Microfluidics
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Lecture – 16
Electrokinetics (Continued...)

Okay, so we have been looking at electroosmotic flow in presence of backpressure. We consider considered the Navier Stokes equation and applying boundary condition. We saw that, you know, we can obtain a solution by dividing the equation into 2 parts were in the first part we will be talking about the electroosmotic flow and the second part we will talk about pressure driven flow and we will add the 2 solutions to obtain the solution for electroosmotic flow with backpressure, okay.

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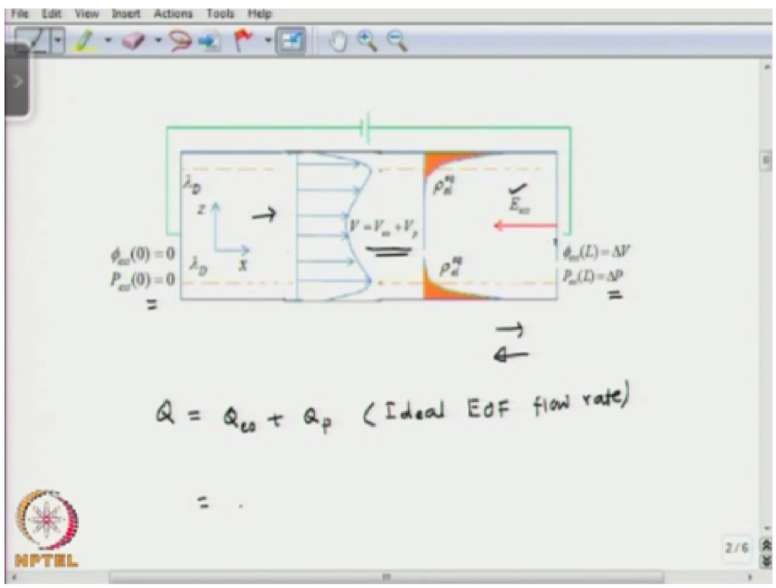

You know, this is the situation we have been looking at. We have been looking at flow in a circular pipe where we have the walls positive zeta potential and the electric field is in this direction as you can see here. So, we would expect the electroosmotic flow to occur from left to right okay in this direction and that the pressure gradient is positive in this direction. We have delta P and P=0 here. So, the pressure driven flow is going to occur from right to left, okay.

These 2 can be added together to obtain a solution for electroosmotic flow in presence of pressure (()) (01:37). So, we obtained a solution for the flow rate, okay. So, the flow rate Q=Q electroosmotic + Q pressure driven and this is for ideal conditions, so ideal electroosmotic flow rate, okay.

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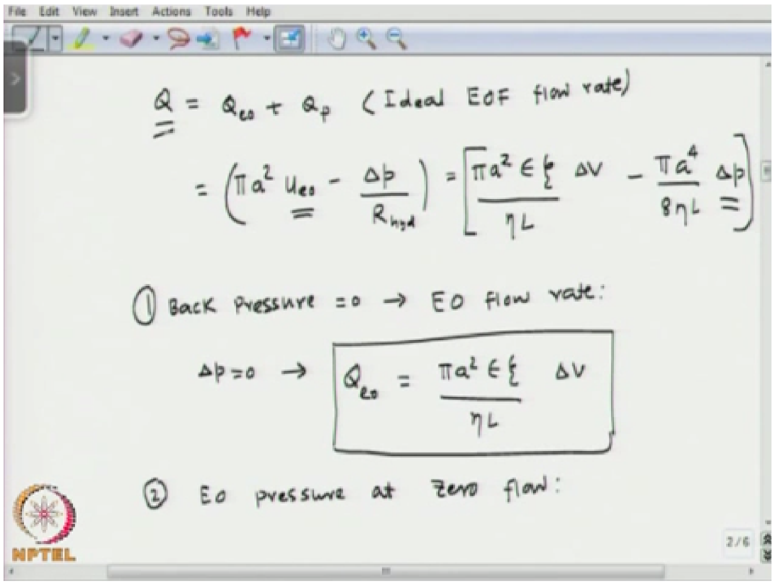

So, we saw this is equal to pia square U electroosmotic-delta p/the hydraulic resistance and we saw this as pia square epsilon xi/eta L*delta v. So, delta v/L is the electric field and epsilon xi electric field/eta is the electroosmotic velocity and pia square is the area. So, -pi a4/8 eta L*delta p. 8 eta L/pi a4 is the hydraulic resistance. So, we get an expression for the total flow rate, okay. So, from here we can find out the pressure flow characteristics of the electroosmotic pump, okay.

So, it is called PQ characteristics. If we say that the 0 flow what is the maximum pressure that we are going to expect from the pump and for 0 backpressure what is the maximum flow rate that we will be expecting from a pump. Pumps are characterised by their pressure and flow characteristics, okay. So, let us look at here. Now if we say the backpressure is 0, so this is the first condition, what is the electroosmotic flow rate, okay.

So, you can find in this equation if you put the backpressure to be 0, okay then let say delta p=0 we can find the flow rate which is the maximum flow rate at 0 backpressure is pia square epsilon xi/eta L*delta v, that is the maximum flow rate we can expect. Similarly, we can do for the electroosmotic pressure at 0 flow, okay. So, now in this equation if you put Q2 to be 0 okay, we can get an expression for the backpressure.

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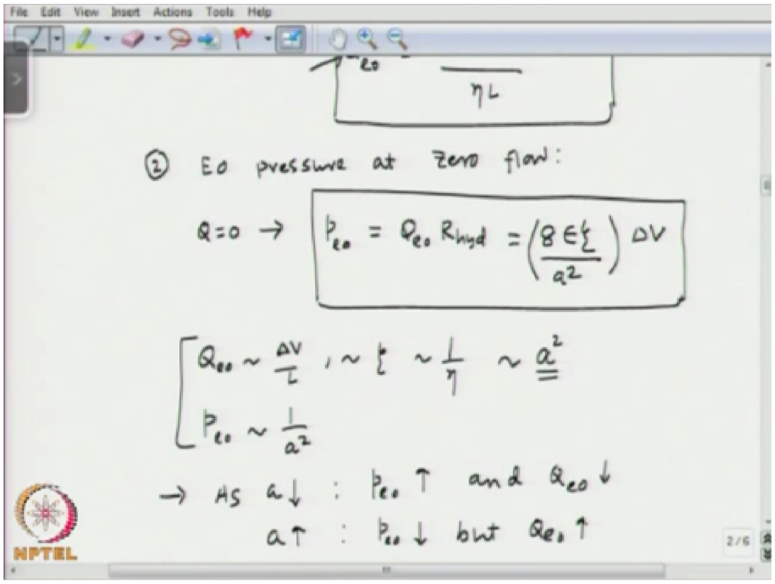

So, if you do that, if you put Q=0 you can get the expression for the maximum electroosmotic pressure to be Qeo*R hydraulic which is from here. So, this the Q electroosmotic and since this side is 0*R hydraulic, okay. So, we will get this as 8 epsilon*xi/a square*delta v, okay. So, what we see here. We see that the electroosmotic flow rate Qeo depends on the electric field is directly proportional to electric field.

So, Qeo is directly proportional to electric field or delta v/L and it is also directly proportional to the zeta potential and inversely proportional to the viscosity and it varies as square of the channel size, okay it varies as a square, whereas as if you look at the backpressure Peo varies as 1/a square. So, what do we mean from here is in electroosmotic flow, if we increase the size of the channel, the electroosmotic flow rate is going to increase but at the same time, the backpressure the pressure capability of the electroosmotic pump is going to reduce as 1/a square.

Whereas if you reduce the channel size, we will get very good pressure capability, okay. So, it will increase as a square, but at the same time the flow is going to reduce, okay. So, what we learn from here is that as a is reduced, the pressure capabilities increased and the Qeo is reduced and as a is increased the pressure capability is reduced but flow rate is increased, okay.

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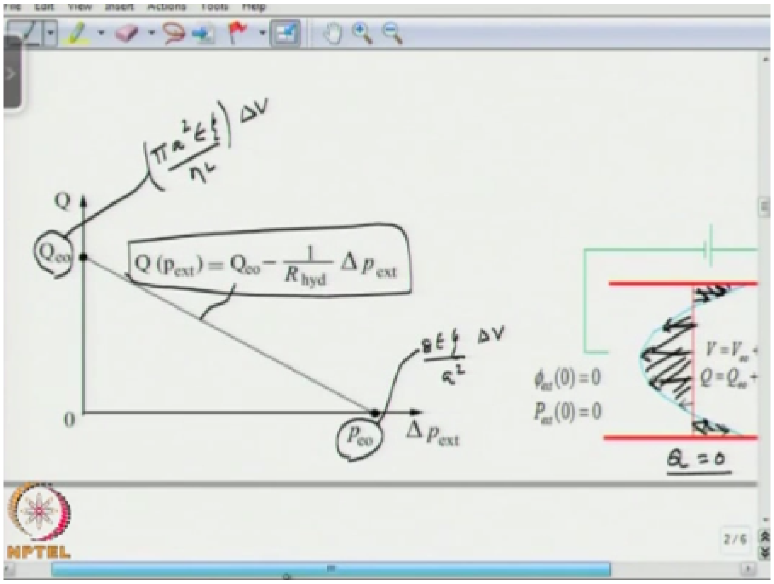

So, here we see the PQ characteristics plotted, okay. This is Peo which is given by 8epsilon xi/a square*delta v and this is the maximum flow rate which is given by pia square epsilon xi over eta L*delta v, okay. So, this is the maximum flow rate when the pressure is 0 and this is the maximum pressure when flow is 0, okay and the curve is given by this equation here. What do you mean by when Q is 0 when you get maximum pressure, what we mean is this, okay?

So, this is a situation where the net flow is 0, that means we have electroosmotic flow occurring close to the wall from left to right. At the same time, the pressure driven flow is at the middle which is from right to left. So, this area is going to be same as these 2 areas added together, okay. So, then we would have ne flow to be 0, right.

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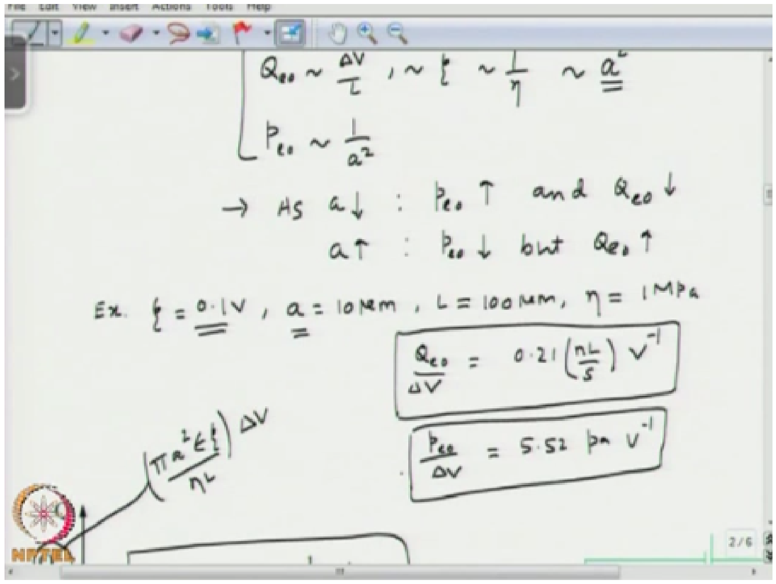

Now, if we take an example, let us consider an example, okay. Let us say zeta=0.1 v and a=10 micron. So, we have zeta potential 0.1 v and the size of the channel is 10 micron and the length of the channel has hundred microns and the dynamic viscosity is 1 Mpa, then we can find Qeo/delta V is about 0.21 nL per second per volt, okay and we can also find Peo/delta v=5.52 Pascal per volt, okay. So, what we see here is the flow rate and the pressure for unit potential difference is not much, okay. So, what we need to do is to use multiple channel pump, okay.

So, in that case we can reduce the channel size so that we can get much better pressure capability. At the same time, you can have multiple n number of such smaller channels in parallel so that would give us n times the flow capability of a single channel electroosmotic pump, okay. So, this single channel electroosmotic pump, you know, do not have practical applications in microfluidics.

For example, if you want to drive some fluid into a microfluidic device such single channel electroosmotic micropump may not be suitable. But these pumps may have some applications for example in electrophoresis where you want to you know carry some sample plug from one location to another because one interesting observations from electroosmotic flow is that the profile is plugged unlike parabolic, so there is no diffusion that occurs between different sections of the sample plug.

So, the sample plug is restored when it is transported from one location to another, okay. So, you know, for some microfluidic applications we are interested in just transporting a sample plug from one location to another, we can use a single channel electroosmotic pump, okay. But for many microfluidic applications, we would need much higher pressure capability and much higher flow capability for the applications, okay.

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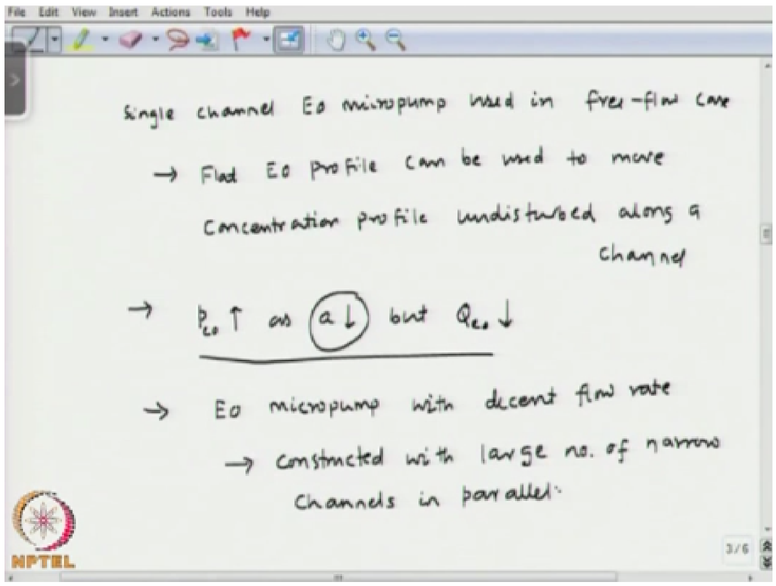

So, what we observe is that the single channel electroosmotic micropump used in free flow case and the flat electroosmotic profile can be used to move concentration profile undisturbed along a channel, okay and as you know that the pressure capability goes up as the channel size goes down but with channels as going down, the electroosmotic flow goes down, okay. So, for that reason if you are interested in decent flow rates, electroosmotic micropump with decent flow rate can be constructed with large number of narrow channels in parallel, okay.

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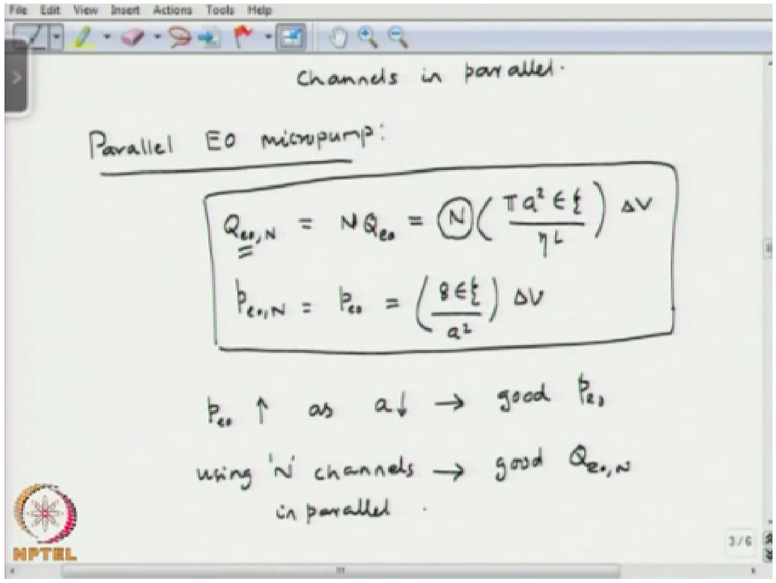

So you can have large number of narrow channels in parallel to construct the electroosmotic micropump and that is called parallel electroosmotic micropump where we can consider n different micro channels each having channel size of A and length Left, okay. So, if you do that we can say the Q electroosmotic or n channels is going to be n times the pressure capability of a single pump, okay equal to N*pia square epsilon xi/eta L*delta v and the flow capability Peon is electroosmotic pressure which is 8 epsilon xi/a square*delta v, okay.

So, what we see here is that the electroosmotic pressure will go up as a is going down, so we can obtain good pressure capability, okay and you know. So, with increase in a even if the electroosmotic flow rate for a single channel will go down, we can have many such channels in parallel. So, you know, using n different channels in parallel, we can also obtain good flow rate and it possible to fabricate such large number of parallel channels using microfabrication technology, okay.

So, it is possible to obtain good pressure capability and good flow rate using electroosmotic micropumps. Now, such micropumps if you want to increase the flow rate or increase the backpressure further, one approach to do that is increasing the voltage, okay and for lab-on-chip applications, increase of voltage is limited, okay. You cannot keep on increasing the voltage to any extent.

So, one approach that could be followed is if you can obtain good pressure and flow rate without increasing the applied voltage significantly and one such approach is known as Cascade electroosmotic micropump where we will have a net voltage drop 0 across one space; at the same time, we should be able to obtain the required flow rate and pressure capability, okay. So, let us see how we can do that.

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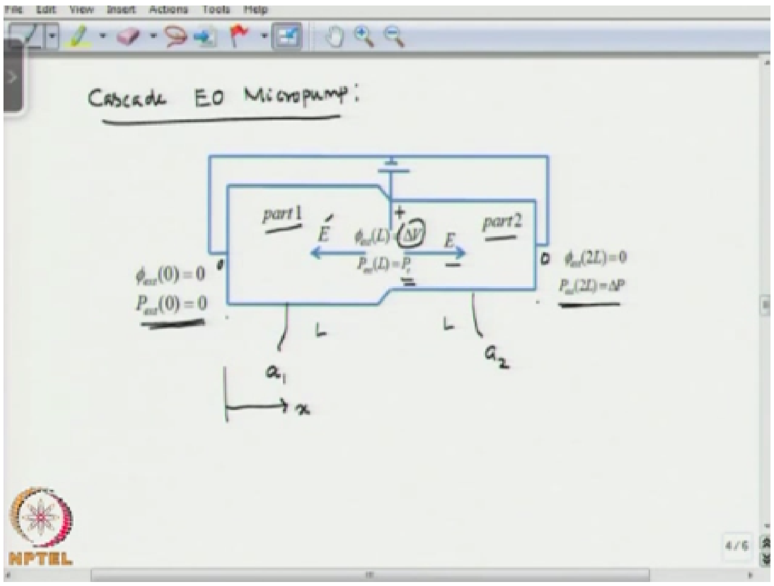

So, what we see here is a cascade electroosmotic micropump, so we call it cascade electroosmotic micropump, okay. So, we have a section of the channel, okay part 1 and part 2. Part 1 has channel size a1and part 2 has channel size a2 and we have the positive terminal at the intersection between 2 parts somewhere in the middle here, so this is positive and the other 2, this here the potential is 0 and here the potential is 0 and the potential here is some delta v, okay.

So, the electric field in part 2 is towards the right as you can see here and electric field in part 1 is towards left as you can see here and the pressure at the entrance of the channel is 0. Let us say here on the left hand side pressure is 0. Let us say pressure at the right hand side where x=2L, let us say the length of the channels are equal, this is L and part 2 is also L. So, the pressure at x=2L is delta P. Let us say x starts from here, okay and pressure at x=0 is 0, pressure at x=2L is delta P and pressure at x=L is some pressure Pc, okay.

So, in that situation the net voltage drop going from x=0 to x=L is going to be 0, okay. So, in spite of having net 0 voltage drop across the stage, we can see how we will be able to generate pumping action, okay.

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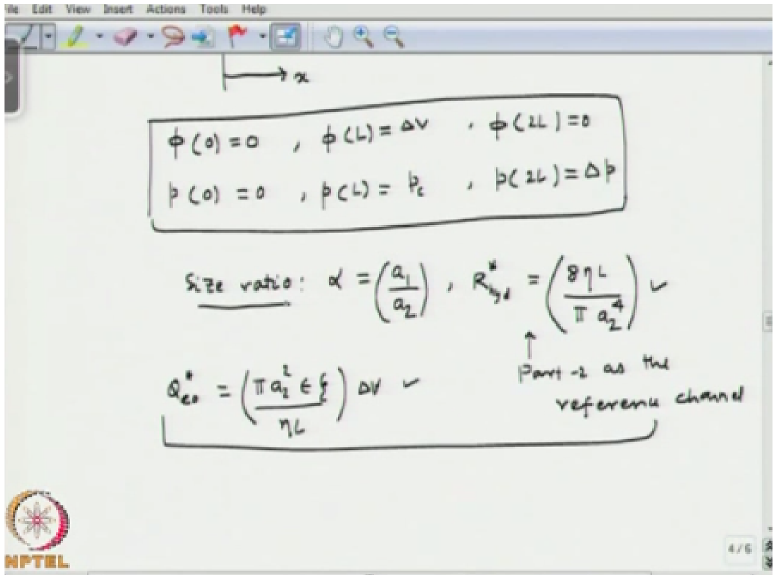

So, you know we see that the phi at x=0 is 0 and we can see pressure 0=0 is 0 and the phi at x=L is going to be delta V and P at x=L is some Pc, okay and phi at x=2L is going to be 0 and pressure at x=2L=delta p. So, these are the boundary conditions and we apply that to this situation here where we have 2 channels in series and we have the potential difference as shown here, okay.

So, we can define something called a size ratio which is alpha and this is 1/a2, okay. The size of the channel in part 1 divided by the size of the channel in part 2 is called size ratio. So, you can define a parameter R hydraulic star which is 8 eta L/pi a2 4. So, let us consider part 2 as the reference channel, okay. So, part 2 if you consider as the reference channel, this considers part 2 as the reference channel, okay and we can find Qeo star=pi a2 square epsilon xi/eta L*delta v, okay.

So, considering part 2 as a reference channel, finding out hydraulic resistance, electroosmotic flow.

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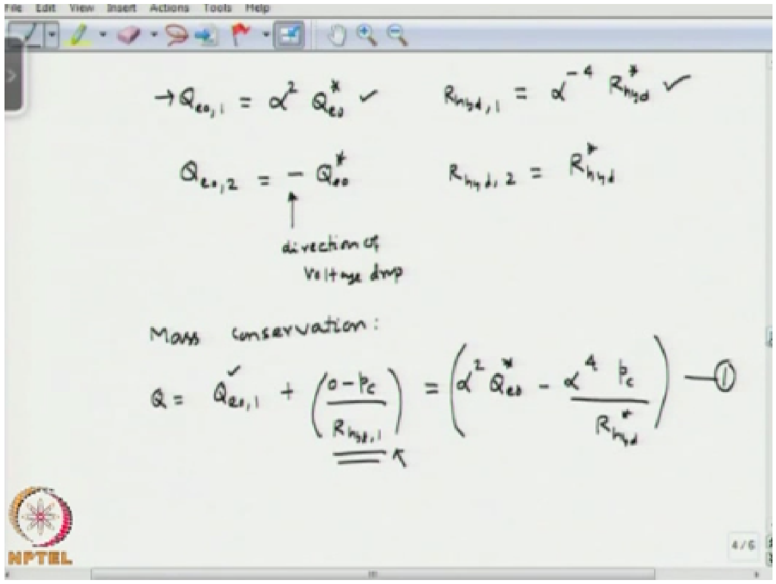

Knowing the size ratio alpha, we can write the electroosmotic flow rate in part 1 of the channel is going to be alpha square*Qeo star, okay. Similarly, you can check Qeo2 and this is going to be – Qeo star. This negative sign here is because of the direction of the voltage drop. So, as I told the zeta potential is positive. So, the electroosmotic flow occurs in the direction opposite to that of the electric field.

So, in channel 2, electroosmotic flow occurs in the negative X direction and in the channel 1, the electroosmotic flow occurs in this direction and that is the reason since it occurs at negative X direction, we get a negative sign there, okay. So, R hydraulic 1, okay the hydraulic resistance of the channel 1 can be written as alpha to the power -4*R hydraulic star and hydraulic resistance of channel 2 will be R hydraulic star, okay.

So, you can write this. Now, if you do mass conservation, we can see here if you see here, you can see that since it is a continuous channel, 2 channels are in series, mass conservation has to be satisfied, okay. If you say mass conservation, then you can write Q=Qeo1+0- Pc/R hydraulic 1, okay. So, the flow rate is going to be the flow rate due to electroosmotic flow plus that due to the pressure.

Since the pressure is 0 here and Pc there, we can accordingly find out what is the flow because of the pressure, okay and this is due to the electroosmotic flow. So, this is going to be alpha square

Qeo star as you can see from here, this will be minus alpha to the power 4 because the hydraulic resistance expression is here, so that we substitute there. So, this will be –alpha 4*Pc/R hydraulic star, okay. So, this is one equation.

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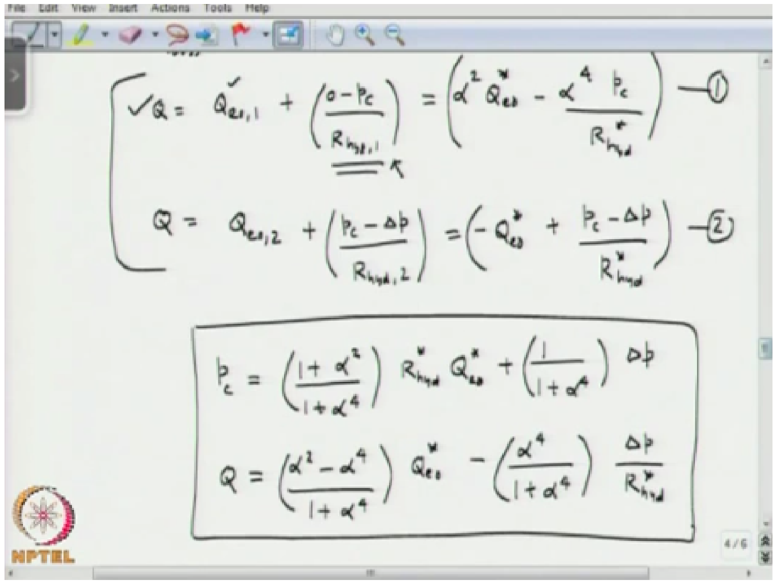

Now the equation 2 we can write for the channel 2 which Q=Qeo2+Pc-delta p. So, this flow rate is what is coming from the left to the intersection of the channel, what is coming from here till this point and then for the channel part 2 what we see it has to be same as what is coming from here to there, okay. So, if you calculate that it comes Pc-delta p/R hydraulic 2 which becomes –Qeo star which we see it here + Pc-delta p/R hydraulic star, okay.

So, becomes our second equation. Now, if you solve these 2 equations together what we get is we get an expression for the central pressure Pc=1+alpha square/1+alpha to the power 4*R hydraulic star and Q electroosmotic star+1/1+alpha to the power 4*delta p. So, we get an expression for central pressure.

We can also obtain an expression for the flow rate. We can say Q is going to be alpha square-alpha to the power 4/1+alpha to the power 4*Q electroosmotic star-alpha to the power 4/1+alpha to the power 4*delta p/R hydraulic star, okay. So, this is going to be the expression for the pressure and the flow.

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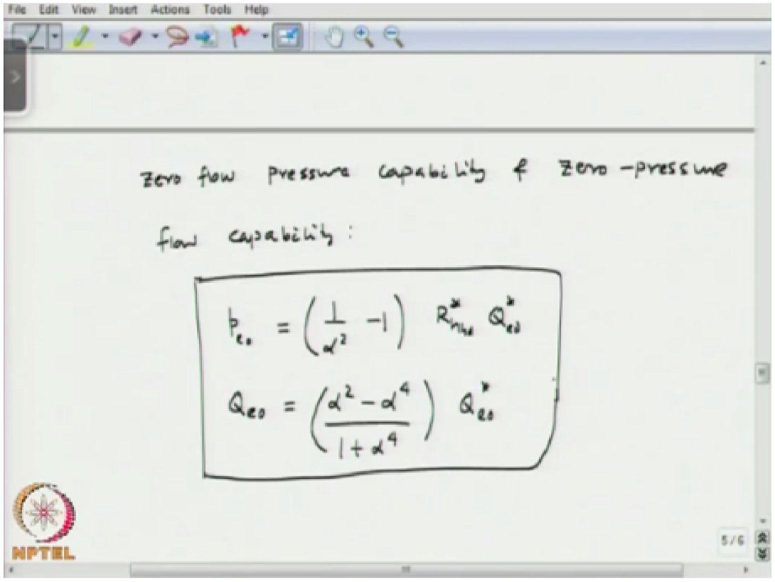

Now, from here we can obtain the expression for the 0 flow pressure capability and 0 pressure flow capability. So, from there we can find the 0 flow pressure capability and 0 pressure flow capacity, okay. So, if you do that, if in one case we put the backpressure to be 0 and the other case we put flow to be 0. If we do that we can obtain the pressure capability Peo at 0 flow is going 1/alpha square-1*R hydraulic star*Q electroosmotic star and 0 pressure flow rate is going to be alpha square-alpha 4/1+alpha 4*Qeo star, okay.

So, what we see here is that despite 0 voltage drop across a single stage, it nevertheless acts as electroosmotic pump. So, we have some pressure capability and we have some flow capability of the pump even if we have 0 net voltage drop across the pump, okay.

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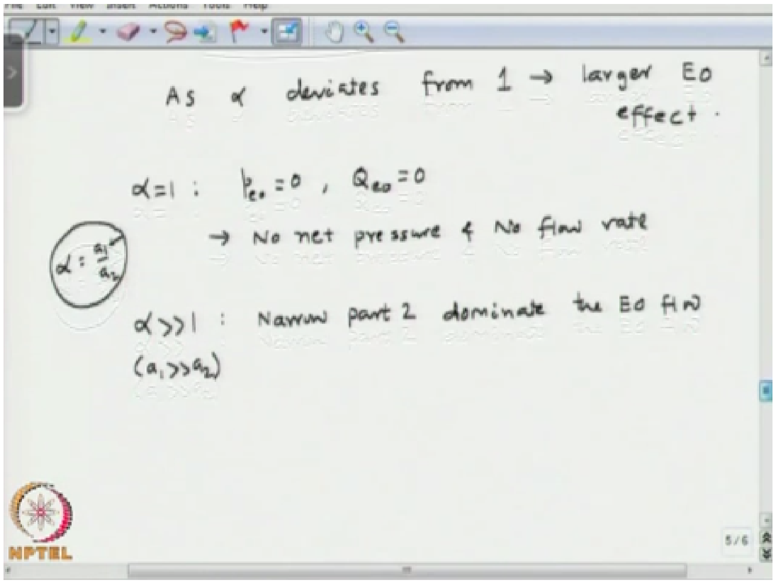

So, if you look at this expression for the pressure capability and the flow capability carefully what we see here is as alpha deviates from 1, we have larger electroosmotic effect, okay. It is very clearly seen, if alpha is very small or is very large then the electroosmotic pressure as well as the flow will be higher, okay and special case would be when alpha=1. So, for alpha=1 if you put in these 2 equations Peo will be 0 and Qeo will be 0.

So, we have no net pressure, okay and no flow rate, okay. So, when we say alpha=1 what that means is that the 2 sections of the channel part 1 and part 2 have equal size. So, in that case both produce electroosmotic effect but they are going to be equal and opposite. So, they cancel each other. As a result, we do not have net pressure capability, we do not have any net flow that is occurring, okay. So, now if you put alpha << 1, so alpha is a1/a2 right.

So, this is what we have defined, alpha is a1/a2, okay. So, alpha is a1/a2 as alpha>>1 that means a1 is much greater than a2, okay. So, it says a1 >> a2. So, in that case in this equation, if you put alpha >> 1, then you would see that the pressure as well as the flow that we get will be close to that of part 2, okay. So, we say that the narrow part will dominate the electroosmotic flow, okay.

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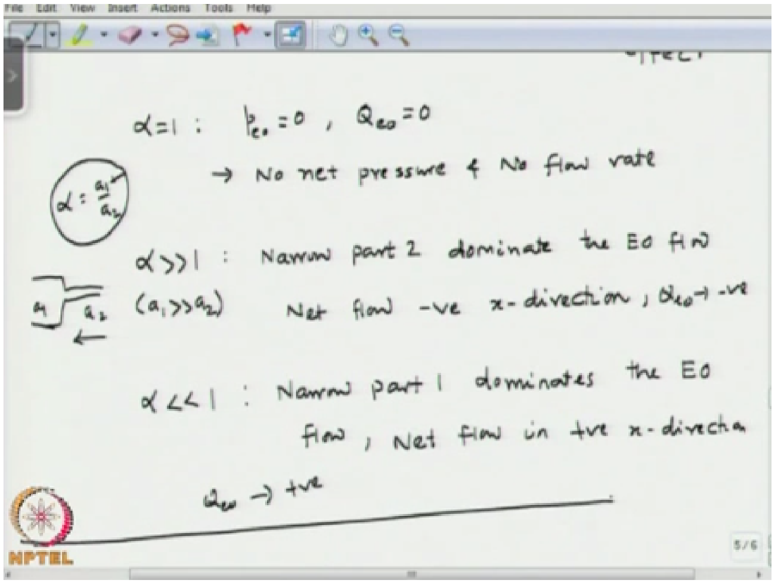

The net flow; so you can see here in part 2, the net flow is in the negative X direction, the net flow is going to be in this direction. So, for alpha>>1, the net flow is going to be in the negative X direction, okay. Now, if alpha<<1, so this is the situation when you are talking about that a1>>a2. So, we have something like this, okay. So, we have a1 is large compared to a2, okay. So, the net flow is going to be in this direction. Now, if alpha<<1, the narrow part 1 dominates the electroosmotic flow, okay. So, we will have net flow in positive X direction. This is something you can observe from this equation when alpha is very large, this is going to be negative.

So, the net electroosmotic flow is going to be in the negative X direction and when alpha<<1, then we would have the Q electroosmotic to be positive, okay. So, in this case the electroosmotic will be negative, okay. Now, we can have such you know 0 net voltage drop pumps in series. We can have many of these in series to build a micropump which can enhance the pressure drop, okay, the pressure capability of a micropump.

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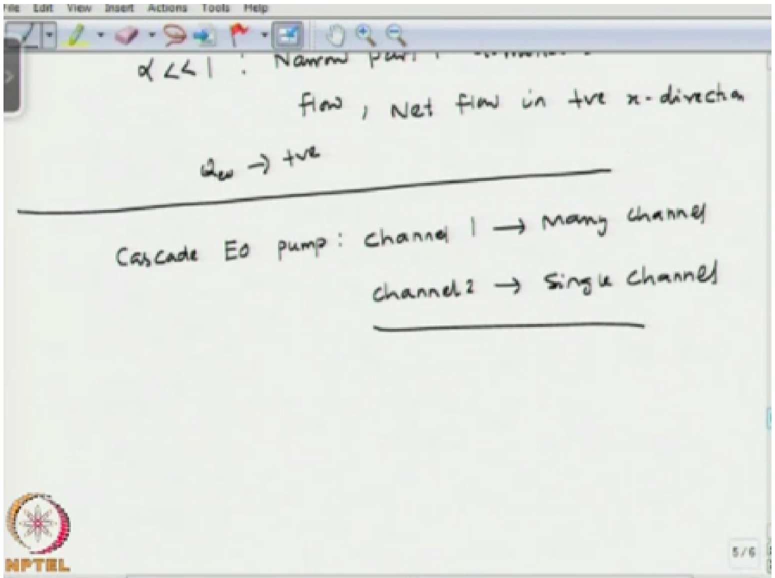

So, that is called the mini cascade electroosmotic pump where we would have the channel 1 was many channel and channel 2 was single channel, okay.

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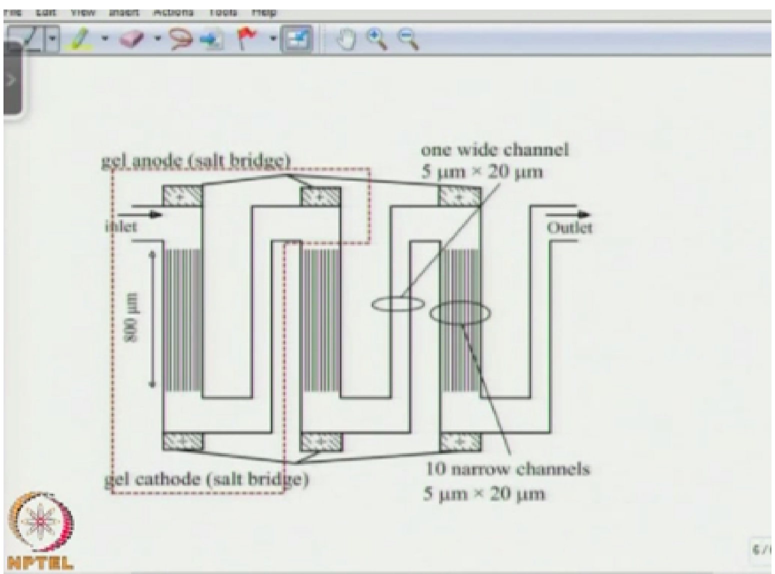

So, this is shown here as you can see in the next page. So, as you can see here, you know we have many such 0 net voltage drop configurations in series, okay. So, here it is positive ground and so on and so forth. So, we have net voltage drop across a stage 0 but with that we will be able to generate electroosmotic pumping action and we have many such stages in series to generate significant amount of backpressure.

At the same time, we have divided the part 1 of the channel into many channels because if we divide them into many channels for each of these small channels, the backpressure is going to be significant, okay and we have the channel 2, it is a larger channel because we want also simultaneously larger flow rate, okay.

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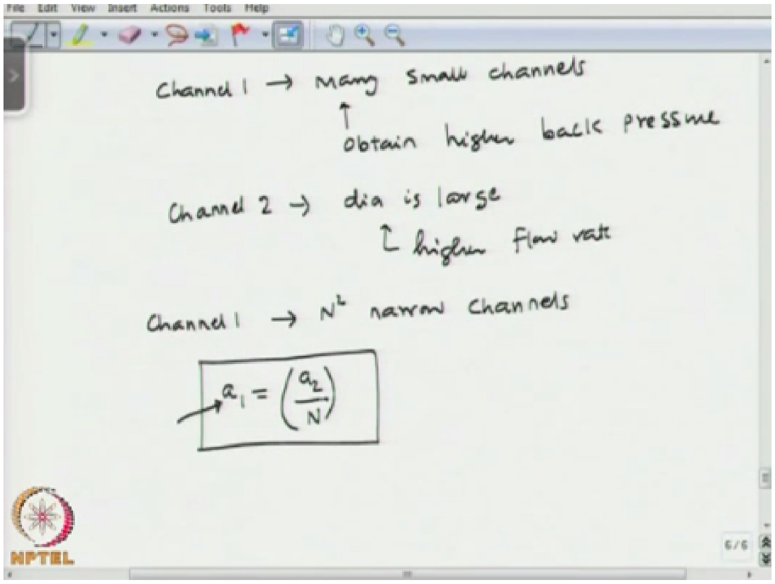

So, the idea is the channel 1 is divided into many small channels and that is to obtain higher backpressure, okay and the channel 2 the dia is large to obtain higher flow rate, okay. So, let us say we have divided the part 1 into n square channels. There are n square different parallel channels here, okay. So, channel 1 has n square narrow channels. So, we say that a1=a2/N. So, the size of the channel in part 1 is a 1/N*the size of the channel in part 2, okay.

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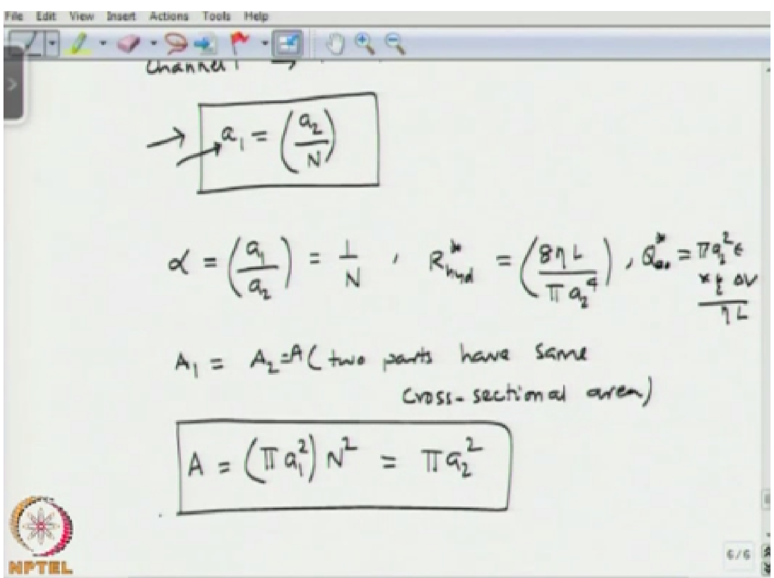

So, you know, if you go back to definition of the size ratio alpha a1/a2, that is going to be 1/n, okay, right. We can say that the R hydraulic star=8 eta L/pi a2 4 and Q electroosmotic star=pi a2 square*epsilon*zeta delta v/eta L, okay. So, the 2 parts of same cross-sectional area, so the cross-sectional area A1=A2, the 2 parts have same cross-sectional area. So, that is A1=A2+A=sum A. So, A=pi*a1 square*N square. So, we say that there are N square channels=pi a2 square. So, a1=a2/N, that is what we have considered here, okay, right.

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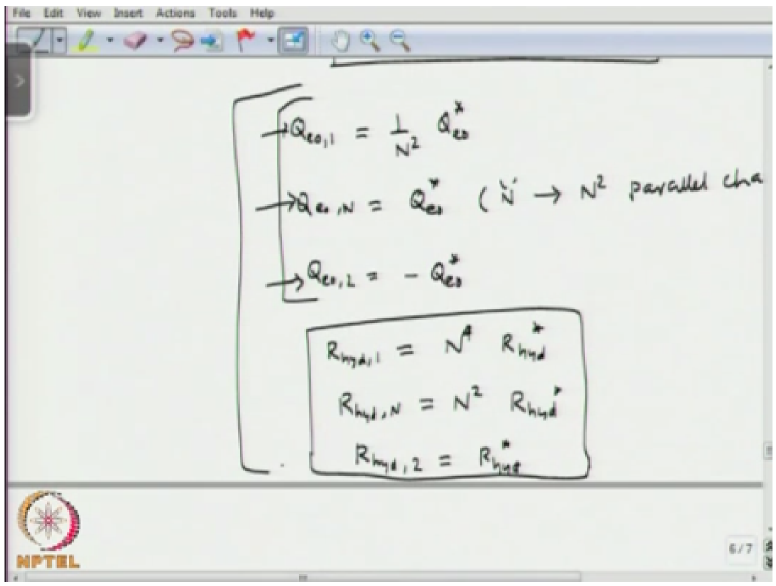

Now, we can define the hydraulic resistances Qeo1=1/N square*Qeo star and QeoN for n different channels is going to be Q, okay. So, the subscript N refers to N square parallel channels, okay and Qeo2 is going to be –Qeo star, okay. Similarly, we can obtain the expression for the hydraulic resistance, R hydraulic 1 is going to be N4*R hydraulic star and R hydraulic N is going to be N square R hydraulic star and R hydraulic 2 is going to be R hydraulic star, okay.

So, we have defined the flow rates and the hydraulic resistances in one small narrow channel, also N different narrow channels as well as in channel 2, okay. So, you know with these definitions what we are going to do is we are going to write the expression for the conservation of mass where we equate the flow rate in part 1 and part 2 of the channels; and by equating the flow rates, we are going to obtain an expression for the pressure and flow rate, okay. So, we will continue that. So, with that let us stop here.