_1$  the leading edge is at a radius of  $R_2$  and the average radius  $R_1 + R_2 / 2$  is  $R_A$ , so this is  $R_1 + R_2 / 2$ . So for this liquid column.

If this is rotated there are two forces that are acting 1 is the centrifugal force which acts outward and the second one is the centrifugal surface tension force which is trying to prevent this liquid column to escape ok so the flow is occurring in this direction from the center to the perimeter and the surface tension is trying to prevent escape of this liquid column okay. So, we can write down the expression for the centrifugal force and surface tension force.

And equalize to determine the speed of rotation or the frequency of the rotation of the CD okay.

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- Pump structure is on a CD platform,  
flow rate  $\rightarrow$  angular vel. of disc

$\left( \frac{dF_c}{dr} \right) = \left( \rho \omega^2 r \right) A$

$\omega = \text{Ang. Vel.}$   
 $A = \text{Cross-sectional area of channel}$

So, here in this case the pump structure is on a CD platform and the flow rate is controlled by the angular velocity of the CD angular velocity of the disc. Here we write down the expression for the centrifugal force we write expression for  $dF_c/dr$  the centrifugal force is changing with the radius so that would be  $\rho \omega^2 r \times$  the area of cross section of the channel so this is basically the centrifugal force per radius so  $F_c$  is  $m v^2/r$  okay.

So, then we have  $dr$  here so that would take  $\rho \omega^2 r \times$  area of the cross section of the channel okay. So, here  $\omega$  is the angular velocity and  $A$  is the cross sectional area of the channel okay.

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Surface tension force:

$$F_s = (\sigma \cos \theta) L_{\text{wetted}}$$

Liquid bursts out :  $F_c > F_s$

$\rho \omega^2 r A \Delta R = \sigma \cos \theta L_{\text{wetted}}$

$\Rightarrow \rho (2\pi f)^2 R_a \times \left( \frac{\pi D_h^2}{4} \right) \Delta R = \sigma \cos \theta (\pi D_h)$



Similarly, we can write down the expression for surface tension force  $F_s$ . So, the surface tension force would depend on the surface tension coefficient\*the weighted perimeter okay and that force directed towards the inside towards the center okay. So, typically if you have the profile something like this and so this is the direction of surface tension. So, we are interested in this component so this is going to be  $\rho \cos \theta$  where  $\theta$  is the contact angle okay.

So, this we have already seen so this would be  $\rho \cos \theta$  and that  $\rho \cos \theta$  is going to act on this weighted perimeter. Okay so the weighted perimeter is going to be  $\pi d_h$ , so let us now we call it  $L$  weighted okay. Now if you balance surface tension with centrifugal force the liquid is going to exit only when the centrifugal force is going to be more than the surface tension force okay.

So, the liquid bursts out when the centrifugal force is there to that and so distance and force so for that to happen we can write the expression for the centrifugal force from here. So,  $\Delta r$  will get multiplied there so we call it  $\Delta r$  so  $\Delta r$  is nothing but the length of the liquid column okay. So, we have  $\rho \omega^2 A \Delta R = \rho \cos \theta * L$  weighted okay. So, now we substitute the value  $\rho \omega^2 \pi F^2$  so we have  $2 \pi F^2 * r$  is  $Ra$ .

This  $R$  is the location of the droplets. So, we call it average radius and area of perception of the channel is  $\pi D_h^2 / 4 * \Delta R = L \cos \theta$  sorry  $\rho \cos \theta * \text{the weighted perimeter}$ , the weighted perimeter is  $\pi D_h$ ,

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$$\Rightarrow f_b = \frac{\sigma \cos \theta}{\pi^2 \rho R_a \Delta R D_h}$$

$w = \text{Ang. Vel.}$   
 $\sigma = \text{Surface tension}$   
 $L = \text{Wetted perimeter}$   
 $R_a = \left( \frac{R_1 + R_2}{2} \right)$

$r = \text{radial coordinate}$   
 $\theta = \text{Contact angle}$   
 $D_h = \text{Hydraulic dia}$

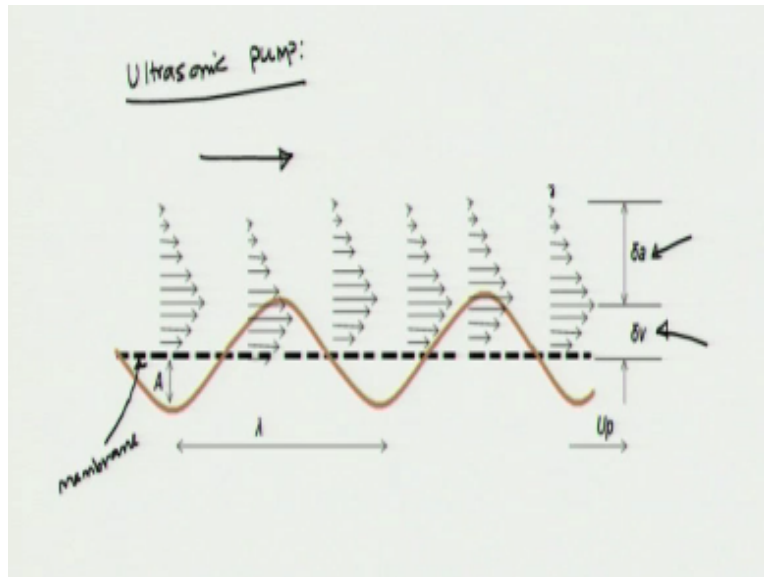
$D_h$  is the hydraulic diameter of the channel okay say  $D_h$  is the hydraulic diameter of the channel then from here we can predict what is going to be the burst frequency ok  $f_b$  is going to be  $\frac{\sigma \cos \theta}{\pi^2 \rho R_a \Delta R D_h}$ . So, the burst frequency depends on the surface tension of the liquid channel interface and it depends on the contact angle it is inversely proportional to the density.

And the length of the liquid column sorry the average radius of the liquid column and radius the length of the liquid column and parallel diameter okay. So, here we have  $w$  is the angular velocity  $\sigma$  is the surface tension and  $L$  is the weighted perimeter  $R_a = \frac{R_1 + R_2}{2}$  and  $r$  is the radial coordinate  $\theta$  is the contact angle and  $d_h$  is the hydraulic dia okay these are the different parameters.

So, with that let us discuss ultrasonic micro pump ultrasonic micro pumps are the mechanical type micro pump but they belong to the dynamic type micro pumps. Okay so in case of ultrasonic micro pump we do not have any moving part. So, what we do is we have a thin membrane and we have transducers on thin membrane coated with P materials and these PZT materials are excited okay using the transducer.

So, they generate what is called acoustic waves ok this is called flexural plates acoustic waves because of the movement of the acoustic waves we generated because of the vibration of the

transducer. We generate a bulk motion of the fluid that is present on the surface of the membrane.  
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So, this is the ultrasonic pump okay so here we have a this is the membrane we are talking about this is the thin membrane coated with the electrodes okay and when you know the electrodes are excited the membrane will undergo oscillation. Okay so these mechanical waves will be traveling on the surface of the membrane okay. So, when the mechanical waves will travel on the surface of the membrane.

Due to this the fluid that is present on the surface of the membrane will start to move in this certain direction okay. So, this is the velocity profile that you see for the fluid that is present on the membrane. So, it will have non 0 velocity on the surface of the membrane because the membrane is always in a dynamic state and so there are 2 parameters to define here 1 is the acoustic evanescent length.

And the other one is a viscous evanescent length the viscous evanescent length is the distance in the y direction from the surface of the membrane to a point where the velocity is maximum ok and acoustic evanescent length is the distance between the point where the velocity is maximum to the point where the velocity in the bulk reduced to 0. Okay so there are two important but important parameters here.

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- Enables pumping without moving parts, pumping is caused by 'acoustic Streaming'  
→ induced by mechanical travelling waves
- mechanical wave excited using interdigitated transducers placed on thin membrane

So, we have you know this ultrasonic pump it enables pumping without moving parts and the pumping is caused by what is called acoustic streaming okay and this is induced by the mechanical traveling waves induced by mechanical travelling with is similar to that of a string okay. So, the mechanical wave these mechanical waves are excited using interdigitated electrodes transducers placed on thin membrane.

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- Velocity profile:  $\delta_a$ ,  $\delta_v$

Acoustic evanescent decay length:

$$\delta_a = \frac{\lambda}{2\pi \sqrt{1 - \frac{u_p}{u_b}}}$$

$\lambda$  = wave length.

$u_p, u_b$  = phase velocity, sound speed

Okay so the velocity profile is characterized by 2 parameters the velocity profile characterized by the acoustic evanescent length and the viscous evanescent length. The acoustic evanescent length acoustic evanescent dk length which is  $\delta_a$  is given by the expression  $\lambda / 2\pi \sqrt{1 - u_p/u_b}$ , so here  $\lambda$  is the wave length okay. So, when you have this flexural wave this is

the wave length okay  $\lambda$  is the wave length and  $u_p$  and  $u_s$  are the phase velocity and the sound speed okay right so  $u_p$  is the velocity of the phase at what speed this phase is travelling okay.

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viscous evanescent decay length:

$$\delta_v = \sqrt{\frac{\nu}{\pi f}}$$

kinematic viscosity  
frequency

$$u_a = \frac{5}{4} \frac{(2\pi f A)^2}{u_p} (K \delta_a)^3 \left[ \frac{1}{K \delta_a} + K \delta_v \right] \left[ 1 - \frac{t}{2 \delta_a} \right]^2$$

And then you have the viscous evanescent decay length viscous evanescent decay length which is given by the expression  $\nu/\pi \cdot f$  square root okay so here  $\nu$  is the kinematic viscosity and the kinematic viscosity and  $f$  is the frequency of the acoustic wave okay. Now the acoustic streaming velocity  $u_a$  can be determined this  $u_a$  can be determined as so this is the velocity of the liquid okay.

So the acoustic streaming velocity is  $\frac{5}{4} \cdot 2 \pi f A^2 / u_p \cdot K \delta_a^3 \cdot [1/K \delta_a + K \delta_v] \cdot [1 - t/2 \delta_a]^2$  okay. So, that is the expression for the acoustic streaming velocity that we use by the flexural waves.

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$$A = \text{Wave amplitude}$$

$$K = \left( \frac{2\pi}{\lambda} \right) = \text{Wave no.}$$

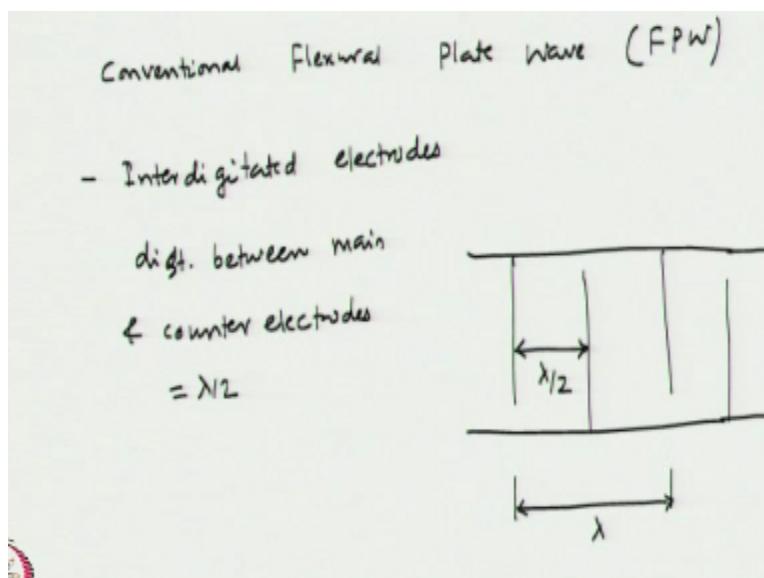
$$t = \text{plate thickness}$$

$$- \text{Acoustic streaming vel} \sim A^2$$

Okay here  $A$  is the wave amplitude so we are talking about this amplitude here okay and  $t$  is the plate thickness. So, the thickness of the flexural membrane and  $K$  is known as wave number which is  $2\pi/\text{the wave length}$  okay so this is called the wave number okay. So, what we see here is that the acoustic velocity  $u_a$  is varying as square of the amplitude okay the wave amplitude okay.

So, what we observe here is acoustic streaming velocity varies as the square of the amplitude. And this amplitude would depend on the configuration of the electrodes so depending on the geometry of the interdigitated pairs of electrodes the amplitude will vary.

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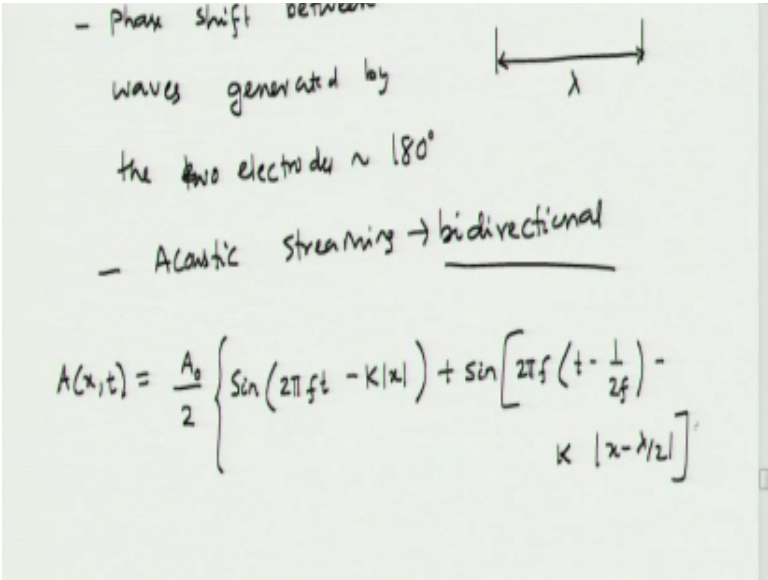


Normally we use okay in a conventional flexural plate wave is FPW flexural plate where we have interdigitated electrodes okay. So, on the surface of this membrane we would have interdigitated electrodes that will cause the acoustic waves and let us say in a configuration we have the distance between we have a configuration something like this okay so let us say this is the separation distance between the main and counter lambda and between 2 electrodes lambda/2.

Okay so if that is the case we have distance between the main and counter electrodes is lambda/2 in that case there will be a phase shift of 180 degree of the waves generated by these 2 electrode pairs okay. So, there will be a phase shift between waves generated by the 2 electrodes will reach 180 degrees and the acoustic streaming is going to be bi directional that means that the acoustic streaming is going to happen in this way as a less in that way.

Acoustic streaming is going to Bi-Directional so the waves are going to go in both the directions okay.

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Handwritten notes on a slide:

- Phase shift between waves generated by the two electrodes  $\sim 180^\circ$
- Acoustic streaming  $\rightarrow$  bidirectional

A diagram shows a horizontal line with arrows at both ends pointing outwards, labeled with the Greek letter  $\lambda$  below it.

$$A(x,t) = \frac{A_0}{2} \left\{ \sin(2\pi f t - K|x|) + \sin\left[2\pi f \left(t - \frac{1}{2f}\right) - K\left|x - \frac{\lambda}{2}\right|\right] \right\}$$

And the resulting wave amplitude is given by this expression so the Axt will be given by  $A_0/2$  \*  $A_0$  is the maximum amplitude so it is  $\sin 2\pi f t - Kx + \sin 2\pi f t - 1/2f - K * x - \lambda/2$  okay. So, that is the expression for the amplitude which will be using in this equation to calculate the acoustic velocity okay. In a case where this gap is lambda/4 instead of lambda/2.

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→ Electrode spacing  $\sim \lambda/4$   
 → Phase shift  $\rightarrow 90^\circ$   
 → Acoustic streaming  $\rightarrow$  unidirectional

$$A(x,t) = \frac{A_0}{2} \left\{ \sin(2\pi f t - K|x|) + \sin\left[2\pi f \left(t - \frac{1}{4f}\right) - K\left|x - \lambda/4\right|\right] \right\}$$

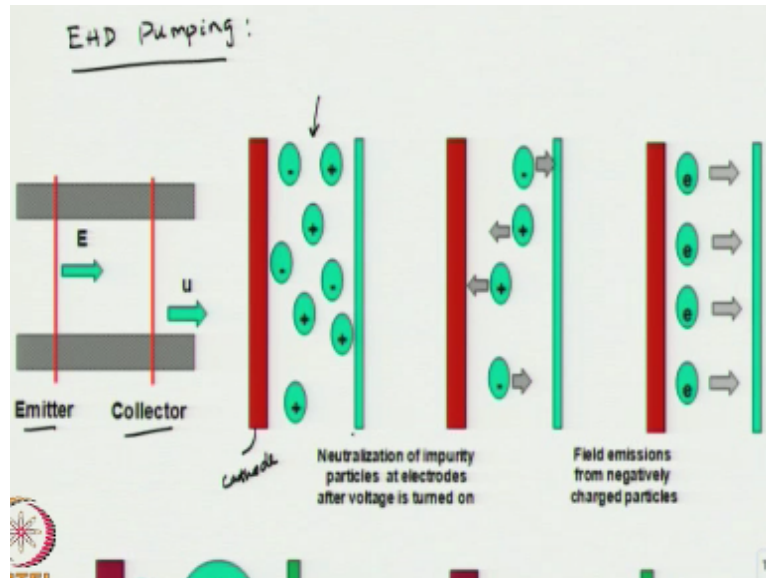
And let us say the electrode spacing is  $\lambda/4$ . In that case the phase shift is going to be 90 degrees and in that case the acoustic streaming will be unidirectional. Acoustic streaming will be unidirectional it is going to be only in one direction and we will have the expression for  $A(x,t)$  okay which is  $A_0/2 * \sin 2\pi f t - Kx + \sin 2\pi f t - 1/4f - Kx - \lambda/4$ . So, this will be the expression for the amplitude if the electrode spacing will be  $\lambda/4$ .

So, here in this case in place of  $\lambda/2$  will be  $\lambda/4$  in place of  $2f$  it is going to be  $4f$  okay. So, with that let us move on and talk about electro hydro dynamic pumps okay so in electro hydro dynamic pumps it is a non-mechanical pump and we use a very high electric field to emit electrons from the emitter towards the collector ok and that ionizes the ion molecules present in the liquid.

And since electrons are negative charged the molecules again and again of negative charge and they migrate towards the collector electrode. So, in doing so they carry the bulk of the fluid creating a pumping action okay.

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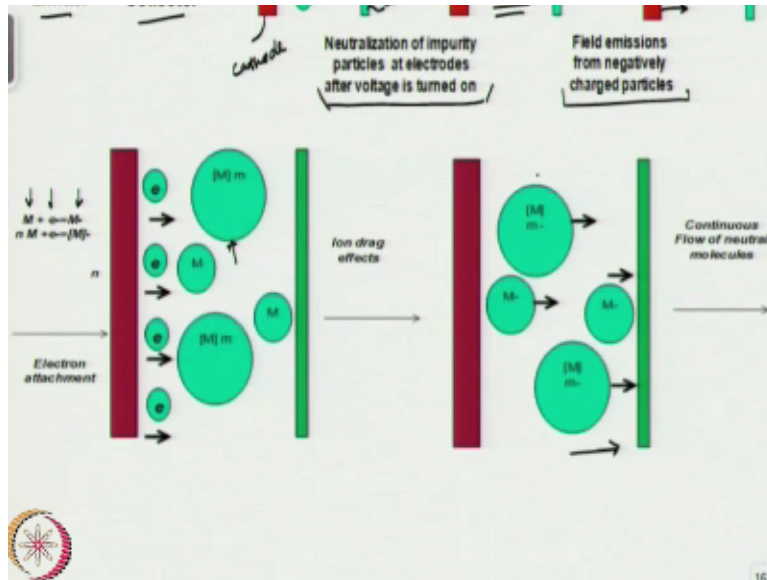


So, we look at EHD electro hydro dynamic pumping which is different than electro kinetic pumping normally in electric kinetic pumping we use relatively low voltage as compared to what to use in electro hydro dynamic pumping okay. So this is the typical configuration of an electro hydro dynamic pump. So, there will be pairs of emitters and collectors so initially the liquid that is present between the emitter and collector may have some impurities okay.

So. Let us say there are some input it is present here in the liquid meaning there are some already in-built charge inside the liquid. Now if you apply a field between the emitter and collector the positive ions will migrate towards the negative. And you know this is the cathode or emitter so this is the cathode and this is the anode okay this is  $+ - p$ , so the positive ions will migrate towards the cathode and negative charges will migrate towards anode.

And they will get electrically neutralized. Okay so this happens when the voltage is turned on and the voltage is relatively in a low then charge neutralization in the impurity present in the liquid will happen. Then if the voltage is increased further to a high value then what will happen is this emitter is going to emit electrons okay so electrons are going to be emitted from the surface of the emitter.

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And these electrons are going to interact with the molecules of the liquid okay and make it negatively charged you can see here the electron is interacting with this molecule to make it negatively charged and so now these molecules become negative charged and so they will migrate towards anode so they migrate towards anode in doing so they carry the bulk of the liquid with them okay.

Because of the drag effect and this is called the ion drag effect so the liquid movement or the pumping action is happening because of the drag that offered by the ions present in the liquid okay.

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- EHD micropump  $\rightarrow$  Ion-drag micropump

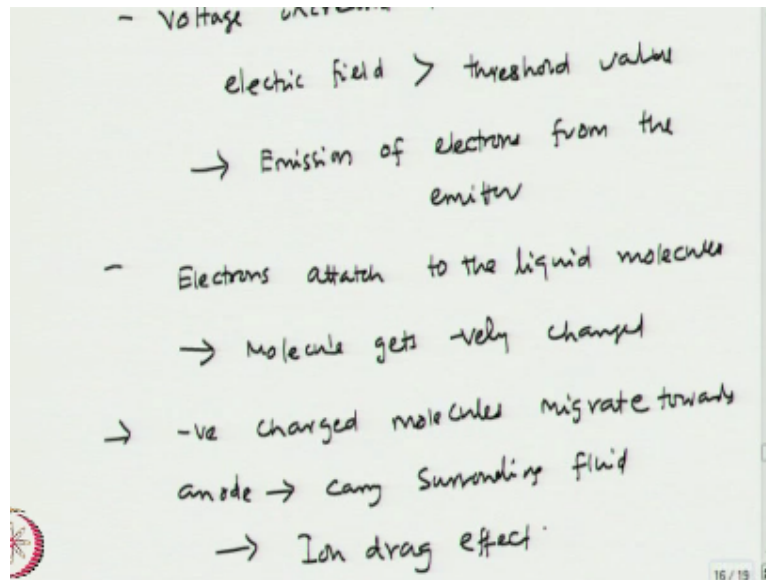
- Pumping between emitter & collector

- Voltage turned on  $\rightarrow$  impurities migrate towards opposite electrodes  $\rightarrow$  process stops after the impurity ions get neutralized

So, here in this case we also call the EHD micro pump we call it as ion drag micro pump here we have a pumping between emitter and collector pair of electrodes and initially there may be some impurity present when voltage is turned on the impurities migrate towards opposite electrodes ok and this is going to be a short process and that they stop as soon as the charge just neutralized okay.

So, this process stops the impurity items impurity ions get neutralized.

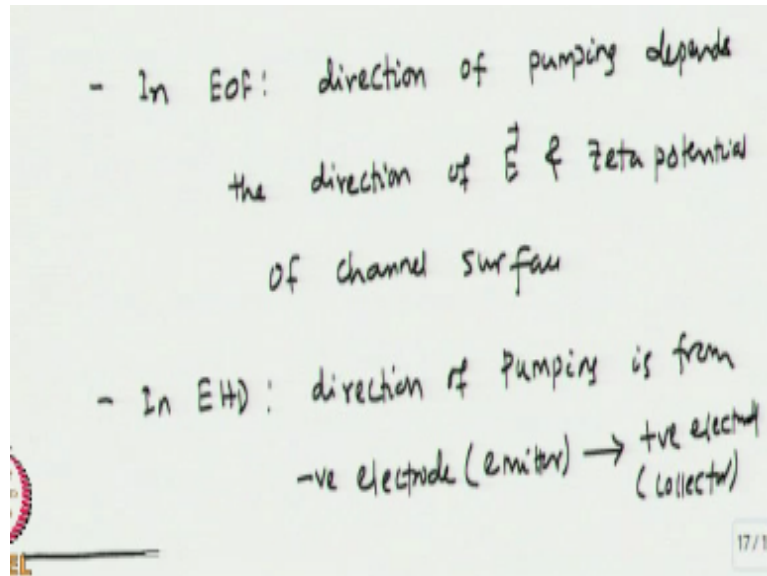
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And if you have voltage increased further then and the electric field  $E$  is greater than the threshold value then you have emission of electrons from the emitter okay and these electrons would attach to the molecules attach to the liquid molecules. And so the molecules get a negative charge okay and these negative charge molecules migrate towards anode. So, they carry surrounding fluid which is called ion drag micro pump.

As you can see here you know the electro hydraulic micro pump or ion drag micro pump the liquid moves from negative electrode towards positive electrode okay whereas in case of electro osmosis it depends on the moment the direction of the pumping would depend on the zeta potential of the surface okay.

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So, we can say here in electroosmotic flow the EOF flow the direction of pumping depends on the direction of the electric field. And the zeta potential of channel surface whereas in HD pumping the direction of pumping is from the negative electrode is also known as the emitter towards the positive electrode and collector okay. So, that discussion we will move on and talk about electro kinetic micro pump.

We have already covered the electro and kinetic micro pump when we were talking about electro osmosis but here we will consider an example problem and tried to design the microfluidic circuit for electro a combination of electro a combination of electro square we talked about electrostatic pumping with capillary electrophoresis okay so will continue our discussion on in the next lecture so with that let us stop here.