Gas Dynamics and Propulsion
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Lecture - 09
Rayleigh Flow

In the last class, we looked at 1-dimensional flow with heat addition where the scenario was something like this.

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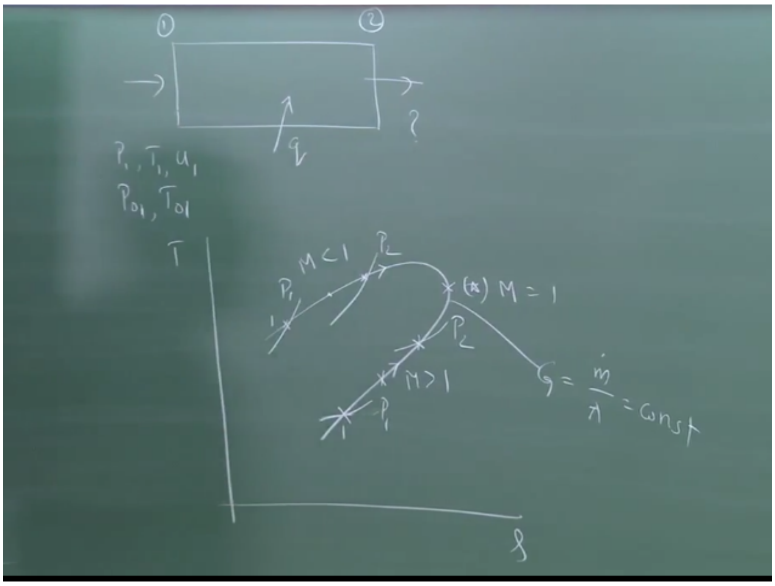

We had a duct of constant cross-sectional area and fluid enters the duct at state 1 and exits at state 2 with certain amount of heat being added in terms heat. This is heat in units of kilojoule per kilogram that is what is being added and starting from state 1 we sketched the possible downstream state. So the state here is given as for example P1, T1 and U1 is known. So consequently, the stagnation pressure and the stagnation temperature is also known.

So we need to determine what the exit state is knowing all the other quantities and we had drawn the states on a TS diagram, which look like this. So this is a TS diagram and we said that starting from state 1 all possible downstream states lie along the Rayleigh line, which we denoted like this. So if state 1 happens to be a subsonic state and we add a certain amount of heat then state 2 say can be somewhere here.

And state 2 can go up to here for a certain value of heat. Now if you increase the amount of heat that we are adding then state 2 can go all the way up to there and if I keep increasing the amount of heat, the state 2 would move further and further down this line until we hit the location, which is the sonic state where the Mach number corresponds to M=1. So this is all the downstream states or when we add heat to a subsonic flow right.

This is what we showed the other day so depending upon the amount of heat I add starting from state 1, my state 2 can be here, here, anywhere along this line until the sonic state. So I can in fact if I add enough amount of heat I can accelerate the flow from a subsonic Mach number M1 all the way to sonic Mach number M=1 with the appropriate amount of heat.

The question of what happens if I add more than this amount of heat is something that we will answer further down after we discuss a few other things; we will talk about this issue. What happens if I add more heat than what is permitted here? Okay and if you remember in the subsonic branch, so this is the subsonic branch of the curve and in the subsonic branch of the curve if you remember the shape of the curves is such that the pressure actually P2<P1 pressure decreases with the addition of heat and density also decreases with addition of heat.

Now if state 1 happens to be a supersonic state, then we start from here this is state 1 and once again depending upon the amount of heat that I add my downstream state can lie anywhere along this until I reach the M=1 point. So if I add heat then I travel along this curve in this direction and if you also remember that in the case of a supersonic flow, this is P=P1 and this will be P=P2.

The Rayleigh line is steeper than the isobar so the pressure increases when I add heat to the supersonic flow and the density also increases when I add heat to a supersonic flow. So this is the supersonic branch of the curve okay. We should also keep in mind that this curve is drawn for a fixed value of mass flow rate in cross-sectional area. In other words, this curve is one along which this quantity G which is m dot/A is a constant.

So if I have a different value of G then the curve will be different, same shape, same inferences, but it will be different okay. So these are the important information that we can get from the TS diagram okay and we started doing the same thing on a PV diagram and this was what we had done in the earlier class.

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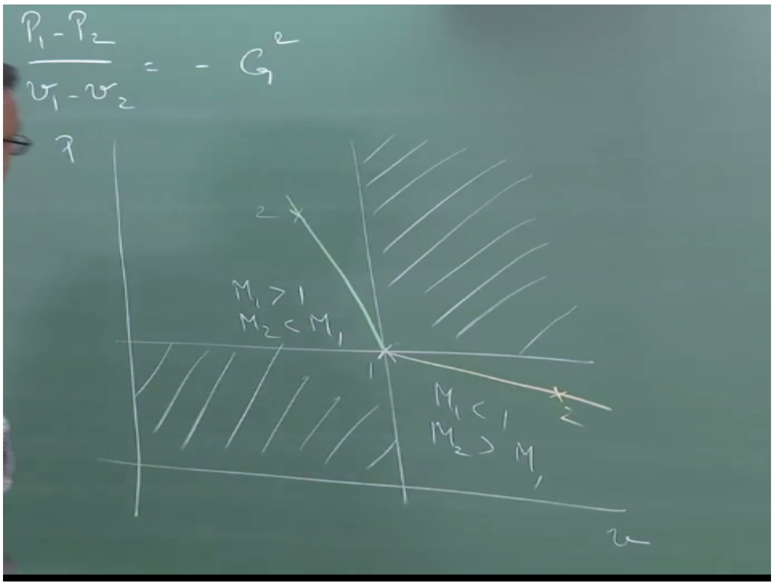

The Rayleigh line remains the same if you remember we wrote down the equation for the Rayleigh line as P1-P2/V1-V2=-G square. So this is the straight line with the negative slope in the PV diagram and if you remember the Rayleigh line itself was so let say this is my state 1. Then the Rayleigh line as a negative slope, which means that the downstream state cannot lie in the second quadrant so this is not allowed or the third quadrant is also not allowed.

So the downstream state 2 can lie in this quadrant or in this quadrant and if it lies in this quadrant then if you remember we said that the downstream state can lie anywhere along this line for example it can be here let say this is 2 or it can also look like this where the downstream state is something like this because we have to make sure that the entropy increases S2-S1 should be > 0 so we need to think about that.

We will discuss that as we go along. Notice that this is the pressure axis, this is specific volume, notice that in this case as I go from 1 to 2, the pressure increases, specific volume decreases, which means density increases so that means this is the supersonic portion of the Rayleigh line. So this is the Rayleigh line on TS diagram. This is the Rayleigh line in a PV diagram. This is the supersonic portion of the Rayleigh line.

This means that M1 in this case if the state lies here then M1>1 and as we know M2<M1, the Mach number decreases in this case right. Pressure increases, density increases, Mach number decreases right. So this then is the subsonic portion of the Rayleigh line and the Mach number increases as we travel along this line. So for different amounts of heat the

downstream state would lie at different parts of this and different points along this curve okay.

That is the information that we are going to bring in next. With this information or with whatever we have here, we do not know where state 2 is going to be, in order to know that, we need to bring in the h line also into this diagram. So this is where we had stopped in the previous class.

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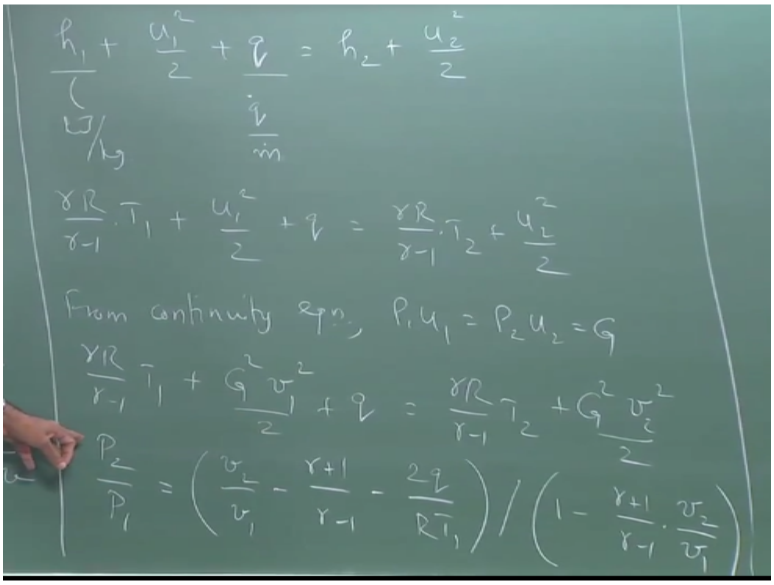

Now we are going to look at the H-curve and if you remember the energy equation for this case looks like this, h1+U1 square/2+q=h2+U2 square/2 and one important note about this quantity q is remember the enthalpy has units of kilojoule per kilogram okay. So this has units of usually kilojoule per kilogram or joule per kilogram. This quantity also has same units.

This is actually q dot/m dot where q dot is kilojoule per second rate at which we are adding heat and m dot is a mass flow rate, which is kg per second so that this comes out to be in units of kilojoule per kilogram. Remember this is a flow situation. So everything has to be a rate quantity so that is why this is kilojoule per second, this is kilogram per second okay that is what this q is. This is important too, keep that in mind okay.

So this is our energy equation and if I do the same thing that we did earlier for example if I write this as gamma R/gamma-1 times T1+U1 square/2+q=gamma R/gamma-1 times T2+U2 square/2 and from continuity equation we know that rho1 U1=rho2 U2=G so I can substitute for U from here. If I do that then I get the following, this equation becomes gamma R/gamma-1 times T1+G square times V1 square/2+q=gamma R/gamma-1 times T2+G square V2 square/2.

And if I go through the same algebraic manipulation that I did earlier with q being = 0, if I do the same algebraic manipulation I can show that this equation can be written finally almost in the same form as earlier, P2/P1=V2/V1-gamma+1/gamma-1-2q/RT1/1-gamma+1/gamma-1 times V2/V1. What is that if I said q=0, I require the equation that I heard earlier in the context of normal shockwaves.

So normal shockwave of course q=0. Now q is not 0, so the shape of the curve does not change, all we are doing is adding one constant quantity to this right. So we are going to get the same type of curves, but curves which are shifted by a certain value depending upon the value of q okay. So let us go ahead and show this. So this is the equation for the H-curve in the case of heat addition correct.

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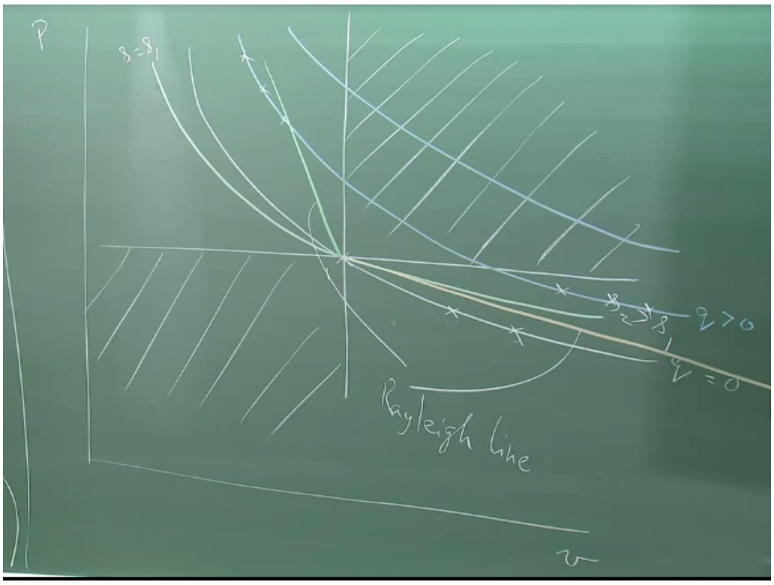

So let us draw the H-curve on a PV diagram. So once again states here are not allowed we know that much so let us hatch these regions. So this is the PV coordinate space. So the H-curve which passes through 1 corresponding to q=0 will look like this. So this is the H-curve corresponding to q=0 that we had earlier. So any nonzero value of q will give me a new curve which looks like this.

So q is positive and nonzero then the curve gets shifted by an amount like this. So the new curve would look like this. So this is q>0 so if I add more heat then the q line will move up even more like that, but note that any state in this quadrant is not allowed okay. The downstream state can lie on this h curve, but in this or this quadrant not in this quadrant okay. So this is what the H-curve looks like for the case with q not being = 0.

Notice that we are talking only about heat addition here that is the most important application so we are focusing our attention on that. So this is state 1, state 2 will lie not on the same curve, but in one of these curve so if this is the amount of heat that I am adding state 2 can lie maybe somewhere here, here or here or it has to lie here, here or here. In the case of normal shock solution, if you recall state 2 also will be on the same h curve.

Because q was 0 so both state 1 and state 2 will lie on the same H-curve in the case of the normal shock solution, but here state 2 will lie on a different H-curve okay. Now we are talking about the entropy for the states and this quadrant. We wanted to ensure that entropy increases. Now if you again recall from our earlier lecture, the H-curve is all steeper than the isentropes right.

So h curves are steeper than the isentropes so the isentrope passing through straight point 1 will look like this which means that any downstream state for example normal shock solution if any downstream state lies on the same H-curve then the entropy for that state will be less than this. So this is s=s1 right.

So if the downstream state lies on the same H-curve then the entropy will actually reduce, which was why we said that the downstream states for that case cannot lie in this quadrant, but notice that in the case with heat addition, the downstream states will have entropy > s1 so any of these states will have entropy > s2, > s1 and so states in this quadrants are also allowed when we have heat addition or heat removal okay.

If there is no heat addition or heat removal, then the downstream states cannot lie here because s2 will become < s1. With head addition, you can see that this isentrope comes like this. All the states will have higher entropy than state 1 okay. So what we are going to do now is combine the H-curve with the Rayleigh line diagram so that the point of intersection of these 2 will tell me where the downstream state is.

That is what we are going to do next. So let us see how we go about doing that so here we have indicated the Rayleigh line. So let me draw a Rayleigh line here in this part right. So that is the Rayleigh line and let me draw one more Rayleigh line, which looks like this okay. So this and this are both Rayleigh lines.

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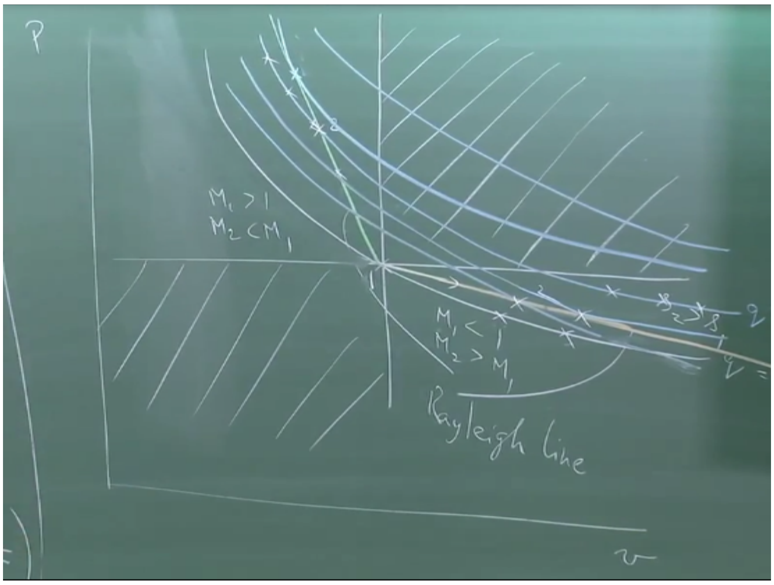

If this is the amount of heat that I add then my downstream state let me remove this s=constant line because this is little bit confusing. So this is my Rayleigh line, then if this H-curve corresponds to the heat that I am adding then my downstream state is going to be right there right. Remember this is the supersonic branch so starting from state 1, which is over here I travel along the Rayleigh line I reach state 2 okay.

One very important thing that you should keep in mind is let say this corresponds to say 50 kilojoules per kilogram okay same (()) (17:02) if I add 40 kilojoule per kilogram then state 2 would lie probably somewhere here corresponding to a different h line right. That h line might look like this right. So the state will lie here. So what this means is that starting from state 1 all the subsequent states will lie along this Rayleigh line until we hit state 2 right.

That is the very important distinction between this and the normal shock solution okay that we must pass through all the subsequent states if we remember we discussed this earlier. So starting from state 1 I add a little bit of heat I get to a new state then I add a little bit of heat I get to a new state so that means I am travelling along the Rayleigh line and cutting across many of the h lines until I reach the exit where the total corresponds to this okay.

So that means I must pass through all the intermediate states from 1 before I can reach 2. I cannot jump from 1 to 2. This is not a wave solution; this is a continuous solution. That is the very important distinction because that brings in certain limitations on the amount of heat that we can add okay, which we will discuss afterwards. Now similarly on the subsonic side for example, I can show H-curve like this, like that and my downstream state will then lie here.

So starting from 1 I go up to 2 so I can see that I am travelling along this Rayleigh line to go from 1 to 2. This is the h line which corresponds to the amount of heat that I have added okay. Is that clear? Now if you remember what we said about the Rayleigh line on the TS diagram, you will remember that starting from 1 if I add an amount of heat which I call as q star then the state at the exit of the duct can become the sonic state that is M becomes = 1.

And if you remember we determine that this was M=1 because we had the equation for the Rayleigh curve, we had dT ds for the Rayleigh curve and we said that as M goes to 1 in the subsonic side dT ds goes to – infinity so which tells me that this is the state where M is indeed = 1 and the slope is becoming –infinity. We did the same thing for the supersonic branch also.

Now here I need to see where the sonic state is going to lie when I draw these types of lines right. Where will the sonic state be? So if you take any one of these branches let us start with the subsonic branch because we have been looking at that so as I travel along this subsonic branch, the Mach number remember M1<1 and M2>M1. So M1 keeps increasing as I add more and more heat M2 becomes higher and higher.

If I add a little bit more heat, then I will go further down here. So at some point right I can see easily that there is a H-curve for which the Rayleigh line is tangential. As I keep going further and further down as I add more and more heat, there is going to be 1 H-curve which is such that the Rayleigh line is tangential at that point okay. Similarly, on the top side if you look at this, there is going to be one H-curve where I become tangential.

So if I extend this line, there is going to be one H-curve where the Rayleigh line becomes tangential to the h curve. Intuitively, you realize that that is the point where we reach the sonic state, we attain the sonic state, but we need to prove that. In the previous case, we clearly showed mathematically that as M approaches 1 dT ds approaches – or + infinity.

So here we need to prove that at this point of tangency, both here and here that the Mach number=1 okay so that is what we are going to do next.

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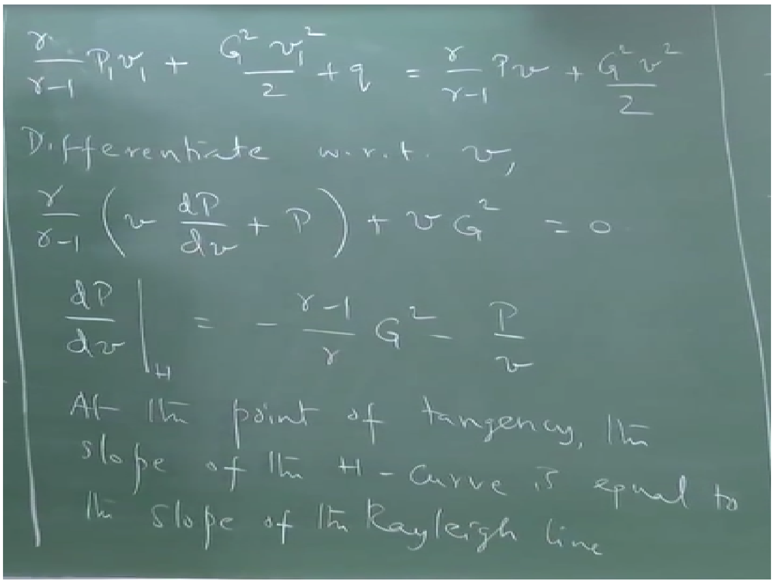

So in order to do that we start by rewriting the h equation in a very generic form okay so I am going to take this equation and write this like this so gamma R/gamma-1 so what I am going to do is I am going to take this RT1 and write the RT1 as P1 V1. So PV=RT so I can take the RT1 and write it as P1 V1 and the second term becomes G square times V1 square/2+q=gamma/gamma-1.

I am going to write this as PV so that the downstream state can lie as we move along that line, the downstream state can be any value of P, V+G square times V square/2. So if I differentiate this with respect to V to get the following, we can do this very easily. It is very easy to show. We can rearrange this like this dP/dV and let me explicitly say that this is dP/dV for the H-curve right.

So this is the slope for the h curve, so dP/dV=-gamma-1/gamma times G square-P/V. So this is the slope of the H-curve at any point. Now at the point of tangency right so if you look at the point of tangency, which is this one here, let me show that with a circle. So the point of tangency both this one and this one, the slope of the h curve=the slope of the Rayleigh line and what is the slope of the Rayleigh line? –G square right.

So we equate this to the slope of the Rayleigh line, so at the point of tangency the slope of the h curve=the slope of the Rayleigh line and the slope of the Rayleigh line itself is –G square. So we equate this to –G square.

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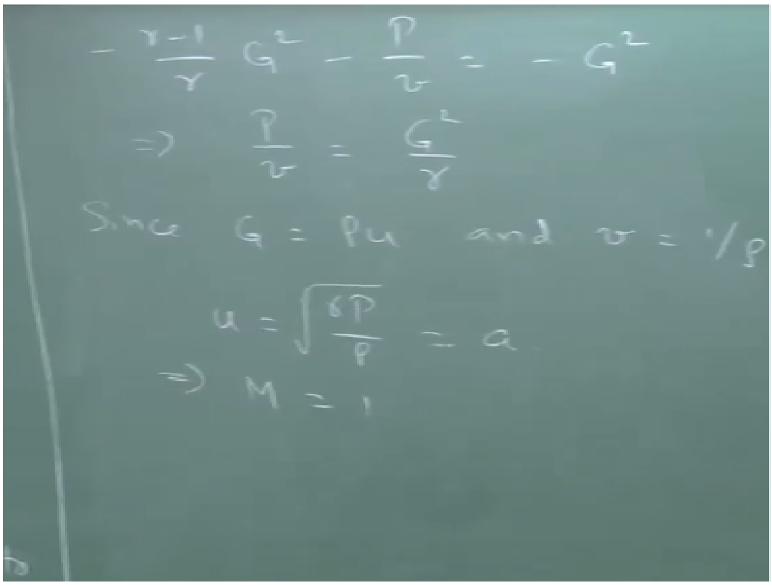

So –gamma-1/gamma times G square-P/V=-G square at the point of tangency. So simplify this and we get this to be P/V=G square/gamma. Now if you use the fact that G=rho times U and V is the reciprocal of the density, I can easily show this thing to be U=square root of gamma times P/rho and what is this equal to speed of sound.

So at the point of tangency, the speed is equal to speed of sound which proves that at the point of tangency M is indeed = 1 right. So at the point of tangency or sonic state, the Rayleigh line becomes tangential to the h curve. Now once again we ask this question earlier what happens if I add an amount of heat, which is more than what is required to reach the sonic state right.

The amount of heat that we add to reach the sonic state if you denote that as q star what happens if I add an amount of heat more than q star right. So in that case, we can see that the H-curve so if this is the H-curve corresponding to q=q star, if I add heat more than q star then my downstream state would have be on this h curve, but how do I get on that H-curve because this has already become tangential right.

How do I get on to that H-curve or similarly here, the H-curve is over here so how do I get on to that H-curve? if this has already become tangential. So some changes have to take place. So that is what we are going to discuss next.

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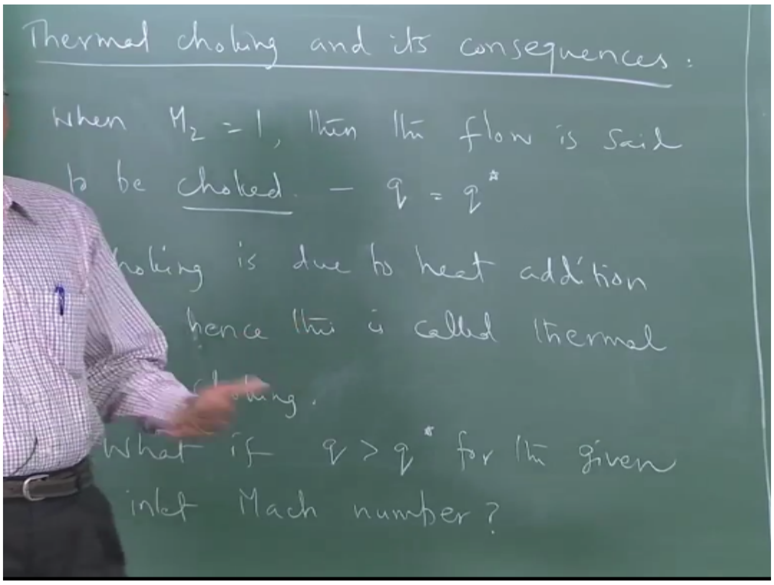

So we answer this question like this so if M reaches the sonic state at the outlet of the duct that means the flow is said to be choked right and M2 becomes = 1 then the flow is said to be choked. Now choking can be due to a variety of reasons okay. In this case, choking is due to heat addition that is why this type of choking is called thermal choking right. So here choking is due to heat addition hence this is called thermal choking.

Any time the flow reaches M=1 at some point in the flow field, we say that the flow is choked okay and if you recall M=1 we discuss that in the first chapter, we said that M=1 is a very important state because it separates regions of flow over which we have control and regions for which we do not have full control okay. So choking is a very important aspect of the flow field and when M just reaches 1 then the flow is said to be choked.

And notice that for M2 to reach this the amount of heat that we add you will denote that as q star. So if we add an amount of heat q star for a given value of m dot and A, the exit state becomes sonic. What if I add more heat than q star? That is the question that we wish to answer. So what if the q that we add is > q star for the given inlet Mach number?

So every inlet Mach number will have a corresponding q star right. So if the inlet Mach number is 0.3, q star will have some value. If it is 0.4, q star will be less. If it is 0.5, q star will be even less. So every inlet Mach number has a q star. So the question is what if I happen to add heat which is > q star? That is what we are going to discuss.

Now we will classify or differentiate our answer into 2 cases, 1 for the subsonic inlet Mach number and for the supersonic inlet Mach number.

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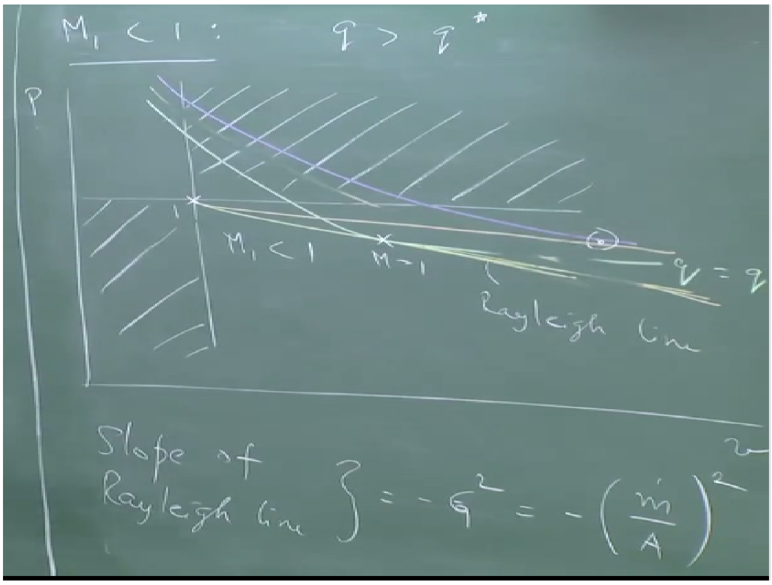

So let us first look at the case when the inlet Mach number<1, subsonic case q>q star, where will the downstream state be? That is the question that we wish to answer. Let us start with the PV diagram and see if we can answer this question somewhat easily there. So let us see. So this is our PV diagram and let me say that this is my state 1. These states are not allowed. So let us say this is my Rayleigh line.

So this is q=q star, but I am trying to add an amount of heat, which is more than q star so that means the H-curve that I want is this. So I am trying to add this much heat to the flow okay whereas the q=q star H-curve is like this so that there is a point of tangency here and this is the sonic state right. So this is the sonic state where M=1 and this is the subsonic branch that we are looking it.

So you can realize that if I keep going along this line, I cannot really go pass this point because if I physically try to go pass this point that means now I am intersecting h curves for which the value of heat is less so I am not removing heat. In order for me to attain this curve, there is one way I can do this that is to change the slope of my Rayleigh line. What if I operate on a Rayleigh line, which has a lesser slope than this Rayleigh line?

Let us say that you know we show it like this right. So let us say that we operate on a Rayleigh line, which looks like this. So let us say that we operate on a Rayleigh line, which looks like this and you can see that I can operate on this Rayleigh line up to this state point right and I can still add this much amount of heat, but now the amount of heat that I am adding will correspond to q star so that it becomes tangential to this h curve.

So that is the only way I can get on to a higher value of heat. So slope of the Rayleigh line if you remember=-G square and G itself is defined as –m dot/duct area square. So how do I operate on a Rayleigh line with the lesser value of slope? Either increase the area or decrease the mass flow rate right.

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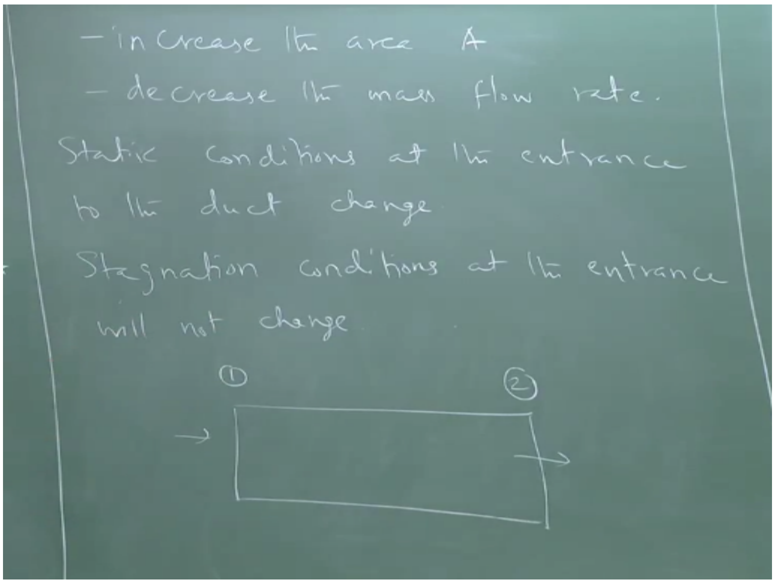

So either I increase the area of the combustor or I decrease the mass flow rate. Notice, that in this diagram I have shown the static state 1 to be at the same location, but when you start to operate on a new Rayleigh line state 1 may not be the same state 1 as what we had earlier right so we had a duct we started with certain inlet state, we are adding a value of heat which is more than q star for the state.

So now when I change the cross-sectional area or the mass flow rate, that state 1 will no longer be the same state 1 as before. Now the state 1 becomes different right. So this means that the static conditions at the entrance to the duct change. So they will be different from before. Notice that stagnation conditions cannot be different right. Why cannot stagnation conditions at the entry to the duct be different? Why do they have to remain the same?

Because stagnation conditions we remember can be changed only by addition of heat or work. So If I want stagnation condition at the entrance of the duct to change that means I must add heat or work before the entrance of the duct so stagnation conditions cannot change, only static conditions will change. Stagnation conditions at the entrance will not change. “Professor - student conversation starts.” Stagnation pressure will also not change?

No, the stagnation pressure at the entry to the duct cannot change or the stagnation temperature because those can be changed only by addition of heat or work. So if you remember if this is my duct and this is the entrance right so the flow comes like this, flow goes out like this, this is my state 2 so if I want to change the stagnation condition here right what should I do?

I should add heat or work before this place, this cannot do that right. I am adding heat here so I cannot change the stagnation conditions here. Stagnation conditions will remain the same, static conditions will be different okay. “Professor - student conversation ends.” So that is what happens, so we go on to a different Rayleigh line on the PV diagram. The same thing if I want to illustrate on a TS diagram, it looks like this.

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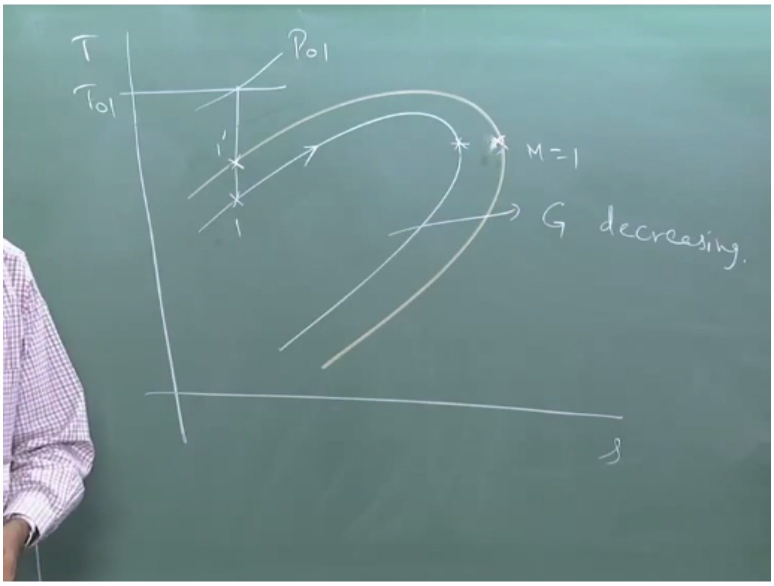

This was the TS diagram that we had so we started with the subsonic state and this is the sonic state corresponding to M=1 with the addition of heat so if corresponding to this value of M1, the heat to be added is more than q star then we can no longer operate along this curve. So we have to get on to a new Rayleigh line on the TS diagram. The new Rayleigh line would look something like this.

And remember G decreases in this direction as you move outwards, so the new condition and remember this is my T01, this is my P01. So the new static condition will be such that T01 and P01 remain the same, but the static conditions are different. So I now get on to a new curve, which I call as 1 prime. So my new operating point is along this line so I operate along this line and the heat that I am adding is such that it is = q star corresponding to the new static state okay.

If you remember this is what we did here also. So I got on to a new Rayleigh line and the q was such that it was = q star corresponding to that H-curve and similarly here also the new q is q star corresponding to this Rayleigh line. It need not actually be = q star, it can be slightly different than q star depending upon the downstream condition.

But for simplicity sake we will simply assume that to be q star corresponding to the new operating condition okay. So this is what is illustrated in the next figure.

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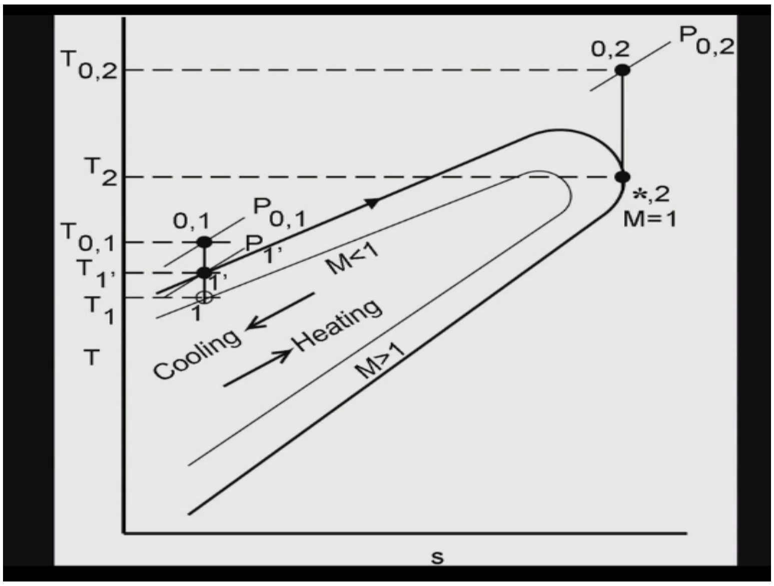

Let us take a look at this figure. So this is what is illustrated in this figure. So you can see that we initially started at state 1 and this was the Rayleigh line that we were operating along and the q that had to be added was more than q star corresponding to this Rayleigh line, which means that the T01 and P01 remaining the same we go from state 1 to a state 1 prime, which lies on a different Rayleigh line, which is shown as a thick line here.

So the amount of heat that we are adding now is equal to q star corresponding to this entry condition okay. For entry condition 1, the amount of heat q would be more than q star corresponding to M1, but now the q is the q star corresponding to M1 prime okay. So that is what happens if we have a subsonic state and we try to add an amount of heat which is more than q star.

Now this kind of adjustment of the inlet condition, so this is the duct so if you add more heat the inlet conditions static conditions adjust itself so that either there is a reduced mass flow rate normally if you keep the area as same or we can do other things right. Changing area is much more difficult practically. So if you add too much heat then the inlet conditions change for a new mass flow rate.

This is possible only because M1 is a subsonic Mach number, so any changes that you try to do here if the heat is too much then the flow can sense it and adjust itself so that it accommodates the new heat release with the reduced mass flow rate. This is possible in a subsonic flow when the amount is < 1. This is not possible in a supersonic flow right. Supersonic flow is traveling at high speed with speed greater than the speed of sound.

So such an adjustment will not be possible in the case of supersonic flow. So something else has to take place in a supersonic flow that is what we are going to discuss next.

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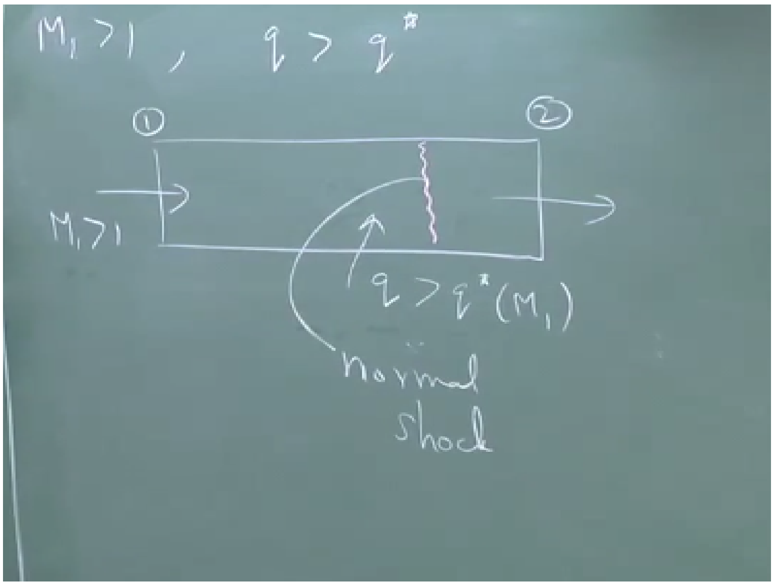

So the next case that we are going to look at is when the inlet Mach number is > 1 and the q that we are adding is > q star corresponding to this inlet Mach number. So once again we are looking at this type of a situation. Now M1>1 this is state 1, exit is state 2, M1>1 and the heat that we are adding is > q star corresponding to M1.

So depending upon how much greater q is in this case as I said earlier there is no option of going back and changing the inlet conditions and the information cannot propagate upstream right. So depending upon how much q is > q star right we tried add too much heat and remember the pressure keeps increasing in a supersonic flow with the addition of heat the pressure keeps increasing.

So depending upon how much q is > q star eventually what will happen is a normal shock is triggered somewhere inside the duct okay. So you have a normal shock at some location in the duct which looks like this. So the Mach number is > 1 before the normal shock and it becomes subsonic afterwards and you continue to add heat. So the same heat can be accommodated, but with the normal shock here.

Because with the normal shock you know that there is a further loss of stagnation pressure, which is undesirable so we have a normal shock here.

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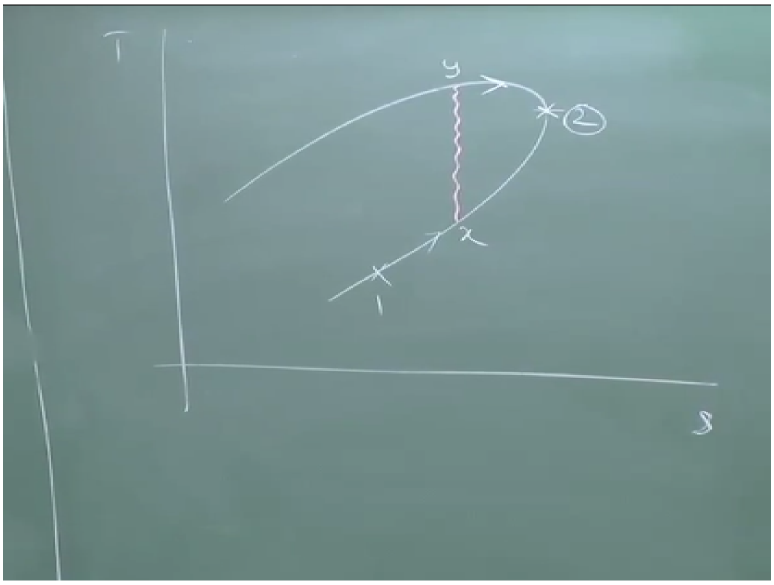

So if I show this process on a TS diagram, it will look something like this. So we are operating in the supersonic branch of the curve and we are adding heat, which is more than the q star corresponding to state 1, so which means that we add heat up to a certain point and then we have a normal shock, which appears like this.

And then we switch to this part of the curve. So if I denote the state just I had of the normal shock as state point as x and just downstream of the normal shock as y then this is state point x, this is state point y and this would be my state 2. So we start from the supersonic portion of the Rayleigh line, we go up to x and there is a normal shock then we operate like this.

The location of the normal shock both here or here will depend upon how much q is more than q star. If q is only slightly more than q star, then this normal shock will be further down. As I keep increasing the q, this normal shock will keep moving further and further up. Eventually, the normal shock may stand at the entrance to the duct itself. As you keep increasing the pressure or the heat addition, it can even move and stand in front of the duct.

So that the flow can become subsonic the entire duct that is also possible. That is not equivalent to a change in inlet conditions right. Remember as you keep increasing the pressure right even the normal shock would have to travel up stream as you keep increasing the pressure. It is an extreme case when it can move out and then stand in front of the duct. “Professor - student conversation starts.” Exit Mach number is always 1?

Exit Mach number need not always be 1, exit Mach number will depend upon what components you have further down the device. If this is the combustor and we have something else down the device and there is a pressure that is applied here, if this pressure is fixed, then the exit condition will adjust itself to suit that exit pressure okay.

If I am prescribing a certain pressure downstream of this, then when the flow comes out the static pressure of the flow will most likely match the exit pressure that we are imposing okay. It need not always be 1, but for the sake of simplicity we are just assuming it to be 1, it can be anywhere else depending upon the pressure that I am imposing okay “Professor - student conversation ends.”

Now the more difficult question is how do we illustrate this on a PV diagram right?

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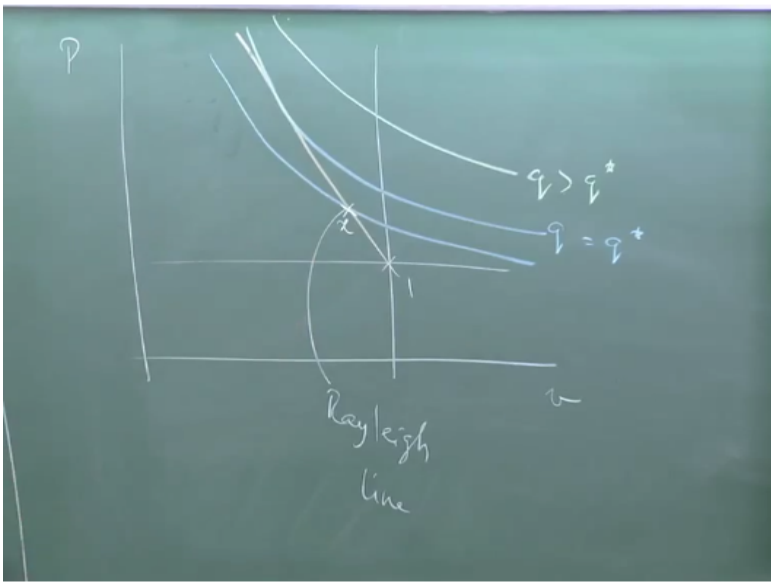

So this is my Rayleigh line, this is my state 1. The important difference now is that I stay on the same Rayleigh line. I am not going from one Rayleigh line to next like I did in the subsonic case. So here we have to be all the states must lie on the same Rayleigh line okay. So let us say that this is the heat that I am adding q>q star. So there is no way I can reach states on this using this Rayleigh line, it is not possible for me to do that right.

So because the Rayleigh line becomes tangential at some point here, so the Rayleigh line becomes tangential to the H-curve at some point here. So this is q=q star. So the actual q that we are adding is more than the q star so I cannot go to those types of states. So what actually happens is I travel along this up to some value of q, I travel along the Rayleigh line up to some value of q.

So this should be my state x. Now each Rayleigh line intersects a H-curve at 2 locations okay though it is not clear in this picture, let me show you a different picture where that is made very clear.

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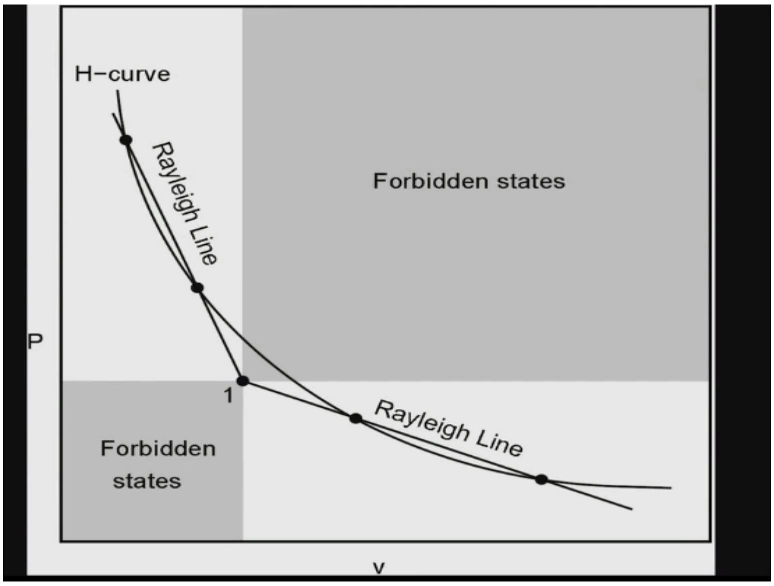

So each Rayleigh line intersects as you can see from here, each Rayleigh line intersects a H-curve at 2 locations, 1 here and 1 there. So what happens in this case is the Rayleigh line intersects the H-curve at this location and then we have a normal shock wave. So state y we are going to jump from here to the other point in this Rayleigh line where this intersects the H-curve again.

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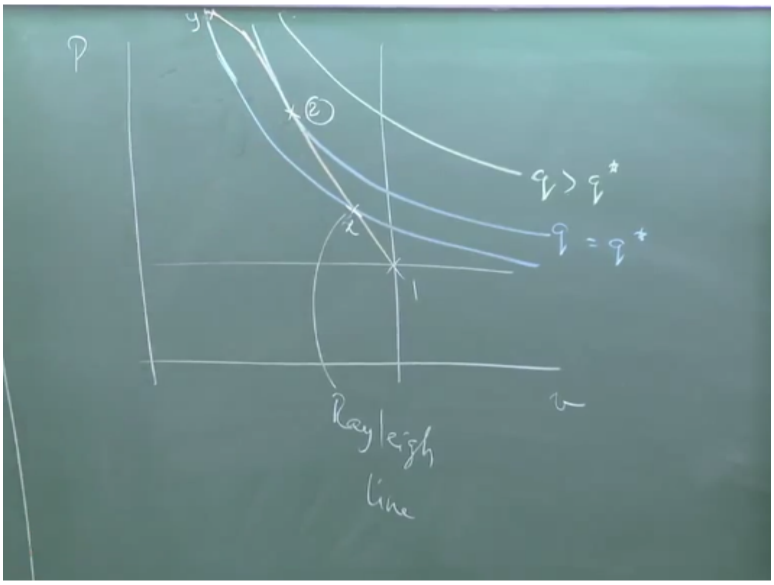

So that is my state y and then I travel up to this point okay. So I jump from this x to another state which is along this Rayleigh line, which will intersect this. If I extend this this will go and intersect this line so that is my state y and then I come back to this point so this is my state 2 okay. I can jump from x to y because the normal shock solution is a discontinuous solution.

So basically what I am doing is I am starting from state 1 going up to some value of q right and then normal shock is triggered so I stay on the same Rayleigh line and the same H-curve. Notice that y must be on the same H-curve, it cannot be on a different H-curve because there is no heat addition into the shock wave right.

So I must stay on the same H-curve. So I go from this state which I label as state x and then we go to state y, which is on the same H-curve and same Rayleigh line, which is state y and then now I begin to move down like this which means further heat is being added now until I reach a location here where state 2 is located. So this is where we are here so we can see clearly that 1 x here corresponds to this portion of the Rayleigh line.

The xy here normal shock solution is a discontinuity so we go from x directly to y without going through this point that is the difference. I do not have to go through this point in a normal shock wave as I can go directly from a supersonic to a subsonic Mach number without going through the sonic state that is the advantage of a normal shock. I do not have to go through the sonic state to go from supersonic to subsonic.

Whereas in the Rayleigh flow I have to go through the sonic state, so I can go from here to here and then I go back here. So that is this portion of the curve where I go back from there to here, which means continuous heat addition further okay. We will do a worked example in the next class and then move on to the next chapter.