Gas Dynamics and Propulsion
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Lecture - 07
Normal Shock Waves / Rayleigh Flow

In the previous class, we looked at depicting the normal shock solution on TS and PV diagrams.

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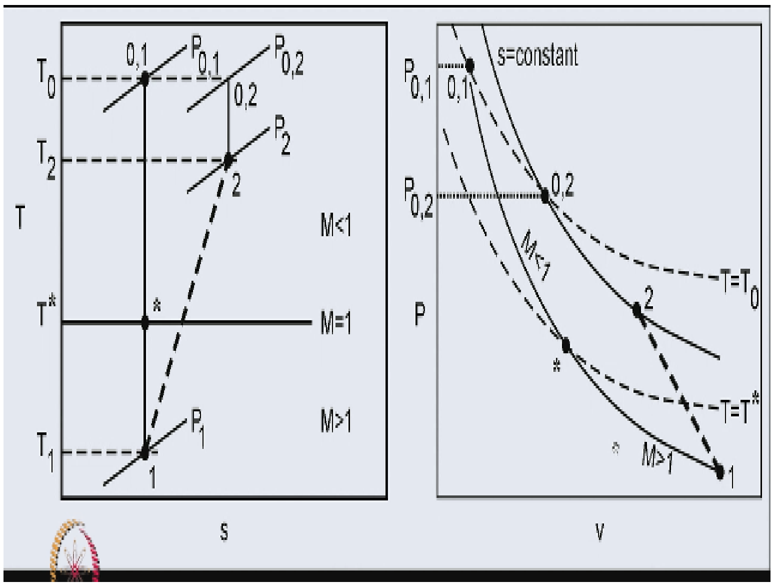

And these are the 2 diagrams that we are looking at the state 1 here is the state ahead of the normal shock and state 2 is the state behind the normal shock this is illustrated on a TS diagram here and we illustrated in the same thing on a PV diagram again. state 1 denotes the state ahead of the shock wave and state 2 denotes the state behind the shockwave. One important aspect about this diagram is that although we have connected states 1 and 2 using a line.

You must keep in mind that states 1 and 2 are actually not connected by a line state 1 and state 2 is a discontinuous solution. So, we have a wave so for the purposes of illustration. We have connected the 2 states with a straight line but notice that the flow actually does not go through any of the intermediate states here it is a discontinuous solution. So, we go directly from state 1 to state 2 without passing through any of the intermediate states.

This is very important because the next solution that we are going to look at for example flow with heat addition we will go from a state 1 to a state 2 through many intermediate states which puts you in more restrictions on what is possible and what is not possible. So, I wish to draw the distinction here that we go directly from state 1 to state 2. Without going through any of the intermediate states.

Right and what we are going to do next is redraw this PV diagram from a slightly different perspective which offers us much more generalized insight into the nature of the flow. So, that we can apply the theory not only for a normal shock wave but for certain other solutions as well. Okay so we are basically going to focus on the PV diagram and then redraw it from a different perspective.

Okay let us see what we can what we can do.

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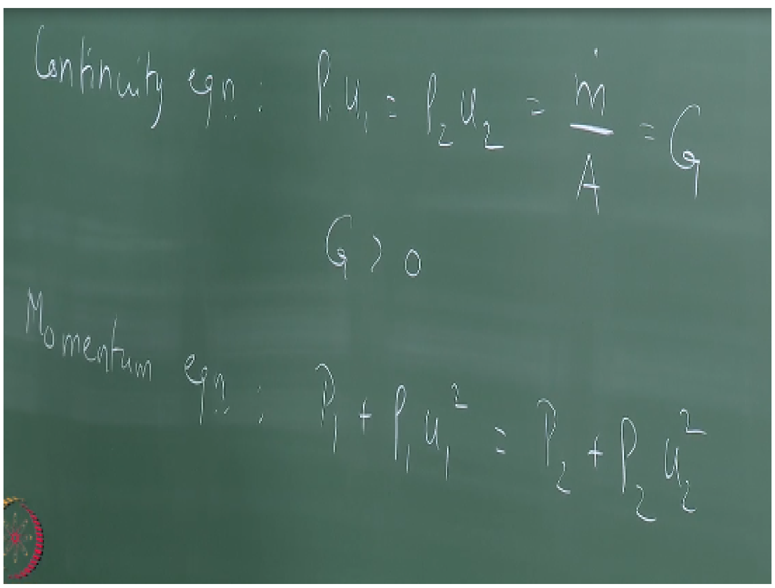

So, basically, we are continuing our discussion with further insights into the normal shock wave solution. So, if you remember the continuity equation that we had written down earlier look like this rho1 u1=rho2 u2 and in fact I can actually write this as mass flow rate say m./ some area A okay rho1 u1=rho2 u2 which can be written as a mass flow rate/ area of cross section and I am going to note this quantity as G.

Okay notice here that G is always positive quantity because you are saying it is mass flow rate / a cross sectional area. Right so this is all continuity equation and the momentum equation look like this p1+rho1 u1 square=p2+rho2 u2 square and if I substitute for u1 from the continuity equation here. So, basically, I am going to write u1 as G/rho1 if I do that now. So, I do the following substitution and u2=G/rho2.

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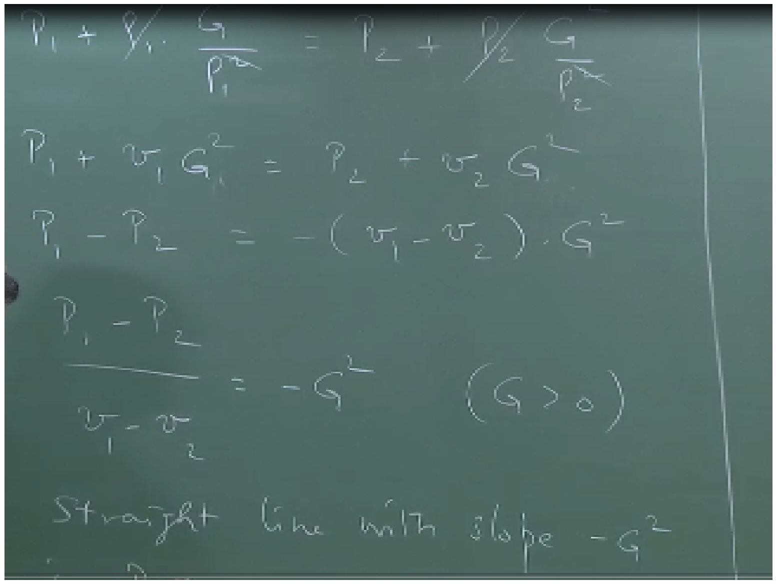

Then the momentum equation becomes p1+rho 1 time G square/rho 1 square=p2+rho2 G square/rho 2 square and if I simplify I get p1+ where I have used the fact that the specific volume is the reciprocal of the density where I have replaced rho 1 with the specific volume rearrange I can write this as I am sorry this is G square there is no G1 or G2 this is G square and if I rearrange this I can write this as p1-p2= -v1-v2 times G square.

Or I can finally write this as p1-p2/v1-v2= -G square and remember G is a positive quantity and let me emphasize that again G is always > 0. So, if you think of a PV coordinate space right this is the equation to a straight line passing through state point 1 and 2 with a slow which is negative remember G is always positive to the slope of this line is always negative. So, we are going to redraw the I am going to redraw this PV diagram with a slightly different perspective.

So, what does that going to look like let us take a look. So, let me complete this and say that this is a straight line with slope-G square in PV coordinates and this straight line is called the Rayleigh line okay.

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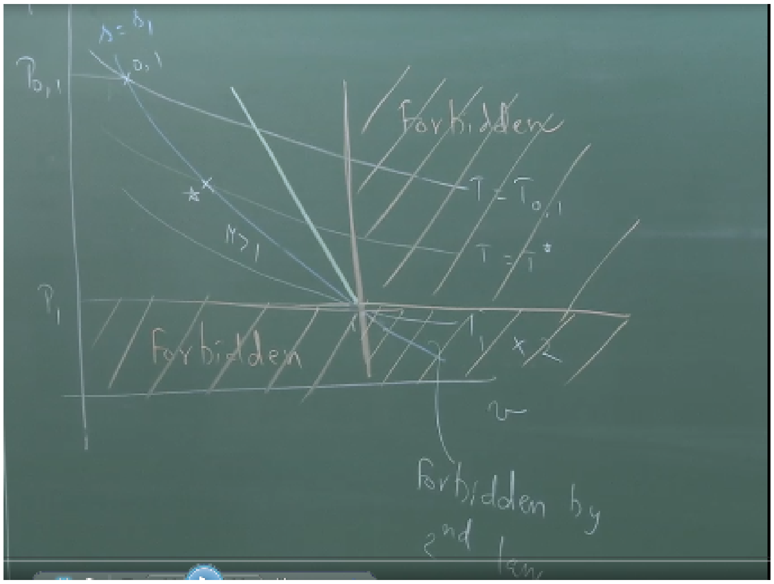

Let us see what this looks like, so basically start in the same way as before. Let us say this is my PV space and let us say p1, let us say this is state point 1. So, p1 is given and let us say that T1 is also given and let us say that this is T=T1 and the isentrope that passes through this point can be shown like this. So, this is s1 so let me do this slightly better so let me state this state right this is state 1. And so now the isentrope that passes through state 1 is given like this.

So, if I go up this isentrope let us say at some point where the let us say this is my stagnation temperature. So, the isotherm corresponding to T=T01 cuts over here so this corresponds to the stagnation said 0,1 with this being be p0,1 this is the stagnation state, and this is the if you since I know T01 I can calculate T star and the isotherm corresponding to T star is going to be something like this.

So, this is the sonic state where M=1 so we are in this branch where M is > 1 we know that M1 is > 1. So, we are in this part of the line where M is > 1. So, the initial flow is supersonic. So, far this diagram is the same as what we have drawn before right now what we want to do is we wish to draw this line. So, this line passes through the straight line passing through state 1 right and it has a slope which is always negative okay.

We do not know where state 2 is that we will do later we know that this is a straight line which passes through state 1 with a negative slope. So, where can this line be it must pass through 1 and it must have a negative slope. So, any straight line that passes through this can look like this like this or like this right this has a positive slope, and this also has a positive slope that means this straight line must lie in this quadrant or this quadrant.

So, I am going to now this is different from what we did earlier, so I am going to now draw a vertical line passing through state point 1. And now I am going to say that since the straight line passes through both state point 1 and 2 and it has a negative slope and we know that the line of the negative slope cannot look like this this tells me that state 2 cannot lie in this quadrant. Right and it also tells me that state 2 cannot lie in this quadrant.

Because the slope is positive if state 2 lies here than the straight line would look like this that has a positive slope. So, state 2 is not allowed to be in this quadrant. So, let us say that this is a forbidden quadrant state 2 cannot lie there and similarly state 2 cannot lie in this quadrant also, So, these 2 quadrants are ruled out so let me just now we are left with 2 choices the line can either be in this second state can be here or second state can be here starting with this state point.

Now, if the second state lies here let us say that state 2 is something like this if this is let us say state 2 this is state 1and this is state 1 and this is state 2 what can we say about p1 and p2 and v2 in comparison to p1 and v1, p2 is less and v2 is more. So, that means this is an expansion shock solution which is as you know is forbidden by second law of thermodynamics because the entropy for this point would actually be less than the entropy here.

Any solution which lies in the quadrant will have s2 less than s1. So, solutions which lie here are also forbidden by second law right they are forbidden but they are forbidden by second law. So, any solution any state point 2 that lies here is not allowed because any solution here represents an expansion shockwave with $s2 < s1$ which is not permitted which means that state 2 can lie only in this quadrant is that clear,

So, let us draw a line let me draw a line. So, this is a line with slope minus G square so starting from this state 2 must lie somewhere along this line we do not know where it has to lie we will determine that next, so this is the Rayleigh line. Okay, now go to the energy equation we have made use of the momentum equation we next go to the energy equation to see what we can do with it and to locate state 2.

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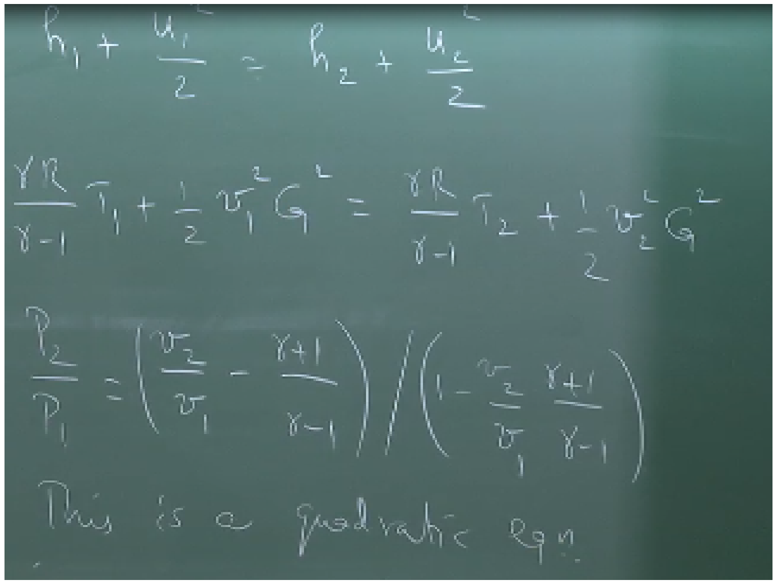

So, the energy equation if you remember was h1+ so we substitute for u1 and u2 in the same way as before and we end up with. Now, if you rearrange this and do-little bit of algebraic manipulation we can finally end up with an equation that looks like this. I am not going to derive this in detail here and it is available in the textbook but if you go through a little bit of algebra you can finally show that this can be written as.

Now, this is actually a quadratic equation in PV coordinates if I replace the p2 with a p I can show that this is a quadratic equation in PV coordinates and notice that it passes through state point 1 and 2. So, the actual state to be said that state 2 can lie anywhere along this line the actual state 2 must then lie at the point of intersection of this straight line and this quadratic. Right there is a quadratic which passes through state point1 and 2.

This is a straight line which passes through state points 1 and 2, So state point 1 lies at the intersection of this quadratic and the straight line and state point 2 also lies in the intersection of this quadratic and the straight line. So, let us draw the quadratic and then see what this looks like this quadratic is also called the H curve. I am going to try to draw the quadratic here but if I am not successful we will actually look at the figure okay.

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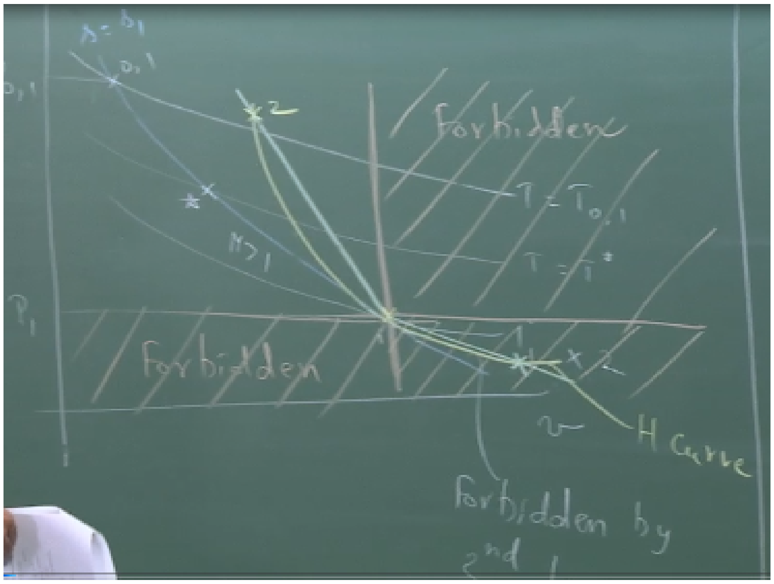

So, this is the H curve. So, notice that this was state 1 so. of intersection of the straight line with this H curve state 1 of intersection of the H curve with straight line and H curve gives me state 2. So, this is state 1 this is state 2 okay what is that there is also a point of intersection on the side of this is another Rayleigh line. You will get another state point 2 here but as we said earlier this solution is not allowed by the second law of thermodynamics.

"Professor- student conversation starts" Yeah, go ahead. 2 points should lie below stagnation point yeah; I will show this picture much more clearly in the diagram okay this is the best I can draw on the board we will look at this on the on the monitor here. **"Professor- student conversation ends"**.

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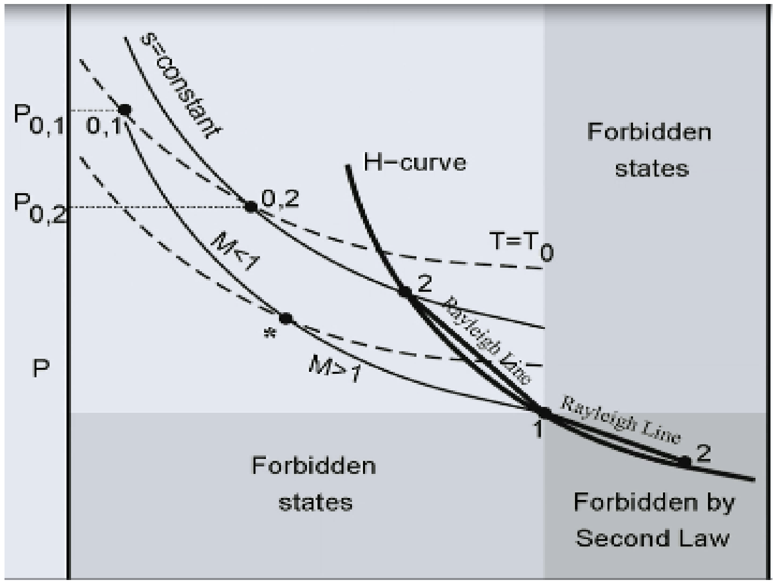

So, this is what the solution looks like, so you can see the features that we already discussed. So, this is state point 1 right and you see the Rayleigh line which is the straight line here on this side and the Rayleigh line which is the straight line on this side and states downstream states 2 are not state 2 is not allowed to lie in this quadrant or in this quadrant. As we discussed earlier, and it can actually lie in this quadrant where such a solution would employ $s2 < s1$.

And so, we do not allow the solution to lie in this quadrant also. So, the only possibility is for state point2 to lie in this quadrant and now if you draw the H curve which passes through state points 1 and 2 it just looks like this you can see that this point of intersection gives me the downstream solution. Once again you must keep in mind that we do not go from state 1 to state 2 we jump from state 1 to state 2.

This is an extremely important fact which we will which will become very clear to you then with this when we go to the next chapter. Okay please bear in mind we do not go from 1 to 2 through the intermediate states. We directly jump from state 1 to state 2 which means that we need not pass through any of the intermediate states. Okay as you rightly said you can see that state point2 lies below the stagnation isotherm and once again.

If I draw a sequel to constant here this is why where it intersects the isotherm $T=T0$ gives me my stagnation state 0,2 so this is p0,2 and as you can see $p02 < p01$. Notice also that the H curve that is drawn here is steeper than the isentrope that passes through state point 2 or the isentrope that passes through state point 1. Because this is steeper than the isentrope for a given change in specific volume.

The compression using normal shock is more effective than compression using an isotropic process because the H curve is steeper than the Isentropic for a given change in specific volume I get less pressure in my isentropic compression and more pressure in my normal shock compression because of that okay we demonstrated this the other day let us just write it down in words and go on.

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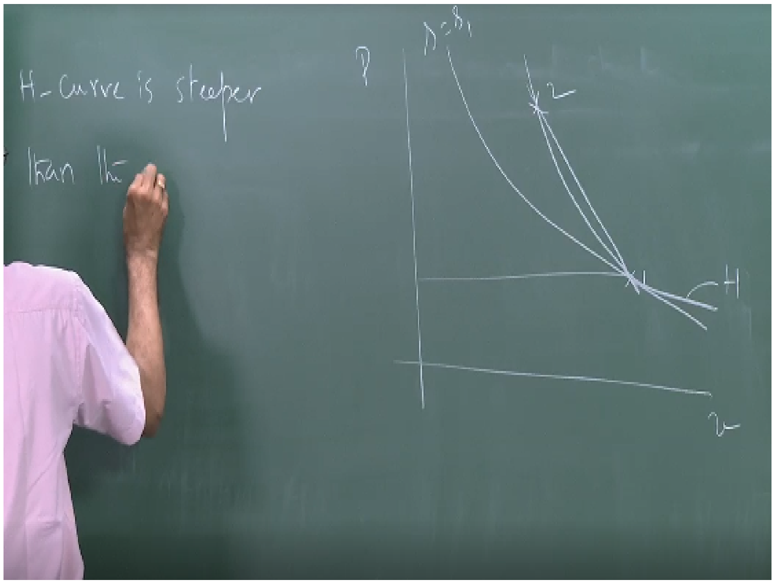

So, let us say that this is state point1 and let us say this is the isentrope which passes through state point 1. Now, remember the H curve is actually steeper than so this is state point 2 and this is the H curve since the H curve is steeper than the isentrope. Since it is steeper for a given change in specific volume v1-v2 the change in pressure this is p1 and this is p2 had we compressed using a isentropic process we would have remained on this is s=s1.

So, I would have come up to here. So, this is the pressure rise let me call this is 2s. So, this is the pressure rise that I would have seen if you had compressed isotopically but since the H curve is steeper or for the same change in specific volume I get much more compression. Okay we discussed this earlier now we are seeing this once again with the H curve also. Okay remember what we have drawn here, and this is the H curve that corresponds to an adiabatic process.

So, I am going to put it down as H q=0 and when we go wrote down the energy equation we said that the process was adiabatic. So, this H curve corresponding to q=0. Okay any questions now this theory will be very useful when we go to the next chapter where we are going to look at flow with heated edition especially this point about jumping from state 1 to state 2 without going through any of the intermediate states is very very important.

Because if you have to go through the intermediate state then you end up with a limiting condition called thermal choking which is not there for the normal shock wave because it is a discontinuous solution I can go from here to there. We will discuss that next.

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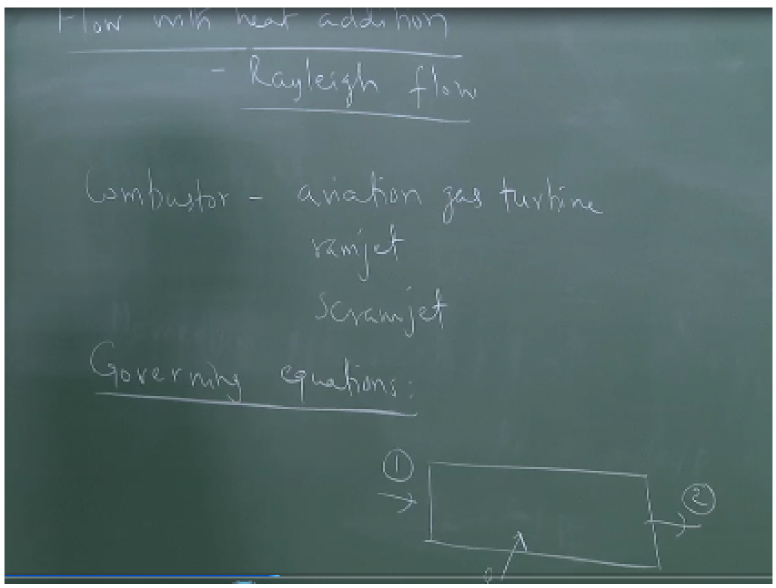

The next chapter that we are going to look at is flow with heat addition which is also known as Rayleigh flow. The theory that we are going to discuss here applies whether you are adding heat to a flow or removing heat from a flow both are okay but removing heat from a flow is very rarely seen in the real-life application. Okay whereas addition of heat to a flow is seen very commonly in combustors.

So, we talked about combustors and what are the combustors that we are talking about, so we are talking about for example aviation gas turbine combustors that is our interest in this course aviation gas turbine combustor or a ramjet combustor or even a scramjet combustor. These are the types of combustors that we are talking about where we add heat to the flow and what we want to know is if compressibility effects are significant then what happens to state points.

How do things change when we add heat to the flow? So, we already mentioned in the very early part of our course that in the case of a gas turbine aviation gas turbine engine in so far as the combustor is concerned compressibility effects were largely absent, but we will now show mathematically why that is so okay that is on something that we will do. So, given this the primary interest is in flow with heat addition we very rarely require heat removal.

Although the theory applies equally well that is our interest. So, let us write down the governing equations for such a flow and the governing equations are the same 1-dimensional flow equation. So, we are looking at a scenario which looks like this so let us say that this is my combustor. So, it is a constant area combustor right so is a constant area combustor flow comes in. Let us call this state 1 flow goes out let us call this state 2.

And we add heat to the flow here and we want to know if I add a certain amount of heat and I have a flow coming in at a certain condition what is going to be the outlet condition okay in other words if for example the flow comes in at a pressure p1,temperature t1 and velocity u1 and let us say stagnation conditions T01 and P01 what are the exit conditions going to be like. So, we want to know say p2 T2, u2 p02 and T02 for a given value of q.

That is what we are trying to find out in this particular chapter. So, the governing equations so you can see that because the area is constant there is only one velocity component, so it is one-dimensional flow. So, the governing equations are almost the same as before is a small modification because you are adding heat to the flow now.

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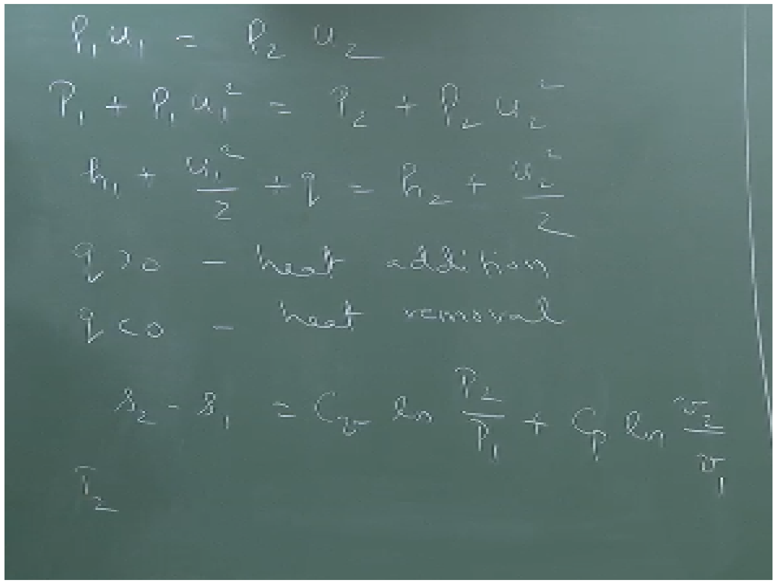

So, the continuity equation looks the same, momentum equation looks the same, energy equation looks almost the same. So, notice that I have now included heat addition term here this is heat added per unit mass or unit mass flow rate if you will apart from this the earlier calculations we assume q to be 0. Now, q is not 0 q can be positive which means heat is added. So, the sign convention is the same as an engineering thermodynamics.

If q is > 0 heat is added and if q is < 0 and of < 0 then we are removing heat from the flow and we supplement this with the entropy equation. So, s2-s1=cv natural log of p2/ p1+cp natural log v2/v1. One important thing that you should realize is that this is not a discontinuous solution that we are seeking. So, the flow enters at state 1 it goes through all the intermediate states before it reaches state 2.

Correct so this is not a wave solution this is a continuous solution okay. So, our strategy is going to be if it enters with this at the state and I add a small amount of heat delta q what will be the change of state then if I add another delta q what is the change of state point So, I keep adding delta q like this this until I reach the exit then I can draw the process diagram on TS or PV coordinates as I like right that is going to be our strategy.

Now, if you look at the energy equation what is that I can re write the energy equation like this if I write it as T2+u2 square/2cp- T1+u1 square/2cp=q/cp. So, I simply rewritten the energy equation here and you should recognize that this is T02 and this is T01. So, the addition of heat results in change of stagnation temperature.

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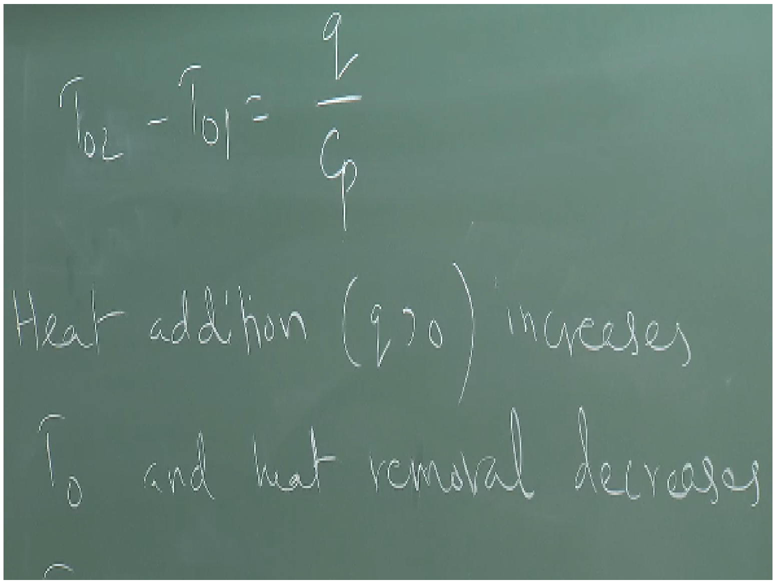

So, which means that T02-T01=q/cp. So, this is also different from the earlier solutions that we saw because in all the earlier solutions since q was 0 the stagnation temperature was constant. Now, because q is not 0 stagnation temperature changes from one point to another. And remember stagnation temperature keeps changing then T star will also keep changing from point to point that is something that we should keep in mind.

So, what we can see here is that heat addition wherein q is > 0 increases T0 and heat removal. So, if I add heat to a flow the stagnation temperature increases if I remove heat from a flow stagnation temperature decreases. So, what we are going to do next is how do we illustrate the process that the fluid under goes on a TS diagram that is what we are going to illustrate next. So, our strategy will be we start from the state.

We add a small amount of heat delta q to the flow and then we see where the next state is then we continue going like this to go from 1 to 2. That is going to be our strategy. So, for this purpose this form of the governing equation is not useful right we need the differential form of the governing questions which we have already written down earlier so that is what we are going to do.

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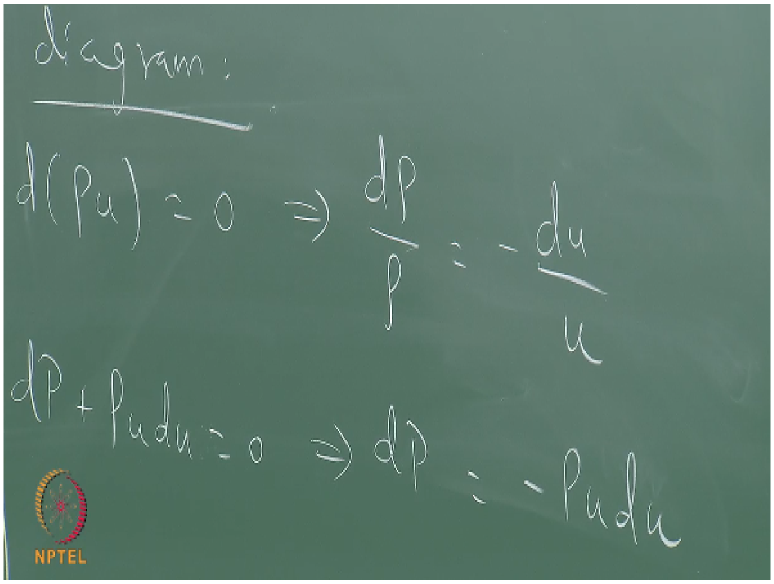

So, we will start with the differential form of the governing equation in order to do this. So, we will illustrate. So, if you remember the differential form of the continuity equation was d of rho u=0 right from which I can write d rho/rho=-du/u. Okay the differential form of the momentum equation looks like this and if I rewrite this I can write like this.

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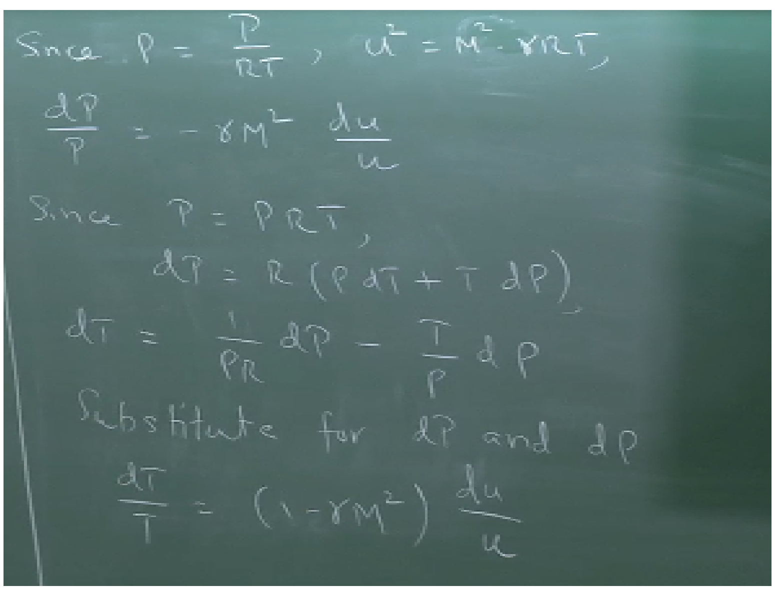

Now, if I use the equation of state to use the equation of state I can rewrite the rho here p/RT and if I use the definition of Mach number I can write u square=m square times gamma RT. So, this allows me to rewrite the equation as dp/=-gamma M square times du/u. Now, from the equation of state I can since p=rho RT. If I take the total differential of this I can write this as dp=R times rho dT +T times d rho and if I rearrange this again I get dT=1/rho R times dp-T/rho times d rho.

And now I can substitute for dp from here and I can substitute for d rho from here and if I substitute for dp from there d rho from here I end up with the following. So, substitute for dp and d rho we get dT/T=1-gamma m square times du/u. By now you must see the pattern in which we are developing the equation notice that we wrote d rho/rho as in terms of du/u we wrote dp/p in terms of du/u involving only the Mach number.

Now, we have written dT/T also in terms of du/u involving only the Mach number we will continue to do this for all the variables.

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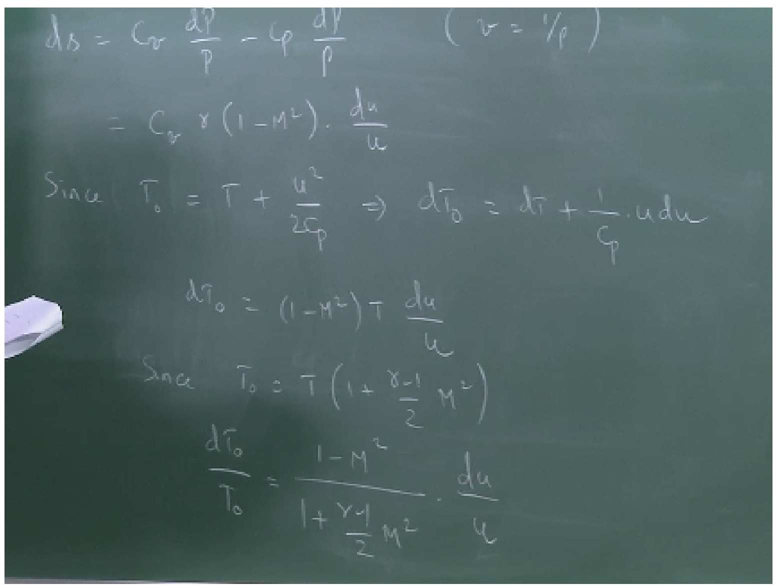

So, we have used the continuity equation we have used the momentum equation and we have used the we have not used the energy equation so far let us proceed with the entropy equations. So, we had ds=cv times dp/p and if I rewrite the dv/v in terms of d rho/rho I can easily write it like this is not difficult to do this right. We use the fact that v=1/the density and if I substitute for dp/p and d rho/rho from the previous equations.

I can easily show this to be ds=cv times gamma times 1-m square times du/u. Now, we will use the definition of the ignition temperature T0 if you remember since these ignition temperature T0= static temperature+u square/2cp.I can write this in differential form if I differentiate I get dT=dT+1/cp times u du and if a substitute for dT and du from the earlier equation I can write this as dT0=1-m square times T times du/u.

But if you look at the way we have been writing the expression notice that on the right-hand side we want to involve du/u and M those are the only 2 quantities we want. So, we would really not prefer to have something like this so would were the preferred to replace this with a T0 because we have a dT0 here. We would prefer to have this with a dT0 and I know that since T 0=T times 1+ gamma-1 /2 times M square.

I can rewrite as dT0/T0=1- M square/1+gamma-1/2 times M square times du/u. Okay the right-hand side involves du/u and the Mach number. Okay, so we need one equation which determines the change in Mach number dM. So, we need one equation for that.

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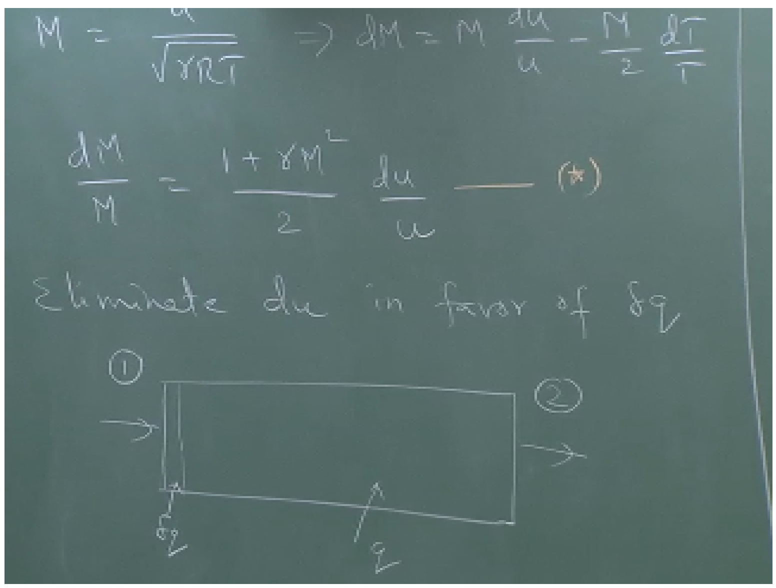

And that we can get from the definition of Mach numbers M=u/square root gamma RT and if I take the derivative of this right I can write dM=M times du/u-M/2 times dT/T. Now once again substitute for du/u and dT/T from before this gives me dM/M=1+gamma M square/2 times du/u. So, the nice thing is what we have done is for an incremental change in du I know how to calculate the changes.

Right I know how to calculate d rho, I know how to calculate dp, I know how to calculate dT, I know how to calculate dT0 from here right now to calculate dT0, I know how to calculate ds and dM. The only problem is everything involves a change in du, if you know the change in the du I can calculate all these other things, but this is a problem where we are looking at flow with heat addition.

So, what I want to know is if I add delta q right what is the change in the state. So, I would prefer to have delta q, or something related to that rather than du/u. So, let us label these equations with an asterisk to show that we are going to these are the governing equation. So, this is governing equation which I am going to denote with a star this is another equation which I am going to denote with a star.

The third equation is this and this and this. So, these are the equations that I am looking at I would just like to involve delta q in these equations rather than a du that is the only thing that I am going to do next. So, we would prefer to eliminate du in favor of delta q. So, basically what we are trying to do here is like this let me redraw the diagram that we drew earlier for the state 1 this is state 1 this is state 2 and the flow comes in at state 1.

And let us say that I look at an incremental amount of heat that is being added here delta q. So, when I add an incremental amount of heat delta q how does the state change from 1 to a new state. So, that means in the new state pressure is going to be p1+dp temperature is going to be T1+dT. But I already have equations that tell me the change I have d rho, I have dp, dT and so on. I just want to relate those changes to delta q rather than du that is what we are looking for.

Now, let us see how we do this.

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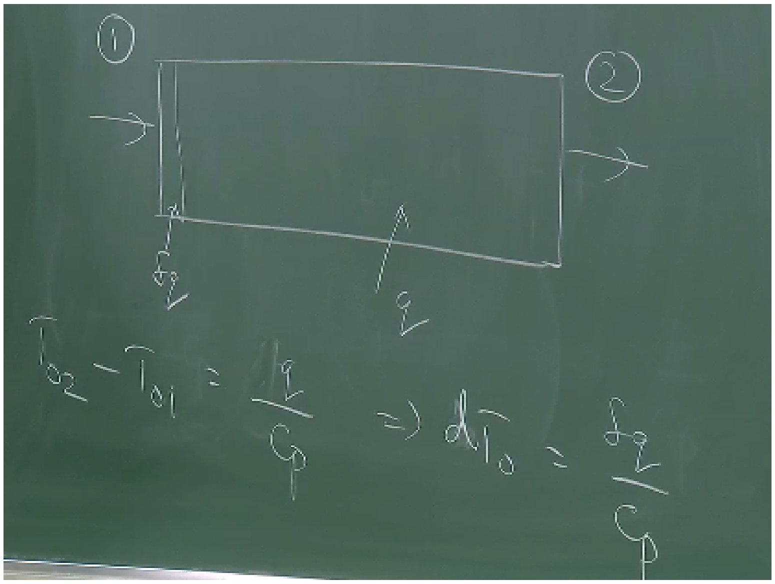

If you remember earlier, we showed that T02-T01=q/cp was something that we derived earlier. Now if I take a differential of this that tells me that dT0=delta q/cp right I am sorry dT0=delta q/cp So, either I get either involve delta q or I can write everything in terms of dT0, dT0 and delta q are the same right there is no problem.

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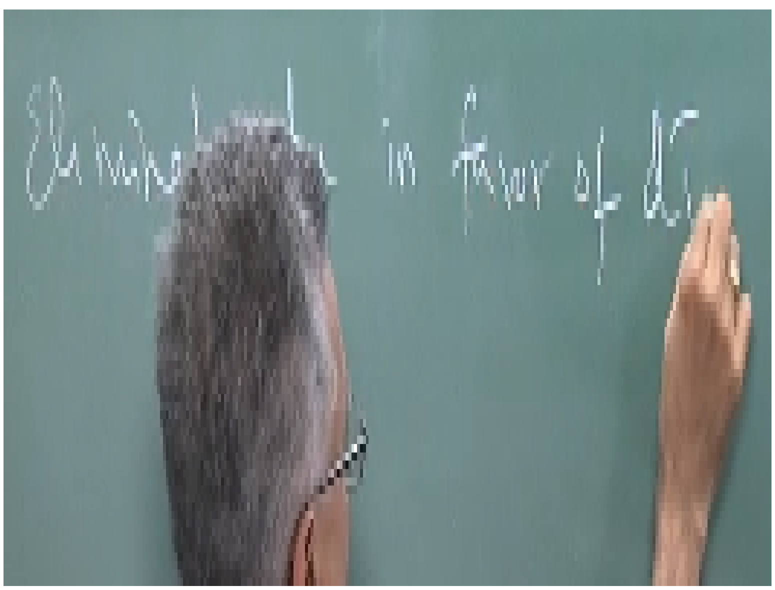

So, what I am going to do is rewrite or eliminate du in favor of dT0. So, what I am going to do is I am going to use this equation here right to eliminate du in favor of dT0. So, substitute for du in terms of dT0 in all the other equations and then I am at the point where I want to be I know how to calculate the incremental change in the properties starting from state 1. If you do that for each one of the starred equation the resulting equations will look like this.

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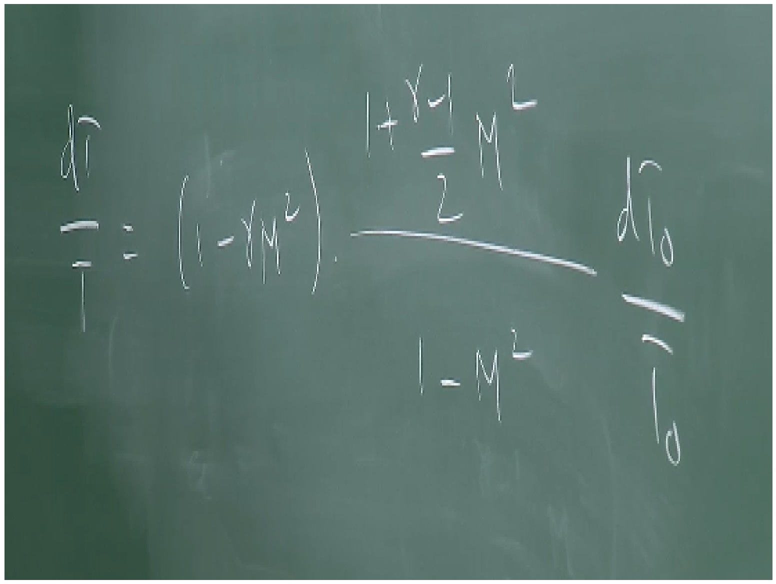

I get d rho/. rho= so when the stagnation temperature changes by this amount dT0 which is the result of an addition of heat which is delta q the density changes by this much. Okay similarly dp/p=-gamma M square times 1+gamma-1/2 times M square /1-M square times dT0 over T0. So, this is how much the pressure changes as a result of an addition of heat which is delta q right. And now I can look at change in static temperature dT/T=1-gamma M square times 1+gamma-1/2 times.

M square/1-M square times dT0/T0 entropy change ds=cv gamma times 1+gamma-1/2 times M square times dT0/T0.

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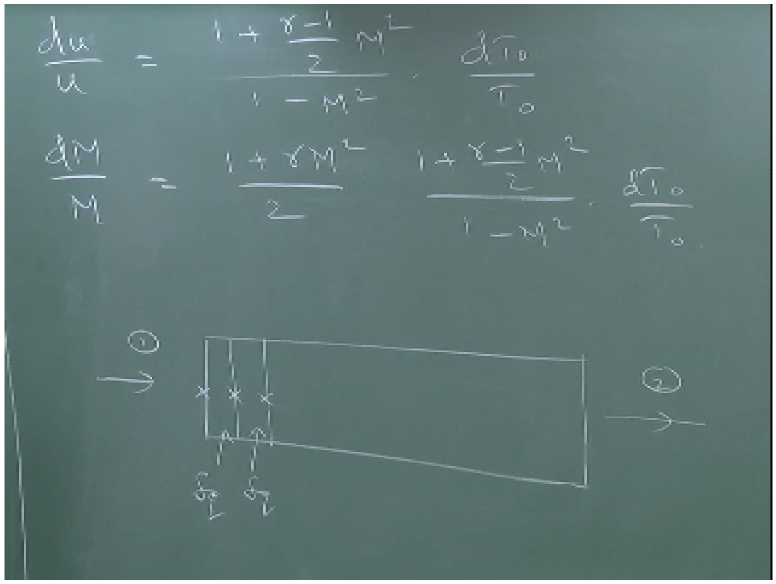

And then du/u=1+and dM/M. Now, the equations are in the form in which they want them I can calculate for a given an addition of heat delta q. I can calculate change in density, I can calculate change in pressure, change in temperature, entropy, velocity, and Mach number. Okay so this is exactly in the form in which I want these equations. So, what we are going to do next is for addition or heat removal in the flow.

We are going to try to see how the state point moves from state 1 how does it move on a TS diagram which way does it go. So, we will track the continuous state so basically as I said as I said earlier so we start with state 1 and we look at say an addition of delta q here and so starting from state 1 we move to this state then again, we look at an addition of delta q. So, starting from here we track the intermediate states until we reach state 2.

And we will plot this process on a TS diagram right that is what we will do in this next class.