Gas Dynamics and Propulsion

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Lecture - 08
Rayleigh Flow

In the previous class, we looked at one dimensional flow with heat addition.

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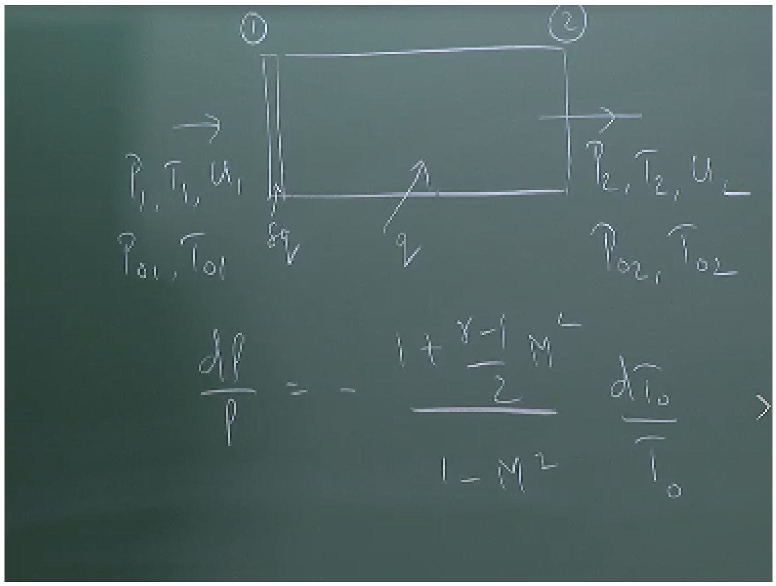

Where we were looking at flow situation like this? So let us say this as duct or a pipe in which heat is added and at entry the flow is at state 1 and at outlet or exit flows is at state 2 and starting from the inlet state which was at p1. Let us say T1 u1 and P01 T01 we wanted to find out the exit state which is P2 T2 u2 and P02 T02. We would like to do this for 2 conditions one when the inlet flow is sub sonic

And another condition when the inlet flow is supersonic both situations occur in real life. So, we would want to look at this and our strategy yesterday was to look at the effect of adding an incremental amount of heat. Let us say delta q which would cause an incremental change in stagnation temperature dT0, so we started from the state we saw where the state would go, or we developed equations.

That would tell us where the state would be with the addition of incremental amount of heat and idea is to the progressively go until we reach the exit that was what we were trying to do I will show the process on a TS and PV diagram to that end, we had developed the following equations we wrote down the following equations. I am going to write them down because we will use them very extensively in today's lecture.

And if you remember we wrote everything in terms of changes in stagnation temperature because that is the effect that we are looking at. This is the change an incremental change in pressure and look at incremental change in temperature which looks like this.

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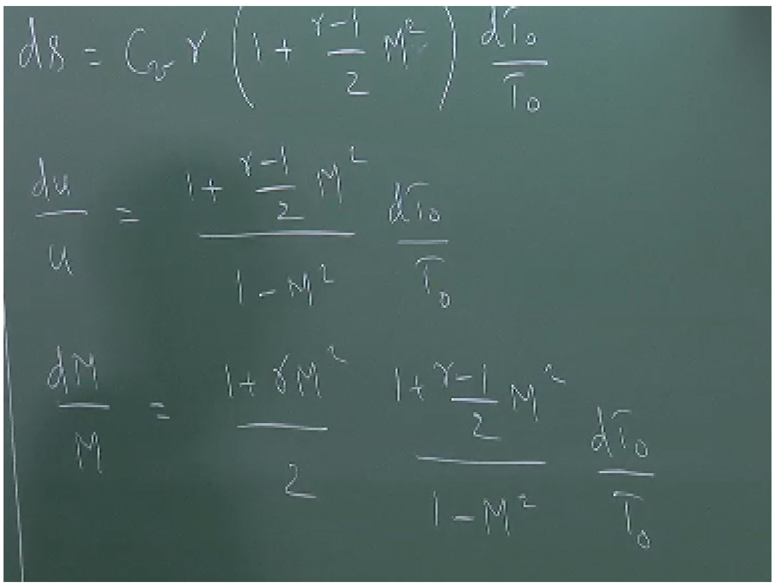

And change in entropy change in velocity can also be written down du/u and change in Mach number finally. So, these are the equations that we have written down additional equation that we are to write down is the change in stagnation pressure remember the stagnation pressure is P0 is 1 here and P02 here and we know that addition of heat increases the entropy. So, we expect P02 to be < P01.

So, that is the other equation that we are going to derive let us look and see how we can do this. So, we really have this relationship ds=this times dT0/T0 and if you remember from our earlier lecture.

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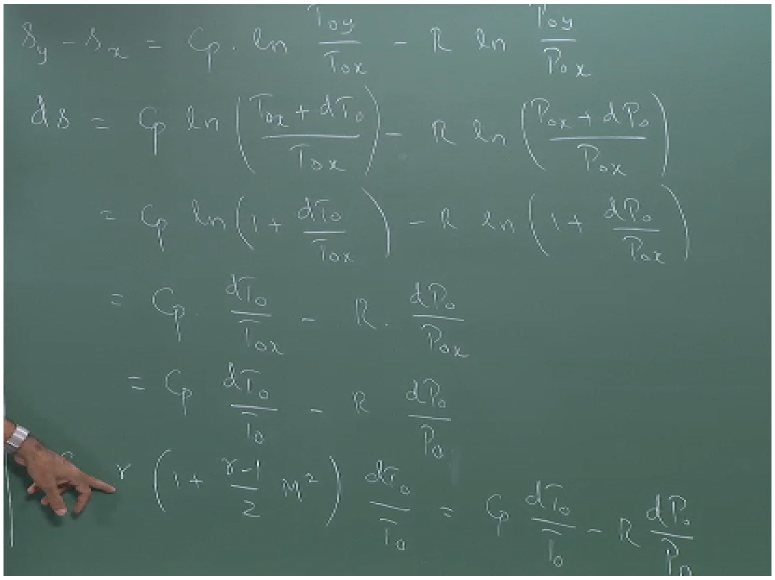

We showed that the entropy change between 2 states x and y can be written in terms of the stagnation quantities as Cp times natural log T0y/T0x-R natural log P0y/P0x this we wrote down earlier and if I take state y to be infinitesimally different from state x then I can write this entropy change as follows. So, ds=Cp times natural log so T0y is infinitesimally different from T0x.So, I am going to write this as T0y I am sorry T0x +dT0/T0x-R natural log P0x+dp0/P0x.

Or if I write this like this I can write Cp times natural log 1+dT0/T0x -R natural log 1+dP0/P0x and I can expand this natural log of 1+ a small quantity remember dT0 is a small quantity. So,1+ epsilon natural log 1+ epsilon can be expanded in a power series and approximated very well like this. is Cp times dT0 /T0x -R times dt0/P0x because I can easily replace x with 1 and I have what I want.

So, basically what we are looking at is the following so if this is state 1 and I add an incremental amount of heat that results in an incremental change in stagnation temperature dT0. So, ds is then is going to be I can easily replace this and write this as Cp times dT0/T01- R dP0/P01.Okay so now all we do is let me just make a small change here let me write this as instead of 1. Let me just write this as without subscripts so that we get a generic relationship relating dP0 and P0.

For an incremental addition of heat. So, I can now equate this expression for ds with this expression for ds and if I equate these 2 expressions for ds I get the following Cv gamma times

1+gamma-1/2 M square times dT0/T0=Cp times dT0/T0-R times dP0/P0.

If I do a little bit of algebra and rearrange this this actually works out to be something very simple notice that Cv times gamma is nothing but Cp right gamma is Cp/cv so cv times gamma is Cp. So, if I do a little bit of algebra I end up with a very simple expression for dPo/Po which I am going to write here along with the other equations, So dP0/Po=-gamma times M square/2 times dT0/T0.

So now everything is in place starting from state 1. I can calculate changes in the quantities that I wanted as I go from 1 to 2 only thing is you want to illustrate this on a TS diagram and see how the process goes as I add heat in a duct like this. Okay let us do that.

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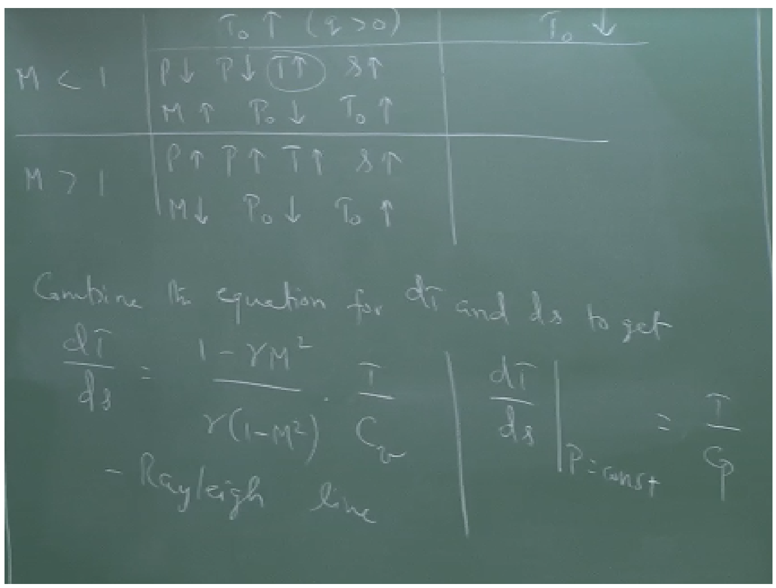

So, what I am going to do next is the following we are going to look at 2 situations. So, I am going to tabulate so we are going to see what happens when I add heat to a subsonic flow into a supersonic flow. Similarly, you can see what happens when I remove heat from a subsonic flow and a supersonic although this is as I said earlier this is not of interest to us generally. So, I am going to concentrate only on this situation.

When we add heat to a subsonic flow or a supersonic flow. This corresponds to a heat addition, so q is positive because the stagnation temperature increases so this corresponds to heat addition.

So, let us look at these quantities one at a time, so dT0 is positive right M is < 1 that means this quantity in the denominator is positive this is also positive. So, that means the density decreases right d rho is negative so that means when I add heat to a subsonic flow the density decreases.

Let us look at pressure positive this term is positive this term is also positive square this is positive so pressure what happens to the pressure P decreases right dP is negative. So, pressure also decreases if I add heat to a subsonic flow is this argument clear. So, we are looking at everything term by term. **"Professor- student conversation starts"** Yeah sir can you repeat again. Which part you want me to repeat density, density point? okay. **"Professor-Student conversion ends"**

So, we are looking at adding heat to a subsonic flow so dT0 is positive correct this term M is < 1 the term in the numerator is positive M is < 1 so this is also positive so that means this entire thing is positive there is a negative sign here, so d rho is negative that means density decreases. Right you are doing the same thing here this is positive this term is also positive for M is < 1 this is also positive for M < 1.

And this is M square there is no problem so dP is also negative P also decreases okay Next we are going to look at temperature temperatures a little bit tricky this term is positive correct this is positive this is also positive M is < 1 but if you look at this expression this is positive only when M is < 1/square root of gamma okay so generally then when M is < 1/square root of gamma dT is positive.

So, I am going to write it like this so T increases then M is < 1/gamma right. So that is very important so let me just put a circle around this and then we will write it here so for M < 1 the static temperature increases if M is < 1/square root of gamma with the addition of heat. And interestingly enough the static temperature actually decreases when M lies between with addition of heat.

So, there is a very peculiar region in the flow field where the flow velocity and the conditions are such that when I add heat to a flow subsonic flow at this Mach number the static temperature actually decreases okay it is not of major significance it is just of academic interest that there can be a region like this okay. So, static temperature now we have generally the static temperature increases we have written that over there.

Now, we go to entropy this is positive this term is also positive, so entropy in this case increases and let us go to Mach number instead of the velocity, we will directly go to Mach number. So, Mach number also if you see positive, positive M < 1 this is positive, this is also positive, so Mach number increases in a flow like this so M goes up and what about stagnation pressure this is positive right this is positive.

So, that means stagnation pressure this is negatives so stagnation pressure decreases in this case so P0 decreases just for the sake of completeness we will write down that T0 is actually increasing which is over there it is redundant. So that is for subsonic flow so for supersonic flow if you try to do the same thing M is > 1. So, that means this is positive this is positive this becomes negative because M is > 1 that is a negative sign here.

So, that means d rho is positive so rho increases and if I look at pressure same thing positive this term is positive this is negative which counteracts this negative sign so dp is positive so that means P Increases static temperature again positive positive negative. Now, if you look at this term when M is > 1 this term is always negative when M Is > 1 this term is always negative this is negative this is negative.

So, that counteract each other so temperature will increase dT is positive in this case also right, so I can write T is positive this case also for any supersonic Mach number and then we look at entropy again positive this term is always positive so s increases in this case also right s increases. And Mach number if I see this is positive this is positive M is > 1 so that means this is negative I am sorry, and I should be looking at this this is positive.

This is positive M is > 1. So, this is negative this term is positive, so we end up with dM decreases because this term is negative this is positive this is positive this is negative this is positive so dM is negative. So, that means the Mach number decreases. And P0 is again positive this is positive but there is a negative sign in front. So, stagnation pressure decreases in this case also and just for the sake of completeness we will say T0 increases.

So, if I summarize what we have said here heat addition to a subsonic flow increases the Mach number. So, if the fluid enters the duct say Mach number 0.4 then when it leaves then Mach number is going to be 0.5 or 0.6 depending upon how much heat we add in the mass flow rate this is for a fixed mass flow rate. So, the Mach number increases static temperature increases but static pressure actually decreases with the addition of heat to a subsonic flow.

If it is a supersonic flow, then the Mach number decreases. Let us say we have the flow coming in at a Mach number 2.5 as a result of heat addition it may go to 2 or 1.5 right and the temperature increases pressure also increases there is a loss of stagnation pressure for both cases and entropy increases in both cases that remains the same whether it is subsonic or supersonic the increase in entropy here.

You must remember that the increase in entropy here is not due to entropy generation remember we said that entropy change is due to entropy transfer plus entropy generation here there are no irreversibility. So, there is no entropy generation so the increase in entropy is solely due to transfer of entropy because we are transferring heat we are also transferring entropy to the system this can be a reversible heating process.

But still the entropy will increase because we are transferring entropy to the system. So, that is what this is due to which is why entropy increases in both the cases subsonic and supersonic case and stagnation pressure decreases in both the cases. Okay that is a summary of the findings what we want to do next is starting from the inlet state. I want to sketch the process that the fluid goes through for each delta q addition.

I want to know where the next straight point is, and I want to keep track of I want to draw the locus of all such state points so that we go from inlet to outlet okay this we are going to do on a TS diagram and a PV diagram that is what we are going to do next. So just like what we did before we are going to combine this equation dT/T and this equation for ds to get the following.

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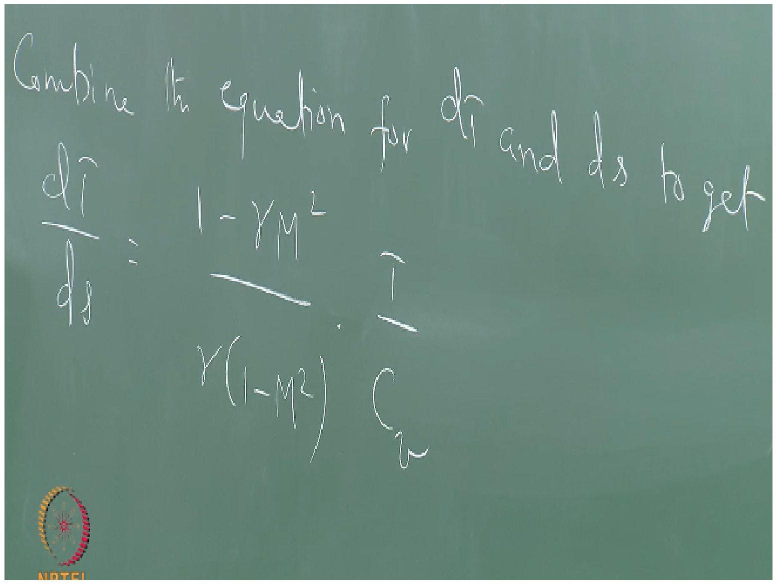

Let us combine, so if I combine the equation for dT and ds and we get the following dT/ds = 1-gamma M square/gamma times 1-M square times T/Cv, So, this is the equation that describes how the state is going to change remember we used this type of equation to draw isobars, isotherms and isentropes and so on we are going to use the same strategy to draw a line like this this line on a TS diagram is known as a Rayleigh line

So, when we complete drawing this line this is known as Rayleigh line. Okay now you will also remember from our earlier lecture that the slope for P=constant line the slope for P=constant line on a TS diagram was given as T/Cp you may recall that. So, this allows me to compare these 2 and we can then sketch the diagram as follows.

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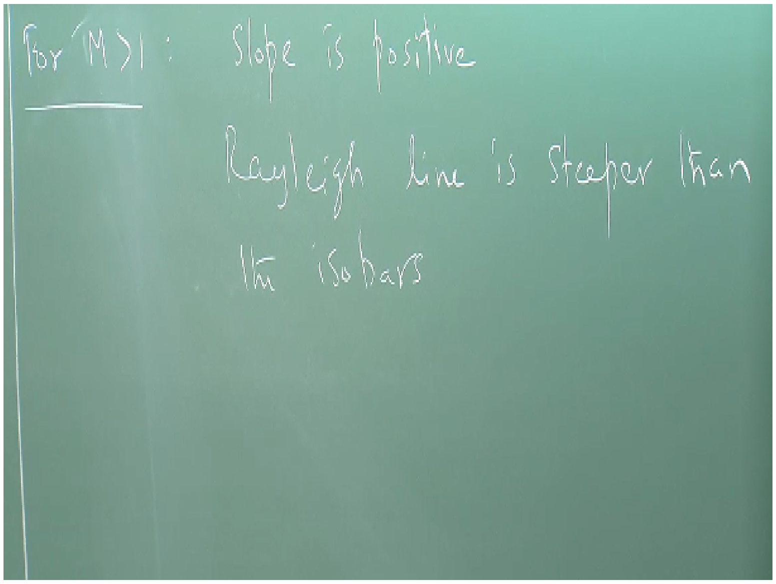

So first let us write down some salient points that we have from these equations before we write down before we draw the TS diagram what are the salient points from this let us see if M Is > 1 it me or do that first. So, if the flow is supersonic then M is > 1 M is > 1. So, the term in the denominator is negative term in the numerator is also negative but dT/ds is positive.

Right the slope is positive for M > 1 and if M is > 1 then a comparison of these 2 equations tells me that the Rayleigh line at any point is steeper than the Isobar which passes through the same point okay. So, I can write that next Rayleigh line is steeper and steeper than the Isobar. Now, as M turns to 1 remember we said that when you add heat to a supersonic flow the Mach number decreases right.

So, you start from Mach number > 1 it keeps decreasing and as M approaches 1 notice that dT/ds approaches +infinity because dT/ds is always positive and as M approaches 1 dT/ds approaches +infinity right as M goes to 1 dT/ds approaches+ infinity. So, I get an idea of what this line is going to look like now at least the the supersonic branch of the line I know what it is going to look like right.

So, that is what we are going to look at see in the diagram so if you look at the diagram right.

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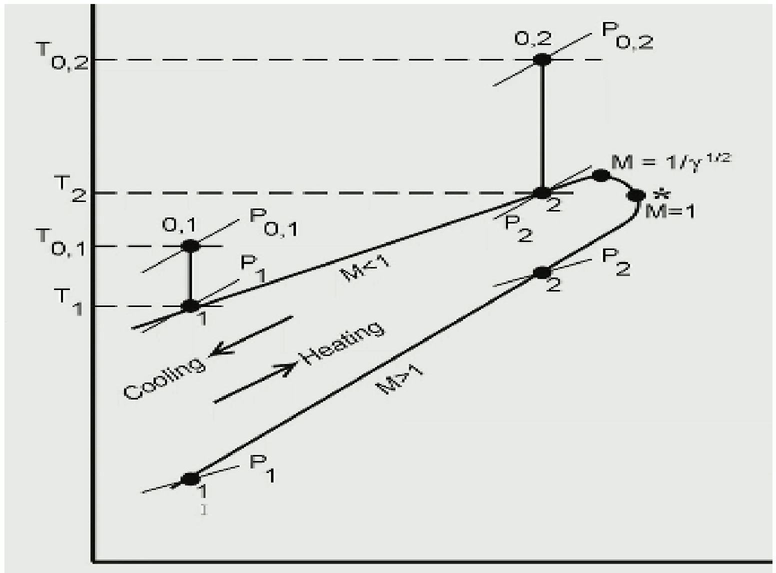

So, we are starting with we are starting with state 1 remember this is a supersonic state which is why it is down here and notice that the the the Isobar is less steep than the than the Rayleigh line, So, the Rayleigh line is shown here using a thick black line as you can see from here and the Isobar which is this one here is less steep than the Rayleigh line and this is the supersonic branch.

So, we keep moving like this the slope is positive as we said earlier right, and it is steeper than the Isobar. So, we keep moving like this and we approach M=1 what is that the slope becomes the slope approaches +infinity. So, the slope is initially positive and as I keep going like this it is not really a straight line because the slope as you can see from here keeps changing with temperature and Mach number.

Okay it is actually a curve and as we approach M=1 this approaches +infinity the slope becomes +infinity that is what is shown in this diagram and approaches M=1 you get + infinity. Okay now we are going to look at the subsonic portion of the curve, so let us see what it looks like.

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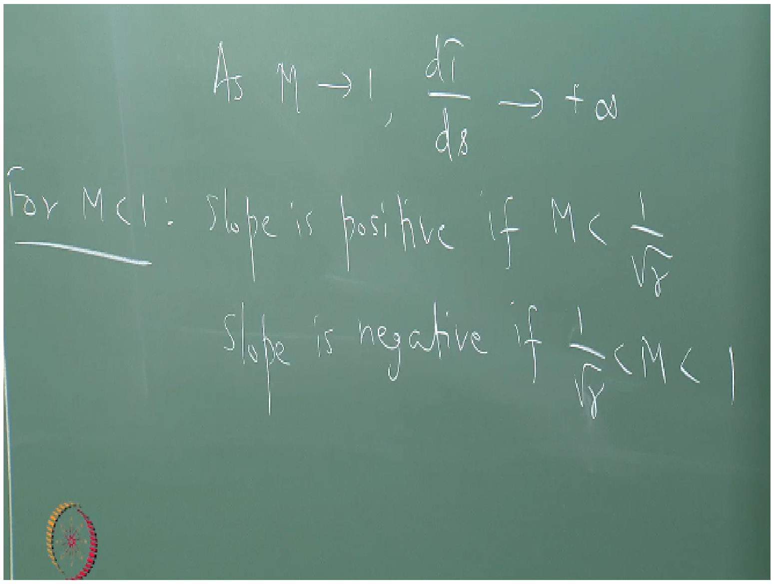

So, we can draw similar kind of inferences on this of subsonic portion of the curve from here right when the flow is subsonic this is positive and if M is < 1/square root of gamma this is also positive otherwise it is negative right. And so, let us write it down slope is positive if the Mach number is < 1/ square root of gamma and the slope is negative if the Mach number lies between 1/square root of gamma and 1.

Okay and it is also easy to show that when M is < 1/square root of gamma it is quite easy to show by comparing these 2 that if M is < 1 the Rayleigh line is actually less steep than the isobars. So, Rayleigh line in this case the previous case it was steeper. So, in the subsonic case the Rayleigh line is less steep than the isobars and as the M approaches 1 remember ones M becomes larger than 1/square root of gamma the slope becomes negative.

So, as M approaches 1 the slope approaches from the negative side so dT/ds approaches -infinity right. So, dT/ds approaches -infinity as M approaches 1 dT/ds-infinity. So, if I summarize this for certain values of M < 1/square root of gamma the slope is positive and as it keeps increasing I get to a value 1/square root of gamma beyond that the slope becomes negative which means it goes like this and then as I approach M=1 it comes like this -infinity.

Right that is what we those of the inferences is that we get from this equation single equation right we are not going to transfer it to the TS diagrams.

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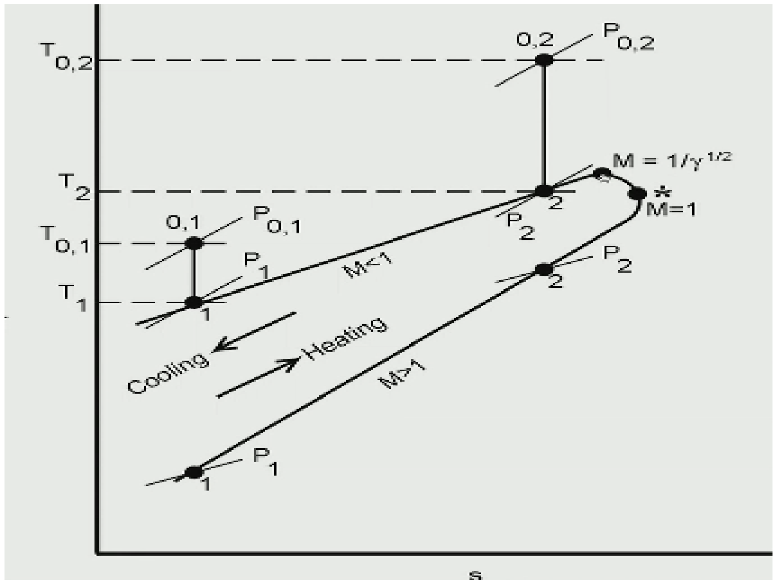

So, to summarize what we said earlier state point 1 if it is supersonic it lies here the Rayleigh line is steeper than the Isobar which passes through this and the slope approaches +infinity as we approach M=1. Subsonic state lies above like this and the Rayleigh line is less steep than the Isobar here is the Isobar corresponding to P=P1 the Rayleigh line is less steep than that and the slope keeps increasing.

Once I reach M=1/square root of gamma the slope will then become negative. So, you can see the curve turning downwards like this and as I approach M=1 we approach dT/ds=-infinity. Okay I am not going to make an effort to show that these 2 curves actually join at the same point, but it is not very difficult to show that. Okay it is also possible that you get one curve which goes like this another one which comes from one of them goes like this.

But that does not really happen they actually merge I am not going to try to demonstrate that in this case and you can see for example that the stagnation pressure in both cases will decrease because the isobars if you remember diverge from each other. So, the stagnation pressure P01 is going to be greater than the stagnation pressure P02. Okay so most of the inferences that we have drawn here and in the table are now available here.

So, you see that P2 for example in the supersonic case because of the slope P2 is > P1 whereas in the subsonic case notice that P2 is < P1 right because the Rayleigh line is less steep than the Isobar here P2 is going to be < P1 for the subsonic case whereas P2 is going to be > P1 for the supersonic case. In both cases T2 is > T1 right so the diagram actually captures all the information that we have put down here.

So, if you remember for subsonic we said pressure static pressure decreases static temperature increases that you see from here right and for supersonic flow static pressure increases static temperature increases that also you are able to see from this diagram, So, diagram captures the essence of what we have done so far. So, there are 2 branches to the Rayleigh line one is the subsonic branch other one is the supersonic branch.

Okay our next task is to draw the same diagram on a PV on a PV coordinate space. Right now, before we do that we want to before we close this there is one important point that I wanted to demonstrate namely heat addition to a compressible versus an incompressible flow remember earlier we said that addition of heat in to an incompressible flow does not cause a change in pressure. Now, we are in a position to actually examine that mathematically.

And see if it is indeed true or not that is what we are going to do next.

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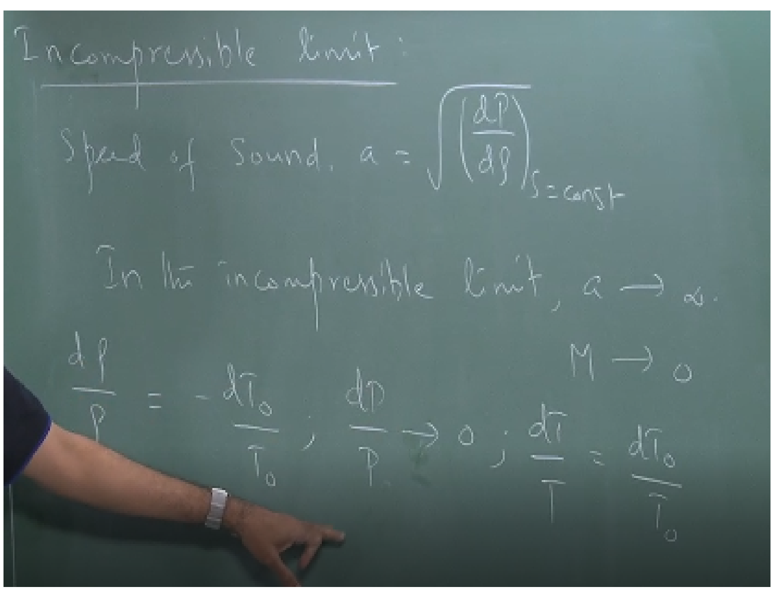

Okay so we are now going to look at incompressible limit if you recall we had defined the speed of sound a=square root of dP/d rho for an isotropic process right and if the fluid is incompressible by definition irrespective of the pressure change the density remains the same. So, I can have whatever delta P I want but delta rho is essentially 0 or very small there is no change in the density of the fluid.

So, the speed of sound for an incompressible fluid by definition is going to be infinity very large right and very large or in the incompressible limit a tends to infinity or very large values. Just for as an example speed of sound in air under normal atmospheric condition is 330 meter per second speed of sound in water with a similar condition is nearly 1.2 kilometer per second. So, it becomes a very large.

Okay under water explosions are much more powerful mainly for that reason okay. So, in the incompressible limits the speed of sound approaches infinity which means that the Mach number what is the Mach number going to do. So, irrespective of u right Mach number is u/a irrespective of u the Mach number is going to approach 0. So, that is the incompressible limit and that is what we are going to do in these equations.

We are going to see how these equations behave when we allow the Mach number to go to 0. Right so let us look at the first equation density so in the incompressible limit when how does the density behave for heat addition. So, dT0 is positive M approaches 0 in both these cases so the numerator becomes =1 denominator also becomes =1 so d rho/rho=-dT0/T0. Next P dP/P dT0 is positive this quantity as M goes to 0 this goes to 1 this also goes to 1 but notice that this goes to 0.

So, dP tends to 0 as M goes to 0 and what about dT/T dT/ T is given over here this term goes to 0. I am sorry this term goes to 1 this term also goes to 1 and this term also goes to 1 so dT/T is dT0/T0. So, you see mathematically that for an incompressible fluid addition of heat dT0 is positive addition of heat does not cause a change in pressure right which was why if you remember.

We drew the temperature variation along the length of the gas turbine engine or aircraft gas turbine engine and you remember that the in the combustor the pressure remain more or less constant This tells you why the pressure remains more or less constant the flow behaves like it incompressible fluid in the aviation gas turbine combustor. Okay so this actually can be used as a check for incompressibility if I add heat and I did notice a change in pressure that means compressibility effects are negligible.

So, your huge fern is there you are adding a lot of heat, but compressibility effects are negligible this is the reason for that. And you also notice that if the flow is compressible I add heat to the flow then definitely there is going to be a change in pressure our TS diagram showed that. And we also know from this equation that the pressure is going to change either increase in a supersonic situation or decrease in a subsonic situation.

Okay so this proves very clearly that what we said in our first lecture regarding compressibility can be proven mathematically also from the proper equations. Okay our next job is to transfer the information to a PV diagram and PV coordinates that is what we are going to do next.

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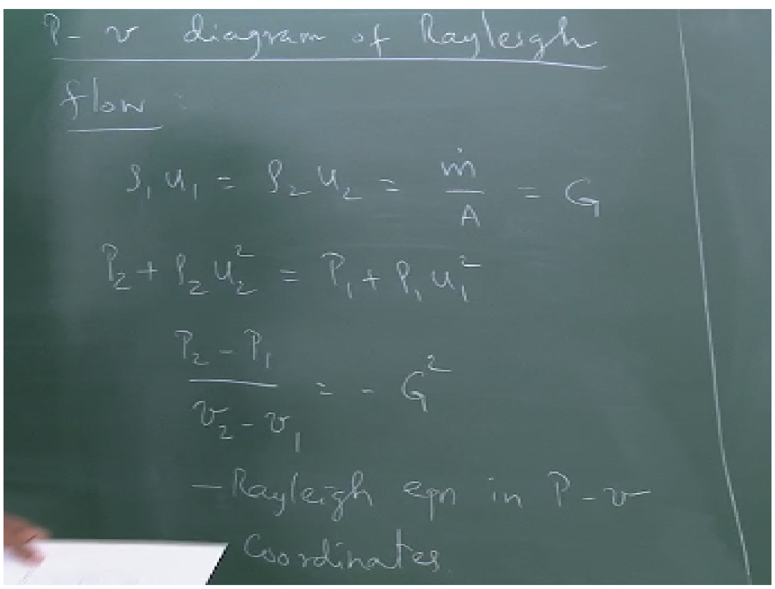

So, we start with the PV diagram of Rayleigh flow and if you recall when we illustrated the normal shock solution on a PV diagram. We rewrote the equations lightly before before doing that so if you remember the continuity equation in this case is rho1 u1 =rho2 u2+m./A and we define this to be a quantity G right that is what we did, and the momentum equation was the same P2 +rho2 u2 square=P1+ rho1 u1 square.

From which we derive the Rayleigh equation which was P2-P1/v2-v1=-G square right this was what we have written down. So, this is the Rayleigh equation in PV coordinates what is that this is a straight line in PV coordinates it is a Rayleigh equation in PV coordinates it is the same whether we are looking at a normal shock solution or flow with heat addition because the governing equations have not changed up to this point.

So, this straight line in PV coordinate is the same as this is Rayleigh line that they are just demonstrated, or we just illustrated this is the Rayleigh line in TS coordinates. Okay so the same Rayleigh line in PV diagram is a straight line remember this Rayleigh line that we have sketched here is also for a fixed value of mass flow rate. And this is also for a fixed value of pass flow rate and if I transfer this information to a PV diagram.

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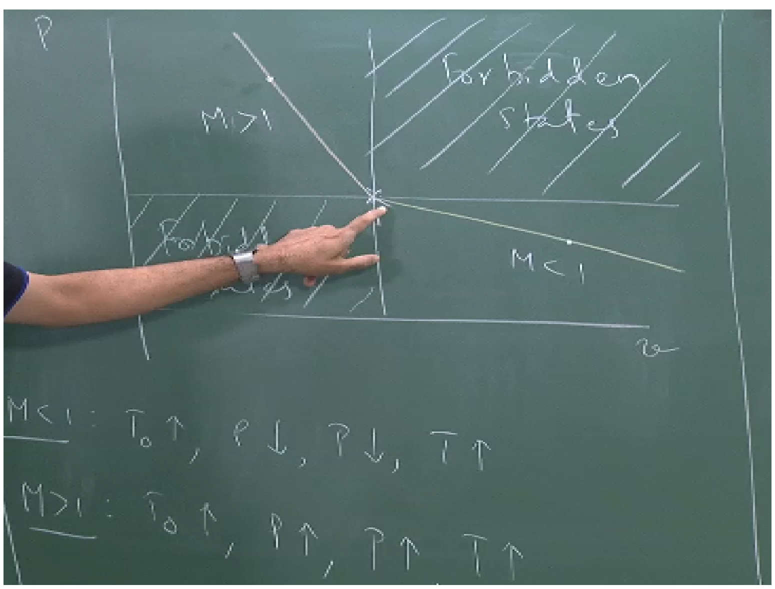

Let us say this is my state 1 since the slope of the Rayleigh line is negative we as argued earlier as the slope of the Rayleigh line is negative you states in this quadrant and this quadrant are not allowed. So, I can either have a Rayleigh line which looks like this or I am allowed to have a Rayleigh line which looks like this. In the previous case normal shock solution. We said that any state here would have a lower value of entropy and then hence it is not allowed.

You need to see whether that is true in this case also we do not know. So, we will say that states in this quadrant and states in this quadrant are allowed for now we will say that when we look at H equation. Also remember state 2 must lie at the point of intersection of the Rayleigh line and the H equation. So, at the time we will decide whether downstream states here are allowed or not for now we will let this be what is that this is a straight line the slope line is-G square.

Now, we have to determine which one is the subsonic branch, and which one is a supersonic branch because if you if you recall the TS diagram that we had here this was the subsonic branch of the Rayleigh line. This is the supersonic branch of the Rayleigh line same Rayleigh line is being drawn here. So, we need to determine which is the subsonic branch and which is the supersonic branch in order to do that.

We make use of the information that we already put down if you remember for M < 1for heat addition with heat addition meaning if T0 increases then we said that rho what did we say about rho we said that rho decreases correct rho decreases P decreases and T increases for a subsonic flow and for a supersonic flow if you add heat then we said that rho increases P increases and T increases.

And so, we can use this information to determine which is the subsonic branch and which is the supersonic branch. So, if for example let us say state 2 lies here then the pressure has increased what about the since we are using specific volume what about the specific volume, specific volume has actually decreased from here to there. So, that means density has increased its specific volume has decreased, density has increased what about temperature.

If you remember the shape of the Isotherms, Isotherms that we drew you will know that the temperature here is higher than this so that means the stagnation temperature has increased. We have added heat pressure has increased right specific volume has decreased which corresponds to this and static temperature has increased. So, that means this is the supersonic branch of the Rayleigh line.

Right that is the supersonic branch of the Rayleigh line, is that clear "**Professor -Student**

conversation starts” Excuse me $M > 1$, thank you “Professor student conversion ends” Is that argument clear should I repeat it clear okay “**Professor -Student conversation ends”.** So, similarly here if state 2 have to lie somewhere here then obviously pressure has decreased right specific volume has increased means density has decreased.

And static temperature has in some cases in this case in so far down static temperature could have actually decreased that depends upon the shape of the Isotherm. In general, the static temperature would increase right depending upon the shape of the Isotherm. So, this corresponds to the the subsonic portion of the Rayleigh line. Okay so this corresponds to the subsonic portion of the Rayleigh line.

Okay and remember the Mach number decreases as I travel outwards from here and the Mach number increases. As I travel outward from here okay on for a given stagnation temperature Mach number increases in this direction and Mach number increases in the supersonic portion of the Rayleigh line. So, you must remember that because then we would need all this information when we transfer our information from any equations to the diagram.

Okay the next thing to do is to draw the H line corresponding to flow with heat addition. Once we do that we can combine the 2 and come up with the information on a PV diagram which we will do in the next class.