Microfluidics
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Lecture - 20
Electrokinetics and Magnetophoresis

Okay we have been talking about dielectrophoresis where you know we manipulate particles inside microchannels that are dielectric in nature and we subject the particles to inhomogenous electric field. Let us continue our discussion on magnetophoresis. Now magnetophoresis is analogous to dielectrophoresis, but here we are talking about particles that are magnetizable okay.

So magnetophoresis is the manipulation of magnetizable particles in presence of inhomogeneous magnetic field okay.

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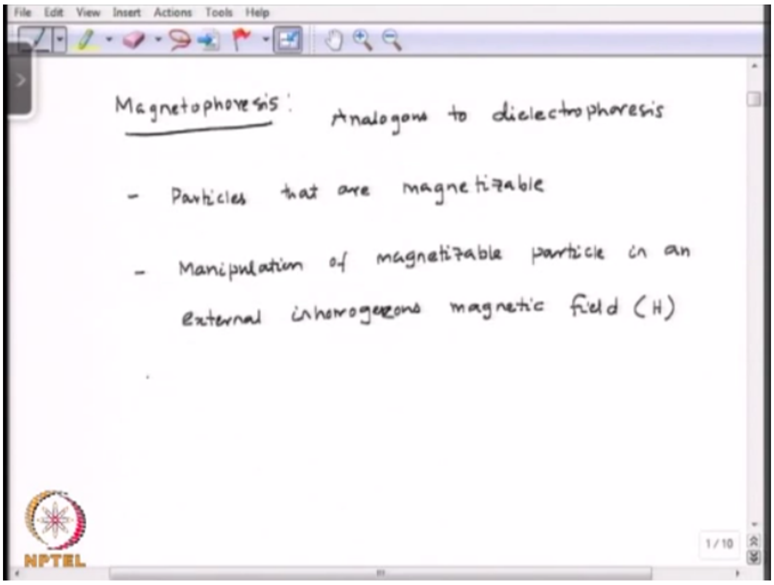

So magnetophoresis is analogous to dielectrophoresis and here we talk about particles that are magnetizable so magnetophoresis means manipulation of magnetizable particles in an external inhomogenous magnetic field and magnetic field is denoted by symbol H okay. Now the dielectric response of most of the biological materials are very strong okay, but the magnetic response of the biological materials are very weak okay.

So in order to achieve good you know magnetic or magnetophoresis in microchannels what we do is we tag or label these biological particles with the magnetic particles okay. So the magnetophoresis can be enhanced by tagging biological particles with magnetic particles so in that way we can selectively tag specific population of biological objects with certain types of magnetic particles and we can separate them using magnetophoresis.

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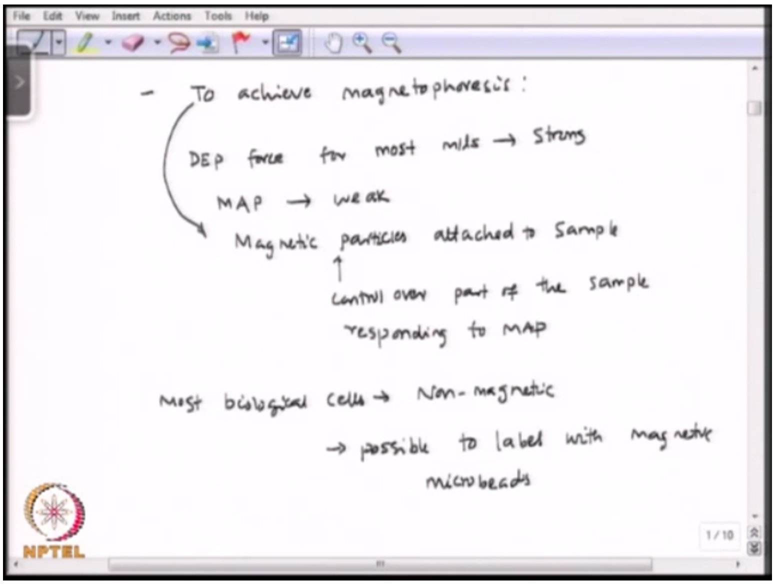

So you know in magnetophoresis to achieve magnetophoresis you know the DEP force for most materials it is strong, but the magnetophoretic force or we call it MAP magnetophoretic force is weak okay. So to achieve magnetophoresis what you do is magnetic particles attached to sample okay so in that way we can obtain control over you know part of the sample responding to MAP responding to magnetophoresis.

So as I said most biological cells are non-magnetic, so it is possible to label with magnetic microbeads okay.

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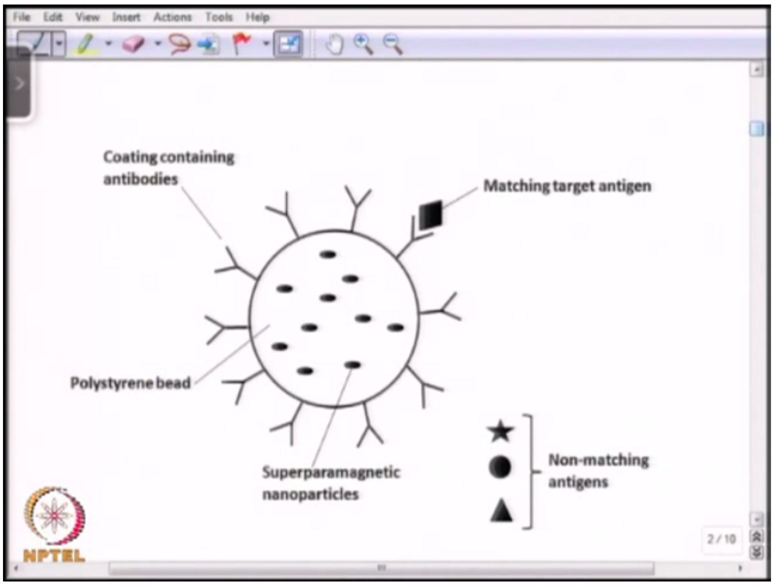

So you know here we see an example so this is the polystyrene bead okay and which is not magnetic and here this polystyrene bead is made magnetic by having these super paramagnetic nanoparticles inside okay. The super paramagnetic means you know these particles will go back to their non-magnetic nature when the magnetic field is taken out that is called superparamagnetic okay.

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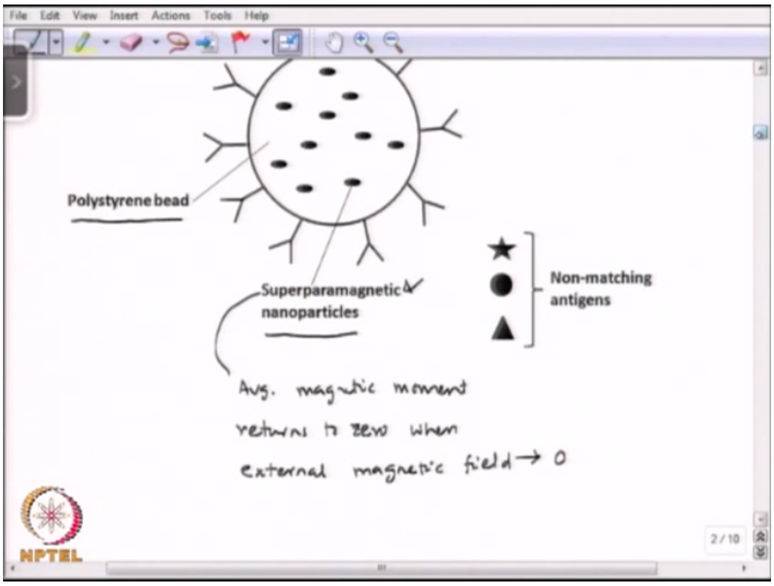

So by superparamagnetic we mean that the average magnetic moment returns to 0 when external magnetic field is removed okay. So when we take out when we have external magnetic field present these nanoparticles will behave as if they are magnetic and when we take out the magnetic field they will become non-magnetic okay so that is called superparamagnetic nanoparticles.

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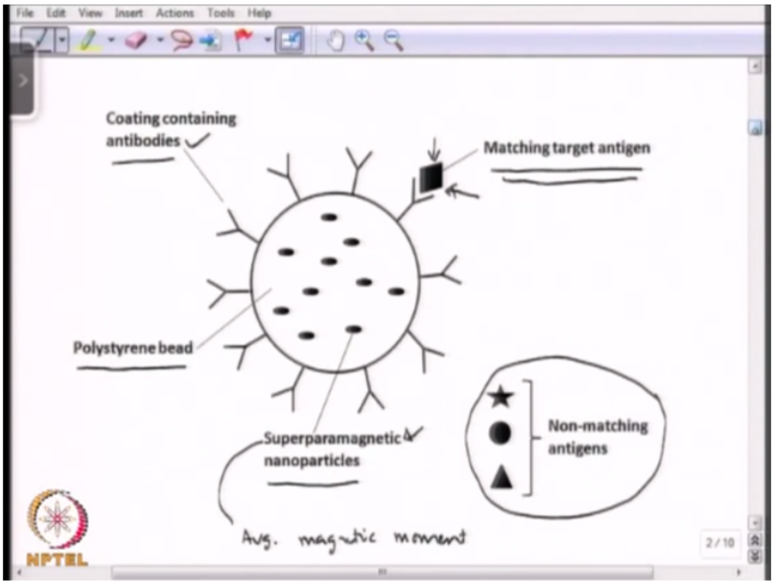

So they are present inside this polystyrene bead so the bead acts as a magnetic particle okay. On the bead, we have a coating of antibodies okay and you know in a sample we may have various different types of antigens and only the matching target antigen will bind to these antibodies okay so these are the antibodies coated onto this magnetic bead and only the matching target antigen will be bound to these antibodies.

And there will be several non-matching antigens that may also be present in the sample which will not bind to the antibodies. So this is one approach how you can you know extract different molecules from the sample using the magnetophoresis method okay so you know it is possible to do magnetophoresis in microfluidic channels where we can have you know these paramagnetic microbeads you know adhering to the wall because of an external magnet.

And you know once these magnetic microbeads with antibodies coated are immobilized onto the wall we can flow the sample and the target antigen can be bound specifically to that antibody, which is attached to the magnetic bead.

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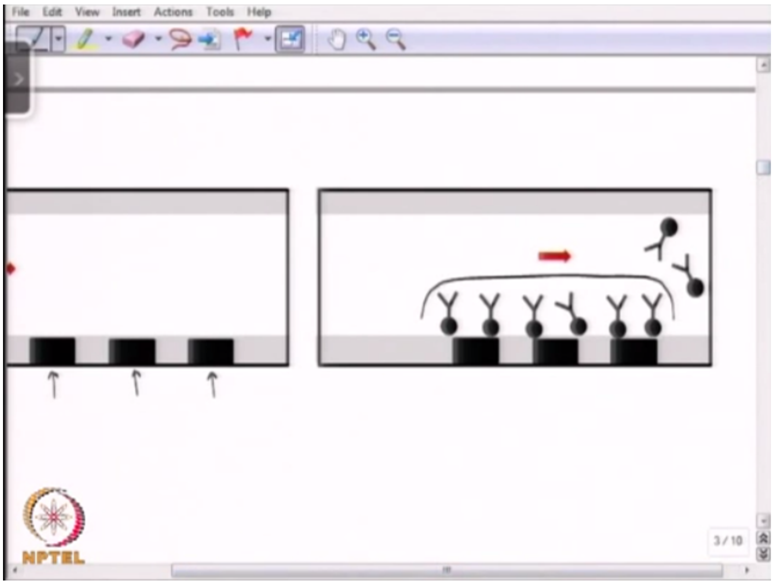

So this is 1 such case shown here. What we see here? This is a microchannel okay so this is the width of the microchannel and we have these magnets present these 3 black rectangles that we see is there you know magnets and we flow these magnetic bead attached to antibody okay what do we see here is this is the magnetic bead attached to the antibody, which is specific to a particular antigen through the microchannel.

And you know when you do so what we see here because of the influence of the external magnet here these beads get attached to the magnet okay. So they get attached to the channel wall here as you see here okay. So these beads are attached to the channel wall okay.

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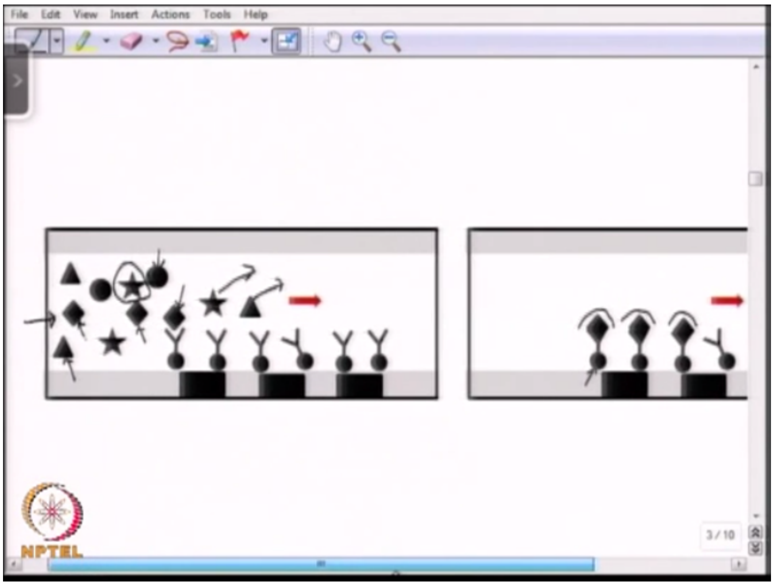

What we do next is you know as you see here we infuse the sample that is having different types of antigens okay and when we do that you know the target antigen, which is specific to the antibody binds okay so this is the target antigen okay and these target antigens bind to the antibodies okay whereas the other antigens okay this, this and this they are not target antigens so they just flow out allowing the channel okay.

So what we end up getting is you know this magnetic bead attached to the antibodies you know these antigens come and bind to the specific antibodies okay.

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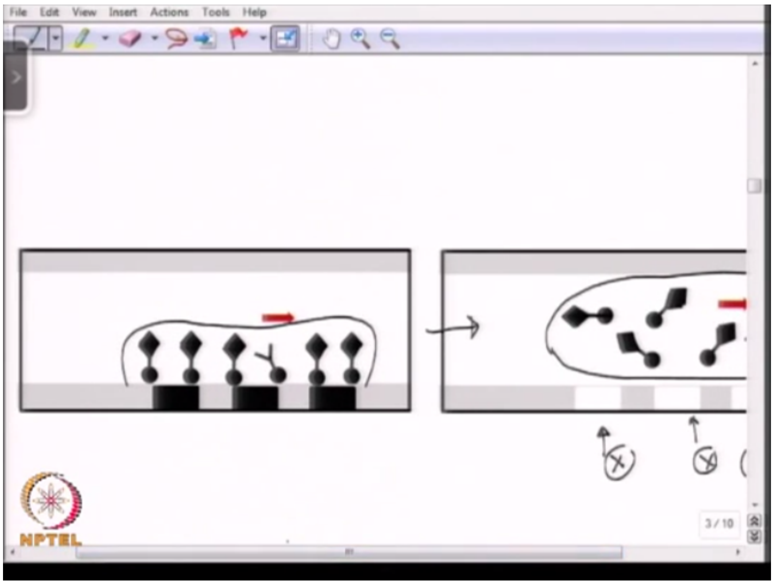

Now once we do that next what we do is you know anything that is not bound is removed okay so we have only magnetic bead with antibody and attached to the target antigen and then we may be interested in these antigen antibodies to do further biology so to do that we can take off the magnetic field so here the magnet is removed and when you do that and we subject this bead antigen to a flow, these antigens and the antibodies can be collected okay.

Because there is no magnetic field, here we have taken off the magnetic field, these antibodies, antigens and the magnetic bead can be released and they can be collected okay. So this is an example of the applications of magnetophoresis what you can use to you know capture antigens and how to release them to collect to process for the biology okay. So let us look at the basics of magnetostatics.

By magnetostatics we mean that the current that we are dealing with is not changing with time so it is invariant with time, so under that assumption will look at the magnetostatic limit of the Maxwell's equation okay.

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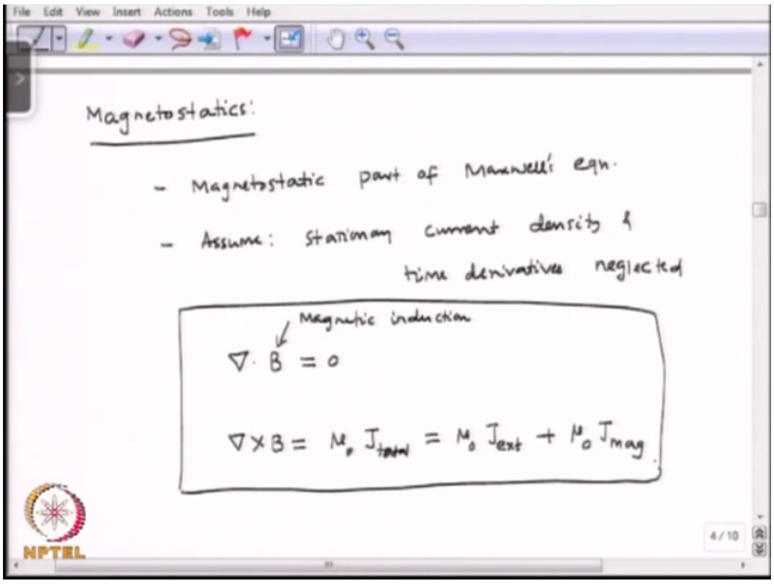

So let us look at magnetostatics okay so we basically look at the magnetostatic part of the Maxwell's equation okay and here we assume that the stationary current density and any time derivative are neglected okay. So under that assumption we can write the Maxwell's equation for magnetostatic in this way so we will have 2 important equations. The first one is the divergence of magnetic induction is 0 okay.

Where B is magnetic induction okay and the second equation will be del cross B will be mu0, which is the magnetic permeability of vacuum okay mu0*total current density, which will be=mu0 external current density+mu0 the current density bound to the magnetic field okay so we have these 2 equations and here we have B is the magnetic induction we already defined here.

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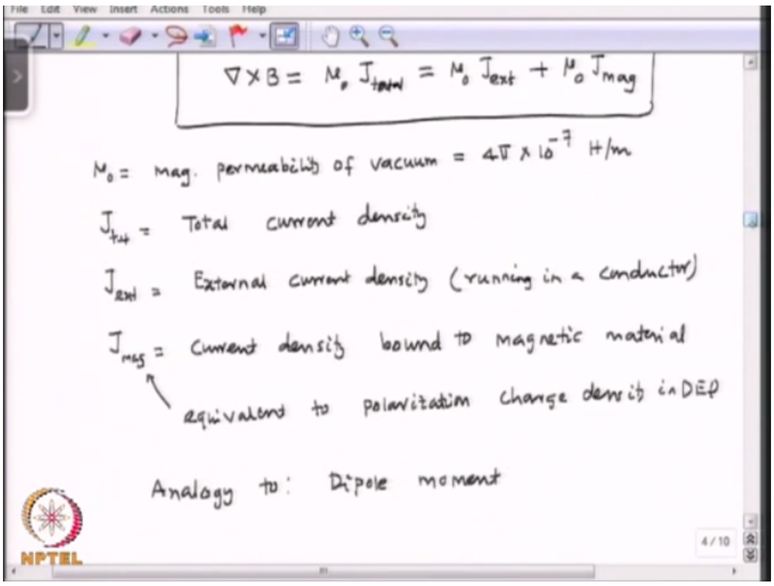

And mu0 is the magnetic permeability of vacuum=4 pi*10 to the power -7 Henry per meter. So this is magnetic permeability of vacuum mu0. J total is the total current density and J external is the external current density for example running in a conductor okay and the J magnetic is the current density bound to the magnetic material and this is equivalent to the polarization charge that we talked in DEP okay.

So this is equivalent to the polarization charge density in DEP okay. Now if you do an analogy to the dipole moment so we do analogy to dipole moment so the electrical dipole moment is charged into the separation distance between the 2 opposite charges, so we can write the electrical dipole moment p as q*d. Similarly, we can define something called magnetic dipole moment okay.

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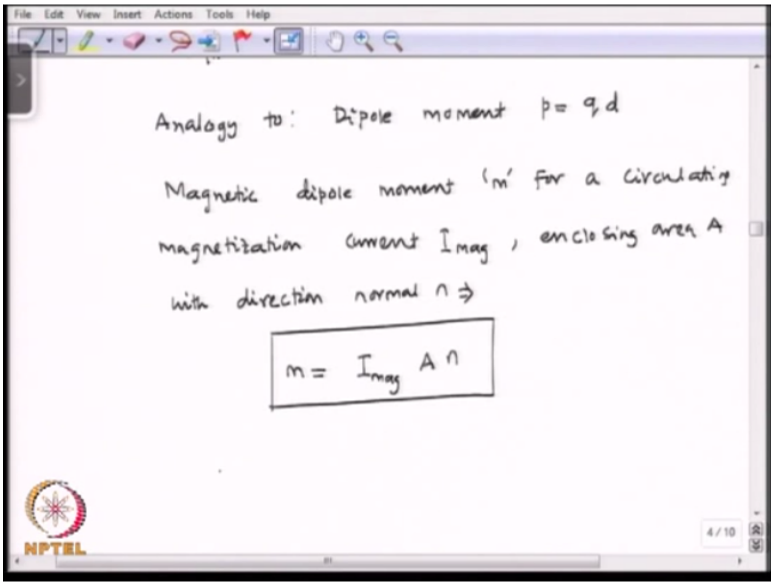

So similar to the electric dipole moment p=q*d we can write something called magnetic dipole moment m for a circulating magnetization current Imag and then closing area A with direction normal n so you can write this expression as m=Imag An so this is along m okay.

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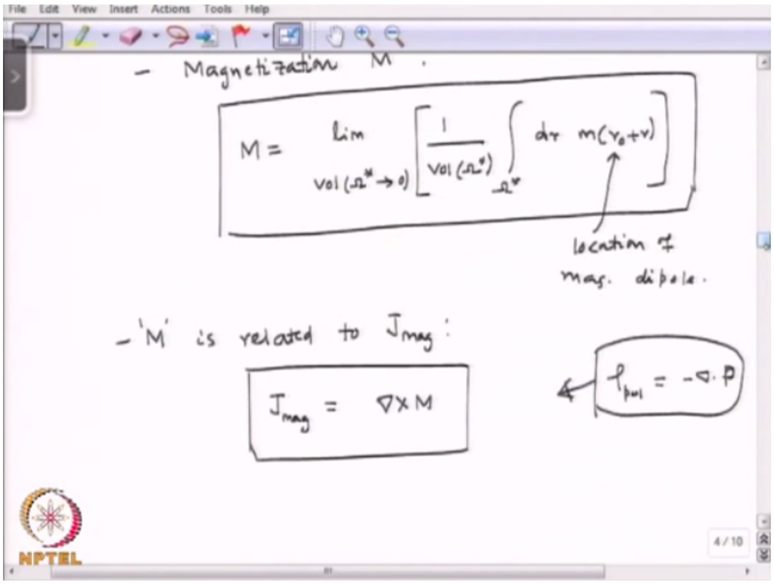

Similarly, we have a term polarization in electric field we can write a term magnetization. So magnetization is equivalent to polarization in dielectrophoresis okay. So you can write magnetization M as limit the volume omega star tending to 0 1/volume omega star integration over omega star dr*m r0+r okay. So this is the expression for the magnetization and r0 is the location of the magnetic dipole okay.

So location of the magnetic dipole okay and r is at any point okay. So with that we can relate magnetization to the magnetic current density so the magnetization M is related to the magnetization current density. We can write Jmag=del cross M okay. So this is analogous to we have electrical charge density the polarization charge density=-del cross p okay. So this is polarization okay so equivalent to this in magnetophoresis we have magnetic current density as del cross the magnetization okay.

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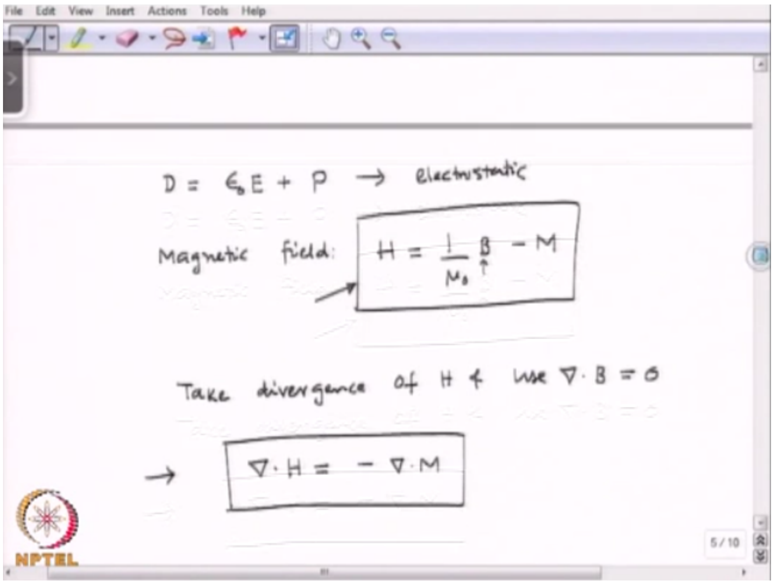

Now we have the electrical displacement=epsilon 0*E+the polarization okay, this is in the electrostatic. In magnetostatic, we have the magnetic field can be written as so H is the magnetic field is 1/mu0*B-M so H is the magnetic field, B is the magnetic induction and M is the magnetization. So now if we take divergence of you know the magnetic field and we take divergence of H and we use that the divergence of the magnetic induction is 0.

We can find that del.H=-del.M so the divergence of the magnetic field is negative divergence of M. This is very easy to show here because this term divergence of the magnetic induction will be 0 so the divergence of the magnetic field will be negative divergence of the magnetization right.

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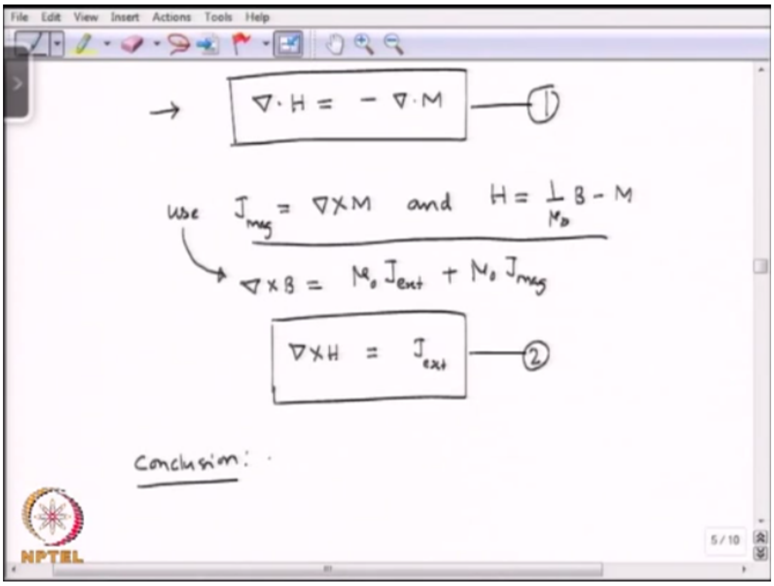

Now if we use the magnetic current density, which is del cross M and you also use the magnetic field as 1/mu0 B, which is magnetic induction-magnetization and you know if you substitute that in the equation del cross B=mu0 J external+mu0 J magnetic okay so use these 2 conditions in this equation you can show that del cross H=J external okay. So what do we see here so this we have you know these 2 equations.

We can say that the conclusion okay is that the source of magnetic field is the spatial variation of the magnetization and the magnetic current density okay.

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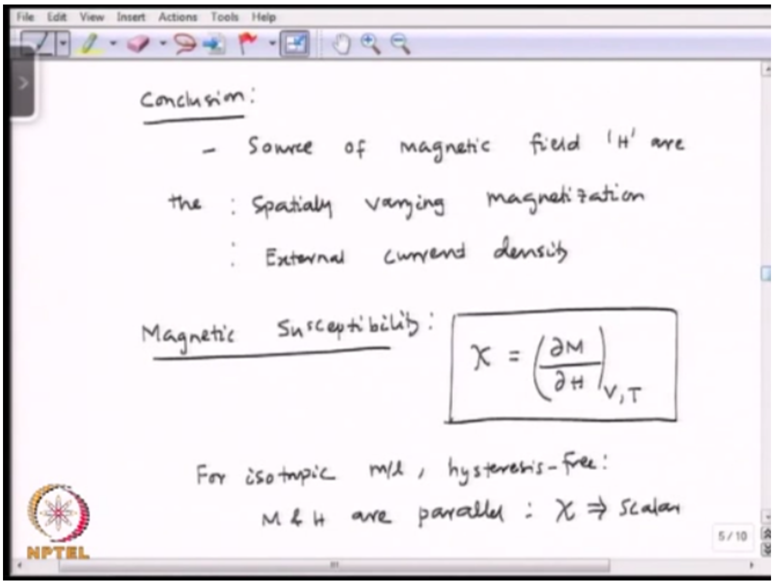

So if we compare these 2 we can say that the source of the magnetic field H are the spatially varying magnetization okay, which is shown in equation 1 okay, this is the spatially varying magnetization and also we have the external current density as you see here in this equation okay. Now with that we can define it on called magnetic susceptibility and this can be defined as the change in the magnetization over the change in the magnetic field at constant volume and temperature okay.

So for isotropic material and with hysteresis-free have no hysteresis then you know the magnetization and the magnetic field M and H are parallel okay so in that case the susceptibility is a scalar okay right.

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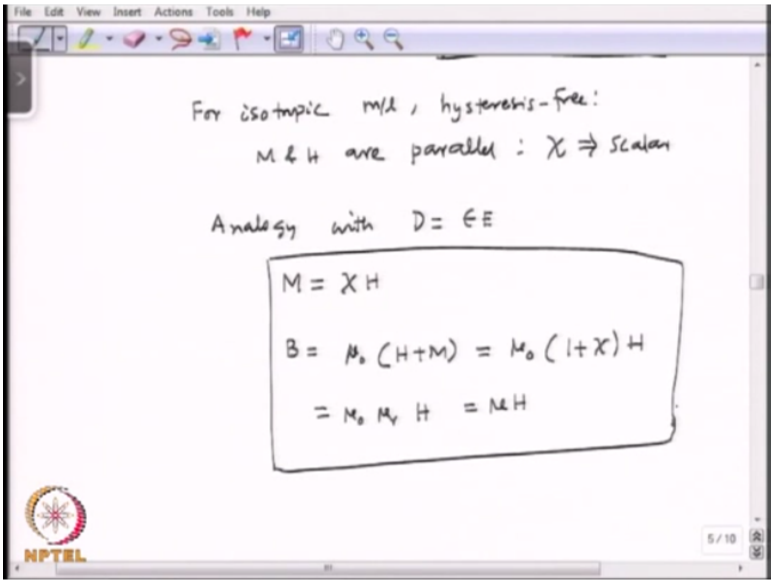

Now if we do an analogy with the electrical displacement as epsilon*electric field okay so we can write magnetization as susceptibility*the magnetic field okay and we can also write that B the magnetic induction is mu0*H+M magnetic field+magnetization, which is mu0*1+susceptibility into the magnetic field, which will be=mu0 mu r*magnetic field, which is mu*H okay.

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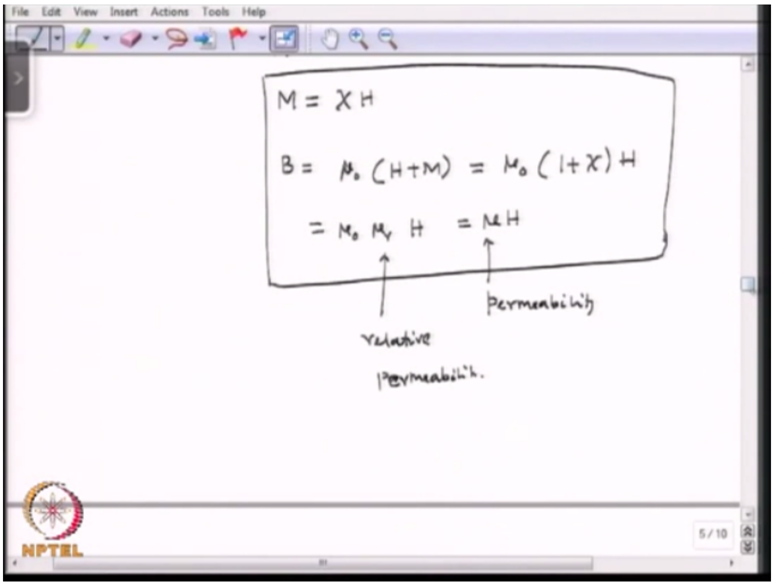

So you have these 2 important equations and here mu is the permeability and mu r is the relative permeability okay. So with that let us move on and try to derive an expression for magnetic force that is acting on a magnetizable particle in an inhomogenous magnetic field okay.

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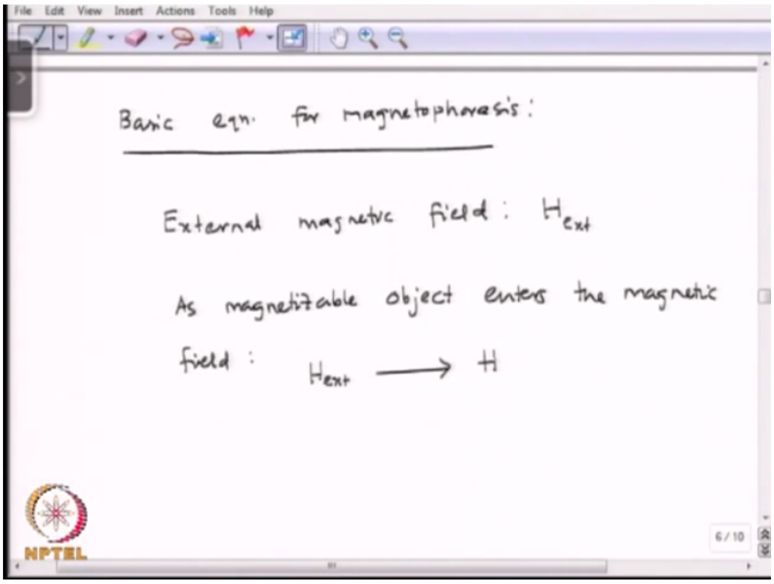

So let us look at the basic equation for magnetophoresis and here we say that the external magnetic field which is on perturbed before we have the magnetizable object in the magnetic field is H external okay so we have a magnetic field which is H external and this is the unperturbed magnetic field before any magnetizable object enters the magnetic field. Now when the object enters the magnetic field.

As the magnetizable object enter the magnetic field the H external the external magnetic field changes to some H okay.

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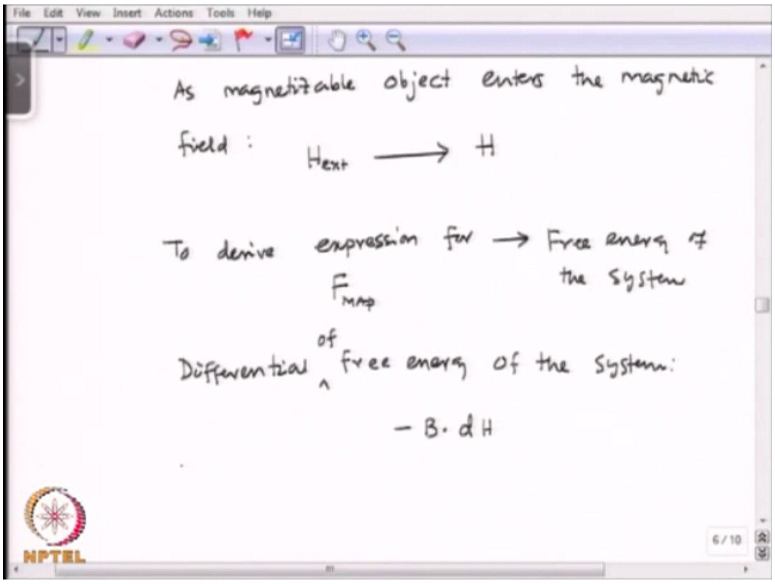

Now we are trying to derive the magnetic force now to derive expression for magnetic force okay F magnetophoresis, we need to consider the free energy of the system okay. So you know we have to consider the free energy of the system and the differential of free energy of the object the magnetizable object is given by –B*dH okay. So that is the free energy of the system okay is –B*dH.

Now when the magnetizable object goes through some displacement in the magnetic field, we going to know what is the change in the free energy that is happening okay.

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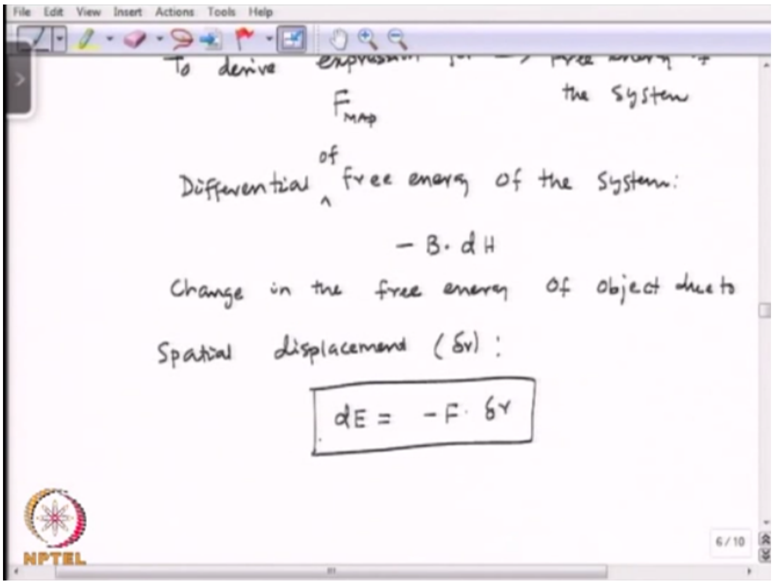

So you know the change in the free energy of object due to spatial displacement delta r so dE the change in the free energy is going to be –F*delta r okay and if you analyze this equation okay.

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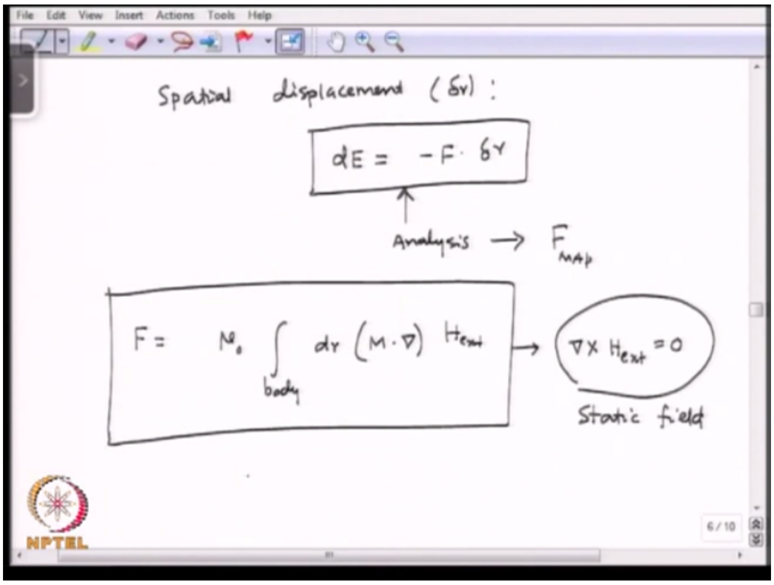

And if you analyze this equation we get an expression for the magnetophoretic force okay. So F is given by mu0*integration over the volume dr*M. radiant of the external magnetic field.

So this is the expression for the force provided del cross external is 0 so the external electric field irrotational. We are talking about a static field okay.

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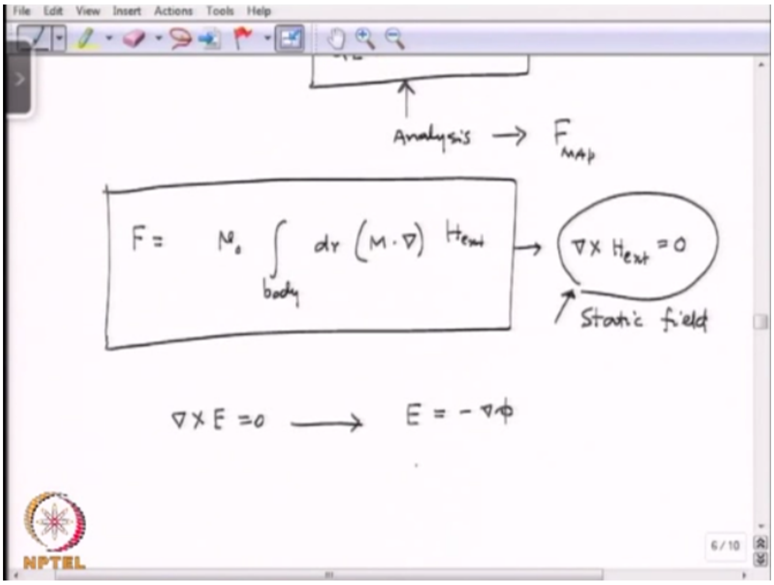

Now you know this is similar to what we have seen in the electric field so for the electric field we have seen del cross E is 0 and from that we can derive that electric field is –del phi okay. So similar to in electric field we have defined electric field is negative of gradient of the potential, here we can define something called magnetic potential okay.

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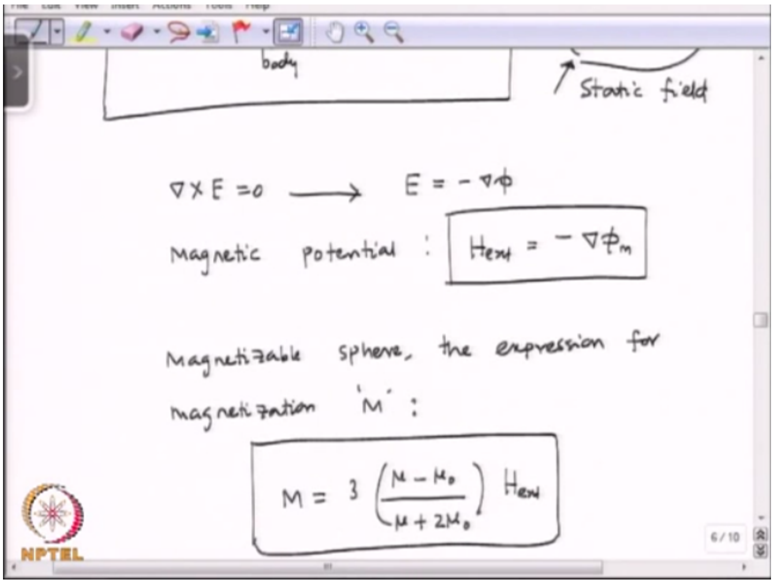

So we define something called the magnetic potential so we can say H external is negative of gradient of del phi m okay and for a magnetizable sphere we have a magnetizable sphere, the expression for magnetization M so the expression for the magnetization would look similar to the polarization in case of the electrostatic okay. So that the expression for the magnetization M is 3*mu-mu0/mu+2mu0*H external okay.

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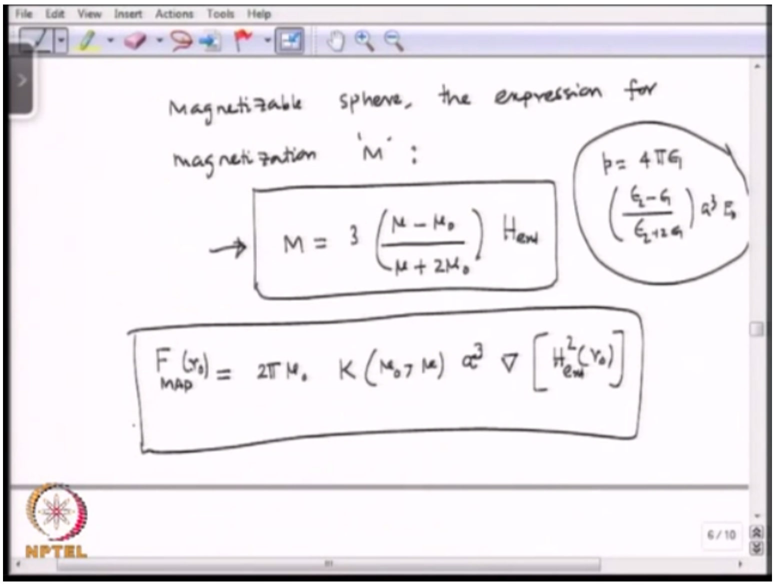

And this is similar to the polarization p which we saw 4 pi epsilon 1*epsilon 2-epsilon 1/epsilon 2+2 epsilon 1*a cube E0 okay so this is what we saw in electrostatic. Here in magnetostatic this is the expression for the magnetization. So we can write down the expression for the magnetic force FMAP will be 2 pi mu0 this is equivalent to the Clausius-Mossotti factor mu0 mu a cube gradient of the square of the external magnetic field okay.

So this is at r0. So here we see that the expression for the magnetophoretic force is very similar to the expression for the dielectrophoretic force. In case of the dielectrophoretic force, we had the dielectrophoretic force was varying as the square of the gradient of the electric field, here the magnetophoretic force is square of the gradient of the magnetic field and similar to the dielectrophoretic force, the magnetophoretic force varies as the cube of the size of the object okay.

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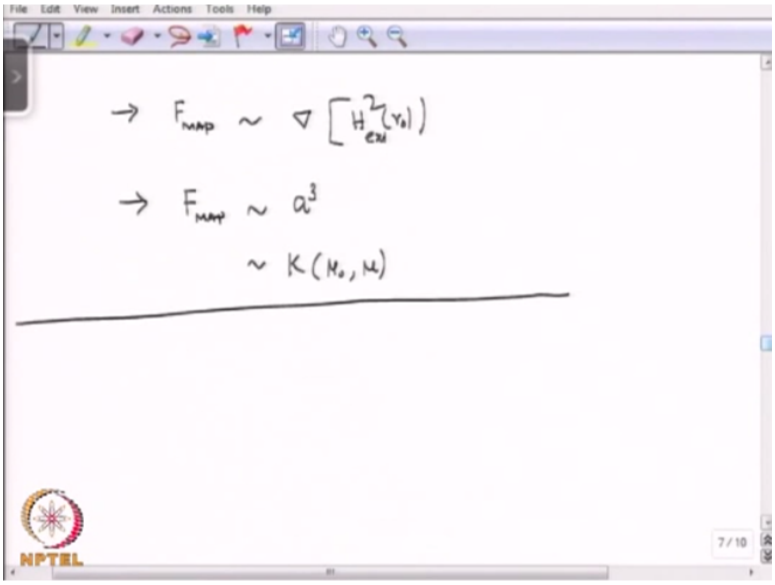

So that is what we observe here okay. So what we see is FMAP varies as the gradient of the square of the external magnetic field and the magnetophoretic force varies as a cube and is also dependent on something similar to the Clausius-Mossotti factor okay, but it is in terms of the magnetic you know permeabilities okay so with that let us move on and talk about the electrocapillary effect okay.

So first of all what we mean by electrocapillary effect? Electrocapillary effect is the electrostatic effect at the interface between different phases okay.

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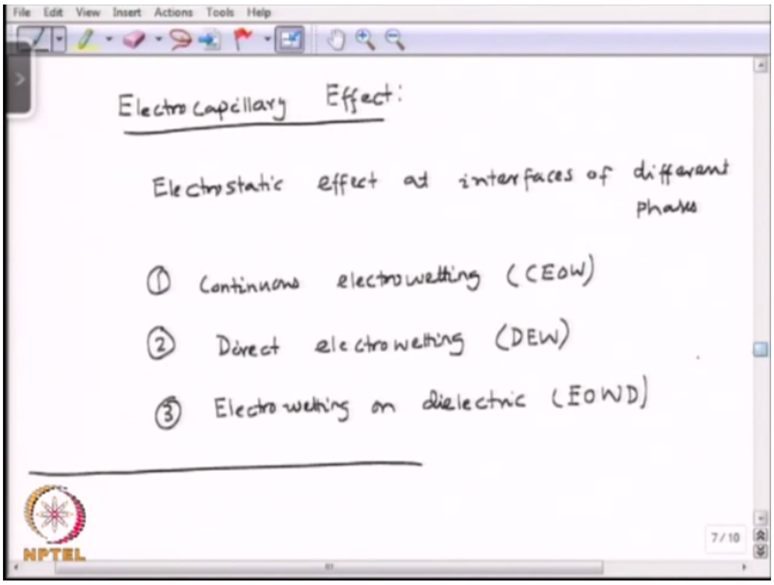

So we look at electrocapillary effect. Electrocapillary effect is the electrostatic effect at interfaces of different phases okay so here we will be talking about the continuous electrowetting CEOW, will be talking about direct electrowetting DEW and we will also talk about the electrowetting on dielectric EO okay right.

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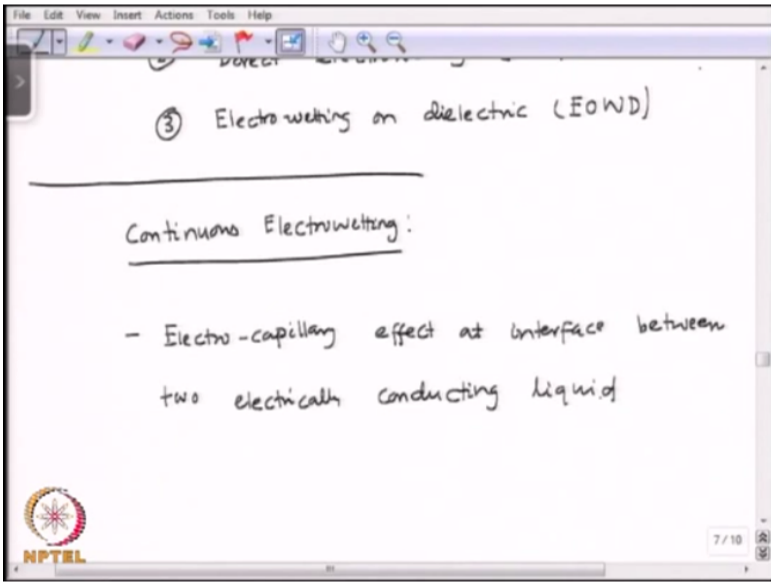

So first let us talk about what we mean by continuous electrowetting. Continuous electrowetting means it is the electrocapillary effect that exists at the interfaces between 2 different you know fluids or liquids okay electrically conductive liquids that have non-zero electrical conductivity.

So in that case we would have an electrical double layer that would exist at the interface and so you know because of the charged situation if we apply an electric field one of the phase would start to move relative to the other okay. So this is what we will be seeing here. So by continuous electrowetting we mean that it is the electrocapillary effect at interface between 2 electrically conducting liquid okay.

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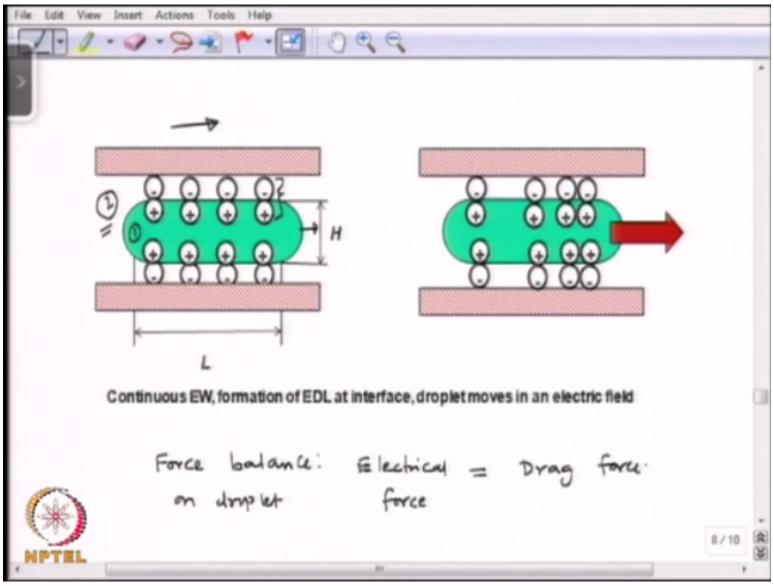

So you can see this here. We are talking about the interface between 2 electrical conductive liquids so you have 1 liquid here which is in the form of droplet and this is the other liquid that we are talking about which is the continuous phase and because of non-zero conductivity at the interface, we would have an electrical double layer that would exist and if we apply an electric field okay.

So this droplet here would act as a charged particle okay and we apply an electric field this droplet would tend to move relative to the liquid, which is phase 2 so phase 1 would tend to move relative to phase 2 okay. Now if you do a force balance you know so the force the electrical force would try to carry this droplet and it will be opposed by the drag force okay so if you look at a force balance on this droplet will have electrical force will be=to the drag force.

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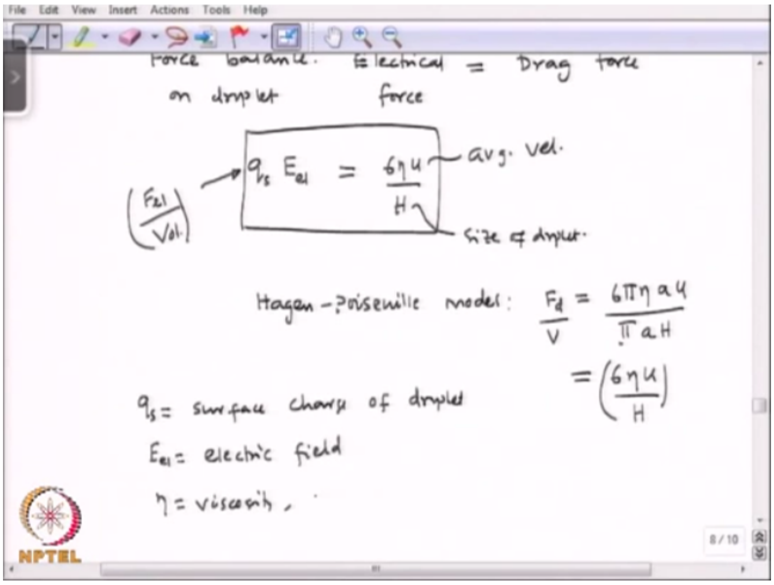

So we can write the expression for the electrical field force is qs is the surface charge on the droplet which is into the electric field okay so that is the electrical force per unit volume okay. So this is the F electrical force per unit volume and so we have to equate drag force per unit volume and we know the expression for the drag force. You know we can assume that Hagen–Poiseuille model can say it is Hagen–Poiseuille model.

So if you do that we can we know that the drag force is 6 pi eta a u so we are talking about per unit volume so here if you look at the droplet, it is looking like an cylinder. Let say a is the radius of the cylinder so we have pi a*H okay so it will be pi a*H so that will be you know 6 eta u/H so this will be 6 eta u/H right. So what are the terms here?

Qs is the surface charge of droplet and E is the electric field, eta is the viscosity and H is the size of the droplet and u is the average velocity okay. So from here we can predict how quickly the droplet is going to move.

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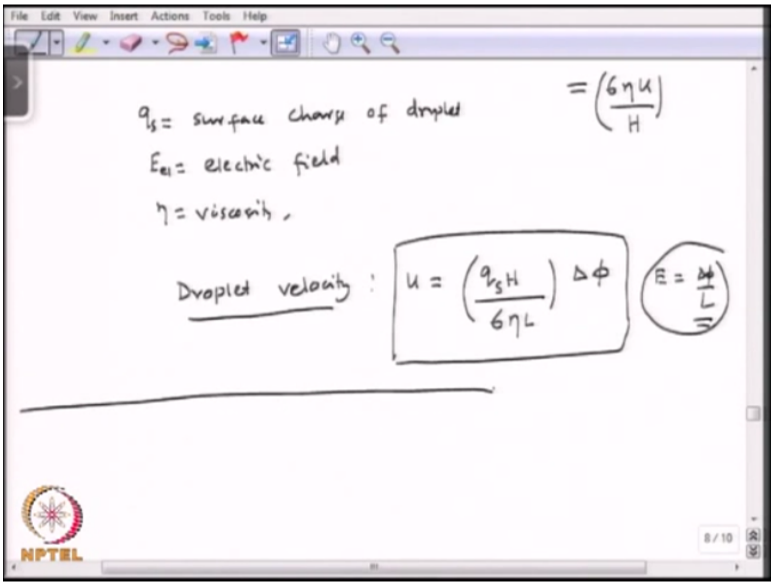

So the droplet velocity u is going to be from this equation we can find what is the value of u okay. So u is going to be qs*H6 eta L*del phi okay. So that is the expression for the droplet velocity so the electric field is represented by delta phi/L okay where L is the length scale of the droplet in this case okay. So let us move on and talk about direct electrowetting.

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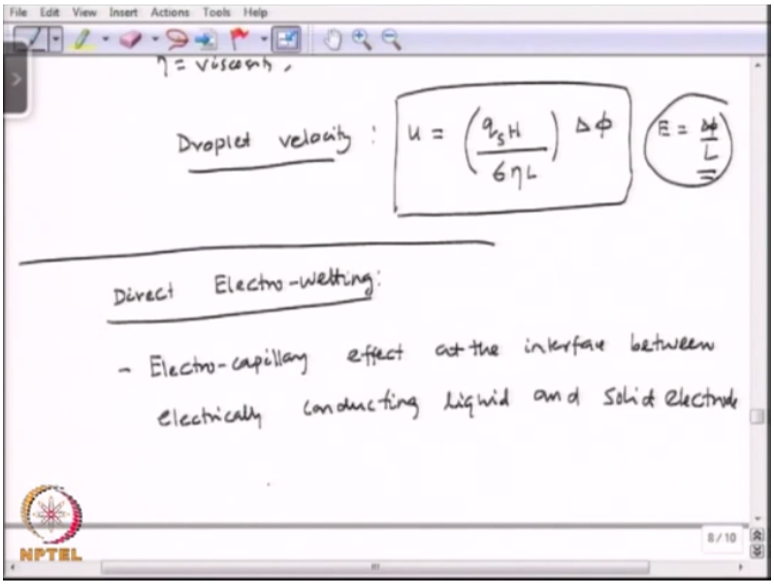

Direct electrowetting is the electrocapillary effect or the interface between a liquid phase and a solid electrode okay. So direct electrowetting is the electrocapillary effect at the interface between electrically conducting liquid and solid electrode okay.

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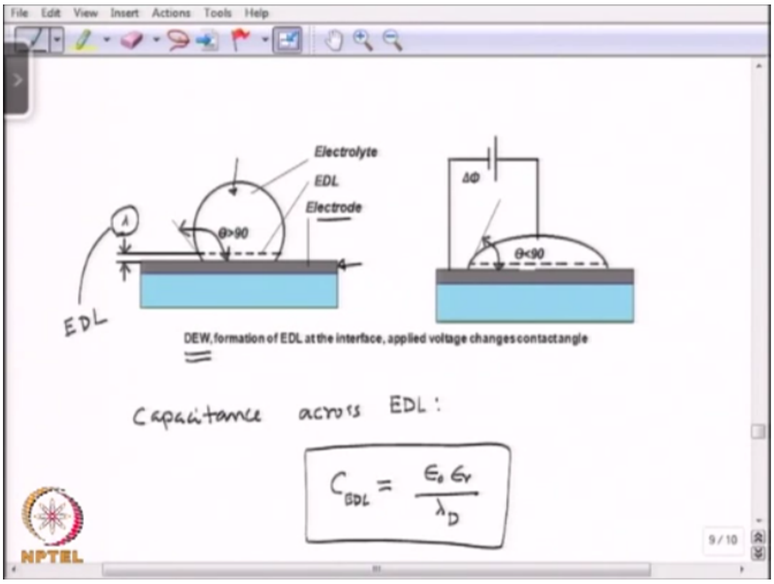

So as you see here we have a solid electrode okay, this is the electrode and we have an electrolyte okay. So in this case we can expect direct electrowetting at the interface between the electrically conducting liquid and the solid electrode. We would have an electrical double layer that would exist okay because the liquid has non-zero conductivity. So we would have an electrical this is the electrical double layer okay.

So we can write the expression for the capacitance across the electrical double layer. We can write down the capacitance across electrical double layer so you can write CEDL will be=epsilon 0 epsilon r/lambda D okay. So what do we see here is that the surface energy is going to reduce because there are some charges which are going to be stored in the form of capacitance inside the electrical double layer okay.

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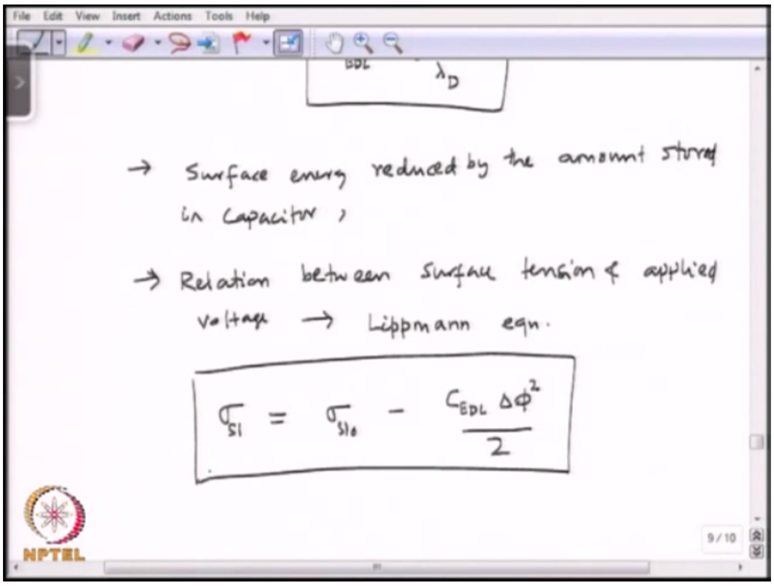

So what would happen is the surface energy reduced by the amount stored in capacitor and so in that case you can write the relation between surface tension and applied voltage and this is given by the Lippmann equation okay so you know this can be given by the Lippmann equation. So the Lippmann equation can be written in this form sigma Sl is going to be sigma Sl0-capacitance across electrical double layer del phi square/2 okay.

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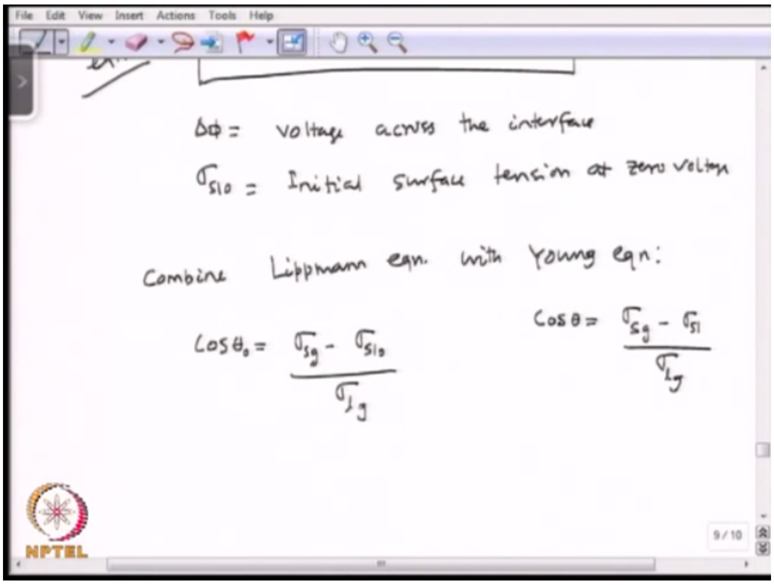

And here delta phi is the voltage across the interface and sigma Sl0 is the initial surface tension at 0 voltage okay. Now if we combine this Lippmann equation okay with the Young equation that we studied in capillary flows. So combine Lippmann equation with Young equation. So Young equation said that cos theta is going to be sigma solid gas-sigma solid liquid/sigma liquid gas.

So cos theta 0 will be=sigma solid gas-sigma solid liquid at 0 voltage/sigma liquid gas okay.

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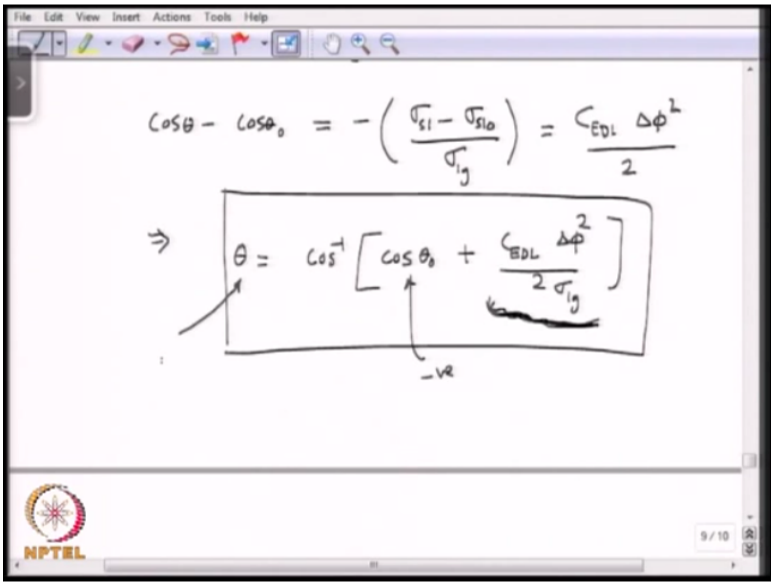

So we can do some math cos theta-cos theta 0 will be=-sigma Sl-sigma Sl0/sigma lg liquid gas, which is nothing but equal to in the form of capacitance and electrical potential you can write this, this is going to be CEDL delta phi square/2 okay. So from here we can predict theta is going to be cos inverse of cos theta 0+CEDL del phi square/2 sigma lg okay. So what do we see here?

We see that a surface, which is originally hydrophobic can be made hydrophilic okay. So you know if the surface is hydrophobic initially, this will be negative right and you can store some energy you can take some energy out of the surface energy and dissipate and store that in the form of the capacitor charged inside the electrical double layer okay. If you do that, the contact angle is going to be reduced okay so cos theta is going to be positive okay.

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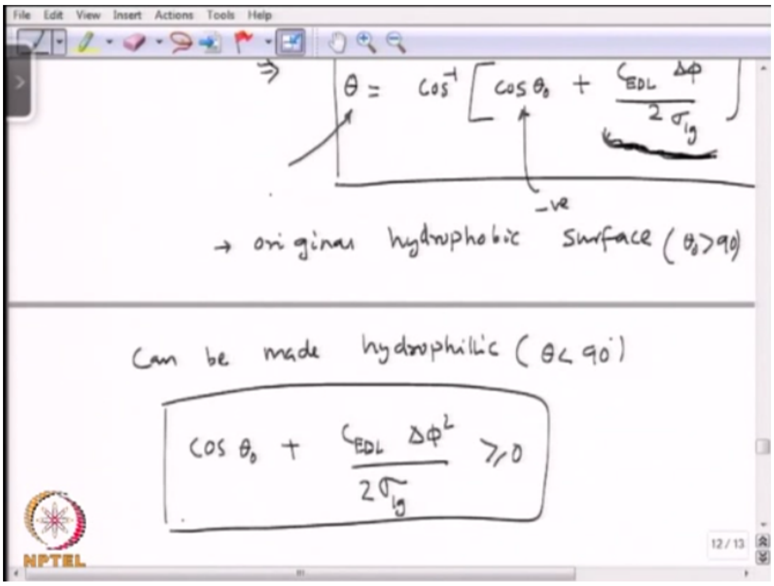

So what we see here that original hydrophobic surface can be made hydrophilic okay. So hydrophilic means theta 0 will be <90 and initially theta 0 will be >90 and theta let us not call it theta 0 this is theta is <90. So what will be the critical potential at which an original hydrophobic surface will become hydrophilic? So in that case cos theta 0+CEDL del phi square/2 sigma lg will be >=0.

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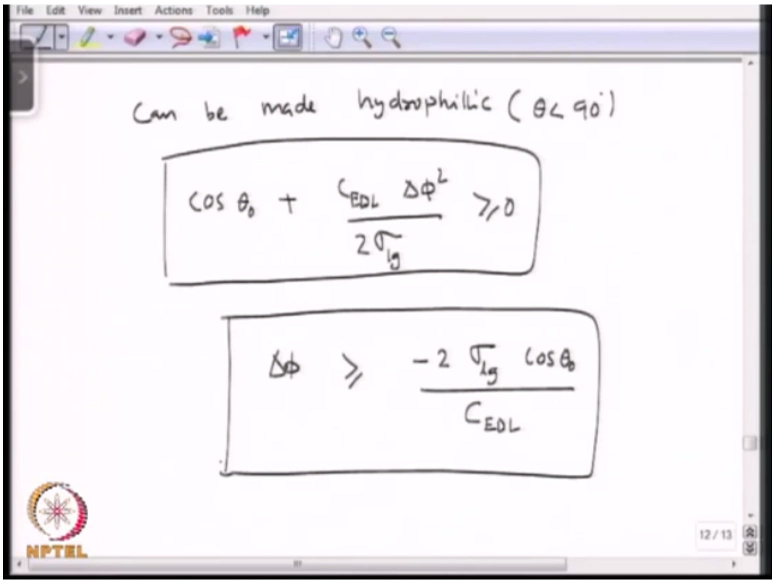

Or if you simplify we can find out that the electrical potential will be>=-2 sigma lg*cos theta 0/CEDL okay. So we found the expression for the minimum potential that is required to make an original hydrophobic surface to hydrophilic. Now the problem with the direct electrowetting is that the capacitance for a given set of liquid and electrode surface is fixed. So what we can do is we can add another insulating layer on the electrode okay.

So that the equivalent capacitance of the electrical double layer plus an insulating layer can be controlled by changing the thickness of the insulating layer okay and that is known as the direct electrowetting on dielectrics.

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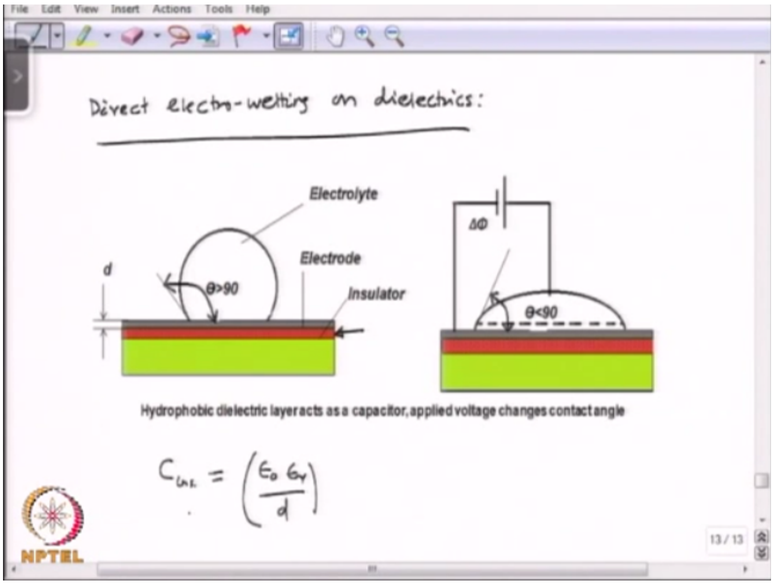

So we talk about direct electrowetting on dielectrics. So as you see here instead of having the electric layer we add one more insulating layer here, so we have the capacitance of this insulating layer plus the capacitance of the electrical double layer. So the capacitance of the insulating layer, which can be controlled is given by epsilon 0 epsilon r/d okay.

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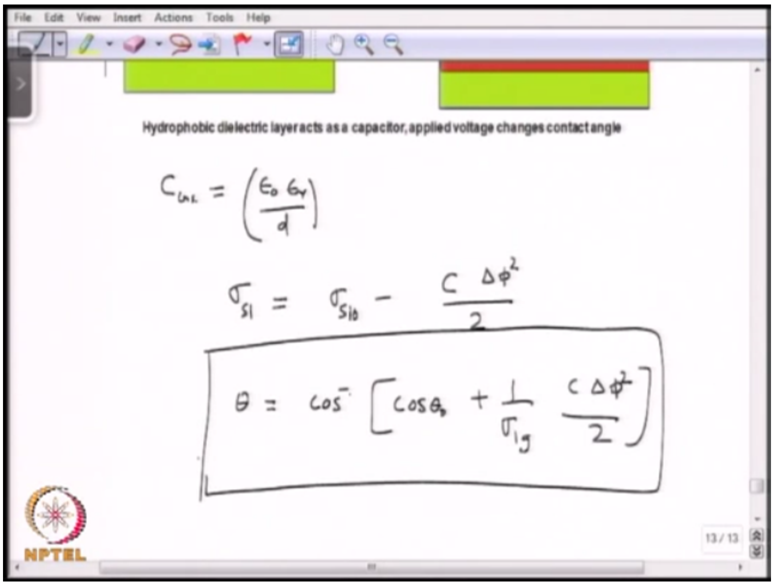

And so in that case we would have sigma Sl as sigma Sl0-C del phi square/2 so you can write theta as cos inverse cos theta 0+1/sigma lg*C del phi square/2 okay. So here in addition to you know making the surface to make an originally hydrophobic surface hydrophilic you would have a control and that control can be achieved by controlling the thickness of the insulating layer okay.

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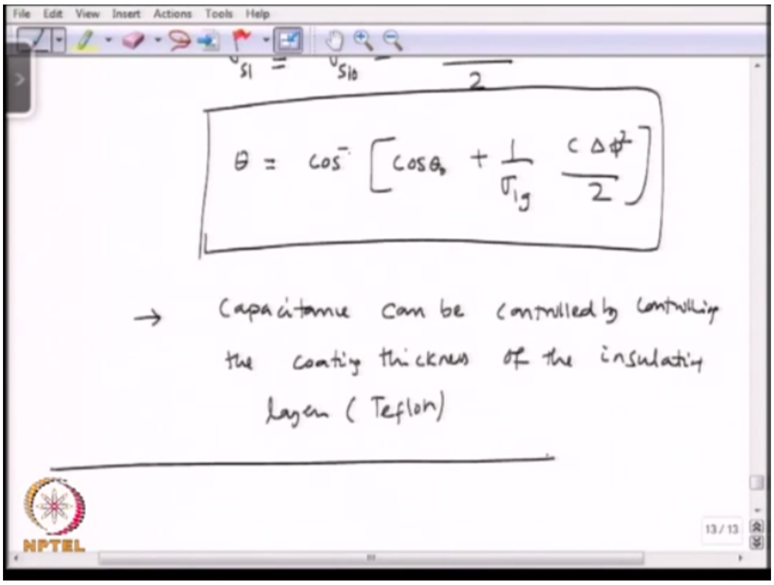

So here the capacitance can be controlled by controlling the coating thickness of the insulating layer which could be for example Teflon okay. So with that let us stop here.