Microfluidics
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Lecture - 02
Scaling

Okay, so we have been looking at Trimmers vertical bracket equation, and here we know that if we have a scaling for different forces, then you know we can find how the scaling is going to work for acceleration time and power to volume ratio.

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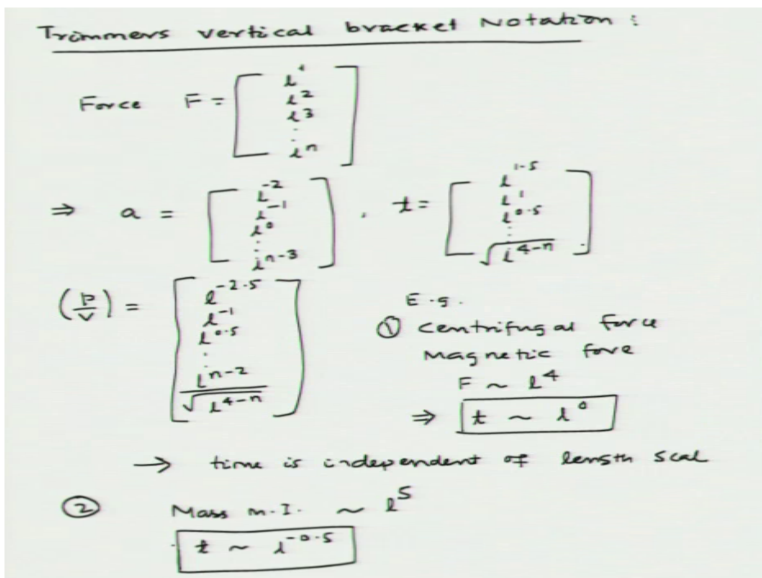

Trimmers vertical bracket notation okay, so here if we know the scaling for force say force F is scaling as l1 l2 l3 up to ln, then we can find the scaling for acceleration is going to be l to the power -2, l to the power -1, l to the power 0, l to the power n-3and similarly the scaling for time is going to be l to the power 1.5, l to the power 1, l to the power 0.5 to l to the power 4-n square root.

And the scaling for power to volume ratio is going to be l to the power -2.5, l to the power -1, l to the power 0.5, up to l to the power n-2 /square root of l to the power 4-1, so that is how the power to volume ratio is going to vary. Now what we see here is that if force scales as l to the power 4, then you know the time will scale as l to the power 0, so for centrifugal force magnetic force where force is scaling as l to the power 4 time will be independent of the length scale okay.

So if you consider example of centrifugal force and magnetic force, so here the force scales as l to the power 4, and in that case time would scale as l to the power 0, so what it means is that the times becomes independent of the length scale okay, so this is contrary to our belief that small things tend to be faster okay. So here we see that you know the time is not going to change with length scale, so that is one observation here.

So the time is independent of length scale okay, so that is one important observation. The second observation is that the maximum scaling is going to be for mass moment of inertia, where it is going to scale as l to the power 5 okay, so for mass moment of inertia going to scale as l to the power 5 then we can see that time t is going to scale as l to the power-0.5 okay, so what it means is that you know if you are talking about 2 different motors one is smaller in size, a small size motor will reach the maximum speed the top speed faster as compared to a larger motor.

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Quantity	Scaling Law
Intermolecular Van der waals force	l^{-7}
Capillary force	l^{1}
Electrostatic force	l^{2}
Force of gravity	l^{3}
Magnetic force without an exterior field	l^{4}
Centrifugal force	l^{4}

So here we have different forces the intermolecular Van der Waals force will scale as l to the power -7, the capillary force scales as l, and the electrostatic force scales as l square, and the force of gravity scales as l to the power 3, the magnetic force without an exterior field scales as l to the power 4, and the centrifugal force also scales as l to the power 4. Now with that we move on and talk about how the scaling is going to work for different microsystems.

The first example that we are going to consider is going to be a parallel plate actuator, where we would have two parallel plates and since there will be oppositely charged there is going to be a force that is going to be exist between them okay, and that can actuate the 2 parallel plate in a fashion that can be used for different microsystem applications.

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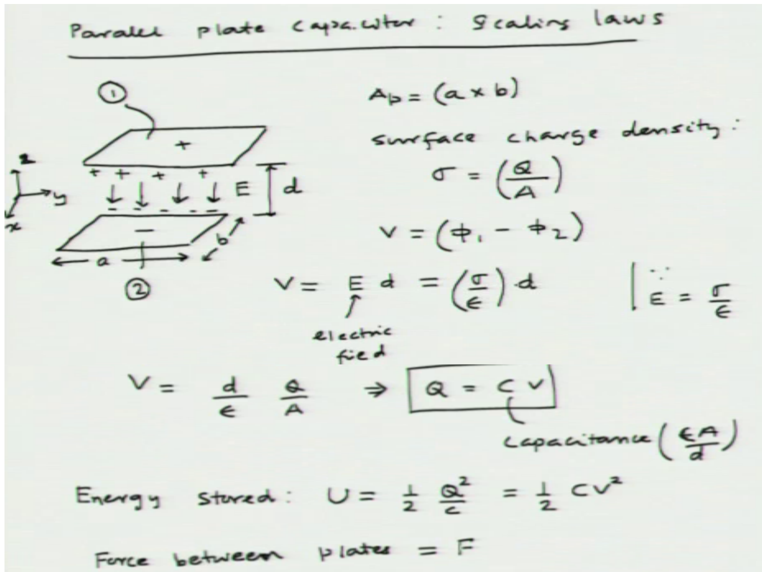

So we are going to look at parallel plate actuator capacitor, and we will look at how the scaling law is going to work? Okay, so you know we can draw 2 parallel plates here this is one plate and this is another plate, let us consider this is y, this is x sorry this is z and this is x, and we say this is plate 1, and this is plate 2, this is negatively charged plate so we have negative charges here, this positive okay. So the electric field is going to be in this direction E.

And the separation distance between the plates let us say it is d, and the length of the plate is a, and width is b okay, so the plate area is a*b okay, so we can write the surface charge density sigma as total charge/area of cross section right, and if the plates are at potential phi 1 and phi 2 the voltage across the capacitor can be written as phi 1-phi 2 okay. And so we can be written as E*d E is the electric field okay this is electric field and d is the separation distance.

So we can write this as sigma/epsilon, so this is you know we have we can write electric field as sigma charge density/epsilon right *d, so we can write V=sigma/you know we can write d/epsilon and sigma we can write Q/a so Q/a, so we can write Q=C*V where C is the capacitance which is given by epsilon a/d okay. So you know we can find the energy stored in the capacitor energy stored is going to be U=1/2 Q square/C=1/2 C V square.

Now since these 2 plates are oppositely charged there is going to be a force between them, and if you are going to move one of the plates by a distance delta Z, then we would need an energy which is F*delta Z.

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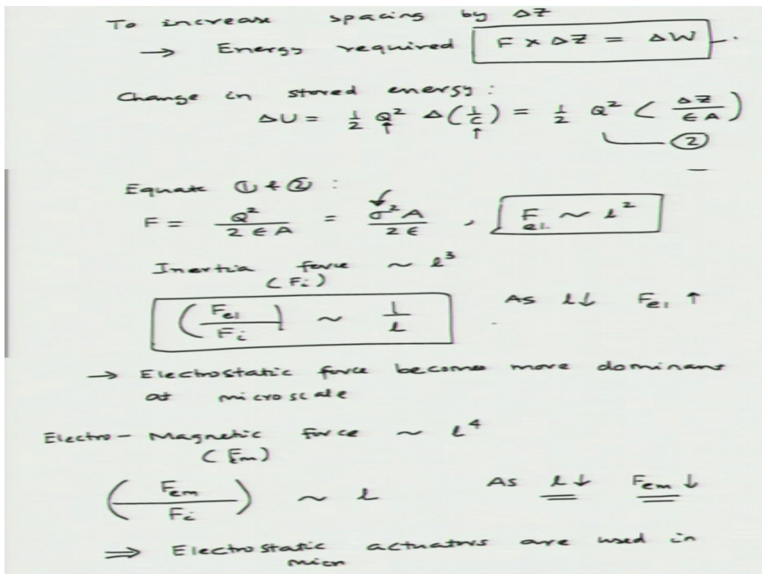

So the force between the plates let us call it F, and to increase spacing by delta Z, the energy required would be F*delta Z, so let us call this delta W as the energy required. Now the corresponding change in the energy stored, so change in stored energy we can find delta U will be 1/2 Q square*change in the capacitance, because the total charge is going to remain unchanged, so the moving the plates would change the capacitance.

So this is going to be 1/2 Q square*delta Z/epsilon A right, so this is let us call this as equation 2, call this has equation 1. Now if we equate 1 and 2 we get F=Q square/2 epsilon A which will be sigma square A/2 epsilon, so you know for a constant charge density the force between the 2 plates is going to scale as l square okay, so this is electrostatic force electrostatic force which is going to vary as l square. Now the inertia force scales as l to the power cube.

So if you take a ratio between F electrostatic/F inertia is going to scale as 1/l right, so what we see here? We see that as we reduce l when you go to microsystems, then the electrostatic force is going to be more and more dominant okay, so that is the reason why you know electrostatic actuators that use parallel plate configuration you know made smaller and smaller, so if you made the electrostatic capacitor based actuators smaller the electrostatic force is going to be dominant over the inertia force okay.

So the sensitivity of the actuation can be enhanced, so what do we see here as l reduces the electrostatic force goes up okay. So what we learn from here is that electrostatic force becomes more dominant at microscale okay. Now if you compare you know how electromagnetic force is going to scale, so the electromagnetic force is going to scale as l to the power 4 okay, so let us all this Fm, so if you Fem, so Fem electromagnetic force/the inertia force is going to scale as l.

So as l reduces electromagnetic force is going to be lesser significant okay less significant, that is the reason why electrostatic actuators are preferred in micro systems, so electrostatic you know we see here that electrostatic force increases as the length scale reduces, where the electromagnetic force is reduces is reducing as length scale reduces. So what we can conclude is that is the reason why electrostatic actuators are used in microsystems.

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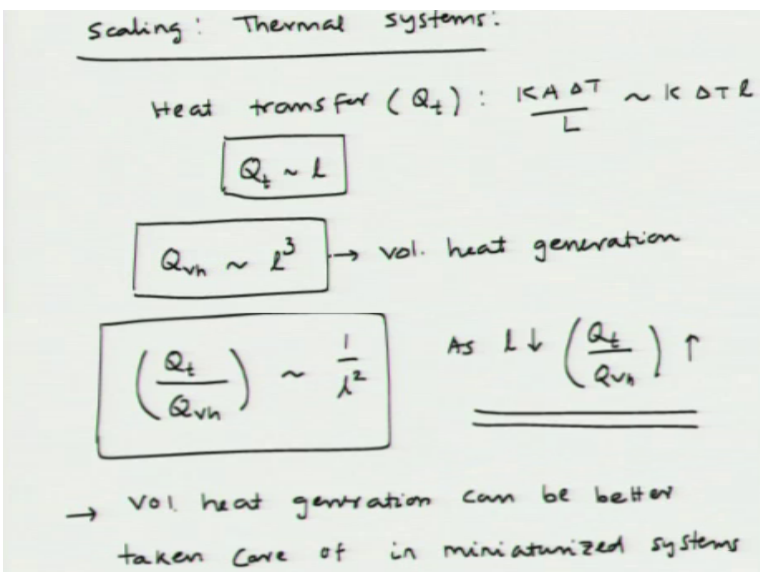

Next, let us look at how the scaling is going to work for a thermal system okay, so we will look at scaling in thermal systems, so here we have heat transfer assuming that it is by conduction, so let us say Q transfer is going to be K A delta T/L, so you are going to scale as K delta T*l, so Q transfer scales as length scale. Now if we consider volumetric heat generation okay heating is being done internally, so the volumetric heat generation will be proportional to the volume of the object.

So we can write volumetric heat generation will be proportional to L to the power cube right, so this is volumetric heat generation. Now if you take a ratio between Q heat transfer/Q volumetric heat generation it is going to scale as 1/l square right, so as l reduces the Q heat transfer/Q volumetric heat generation reduces okay sorry this is going to increase okay. So what we see here is that the microsystems are capable of handling internal heat generation in a much better way as compared to larger system okay.

So what we learn from here is the volumetric heat generation can be better taken care of in miniaturized systems okay. So next we consider a case where we talk about you know transient unsteady heat transfer, so you know if you place a hot object in a cold ambient the heat is going to be dissipated from the object to ambient, as a result the temperature of a hot object is going to change with time, and we see by miniaturization how the heat transfer the temperature characteristics are going to vary as we you know change the length scale.

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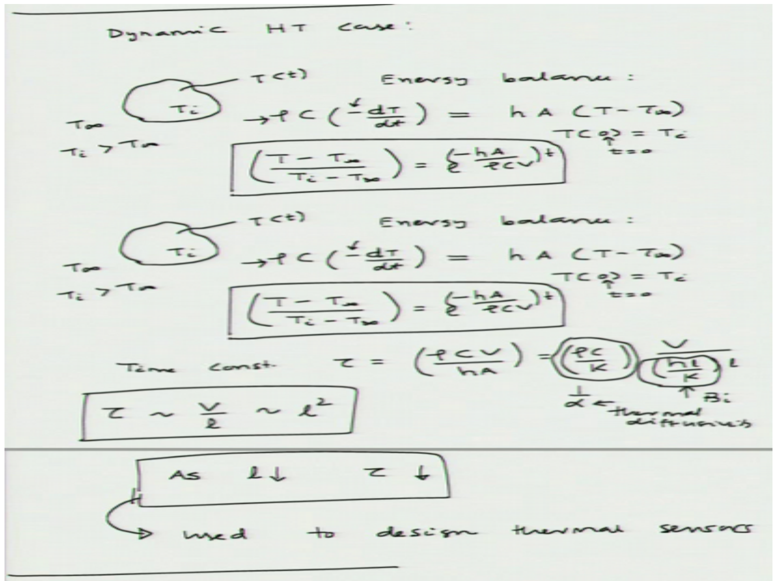

So we consider dynamic heat transfer case, here we have a hot body let us say initially at temperature Ti and in an ambient at T infinity, so Ti>T infinity, and the temperature of it is going to vary as T with time okay, so we can write you know the energy balance we can write rho density of the object*C specific heat-dT/dt because the temperature is going to reduce with the time, so there is a negative sign here is going to be h A*T-T infinity.

So we can write a solution for this, if you solve this simple differential equation the solution can be written in this form T-T infinity/Ti-T infinity, so with the initial condition that temperature at time T=0 is going to be Ti right so t= 0, so this will be e to the power -hA/rho C V*t okay, so that is how the temperature is going to vary with the time. So we can write the time constant tau is going to be rho C V/h*A okay.

So you can write this as rho C/K*V/hl/K*l, so this term is 1/alpha, alpha is called thermal diffusivity, which is a property of material and this is known as Biot number okay which is a constant, so the time constant is going to vary as V/l or it is going to vary as l square okay. So what we see here is that you know as the system is reduced in size as l is reduced the time constant is going to reduce.

So you know this is a very important conclusion where the length scale l reduces the time constant reduces, so this is the reason why we try to make the thermal sensor smaller and smaller, as the thermal sensors become smaller, because of the small length scale the time constant is going to be reduced for which if you expose a sensor to an ambient it is going to be very quickly attend the ambient temperature and read it accurately okay. So this used to design thermal sensors.

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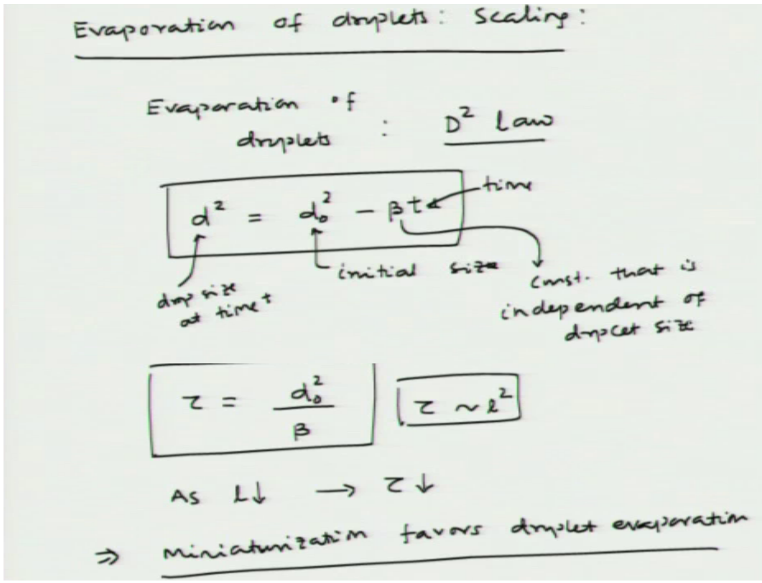

Now let us look at another microsystem, how you know the evaporation of the droplet is going to scale with the length scale okay. So we look at evaporation of droplets how the scaling is going to work, so we know that the evaporation of the droplets is going to follow what is called D Square Law okay, so evaporation of droplet going to use D Square Law okay, what it means is that d square=d0 square-beta t okay.

So let us say we have a droplet, so this is the initial size of a droplet and this is droplet drop size at time t, and so this is time, and beta is a constant that is independent of droplet size, so you know if you want to find the times scale at which the droplets would disappear okay, so in that case you can say that you know the time constant will be d0 square/beta okay. So the time constant is going to scale as l square right.

So what it means is that as l is going to reduce the time constant is also going to reduce, so what we see here is that miniaturization favors droplet evaporation right, as the droplet size is smaller and smaller it is going to evaporate very quickly okay, and this is going to be important in case of applications where we are talking about you know polymer drug delivery for example, so there we generate you know small size droplets a gloom of small sized droplets.

And we have to when you design a system we have to understand that if the droplet size is going to be smaller it is going to evaporate very quickly following D Square Law okay. So next we move on and talk about you know the scaling in fluidics.

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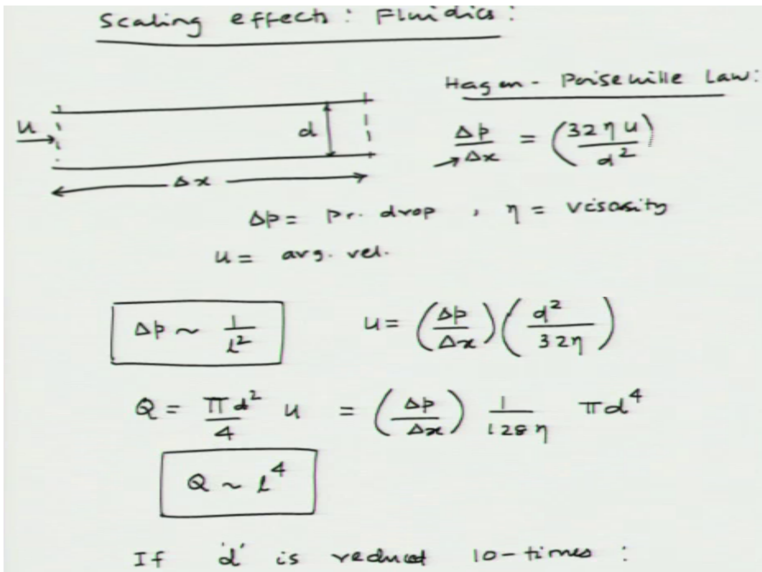

So we talk about scaling effects in fluidics, so here we consider a straight channel of diameter d let us say this d and the velocity is u, let us say delta x is the length of the channel okay, so you know we know that in case of laminar flow through a channel or a tube we can apply the Hagen–Poiseuille law, so if you apply the Hagen–Poiseuille law, we can write delta p/delta x is going to be 32 eta u/d square.

So here delta p is the pressure drop okay, delta x is the length of tube, eta is dynamic viscosity, u is the average velocity, d is the diameter. So what we see here is that the pressure drop is scaling as 1/l square okay, and so now from here we can rearrange the term to write u=delta p/delta x*d square/32 eta okay, so this what we can write. Now we can write the expression for the flow rate Q will be the area of the section which is pi d square/4*u which will be delta p/delta x*1/128 eta*pi d to the power 4.

So what we see here is that the flow rate will scale as l to the power 4 right, now if we reduce the diameter of the channel by let us say 10 times okay, so if d is reduced 10 times then what is going to happen? The delta p is going to increase by 10 to the power 4 times okay, so if the channel is going to reduce 10 times the delta P is going to increase by 10 to the power 4 times, the pressure drop is going to increase which is an undesirable effect.

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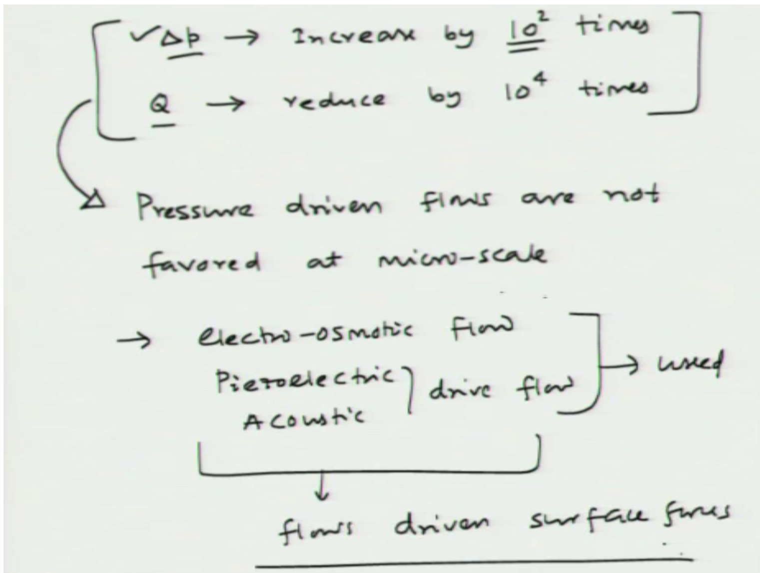

On the other hand, the Q is going to reduce by 10 to the power 2 times okay right, this is 10 to the power 2 times because pressure scales as l square right delta p scales as l square, so you are going to reduce 100 times as l is reduced 10 times, then Q delta p is going to increase by 100 times when d is reduced 10 times, and the flow rate is going to reduce by 10 to the power 4 times, so both effects are undesirable effects.

When we are talking about scaling down the size of a microchannel okay, if you scale down by 10 times the pressure drop across the channel is going to increase by 100 times, and the flow rate across the channel is going to reduce by 10 to the power 4 times okay, so you know scaling down is not helping the pressure drop or the flow rate okay. And this is the case when we are talking about pressure driven flow, pressure driven flow you know it is basically a volume force okay.

So at microscale you know the volume effects are very small, whereas the surface effects are dominant, so you know rather than using Poiseuille flow, pressure driven flow at microscale you know things like electroosmotic west flow or piezoelectric west flow are used, because these are surface driven forces you know at microscale these effects become more and more dominant. So because of this region pressure driven flows are not favored at microscale.

And the electrostatic flow electroosmotic flow, piezoelectric or acoustic driven flow, so these are used, this is because this flows are flows driven by surface forces, so this would require you know alternate pumping techniques at microscale in microchannel okay, so to drive fluid in microchannels instead of going for pressure driven flows, we would be using electroosmotic west flows, acoustic flows, piezoelectric driven flows, which will be discussing in the later part of the course okay. So with that let us stop here.