**Microfluidics**
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**Lecture – 18**
**Electrokinetics (Continued...)**

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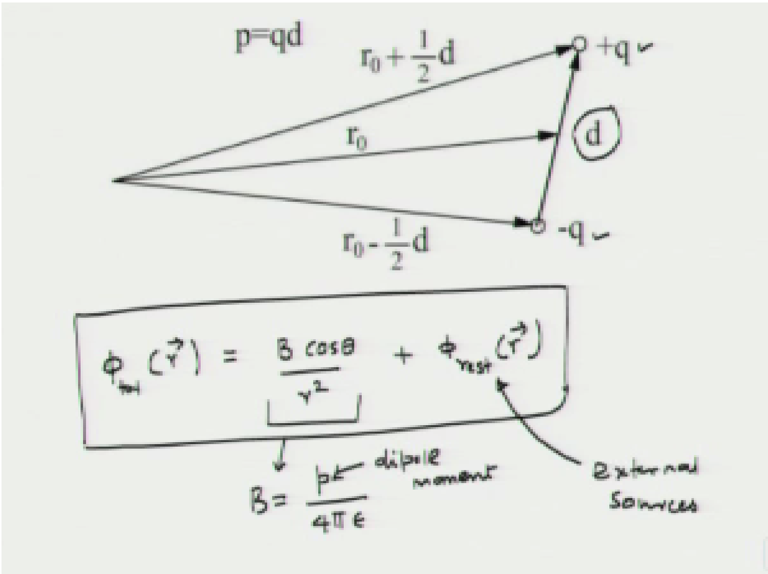


Okay, so we are talking about a point in a; dipole in an electric field and we saw that the dipole strength is inversely proportional to the square of the distance okay. So, this is what we are talking about; so we are talking about a dipole, so you have +q here, -q here separated by a distance d and we are seeing how the potential distribution because of this dipole is looking like okay.

So, we saw that the potential distribution; phi total are some distance r will be B cos theta over r square + phi rest r, okay so this is the expression for the potential at any point and this contribution is coming from the dipole okay, here B is = p, which is the dipole strength divided by 4 pi epsilon okay, so p is the dipole moment and this may be because of the external sources okay, so this is what we saw.

We saw that if we have a dipole; you know this effect at any point are in terms of the potential distribution is going to vary as the inverse of the square of the radial distance okay. So, with that let us move on and talk about a dielectric sphere in a dielectric liquid okay. So, if you have an electric field present in the dielectric liquid, the presence of the dielectric sphere is going to modify the electric field.

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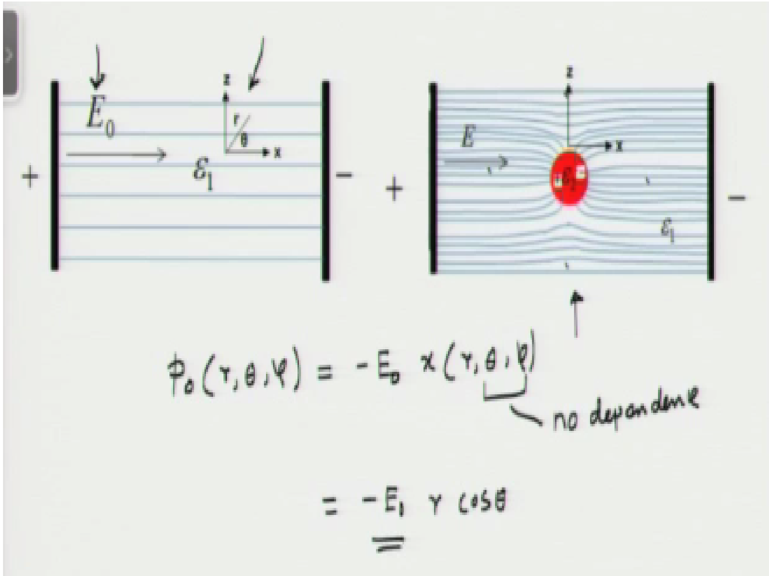


And we would see how the dielectric moment; you know the dipole moment is going to vary in space okay. So, this is the situation will be talking about, so here we have a positively charged plate and negatively charged plate, so we establish an electric field okay and let us say the dielectric constant of the fluid is epsilon 1 okay, so you know that the dielectric fluid here is being penetrated by the electric field epsilon 0.

So, we can write the potential variation phi 0 as a function of r, theta and psi, which is the azimuthal coordinate to the minus E0 * x r theta psi, so in this case the electric field is varying only along x direction, so the theta and psi, there is no dependence okay and so this is going to be –E0 8 x; x is nothing but r cos theta in this coordinate system here x is r cos theta and here we have a negative sign, this indicates that as x increases, the potential is going to reduce okay.

So, that is denoted by the negative sign there and you know if you have; now, we have a dielectric sphere. If you look at the right side figure here okay, so we have a dielectric sphere which dielectric constant epsilon 2 is introduced into the liquid okay. So, you know we have an electric field, which is maintained because of 2 charges; oppositely charged plates and so we know the electric field okay, we know the potential how it is varies along x okay.

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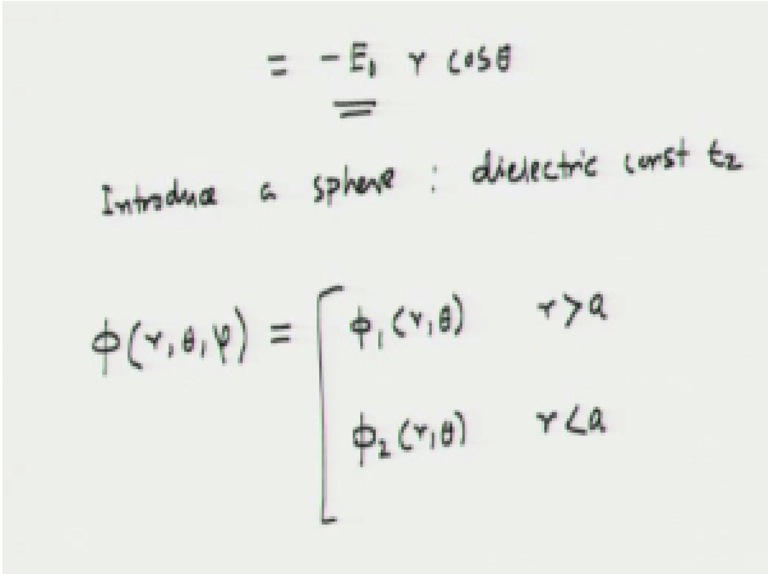


Now, we introduce a dielectric sphere into the medium okay and whose dielectric constant is epsilon 2. So, we introduce a spear and the dielectric constant is epsilon 2, as you can see in this figure here and when you introduce this sphere into the uniform electric field, the electric field itself gets better okay, so because of the presence of the electric field, this dielectric sphere is going to be polarized.

And the polarization of the dielectric sphere is going to perturbed the electric field okay, so and that is what we see here, then uniform nature of the electric field is perturbed and the whole electric field gets modified as you see it here on the right side okay. So, we can say that because of the modification of the electric field, potential distribution also changes, so let us say phi is r, theta psi is phi1 r theta.

So, here we are talking about a sphere in an uniform electric fields, so there is no azimuthal dependence okay, psi; there is no variation along psi, so the 2 coordinates which are important is r and theta okay. So, let us call the potential inside the liquid is phi 1 r theta when r is > a, okay and the potential is phi2 r theta, when r is < a, okay. Now, to obtain the distribution of this potential, we have to solve the Laplace equation.

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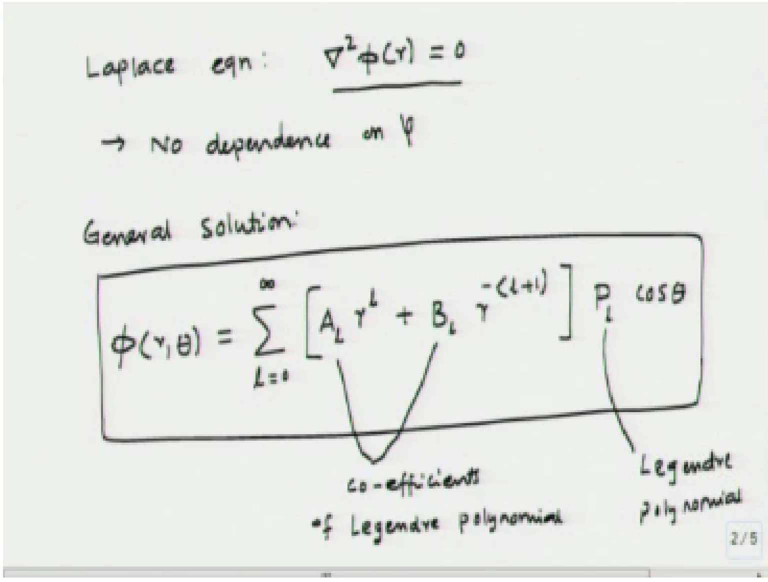


So, the Laplace equation which is nothing but del square phi r is going to be 0 and so here as you said before, there is no dependence on azimuthal coordinate; no dependence on psi, okay. So, we can write a general solution and the general solution for this Laplace equation in this case, it considers Legendre polynomial in the solution okay, so the phi r theta is going to be summation L = 0 to infinity AL r to the power L + BL r to the power –L + 1 * pL * cos theta.

So, this is the general solution of the Laplace equation for this case and so, L is; so AL and BL are coefficients of the Legendre polynomial; of the Legendre polynomial, pL is the Legendre polynomial and L varies between 0 to infinity okay. Now, these coefficients of Legendre polynomial can be determined using the boundary conditions okay, so let us look at what are the boundary conditions that we use to evaluate the solution.

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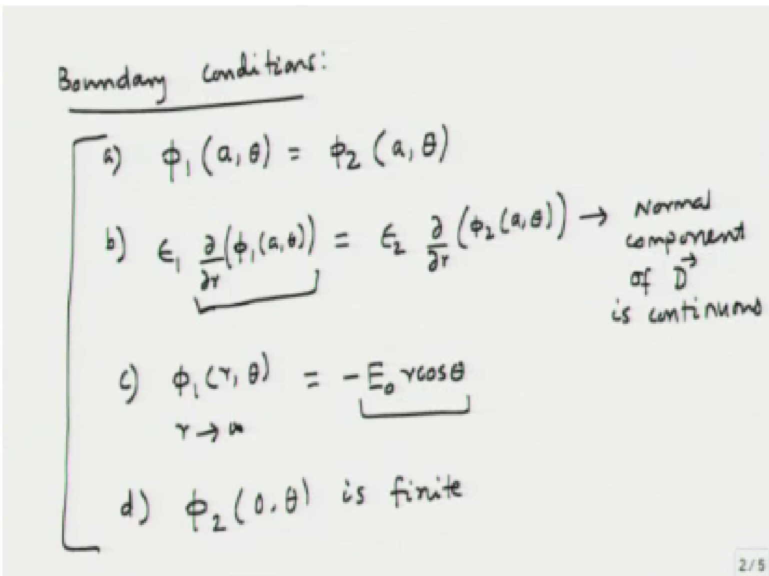

So, the boundary conditions; the first boundary condition is that the potential phi1 at r = a theta will be = phi2 at r = a and theta, so there will be no discontinuity in the potential okay. So, here we say that phi1 is outside the sphere and phi2 is inside the sphere but here we say that the boundary conditions the phi1 at r = a, should be = phi2 at r = a, so there is no jump in the potential.

The second condition is that epsilon1 * del over del r * phi1 del over del r of phi1 a theta is going to be epsilon2 del over del r of phi2 a theta okay, so this is saying that the normal component of the electric displacement is continuous okay, electric displacement you know; we have seen earlier can be written as epsilon * electric field, so this gives us the normal component of the electric field.

So, you know we say that the normal component of the electric displacement is continuous. The third boundary condition is that the potential phi1 r theta, when r tends to infinity and this when r tends to infinity; if you see here at r tends to infinity, the electric field is unperturbed okay, so the modification of the electric field or modification of the potential distribution is around the sphere, as you move away from the sphere; far away from the sphere the electric field is still unperturbed, the potential distribution is still unperturbed okay.

So, we can say that r tends to infinity, the potential distribution is same as the unperturbed potential, so r cos theta and this is the value when before we introduce the sphere into the electric field; uniform electric field and the last boundary condition is that phi2 0 theta is finite okay, that means the potential at r = 0 has a finite value. So, with these 4 boundary conditions, we can use to evaluate the unknowns in this solution; expression for phi r theta.

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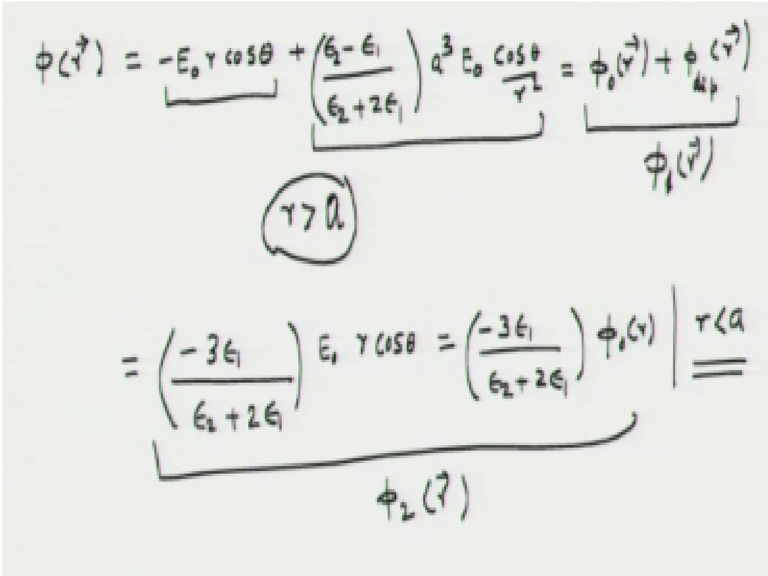


So, if you do that; if we do that, we can write the expression for phi, so phi r okay, is going to be – epsilon 0 * r cos theta + epsilon 2 – epsilon 1/ epsilon 2 + 2epsilon 1 * cube E0 cos theta or r square okay, which is =; so this is nothing but the unperturbed potential when you do not have the sphere in the uniform electric field, so phi 0 r and this potential is because of the dipole okay.

Since we introduced a dielectric sphere, in which the dipoles get developed, so this potential comes from the dipole that gets introduced inside the sphere, so the dipole potential okay and this is valid for r = r > a, so this is r > a, okay. Now, we have another solution when r is < a okay, so that would be -3 epsilon 1/ epsilon 2 + 2epsilon 1 * E0 r cos theta, which will be = -3 epsilon 1, epsilon 2 + 2epsilon 1 * phi0 r, r < a, okay, so this is for r < a, okay.

So, what we see here is that the potential phi2, so this is nothing but phi2, right; this is phi2, this is inside the sphere and this is nothing but phi1, okay; phi1 right, so what we see here is that the potential distribution inside the sphere is directly proportional to the unperturbed potential okay, phi0 r. So, phi 2 r is going to be proportional to phi0 r, which is the unperturbed potential and the potential distribution outside the sphere okay, phi 2 r is going to be related to the dipole potential okay.

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Observations:

1. Potential $\phi_2$ inside sphere is proportional to unperturbed potential
2. Potential $\phi_1$ outside sphere depends on the dipole potential

$$\phi_{dip} = \left(\frac{\epsilon_2 - \epsilon_1}{\epsilon_2 + 2\epsilon_1}\right) a^3 E_0 \frac{\cos\theta}{r^2}$$

So, the 2 things we observed from here, so we observe that the potential phi2 inside the sphere is proportional to unperturbed potential okay, so this is what we see here right, so phi2 r is directly proportional to phi 0 r and the second observation is that the potential inside; sorry, in the liquid outside the sphere is related to the dipole potential okay. So, the potential phi1, which is outside the sphere, depends on the dipole potential.

Now, if you compare this term here, which we say that it is because of the dipole that introduced; gets induced inside the sphere with the potential because of the point dipole okay and that is something we saw here. If you compare that with this expression, which is the potential distribution because of a dipole okay, then we can express, what is the dipole moment for the sphere case? okay.

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dipole potential

$$\phi_{dip} = \left(\frac{\epsilon_2 - \epsilon_1}{\epsilon_2 + 2\epsilon_1}\right) a^3 E_0 \frac{\cos\theta}{r^2} \rightarrow$$

$$\phi_{dip} = \frac{B\cos\theta}{r^2}, \quad B = 4\pi\epsilon p$$

Induced dipole moment:

$$\boxed{\vec{p} = 4\pi\epsilon \left(\frac{\epsilon_2 - \epsilon_1}{\epsilon_2 + 2\epsilon_1}\right) a^3 \vec{E}_0}$$

So, the phi dipole is epsilon 2 – epsilon 1/ epsilon 2 + 2epsilon 1 * a cube E0 cos theta over r square, so that is the solution what we saw here okay. Now, if you compare that with the dipole potential which is we derived earlier is B cos theta over r square, where B is 4pi epsilon * p. So, from there, we can obtain for this case; for the sphere case what is the dipole potential; a dipole moment; sorry.

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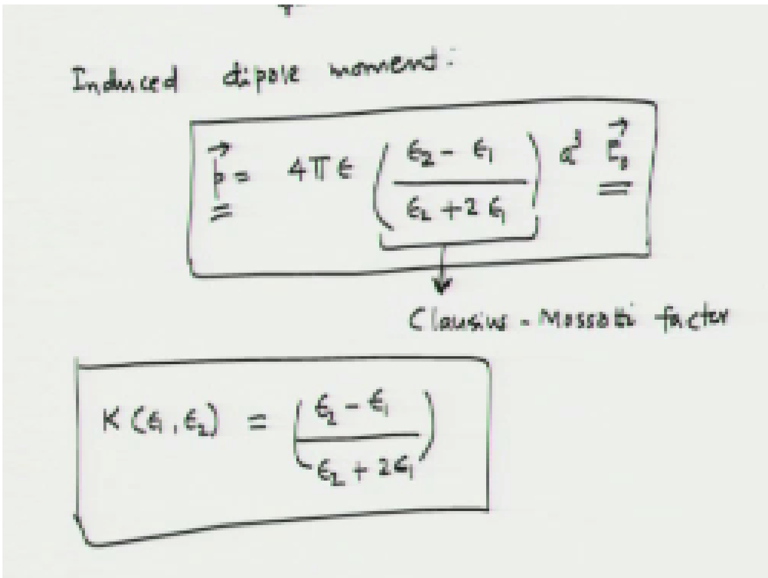


So, we can say the dipole induced dipole moment; moment p is going to be 4pi epsilon okay * epsilon 2 – epsilon 1/ epsilon 2 + 2epsilon 1 * a cube * electric field okay, so that is going to be the expression for the dipole moment p, right and this parameter here; epsilon 2 – epsilon 1/ epsilon 2 + 2epsilon 1 is called Clausius mossotti factor, so you can write the Clausius mossotti factor K epsilon 1, epsilon 2 is epsilon 2 – epsilon 1/ epsilon 2 + 2 epsilon 1, okay.

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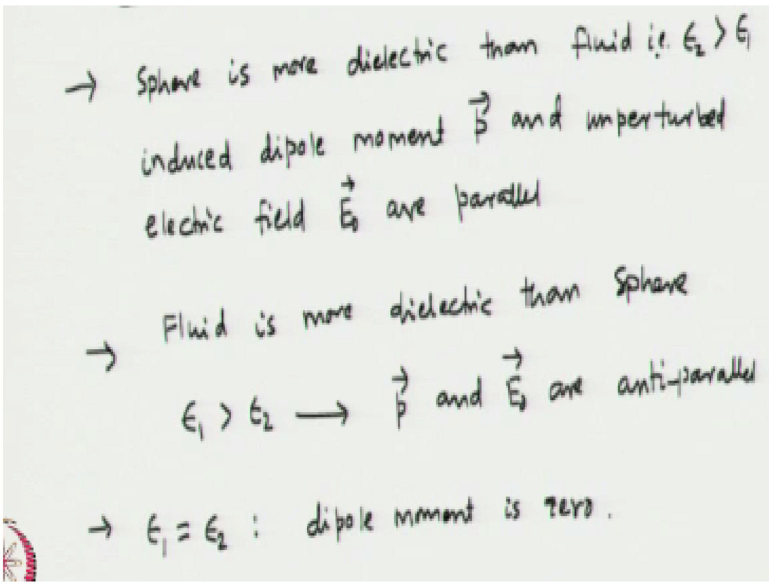

Now, what we see here is that the direction of the induced dipole moment is dependent on the; with respect to the direction of the electric field is dependent on the Clausius mossotti factor okay, so if the sphere is more dielectric than the liquids, so epsilon 2 is > epsilon 1, then the dipole moment and the electric field are going to be parallel okay. So, what we observe here is if the sphere is more dielectric than fluid, so that is epsilon 2 is > epsilon 1.

Then, the induced dipole moment p and the unperturbed electric field, which is E0 are parallel okay, so they are going to be parallel and if you see that the medium is more dielectric than the sphere, then the direction of the induced dipole moment and electric field are going to be antiparallel. So, if the fluid is more dielectric than sphere that means, epsilon 1 is > epsilon 2, so the dipole moment and the electric field are antiparallel.

And if the dielectric constant of the fluid and the sphere are equal, then the dipole moment is going to vanish, okay. So, the dipole moment is 0, okay, so with that discussion on the dipole moment, let us move on and talk about what is the dielectric force that can act on this dielectric sphere, when it is present in an electric field okay. We have discussed how the induced dipole moment is going to look like when it is placed in an uniform electric field.

So, you know the expression for the dipole moment is relatively simple but when this is placed in a non-uniform electric field okay, in that case we would only realize that there is a dielectric force that exists, so in case of a non-uniform electric field, the calculation of the dipole moment is more involved because in that case, we will not only have dipole moment but we also have multiple moment.

Meaning; you would not only have only a cluster of positive and negative charges, which would give you a; in a single positive clusters, single negative cluster that will give you a dipole but we would have multiple moments okay but if we assume that the radius of the sphere is very small as compared to the length scale of the field still, we can assume that we would have only dipole moment existing okay.

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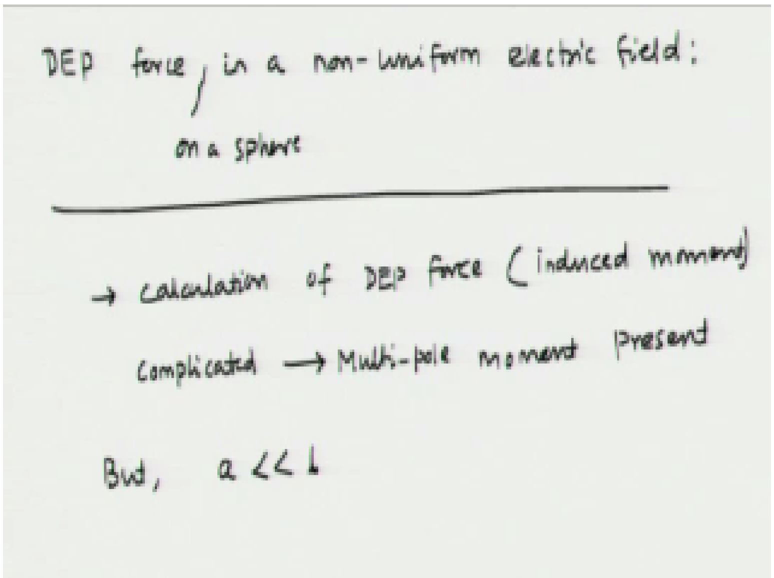


So, let us look at the DEP force or the dielectric; for a dielectrophoretic force; DEP force in a non-uniform electric field, so DEP force on a sphere in a non-uniform electric field. So, in this case, the calculation of DEP force or the induced moment is complicated, so this is because we have dipole moment present okay but if we say that the radius of the sphere is << the length scale of the field L, then we can expand the electric field around the centre of the sphere okay.

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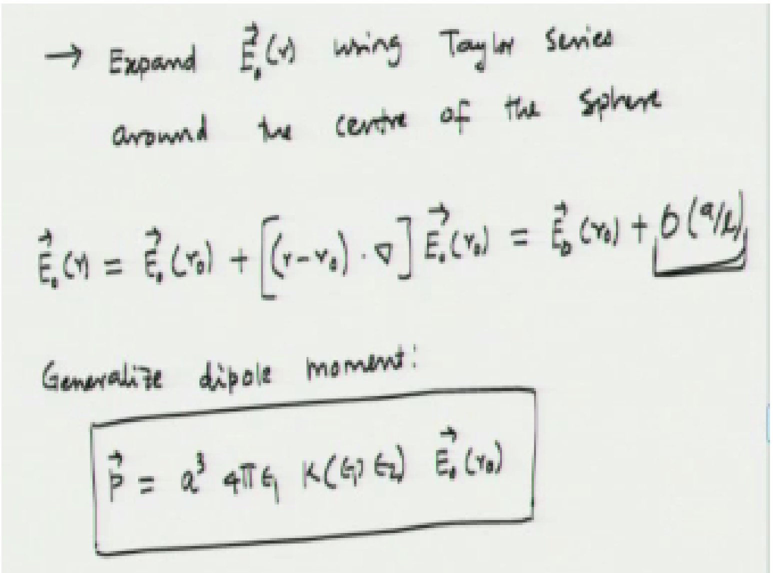


So, in that case we can expand the electric field using Taylor series around the centre of the sphere okay. So, if we do that, we will say epsilon 0 * r is going to be epsilon 0 r0, so that is the centre of the sphere + r – r0 * the gradient of the electric field okay, so which is nothing but E0 r0 + a term, which is of the order of a over L and since, a over L is negligible, this term can be dropped up, okay.

So, we can generalize the dipole moment; the dipole moment okay as p, we can write it as; a cube * 4pi epsilon1 * the Clausius mossotti factor; epsilon 1 epsilon 2 * E0 r0, okay, so what we see here; this is what we derived right, so this is the electric field. So, instead of writing the general electric field, we are writing this electric field around the centre of the sphere and this is when we are talking about a non-uniform electric field okay.

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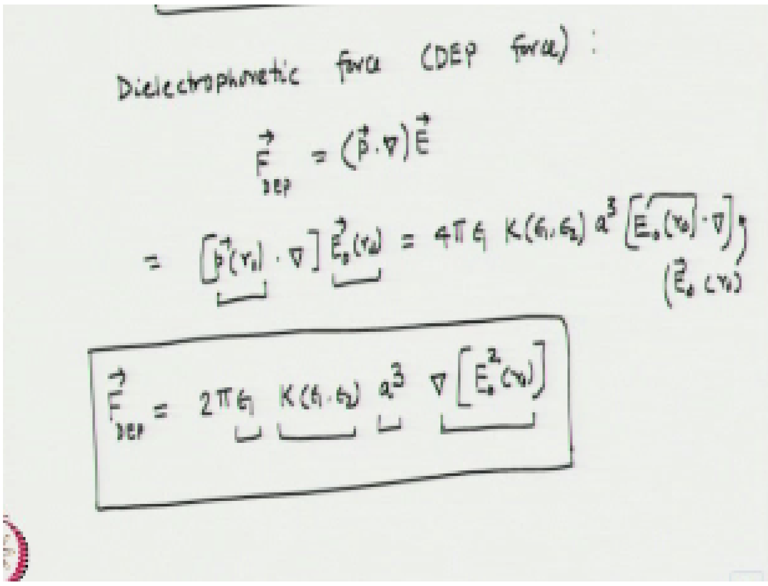


So, now we can determine the dielectrophoretic force or the DEP force and we had looked at the formula to do that. So, F DEP is going to be the dipole moment, to the gradient of the electric field, right, so this is going to be = p r0 * radiant of the electric field rj, okay. So, this is vector, so that is going to be 4pi epsilon 1 * the Clausius Mossetti factor * a cube, so we have; you know one electric field coming from there and there is one more here.

So, that becomes; you can write it as; E0 r0 at gradient of E0 r0, okay, so this would become 4pi epsilon 1, now this we can take this expression for the electric field, we can take it inside the gradient and so that will become E0 r0 square, now when it comes out of the gradient, it becomes twice of the electric field, so here we can observe 2 in there, so this becomes 2pi epsilon 1 * K epsilon1 epsilon 2 * a cube * gradient of the square of the electric field okay.

So, this is going to be the expression for the dielectrophoretic force okay, so that dielectrophoretic force would depend on the Clausius mossotti factor, the dielectric constant of the fluid, the size of the sphere and the gradient of the square of the electric field okay, so what we learn from here is that the dielectrophoretic force is independent of the sign of the electric field okay.

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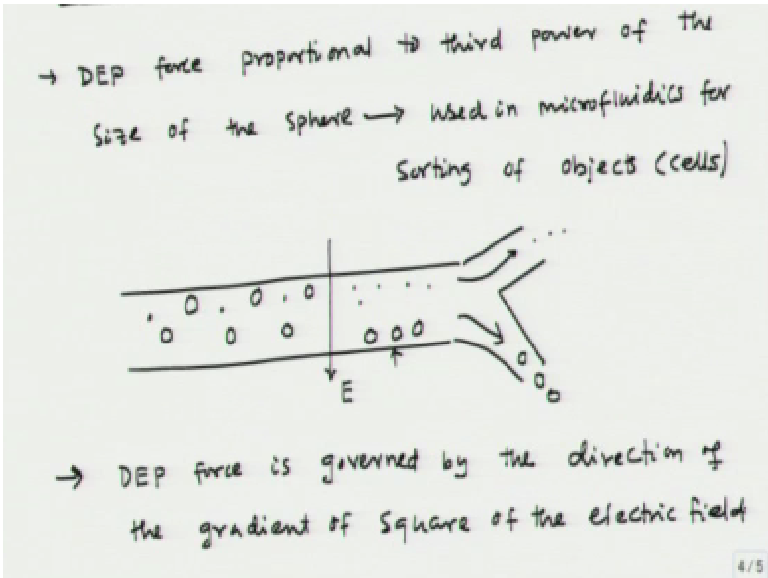


And it is dependent on the square of the gradient of the square of the electric field, it is directly proportional to the third power of the size of an object and it is the sign of the dielectrophoretic force is dependent on the sign of the Clausius mossotti factor and it is going to be proportional to the dielectric constant of the fluid in which the object is present okay. So, what we observe from here is that the DEP force is proportional to third power of the size of the sphere.

So, although here we are talking about a sphere; in general, the DEP force is going to be proportional to the third power of the size of an object, so this can be used in microfluidics to sort objects of different sizes okay, where for a smaller object the dielectrophoretic force will be less as compared to that for a larger object okay. So, this can be used in microfluidics or sorting of objects like cells, okay.

For example, you know if we; let us consider a channel and we are able to establish an electric field in this direction okay and here we have a mixture of small and large objects okay and assuming that they have the same dielectric properties then, because of the electric field, the larger objects will be subjected to larger dielectrophoretic force, so they will move towards the wall okay, irrespective of their initial position, they will be moving towards this wall.

And the smaller object will tend to remain in their original position okay, so they can be separated okay. So, where you can take out smaller particles and separate it from large particles okay and we also observe here that the DEP force is governed by the direction of the gradient of square of the electric field okay. So, even though the dielectric force will not depend on the sign of the electric field, it will depend on the gradient of the square of the electric field, okay.

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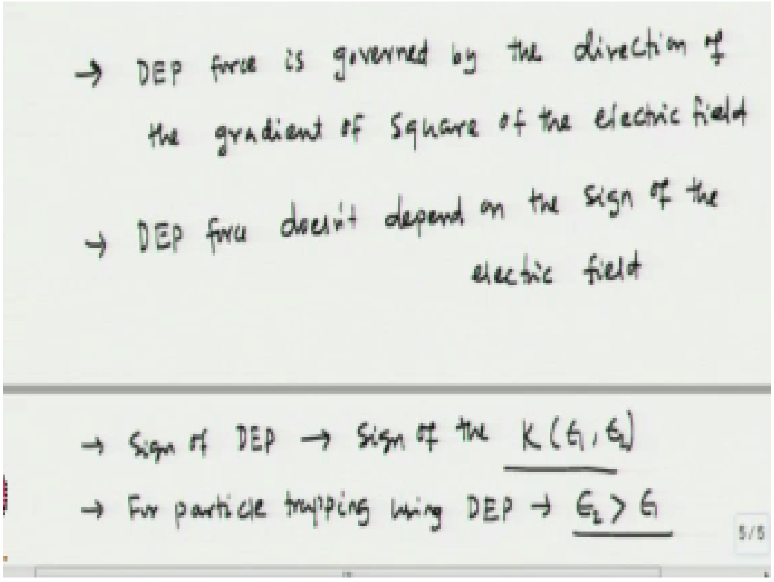


And the other important observation is that; so here we say that, you know DEP force does not depend on the sign of the electric field okay, so that is; so the other important observation is going to be that the sign of DEP dielectrophoretic force is given by the sign of the Clausius mossotti factor, okay. So, you can see from this equation here, the direction of the electrophoretic force is very much dependent on the sign of the Clausius mossotti factor.

And if the sphere is more dielectric than the liquid, the dielectrophoretic force is positive okay and this is the requirement when you are talking about there is a trapping of particles okay. in microfluidics; in some applications we are required to; you know pull particles from the bulk of the liquid towards an electrode, so in doing so you would require the dielectrophoretic force to be positive, so they can come towards a specific point or towards the electrode.

So, in order for the trapping to occur, the dielectric constant of the objects or the cells or the particles has to be higher than the dielectric constant of the liquid okay. So, for particle trapping using DEP, the dielectric constant of the sphere has to be greater than the dielectric constant of the liquid okay. So, with that let us move on and talk about trapping of particles in microfluidics, okay.

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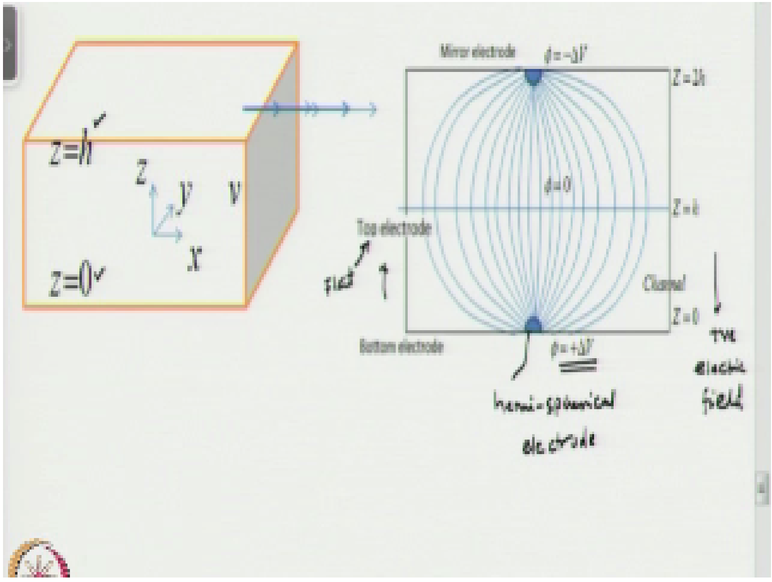

So, this is the case we have here; you know we are talking about a rectangular channel, so you know the bottom wall is at z = 0 and the top wall is at z = h, so you know the flow is in the x direction; the positive x direction. So, what we have here; you know as I told for the dielectrophoretic action to come into play, there has to be a non-uniform electric field okay, so if you look at here, you know we have an asymmetric electrode configuration.

On the bottom surface of the channel, we have a; you know hemispherical electrode, so this is a hemispherical electrode and our z = h; we have a flat electrode, so this top electrode is flat. So, we create an electric field having; you know phi = 0 at the top electrode and phi = some positive voltage at the bottom hemispherical electrode, so we create an electric field however, this electric field gradient is positive in this direction okay.

So, the lines are more concentrated close to the hemispherical electrode, so in this direction we maintain a positive electric field okay. So, you know when the particles are flowing okay, the 2 forces that will come into picture when you are talking about trapping; one is the dielectrophoretic force, which will come into picture. If you are trapping towards the electrode, it has to be positive okay.

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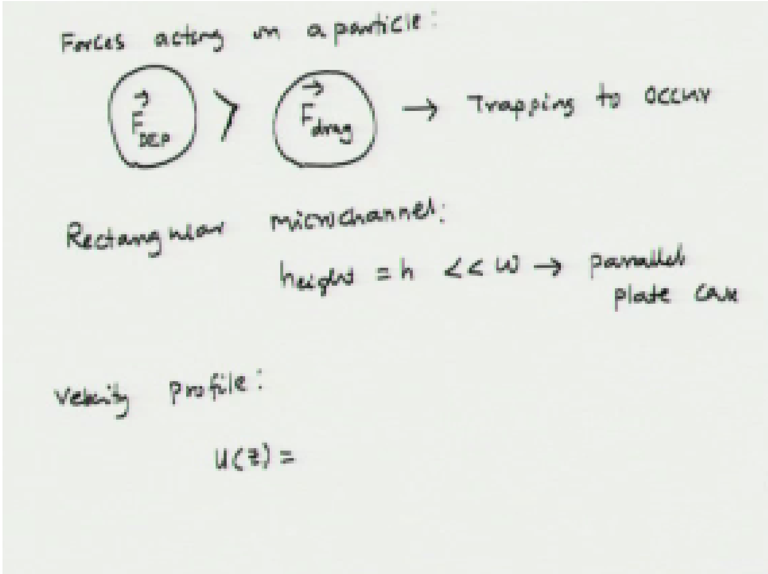


And for that to happen, the dielectric constant of the sphere has to be or the particle has to be more than that of the liquid, so that is how you know the first force, the second force that comes into play is the drag force, which is trying to carry the particle along with the flow, okay. So, we are talking about; you know forces acting on a particle; one is the dielectrophoretic force, F DEP and the other one is the drag force.

So, you would calculate you know the DEP force and drag force individually and for the balance for the trapping to occur, the dielectrophoretic force has to be greater than the drag force, so this has; the dielectrophoretic force has to be greater than the drag force for trapping to occur, okay right. Now, we for the rectangular micro channel, so here we say that the height is h and the height h is <<, w which is the width.

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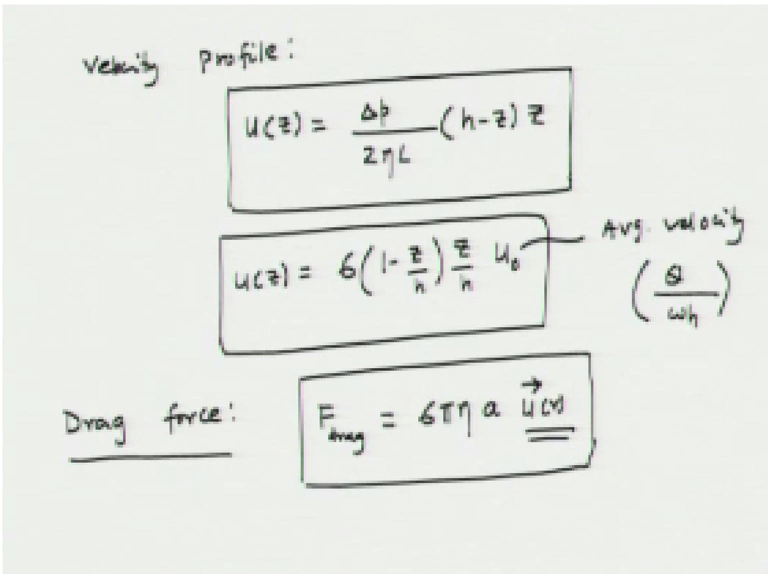

So, here we are talking about; so this is h okay and this is w, we say that h is << w, so it is almost a parallel plate flow situation, so this is parallel plate case and in that case we know the velocity profile; velocity profile looks like; so u * z; z is here; so this direction is z, so the velocity profile uz is given by delta p/ 2 eta L * h - z * z, so this is how the velocity of the fluid containing the particles is going to vary, right.

So, we can also write this in another form, we can write uz is going to 6 * 1 – z/h * z/h * u0; u0 is the average velocity okay, so u0 is the average velocity, which is Q over wh, so we can find an expression for the drag force. Drag force; F drag is going to be 6pi ETA * radius of the particle * ur, okay. So, we do not know where the particle is; so, let us say, a particular is present at some location r, where the velocity is ur, okay.

So, now we have to first find a potential distribution between the 2 electrodes, since we have 1 spherical electrode, 1 flat electrode to find the potential distribution will be using the method of mirror okay, so in; what will be doing is; we will be considering another in a spherical electrode as a mirror image okay on the opposite side of the top electrode and considering that will be finding out the potential distribution.

And knowing the potential distribution, we will try to find an expression for the dielectric force, will compare the dielectric force with the drag force for the trapping dielectrophoretic force has to be greater than the drag force, so you try to find out there has to be a limit on the maximum velocity of the particle above which trapping may not be possible. So, we are trying to find the limiting velocity of the fluid okay, so we will continue this with that let us stop here.