Microfluidics
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Lecture - 13
Capillary Flows and Electrokinetics

Okay. So, let us talk about interfacial boundary condition, that is the boundary condition at the interface between 2 fluids okay. So, we will talk about the kinematic boundary condition as well as the dynamic boundary condition. The kinematic boundary condition would require that the normal component of the velocities across the interface are the same and the dynamic boundary condition will require that the tangential component of the velocities across the interface, they are the same.

Also it would require balance of the stresses at the interface okay. The balance of the fluid stresses with the surface tension stresses okay.

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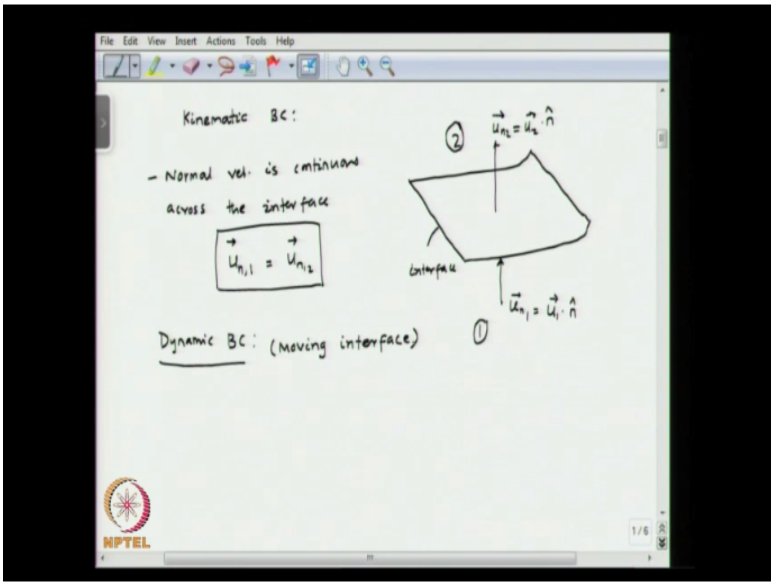

Kinematic BC:

- Normal vel. is continuous across the interface

$$\boxed{\vec{u}_{n,1} = \vec{u}_{n,2}}$$

Dynamic BC: (Moving interface)

Interfacial boundary conditions. So let us draw a surface interface so, let us say this is an interface between 2 fluids okay. So, this is the interface and let us say this is fluid 1 and this is fluid 2. So, let us say this is Un1 and which = U1.unit vector along the normal. This is Un2 which is U2.n cap okay. So, this is the vector sign okay. So, you know the kinematic boundary condition would require that the normal velocity is continuous across the interface so, we would have Un1 = Un2 okay. So, Un1 will be = Un2.

Then we talk about dynamic boundary condition which is important when the 2 fluids are moving okay. so say moving interface.

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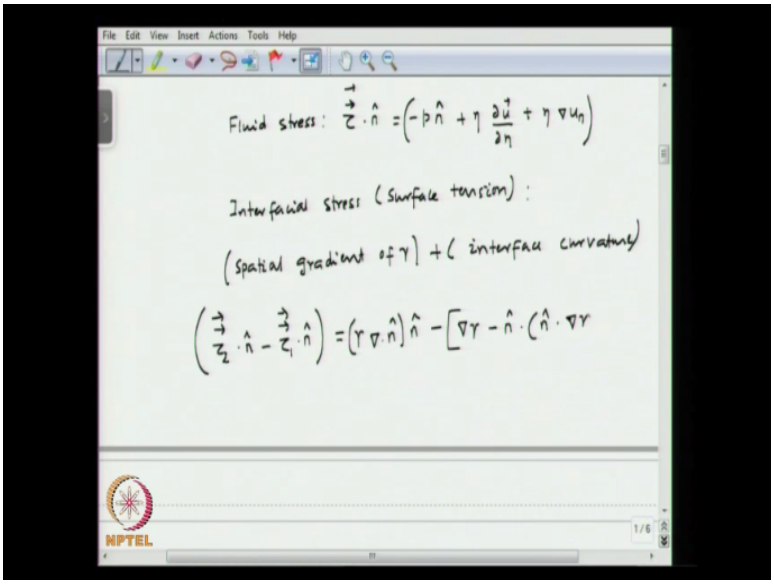

Fluid stress: $\overset{\rightarrow}{\vec{\tau}} \cdot \hat{n} = \left(-p\hat{n} + \eta \frac{\partial \vec{u}}{\partial \eta} + \eta \nabla u_n\right)$

Interfacial stress (Surface tension):

(spatial gradient of γ] + (interface curvature)

$\left(\overset{\rightarrow}{\vec{\tau}}_2 \cdot \hat{n} - \overset{\rightarrow}{\vec{\tau}}_1 \cdot \hat{n}\right) = (\gamma \nabla \cdot \hat{n})\hat{n} - [\nabla\gamma - \hat{n} \cdot (\hat{n} \cdot \nabla\gamma$

So in that case, we will say that the tangential component of the velocities Ut1 will be = Ut2 right. So, the tangential velocity is continuous across the interface okay. Let us talk about dynamic boundary condition for stresses okay. So, the dynamic boundary condition for stresses. So, we know the fluid stress is given by this expression so tau.n cap, this is actually a tensor so we have double arrow-Pn dart +eta del u over del eta+eta del Un okay. So, that is the expression for the stress tensor.

Now the interfacial stress due to surface tension. So the interfacial stress due to surface tension will have 2 different components, 1 will be in the tangential direction okay which will be due to the spatial variation in the surface tension okay. If there is variation in the surface tension along the interface and the other component is in the normal direction which is because of the radius of the curvature of the interface okay.

So, the interfacial stress due to surface tension will have 2 components, 1 is the spatial gradient of surface tension right and the other 1 would be due to the interface curvature. So, these 2 will contribute to us the interfacial stress due to surface tension. So, we can write the dynamic boundary condition to be tau 2, so this is on the fluid 2 side - tau 1.n okay will be = gamma del.n * n cap - gradient of surface tension - n cap. N cap. gradient of surface tension.

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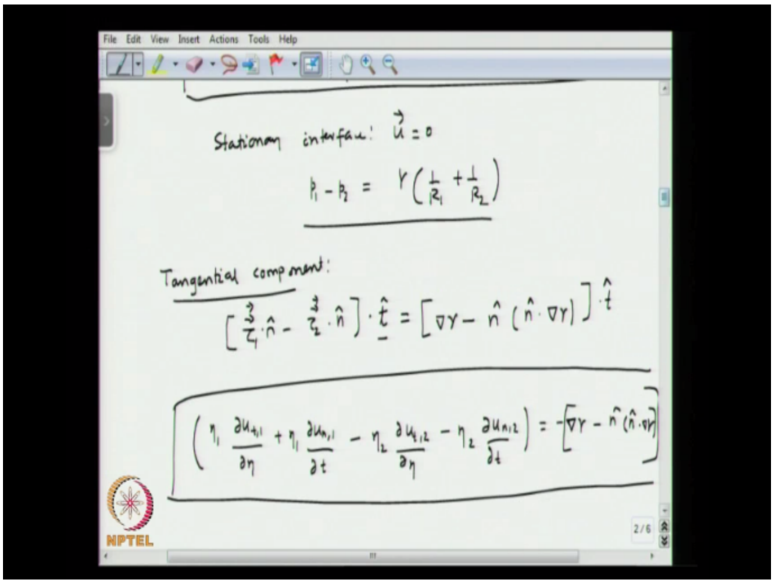

Stationary interface: $\vec{u} = 0$

$$p_1 - p_2 = \gamma\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

Tangential component:

$$\left[\vec{\vec{\tau}}_1 \cdot \hat{n} - \vec{\vec{\tau}}_2 \cdot \hat{n}\right] \cdot \underline{\hat{t}} = \left[\nabla\gamma - \hat{n}\left(\hat{n}\cdot\nabla\gamma\right)\right] \cdot \hat{t}$$

$$\boxed{\left(\eta_1 \frac{\partial u_{t,1}}{\partial \eta} + \eta_1 \frac{\partial u_{n,1}}{\partial t} - \eta_2 \frac{\partial u_{t,2}}{\partial \eta} - \eta_2 \frac{\partial u_{n,2}}{\partial t}\right) = -\left[\nabla\gamma - \hat{n}\left(\hat{n}\cdot\nabla\gamma\right)\right]}$$

So that is the boundary condition at the interface where we have tau1 and tau 2 are the fluid stresses in domain 1 and 2 so, this is due to pressure and viscosity okay and the normal stress due to surface tension is this component here. So this is you know del.n so which is equivalent to 1 over R1+1 over R2 okay for a simple case. And the tangential stress is the gradient of the surface tension - n cap n cap.gamma, which is the tangential component of the surface tension stress is the normal component of the surface tension stress.

So, this the stress due to the surface tension is balanced by the fluidic stress. The stress we see here. Now, if we split this into 2 different components, this stress balance into 2 different components, so we can write in the normal component, we can write tau 2.n cap-tau 1.n cap.n so that is making it in the normal component right will be = gamma del.n. so this can be simplified and written as P1-P2+2 eta 2 * del Un2 divided by del eta - 2 eta 1 * del Un1 over del eta will be = gamma * 1 over R1+ 1 over R2.

So this is the normal component of the stress balance at the interface. Now, for a static interface, if the fluid is not moving then the velocity will be 0 so stationary interface U will be = 0, so we will have P1-P2 as gamma * 1 over R1+1 over R2, which is nothing but the Young Laplace draw okay. And you can write the tangential component also, so we can write tau 1.n-tau 2.n.t will be = so that makes a tangential component gradient at the surface tension - n cap n. gradient of surface tension * t okay.

So, you know if we simplify this, we can write eta 1 del ut1 over del eta + eta 1 del un1 over del t - eta 2 del ut2 del eta - eta 2 del un2 over del t is going to be - gradient of surface tension

- n cap * n. okay. So, this is the tangential component of the stress balance at the interface okay. So, next we move on and talk about Marangoni force. Marangoni force comes into play when we have gradient of surface tension okay.

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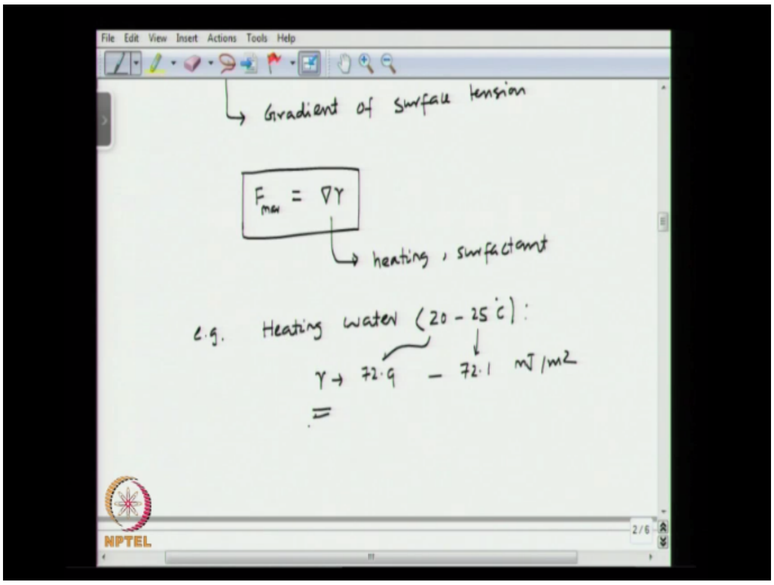

↳ Gradient of surface tension

$\boxed{F_{mar} = \nabla \gamma}$

↳ heating, surfactant

e.g. Heating water (20 - 25 °C):

$\gamma \rightarrow 72.9 - 72.1$ mJ/m2

So, we can say Marangoni force and this is because of the gradient of surface tension. We can define Marangoni force so F Marangoni is gradient of surface tension okay. So let us say you know we heat water okay from let us say 50 degree to 70 degree, the surface tension will change okay at different locations depending on how we are heating okay. So, because of the heating, the surface tension can change and which can induce the force for the fluid to move in certain direction okay and this is possible because of the Marangoni force okay.

So, this gradient of surface tension is possible because of heating, it is also possible by addition of surfactant okay. So, let us say one example we consider is if we heat water, let us say you know between 20 degrees to 25 degree c, the surface tension gamma will change from 72.9 at 20 degree c to about 72.1 at 25 degree c okay. So, its millijoule per meter square. So, you can achieve a gradient in the surface tension if you can manage the way you are heating water okay.

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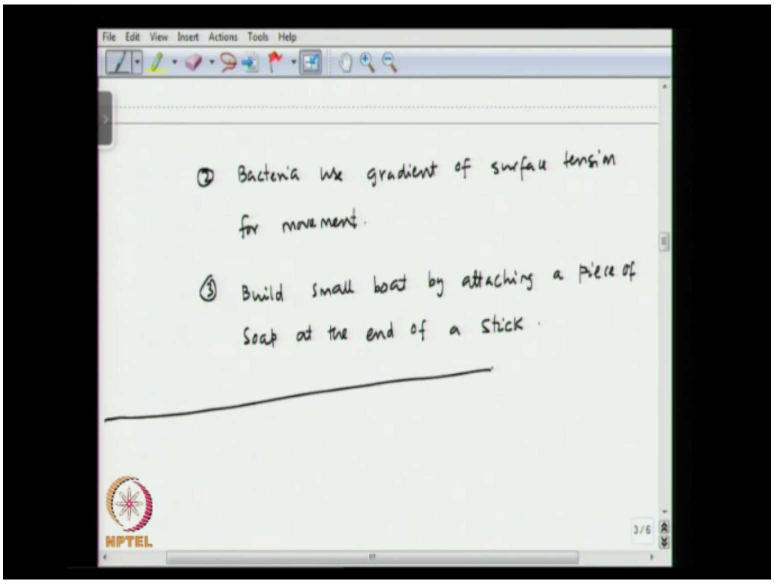

② Bacteria use gradient of surface tension for movement.

③ Build small boat by attaching a piece of soap at the end of a stick.

The second example we would consider is how bacteria propel themselves okay. So, bacteria cannot move of their own by applying some force themselves because they are very tiny. So, what they do is they eject some fluid at their back so that ejection of the fluid which acts as a surfactant causes a gradient in the surface tension okay and since the gradient of the surface tension is positive in the forward direction, they propel themselves okay in the forward direction okay, that is how bacteria move.

So, another important consequence of the Marangoni force is bacteria use gradient of surface tension for movement. The other example we can consider is if you have a stick and you can attach a bulb of soap at the end of the stick and you put it on a water surface, then because of the soap you know dissolving in water and reducing the surface tension locally, you can induce a gradient of the surface tension which would cause the stick to move in the forward direction.

So as another example, you can use you know build a small boat by attaching a piece of soap at the end of a stick okay. That is possible to do that right. So with that we finish the chapter on capillary flows and surface tension. We move on to the next chapter which is on electrokinetics okay. So, for many microfluidics applications, we would require to you know move bulk of the fluid inside a channel or we need to move particles inside the micro channel.

The particles may be either charged or uncharged okay. So, it is important to study electrokinetics. So electrokinetics can be studied if we talk about electro hydrodynamics, where we talk about the coupling between electrodynamics and hydrodynamics okay. So, in the electrodynamics, we will briefly talk about electro statics okay. So, this is what we will be concerned about in this particular course you know about electrostatic and we will not consider the magnetic effects when you talk about electrokinetics.

This is because you know we will be dealing with charges, which move you know relatively very small as compared to the movement of electrons inside wire. So, the magnetic field that will be created in case of microfluidic you know channel flows, where the charges will be fluing through will be very small, it will be negligible. So, our assumption that you know magnetic effects are not important is valid in this case okay. So, let us very briefly talk about electrostatic okay.

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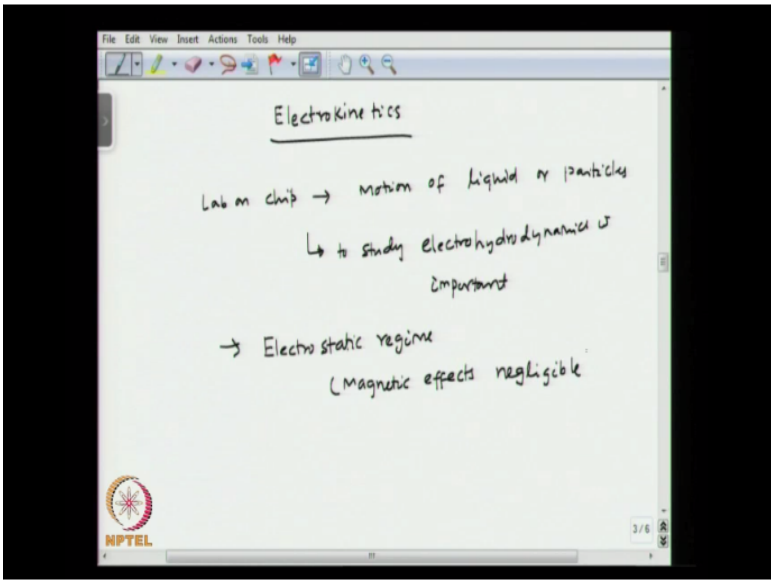

Electrokinetics

Lab on chip → motion of liquid or particles

↳ to study electrohydrodynamics is important

→ Electrostatic regime

(Magnetic effects negligible

So we talk about electrokinetics and this is important because in many Lab on chip applications, we need motion of liquid or particles okay. So, you know to study electro hydrodynamics is important and here we will be studying the electrostatic regime and the magnetic effects negligible okay. So, briefly we will talk about electrostatic. So, let us say if we consider a point charge Q in a space one important consequence of it will be the influence it will have on the surrounding okay.

And which can be represented in terms of the electric displacement okay.

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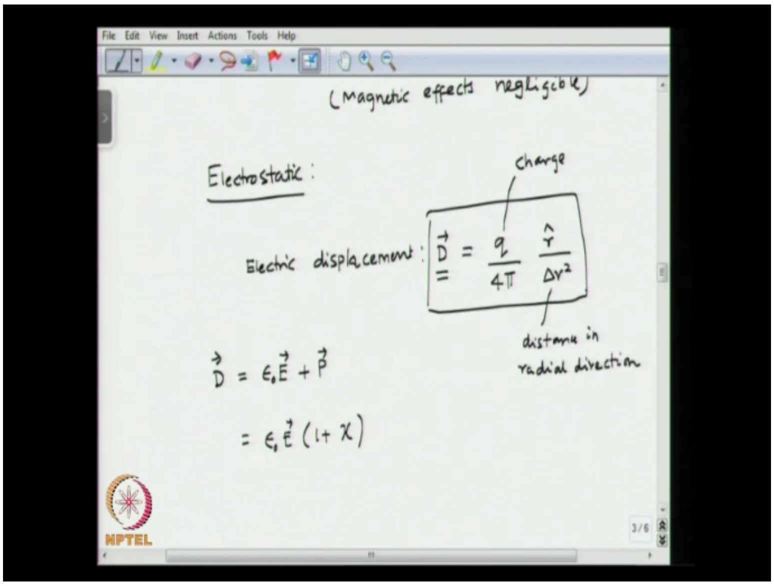

(Magnetic effects negligible)

Electrostatic :

Electric displacement : $\boxed{\underline{\underline{\vec{D}}} = \frac{q}{4\pi}\frac{\hat{r}}{\Delta r^2}}$

charge

distance in radial direction

$\vec{D} = \epsilon_0\vec{E} + \vec{P}$

$= \epsilon_0\vec{E}(1+\chi)$

So, the electric displacement which is given by the symbol D will be q divided by 4 pi * unit vector r cap divided by delta r square okay. So, what electric displacement represents is, if you have a point charge in a space what it's effect is going to be on the surrounding okay and its effect in terms of electric displacement is going to look alike q over 4 pi * r cap over delta r square, where delta r is the distance between the origin of the charge to a particular point where you want to see its effect okay.

And this electric displacement is going to be radially outward okay. So you know this electric displacement, this is the charge and this is the distance in radial direction okay. So, now this electric displacement has got 2 components okay. One is the electric field it will induce okay and the second one is the polarization okay. So we can write electric displacement D in terms of epsilon 0 * electric field + the polarization okay and we can write this as epsilon 0 * E * 1 + a term called chi okay.

So, this chi so here we have different terms this is the electric field right we will define it here.

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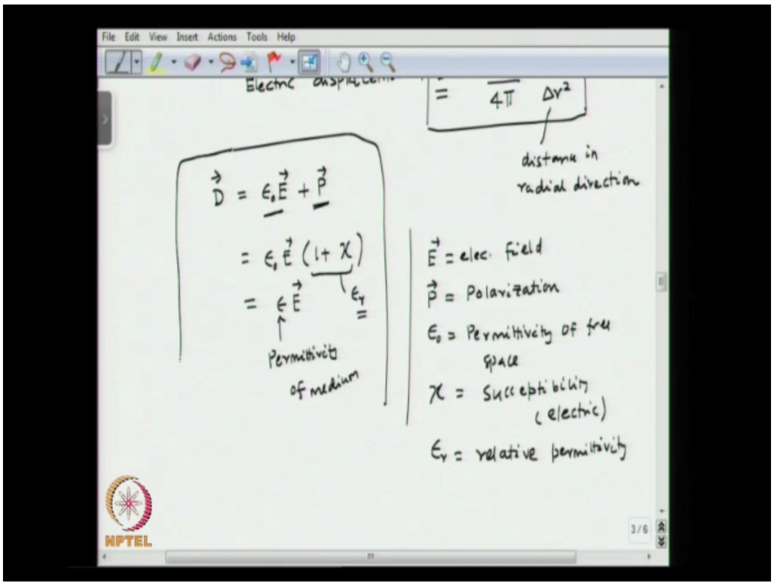

$= \quad 4\pi \quad \Delta r^2$

distance in radial direction

$$\vec{D} = \underline{\epsilon_0 \vec{E}} + \underline{\vec{P}}$$

$$= \epsilon_0 \vec{E} \, (\underbrace{1+\chi}_{\epsilon_r})$$

$$= \underset{\substack{\uparrow \\ \text{Permittivity of medium}}}{\epsilon} \vec{E}$$

$\vec{E}$ = elec. field

$\vec{P}$ = Polarization

ϵ_0 = Permittivity of free space

χ = Susceptibility (electric)

ϵ_r = relative permittivity

E is the electric field and P is the polarization and epsilon 0 is permittivity of free space and chi is the electrical susceptibility. We have equivalent magnetic susceptibility also. So, you know if you look at here as I said electric displacement has 2 components one is the electric field, that the point charge will create and the other one is the polarization. Now, if you consider this you can also write it as epsilon * E okay. We have 1 + chi is known as the relative permittivity okay.

Epsilon r is relative permittivity okay. So, epsilon 0 * relative permittivity is the permittivity of the medium. The medium that we are talking about. Permittivity of medium okay. So this is an important equation in electrostatic. Now if you talk about you know in free space okay, talk about vacuum, we don't have anything present okay. So, if you put a point charge in vacuum, there will be nothing to polarize okay. So all its effect will be to create to induce an electric field okay.

So here you can see that you know for vacuum, for free space, the polarization is going to be 0 right. And so as a result you can see here the chi okay will be 0 for free space right. Now, if you talk about a charge in water okay because water molecules are present, the charge would have effect in terms of inducing an electric field as well as polarizing the medium okay. So, if you use let us say water, so you know water as the medium, then the charge would induce electric field + it will also polarize okay.

Now, if you consider the permittivity of water is about 80 times that of the free space okay. Now, how do we compare you know if the permittivity is 80 times that of free space, if you put a charge in water, whether it is going to you know create more field or it is going to polarize more okay. And this can be answered if you look at this equation here. If you look at this equation here okay, then we can see very clearly that you know since this is going to be 80 times epsilon 0 okay, so this is 80, this is 1, so chi will be about 79.

So, about 79 part of the influence of the charge would be to polarize the medium and only one part of it would create an induced electric field okay.

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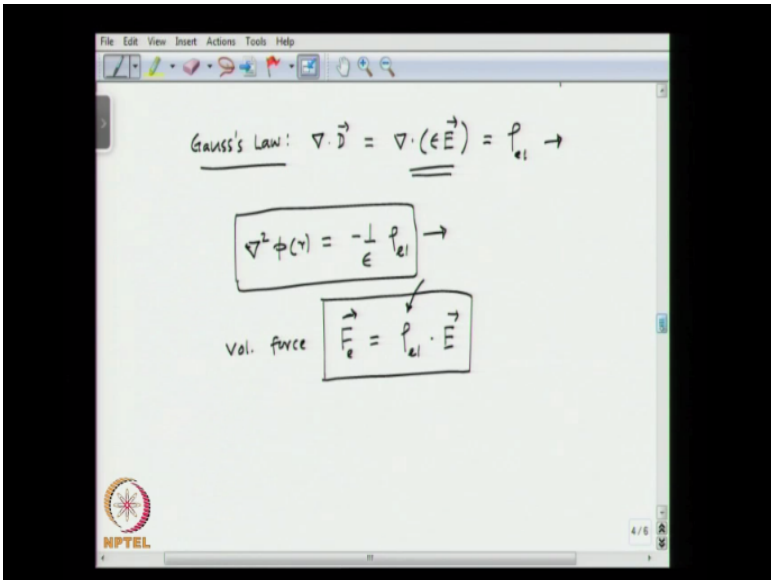

Gauss's Law: $\nabla \cdot \vec{D} = \nabla \cdot (\epsilon \vec{E}) = \rho_{el} \rightarrow$

$$\boxed{\nabla^2 \phi(r) = -\frac{1}{\epsilon} \rho_{el}} \rightarrow$$

Vol. force $\boxed{\vec{F}_e = \rho_{el} \cdot \vec{E}}$

So, what this means is that the water medium gets more polarized than the induced electric field okay. So apart from these 2 equations, we have few more important equations to write. So, the one equation is the electric field is rotational so, del * E will be 0 okay. So, from which we can also say that the electric field will be negative of the gradient of phi's the electric potential okay.

Then we can also write the Gauss's law which tells that the divergence of the electric displacement will be the divergence of epsilon * E, that is something we know and so that will be = the electric charge density. This is also an important equation okay. So, divergence of the electric displacement is the electric charge density. Now, from here E = - del phi if you substitute in this equation here what you would get is del square phi r will be = - 1 over epsilon * the charge density okay.

So, this is more easy to solve as compared to this, considering a physical situation because you know in physical situation, we have the voltage boundary conditions, that is specified for a given case, whereas charge density is you know it is very difficult to specify okay. So this equation is very important. From here, we can write the expression for the force for the electrical force is the charge density * electric field okay. So this is the expression for the electric field in terms of the charge density and the electric field okay.

So, if you know the charge density present in an electric field we know how much is the electrical force that is acting. Now, the effect in electro hydrodynamics, we have to consider the couple electric field as well as the hydrodynamics. The effect of the electric field is taken into account by considering this electrical force as a body force in the Navier stokes equation. Similar to you know in case of ync, we have the gh term, the gravitational term will be important in this case, the electrical body force term will be important.

So, the coupling the electric field and hydro dynamics coupling is achieved by using electrical body force in the Navier stokes equation. So we can write this, this way we can write rho del v am using t notation here as the gradient of del v over del t + e . del * v is the convection term will be = - del p + eta del square v + rho * g + rho electric, the charge density * electric field. So, that is the Navier stokes equation for electro hydro dynamics okay. And this is the electrical body force term okay.

So with that introduction, let us move on and talk about 3 different important electro kinetic effects, one is electro osmosis, second one is electrophoresis, and the third one is dielectrophoresis. So, before we go on and talk about this 3 effects in detail, we will very briefly see you know what these 3 effects mean okay. So the first one to talk about is electro osmosis. Electro osmosis is the bulk movement of a liquid relative to a charge surface by application of electric field okay. So, we have 3 different electro kinetic effects okay.

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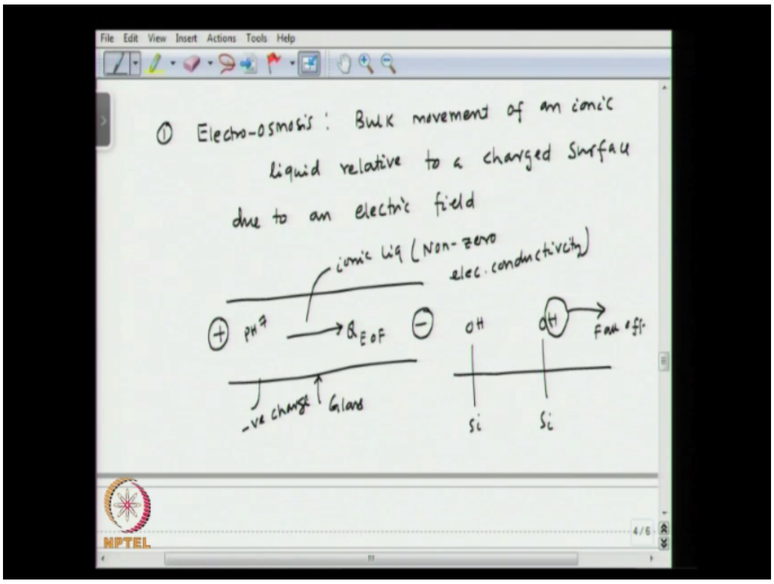

① Electro-osmosis : Bulk movement of an ionic liquid relative to a charged surface due to an electric field

So, the first one is electro osmosis, second one is electrophoresis and the third one is dielectrophoresis okay. So, let us look at the first one, which is electro osmosis and this is nothing but bulk movement of an ionic liquid relative to a charge surface due to an electric field okay. So, in this case let us consider you know we have a glass capillary okay. So this is made out of glass and we have an ionic liquid present that means it has non 0 electrical conductivity okay.

And if we apply a potential difference across the glass capillary or glass channel, then we will induce a flow in this direction okay. So this is the electro osmotic flow okay. So, this is because if you look at the surface of the glass, it will have this SioH bond Si oH bonds okay and if you have a PH7 liquid here, let us say this is PH7 liquid what happens is these protons they get knocked of okay. So these protons fall of and when they fall of the surface gets a net negative charge okay.

So, if you look at the microscopic layer close to this wall, what happens is this.

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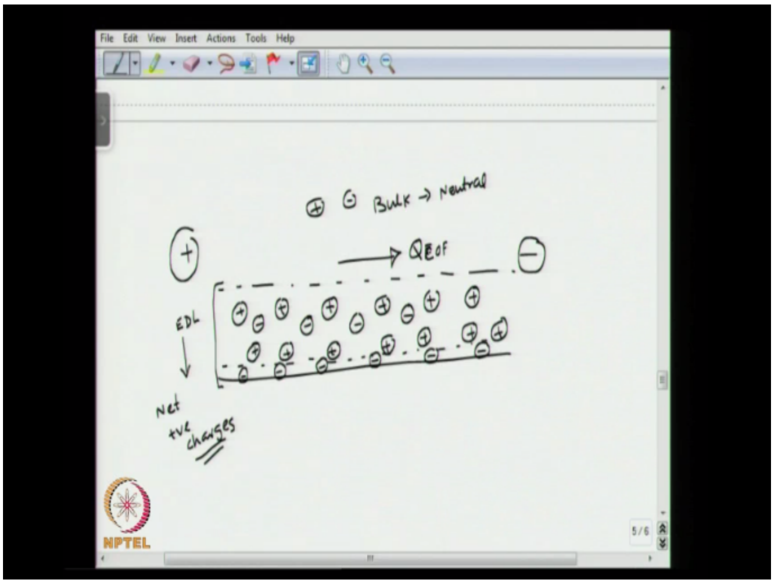

What we see here is the wall is negatively charged okay. So, this is negatively charged and these charges do not move okay. So this are almost fixed to the wall and so we have positive charges which gets attracted and come towards the wall positive charges from the ionic liquid. So, if you look at in a layer close to the wall, we would have more positive charges as compared to the negative charges okay. And the bulk will be uniform.

So you will have equal number of positive and negative charges in the bulk. So bulk will be neutral. And across this layer which is called dielectrical double layer as we would be discussing later. The electrical double layer in this case will have a net positive charge okay. Because the wall is already negative. So you will have more positive charges in the fluid in the electrical double layer.

Now, if you apply a potential difference here, this positive charges will tend to migrate towards the negative electrode, so we will have a bulk motion of the liquid in this direction okay. This is the electro osmosis phenomena. The second electro kinetic phenomena that we will talk about is called electrophoresis. In electro osmosis, we were talking about an ionic liquid in a capillary or in a channel. So we are not talking about any charges here.

However, in electrophoresis, we will have charges present inside the channel in some liquid and if we apply a potential difference between the ends of the capillary or the channel, depending on whether the charge is positive or negative, they will migrate towards opposite electrodes okay. And this effect is known as electrophoresis okay.

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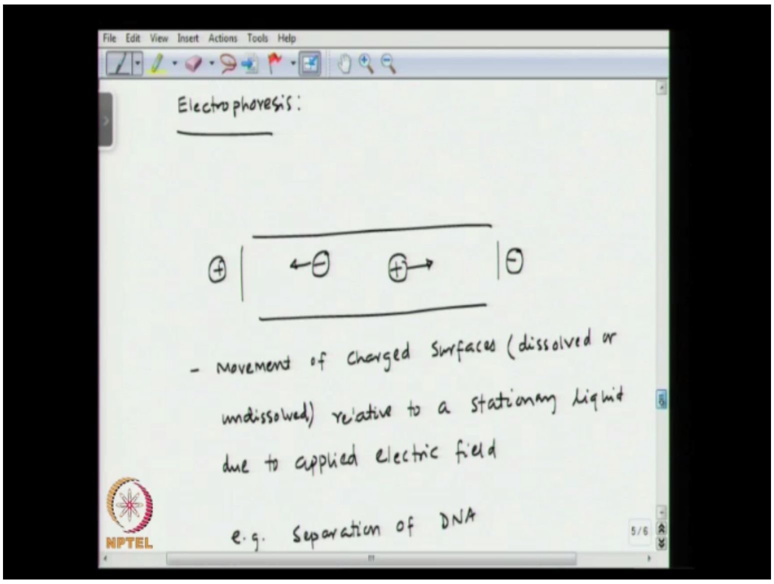

Electrophoresis:

- Movement of charged surfaces (dissolved or undissolved) relative to a stationary liquid due to applied electric field

e.g. Separation of DNA

So, we talk about electrophoresis, here we have a capillary or a channel, which has some charge particles let us say negative charge, positive charge and we have electrodes, let us say this is anode and this is cathode. So when we apply this electric field, the charges will migrate towards opposite electrode okay. So because of that, these charges would get separated. So, the definition of electrophoresis would be movement of charged surfaces.

So, these charged surfaces they may be dissolved in the liquid or they may be undissolved. So, for example in case of DNA in some buffer because DNA has got a negative charge because of the first met in the backbone. So, it is not dissolved in the buffer, it can be separated. Different DNA's depending on their mass to charge ratio they can be separated. Whereas things like the dissolved charges in sodium chloride for example okay.

So this charge surfaces can be either dissolved or undissolved, so movement of charged surfaces relative to a stationary liquid due to applied electric field okay. So, one example is separation of DNA. So what we will see later is in addition to movement of the charges, the bulk of the liquid is also moving okay. Because electro osmosis is much stronger effect as compared to electrophoresis. So, always there will be movement of the liquid itself going towards a specific electrode.

But, because of the charges in the charge particles, they will tend to migrate towards opposite electrodes. So there will be a net effect. If something if the fluid is, if the capillary is made out of glass, the zeta potential is negative, the electro osmosis will happen from positive to negative electrode and if we talking about 2 different charges positive, negative. The positive go towards the negative electrode, that negative will tend to go towards the positive electrode.

So they will get separated okay. So the application of electro osmosis we can write is electro osmotic flow in micro channels. Now, let us talk about what is dielectrophoresis. So we will talk about dielectrophoresis.

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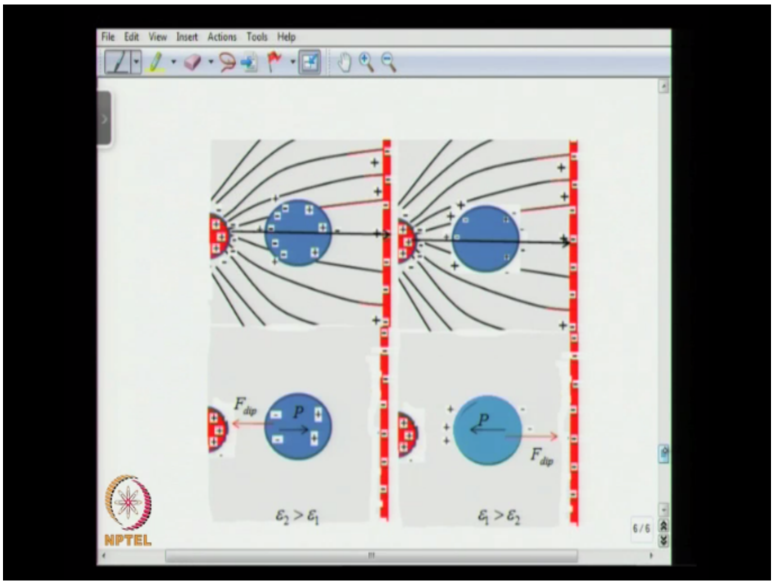

So dielectrophoresis is the force exerted on dielectric or neutral particles subjected to non uniform electric field okay. So one application of dielectrophoresis would be to separate neutral objects like cells based on their physical properties. Dielectrophoresis as I said is the movement of these neutral objects or dielectric objects when they are subjected to a non uniform electric field okay.

So if you look at this image here, here we talk about an object okay shown in the blue color and present in a medium okay and we have a voltage difference applied between a spherical electrode here and a flat electrode there. And because these 2 electrodes shapes are different, we have a non uniform electric field present here. So the electric field is more positive in this direction. So the gradient of electric field is positive in this direction.

So, here this is the first case okay, this is case 1, this is case 2 in the first case, we see that the permittivity of the object is more as compared to the permittivity of the medium okay. So, this is the electric permittivity of the relative permittivity of medium and this is for the object okay. So, since the relative permittivity of the object is more as compared to the medium, you can see that the object is more polarized as compared to the medium.

You have more charges present here as compared to in the outside medium okay. So, as a result we have a net polarization which is happening in this direction okay. So here, the polarization is in this direction but the gradient of the electric field is positive in this direction. So in this direction, the gradient of the electric field is positive okay. Now, as per definition of the dielectric force which will be discussing later.

The F DEP is the polarization. gradient of the electric field. So, since here, the polarization and the gradient of electric field are opposing each other okay. So let us consider this as the positive direction okay x positive, so this is the polarization is in the positive direction but the gradient of the electric field is in the negative direction so the dielectric force will be in the negative x direction. So, the dielectric force is going to be in this direction okay.

So, what it means is that the object is going to go towards the higher electric field or the positive electric field gradient okay. So this is known as positive DEP dielectrophoresis, where the object or particle moves towards higher electric field okay. And if you look at the situation on the right, the second case here we say that the medium is more polarizable than the particle so we have more charges on the outside in the medium as compared to that inside the object okay.

So we have a net polarization which is in this direction okay. And the gradient of the electric field is also in this direction is positive. So you would have the dielectric force in the positive direction okay in this direction. So we can say that according to this, the dielectric force will be in this direction. So here, this is known as negative DEP and here in negative DEP, we mean that the object or particles moves towards lower electric field okay. So that is what is happening.

The lower electric field is in this direction and the object gets a dielectrophorotic force which takes it towards the lower electric field okay. So, we know the application of electric field, it can be applied to neutral objects like cells okay depending on its physical properties okay. And if we talk about positive DEP, the particles of the objects will be moving towards positive electric field, gradient of the electric field and if we talk about negative DEP the particles or cells will move towards the lower electric field okay.

So, the application of DEP application is into sorting of cells based on physical properties okay. So with that let us stop here.