Gas Dynamics and Propulsion
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Lecture – 06
Normal Shock Waves

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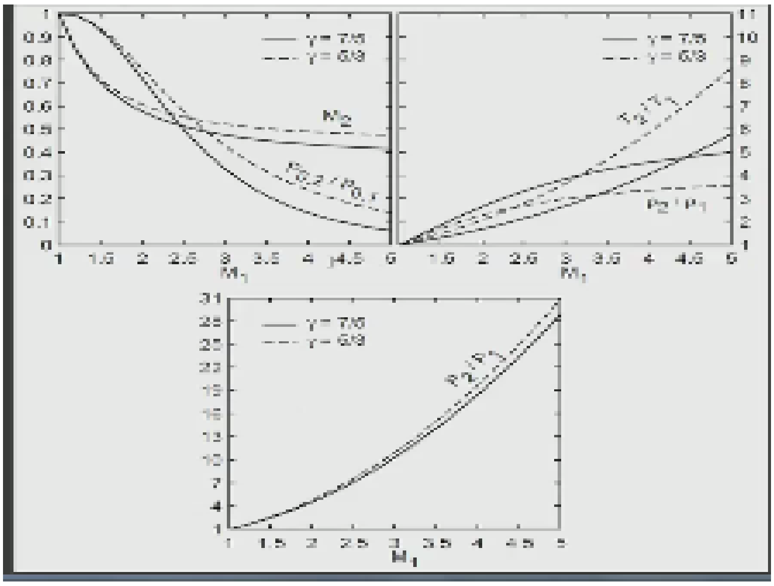

In the last class, we looked at calculation of Mach number, downstream of the normal shock wave and also variation of static pressure, static temperature and other downstream of shock properties, wave. This was the graph that we were looking at, as you can see all the downstream quantities for example, Mach number M2, static pressure, ratio of static temperatures T2/T1, ratio of densities and ratio of stagnation pressures are all marked here.

And here, ratio of static pressure downstream of a shock wave to upstream of the shock wave is also indicated here and one of the things that we are going to do today is to look at limiting cases for this solution. We are going to look at 2 limiting cases; one limiting case, where the Mach number M1 tends to 1 and one limiting case where, the Mach number M tends to infinity okay.

The first case as you can see, as you can imagine when M1 tends to 1, we can imagine that corresponds to the solution of an acoustic wave, which is propagating through a compressible medium and the other one corresponds to a very, very strong normal shock wave propagating through the medium okay. So, we start this discussion by writing down the expressions for the solution that we derived.

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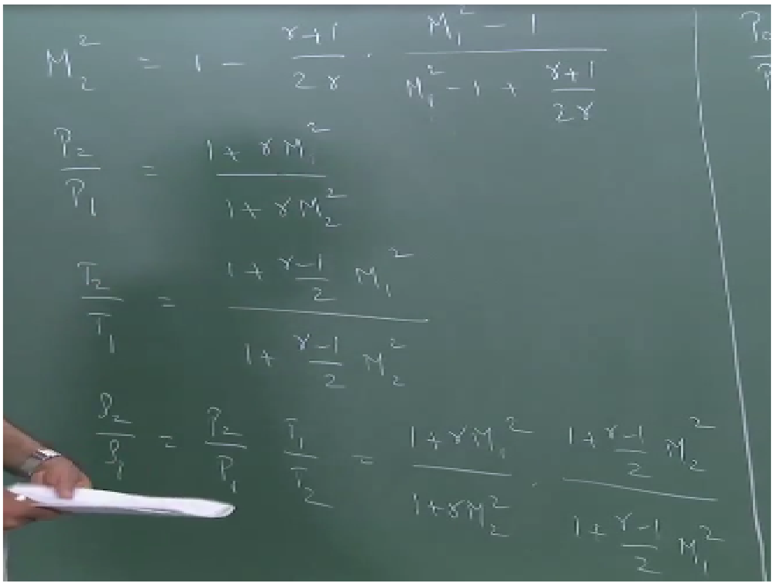

And if you remember, the solution looks like this, we wrote down M2 square downstream of the shock wave as 1-gamma +1/ 2 gamma times M1 square-1 divided by M1 square-1+gamma+1/ 2 gamma. This was the solution for M2 for a given value of M1 and the other quantities, we wrote down earlier, so let me just rewrite them here; P2/P1 = 1+gamma times M1 square divided by 1+gamma times M2 square.

And T2/T1 was given as, 1+gamma – 1/2 times M1 square divided by 1+gamma – 1/2 times M2 square and we also said that the ratio of densities; rho2/ rho1 = P2/P1 divided by T2/T1, so if I substitute from above, I get this to be =, 1+gamma M1 square divided by 1+gamma M2 square times 1+gamma – 1/2 times M2 square divided by 1+gamma – 1/2 times M1 square.

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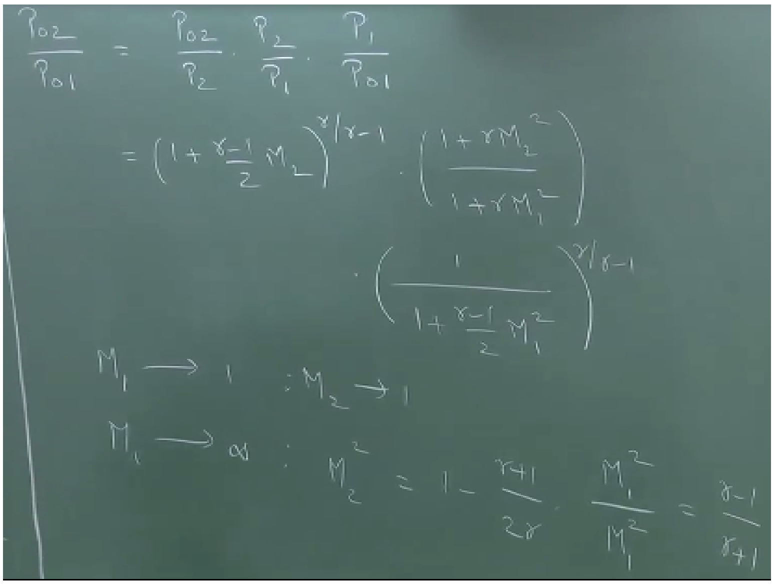

One additional quantity which is of importance in normal shock calculation is the ratio of stagnation pressures, downstream of; or upstream of the shock wave and if you recall, this can be written as P02/P2 times P2/P1 times P1/P01 and P02/P2 can be written as 1+gamma – 1/2 times M2 square raised to the power gamma over gamma -1 and P2/P1, we can write from here, so that is =, 1+gamma M2 square divided by 1+gamma M1 square and P1/P01 can be written in the same way.

So, this is multiplied by 1+gamma – 1/2 times M1 square to the power gamma over gamma – 1. So, this is the expression for the ratio of stagnation pressures across the normal shock wave. Now, we are going to look at what these values become in the 2 limits that we talked about; one limit, where M1 goes to 1 and another limit, where M1 goes to infinity or M1 becomes very large.

The limit, where M1 goes to 1, we should recognize that this is the limit in which we obtain the normal shock solution actually; the normal shock becomes an acoustic wave because an acoustic wave travels with the speed of sound, so the Mach number for the wave is 1. So, the limit M1, going to 1 should represent an acoustic wave, where the changes in properties across the shock wave or infinitesimally small okay.

But, if you let M1 go to1 in any of these relationship, for example, if I let M1 go to in this relationship okay, notice that when M1 goes to 1, M2 goes to 1 as well, right. So, if M1 goes to 1, then in this limit M2 goes to 1 and if M1 and M2 go to 1, then you can see that all these expressions revert to being 1; P2/P1 = 1, T2/T1 = 1 and so on. So, the reason why we are getting P2/P1 to be = 1, is that P2 in this case, is only infinitesimally different from P1.

So, P2 is actually, if you recall P1+delta P divided by P1, so if P1 is a large number, then the delta P is going to be a very small number, which is why we are getting the ratio to be = 1 in all these cases, okay from a normal shock solution perspective and if you look at the; if you look at the graph here, you will see the same thing. If you look at the graph here, you can see that as M1 goes to 1, you notice that M2 goes to1 and P02/P01 also goes to 1.

And as M1 goes to 1, you notice that T2/ T1 goes to 1 and rho2/ rho1 also goes to 1 and so does P2/P1 and also goes to 1 in this; in this limiting case, okay. Let us take a look at that again, as M1 goes to 1, so you can see M1 here as M1 goes to this limit M2, you can see goes to 1, irrespective of the value for gamma M2 goes to 1 and as M1 goes to 1 in this limit, you can see that T2/ T1 for both values of gamma tend to1 and rho2/rho 1 also tends to 1.

And P2/P1 also tends to 1, in the limit as M goes to 1 and in this case, because the process is an isentropic process, there is no change in entropy; you can see that P02/P01 also goes to 1 in this case. Remember for an acoustic wave, the process across the wave is an isentropic process, so in this case, there is no loss of stagnation pressure P02/P01 also tends to 1, okay. So, that makes sense, so it tells us that the normal shock solutions are consistent with whatever we derived earlier in the M = 1 limit.

Now, we are going to look at the other limit, which is the M tending to infinity limit. Now, as M1 becomes very large, I can see from this expression that if M1 becomes very large, then M1 dominates in the numerator and M1 also dominates; M1 square dominates in the denominator, see the other 2 terms as M1 becomes very large, the other 2 terms become much smaller by comparison.

So, I can then write in the limit as M1 goes to infinity, we can see that M2 square goes as 1-gamma+1/2 gamma and M1 square dominates in the numerator, so I just do that and M1 square again dominates in the denominator and so I can do this, as M1 goes to infinity subtracting 1 or adding this kind of number, will make no difference to the denominator, right and M1 becomes very large, right.

Let us say M1 is 10, so M1 square is 100, so subtracting 1 from 100 is still 99, very close to being 100, right that is why the term M1 square dominates in both the numerator and the denominator, so that M2 square tends to; in this case M2 square tends to; if I do the algebra, I can see that this is nothing but gamma-1 divided by gamma+1, so that is the limiting value as M1 becomes very large, okay.

And you can see that in this graph also, so if you look at this graph, you can see that as M goes to infinity, M2 tends to a finite value depending upon the value of gamma, M2 tends to a finite value, M2 does not go to 0, M2 tends to a finite nonzero value as M1 goes to infinity, okay. So, that is clear from this picture, we can do similar things for the other quantities also. So, if you look at the expression for P2/P1 as M1 becomes very large, we can see that this quantity is going to keep on increasing.

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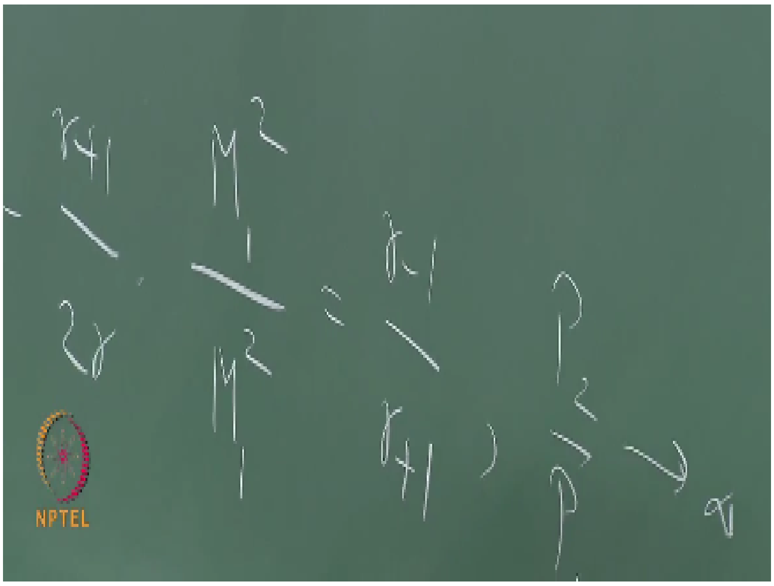

It is not going to tend to a finite value because M1 keeps becoming larger and larger, so we can see that as M1 tends to infinity, P2/P1 is going to tend to infinity as well, right. So, P2/P1 in this case will tend to; will tend to infinity and that is what this graph is also telling you, so you can see from this graph that as M becomes; M1 becomes very large, P2/P1 keeps increasing without showing any sign of levelling off, so P2/P1 tends to infinity in this case.

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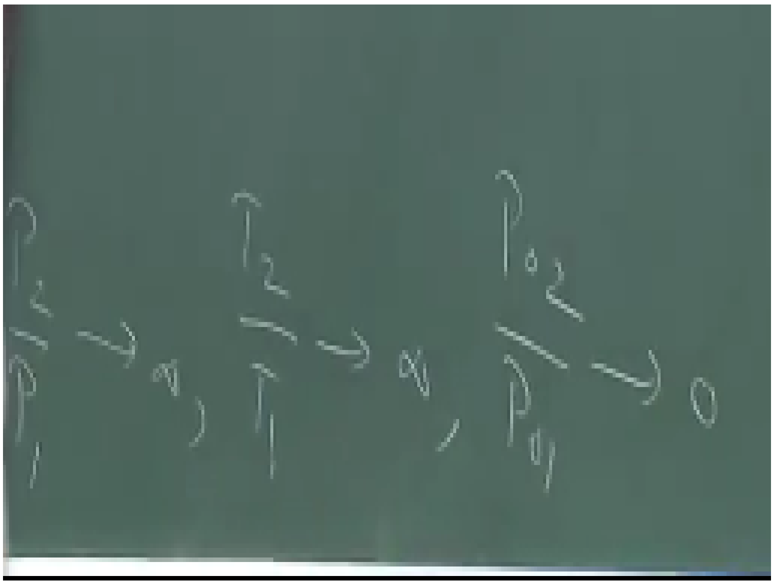

And if I look at the expression for T2/ T1, what do I get? So, if you look at the expression for T2/T1, you see a similar behaviour as M1 becomes very large, this term dominates and it continues to grow. So, T2/T1 also tends to infinity as M1 becomes very large, so I can write the following and that trend is visible from this graph also, as the Mach number increases, we can see that T2/T1 in this case, for both these values of gamma, we can see that T2/T1 continues to increase, does not show any sign of levelling off; continues to increase to large values.

Now, what about P02/P01? Okay, if you look at P02/P01, once again if you apply the limiting process, we actually get 2 M1 square terms in the denominator, so as M1 becomes very large, P02/P01 goes to 0, this means that this loss of stagnation pressure is 100 %, right. So, Po2/P01 tends to 0, as M1 becomes very large and that trend can be seen again once again from this graph.

So, as M1 increases, we can see P02/P01 keeps becoming smaller and smaller and asymptotically, we can see that this is going to reach zero 0, as M1 becomes very large.

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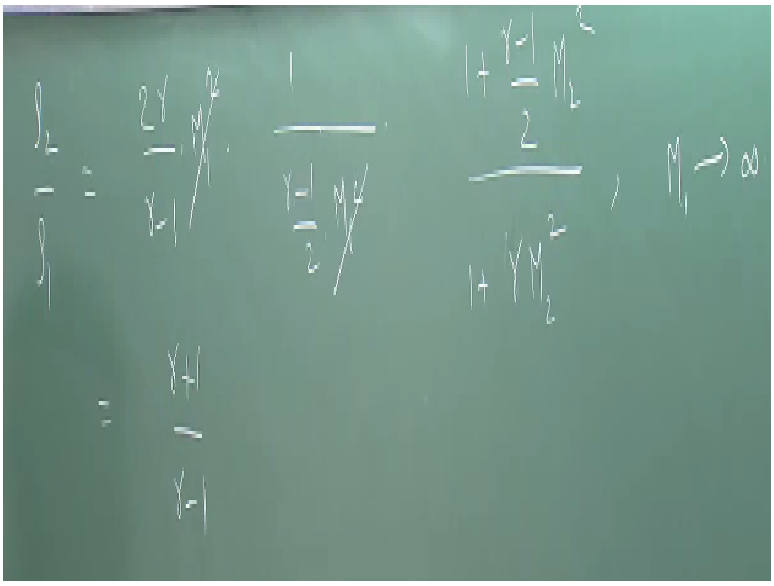

One additional quantity that we want to talk about is, rho 2/ rho 1, now it is quite possible that rho 2/ rho 1may tend to a finite value because we have M1 square in the numerator and another M1 square in the denominator okay, so if you do this limit properly, we can actually show the following. Let us try to do this limiting process properly, as M1 goes to infinity. So, as M1 goes to infinity, we can see from here that this term will dominate in the numerator, right.

And again this term will dominate in the denominator, so if I write it that way, I can write this as 2 gamma/gamma-1 times M1 square and for the second term, this is going to dominate, so I am going to write this as 1/ times 1+M2 square, so we are writing this for M1 becoming very large. So, I can cancel out these 2 and remember, this as M1 becomes very large, M2 is going to; tend to to this value here, right, we have already written that down; M2 is going to tend to this value, right; gamma -1/ gamma+1.

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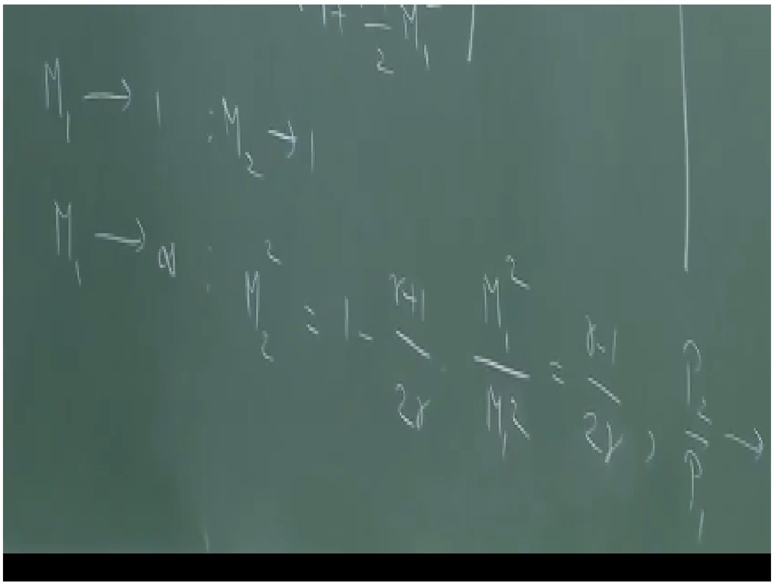

So, I substitute that value into this expression and if I simplify, I get the fact that rho2/ rho 1 tends to be gamma+1/ gamma – 1, okay. So, I substitute for this M2 square from here and if I put this in, I am sorry; they do this correctly, oh! I am sorry; yes, please make a small correction, this one should be gamma -1/ 2 gamma this is also M1 square but this is gamma -1/ 2 gamma.

So, if I substitute that expression into this one here, I end up with this; so, rho2/ rho 1 tends to a finite value as M1 becomes very large okay and that trend is visible from this graph here, so you can see that as M1 increases, rho2/ rho 1, we can see tends to a finite value, so they curve levels off and it tends to a finite value for both the cases, when gamma is 7/5 or when gamma is 5/3, so you can see that this curve levels off as M1 goes to infinity.

So, it tends to a finite value okay, this is only of academic interest because as T1; I am sorry; as Mach number increases, T2/ T1 keeps increasing and if you remember, all the equations that we have written down, made use of the assumption that the gas is calorically perfect and in the first chapter, we said that this assumption is true for temperatures below 600 Kelvin, so as your M1 increases, T2 will very easily go beyond that and so the calorically assumption is no longer valid.

So, from that perspective these solutions or this limiting solutions are of interest academically but they may not be accurate in a realistic sense because of the assumptions that we have made use of to derive this okay, but they are definitely of interest to know that as M1 tends to a large value, M2 tends to a finite value and density ratio tends to a finite value but not some of the other quantities, okay.

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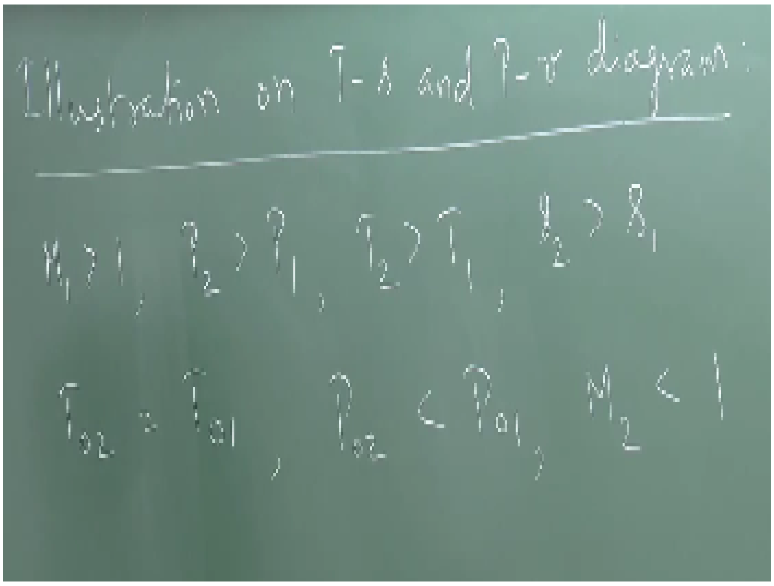

What we are going to look at next is illustration of the normal shock solution on TS and PV diagrams, we have already seen that, so we are going to illustrate the normal shock solution on TS and PV diagrams. So, we start with the TS diagram first and before we do anything, you need to remember the fact that for a normal shock solution, M1 is > 1, correct and the pressure increases across the normal shock P2 is > P1 and the temperature; static temperature increases across the shock wave, T2 is > T1.

And the process is also irreversible, so that s2 is > s1 and since there is no heat addition or work addition, right T02 = T01 but because of the increase in entropy, P02 is going to be < P01, correct, so these are the features of the normal shock solution because s2 is > s1, P02 is going to be < P01. We should remember these things, when we do this.

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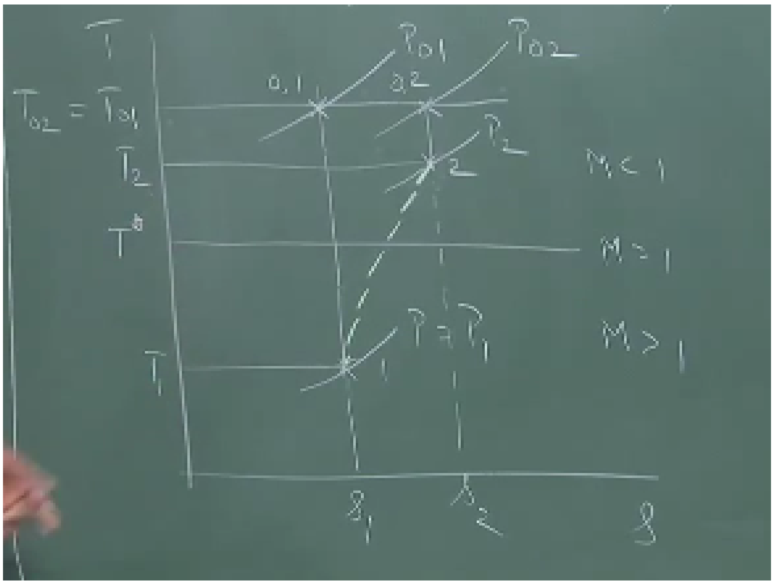

In addition, we know that downstream on the shock wave, the flow is subsonic. So, when we illustrate these state points on the TS diagram, all these information must be put in. So, let us see how we go about doing that. So, first we do this on a TS diagram, let us say that for the given state P1 and T1, say that this is T1, this is the isobar corresponding to P1, so that is the initial state 1 and let us say that this is s1.

Now, we draw a vertical line from here and this we take to be T01 and so this is my stagnation state0, stagnation state corresponding to the static state 1, so that is T0, 1, correct and the isobar passing through this state point, which will look like this is P = P01. Now, if I calculate T star corresponding to this T 01, I can show that say, like this so this is T = T star corresponding to this stagnation temperature.

So that, this represents the sonic line, anything below this is supersonic and anything above this is a subsonic state, okay. So, you are completely illustrated state 1 now. Now, we need to show state 2 on this diagram, so we have taken care of this, right, this has been demonstrated here. So, state 2; P2 is > P1, T2 is also > T1, so that means state 2is going to be above this, okay and since M2 is < 1, state 2 is going to be even above this line, correct because M2 is < 1.

Now, the only question is, does it lie on this line to the left of this line to the right of this line? Since s2 is > s1, state 2 is going to lie to the right of this line over here, okay. So, let us denote that, so this is state 2 and the 2 states are connected by; I am going to show this as a dashed line to denote; indicate the factor, this is an irreversible process, so that is static state 2 and let me just show this as P2and I can show this as static temperature T2.

Now, the stagnation state 0,2 corresponding to static state 2 can be obtained by just drawing a vertical line like this, notice that this is s2, so s2 is > s1. So, this is going to be my stagnation state corresponding to the static state 2 because T01 = T02, where this line cuts this gives my stagnation state because T01 = T02, that is my stagnation temperature line, so the stagnation pressure line P02 will look like this.

And if you recall, when we drew isobars on a TS diagram, if you remember, we said that isobars diverge from each other, right and so you can see from this construction that P02 is indeed < P01. In fact, any time there is irreversibility or an increase in entropy and you move from one state to the right of that state, there will always be a loss of stagnation pressure because the isobars diverge from each other.

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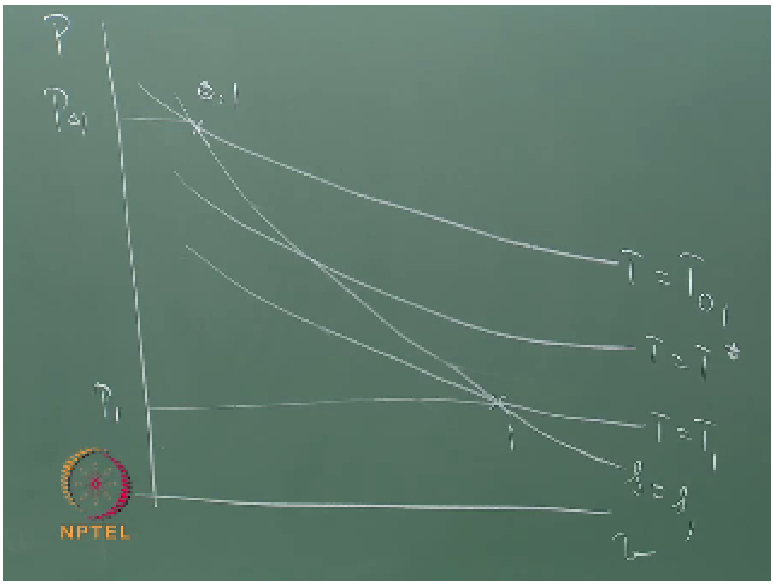

Anytime you move to the right, there will be a loss of stagnation pressure okay and that is what this solution indicates. So, this indicates the normal shock solution completely on a TS diagram. What we will do next is demonstrate the same thing on a PV diagram okay. So, we start with the PV diagram in the same way, we draw the PV diagram in the same way, so let us say that this is the initial state.

Let us say, this is T1 and let us say that the isotherm corresponding to T1, looks like this, let us say this is T = T1, so that this is state 1. Now, you recall that isentropes on a PV diagram are steeper than isotherms right, so the isentrope s = s1 that passes through this, will look something like this, so this is the isentrope corresponding to s = s1 and the stagnation state, let us say, this is my stagnation state 0, 1, so that this is P01.

And the stagnation temperature line can be shown like this, so this is T = T01 and the sonic temperature line or the sonic line can be shown here like this, T = T star will look like this, right. So, whatever information we put in here, we have now shown on the PV diagram. Now, we need to show state 2 and we show state 2 again by looking at this, P2 is > P1. In addition to this, now we need to know, is rho 2 > rho 1 or not?

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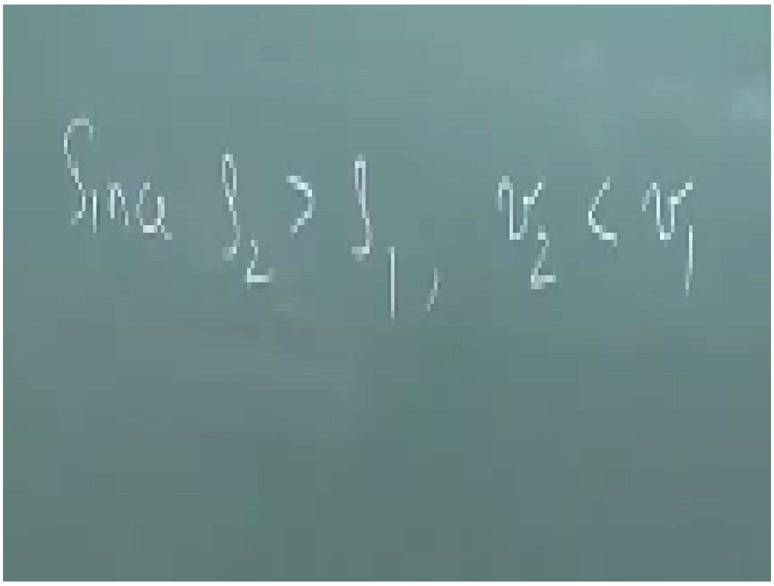

Rho 2 is > Rho 2, that we have already shown, so since rho2 is > rho1, v2 is going to be; the specific volume is going to be < v1, so that means that state 2 is going to lie above and to the left of state 1 but it is going to lie on a different isentrope because the entropy increases across the shock wave. So, we draw a different isentrope, which looks like this.

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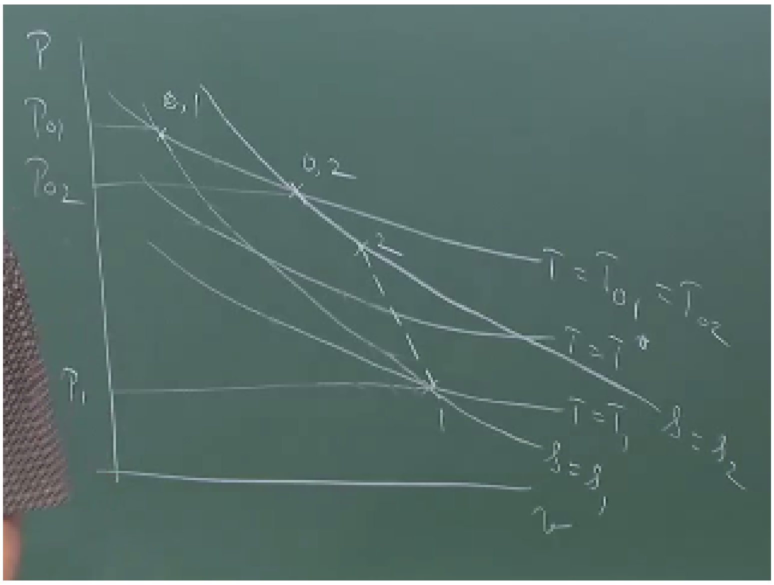

So, let us say that this is my s = s2 and so state 2 will lie over there and I am going to connect these 2 with a dashed line that looks like this. Now, remember the stagnation temperature remains the same, right, so the point of intersection of this isentrope, remember the point of intersection of s = s2 with the isotherm corresponding to T = T02 gives me my stagnation state, okay. So, similarly the point of intersection of my isentrope; s = s2 with the isotherm T = T02 which is this one, right.

So, this is T = T01 = T02, so the point of intersection of s = s2 with T = T02 over here gives me my stagnation state 02, right and the corresponding stagnation pressure is then going to be T02 and it is clear from this that P02 is < P01 because the isentropes are steeper than the isotherms P02 is < P01 because we are climbing onto a isentrope which is at higher value and because isentropes are steeper than isotherms, P02 will always be < P01.

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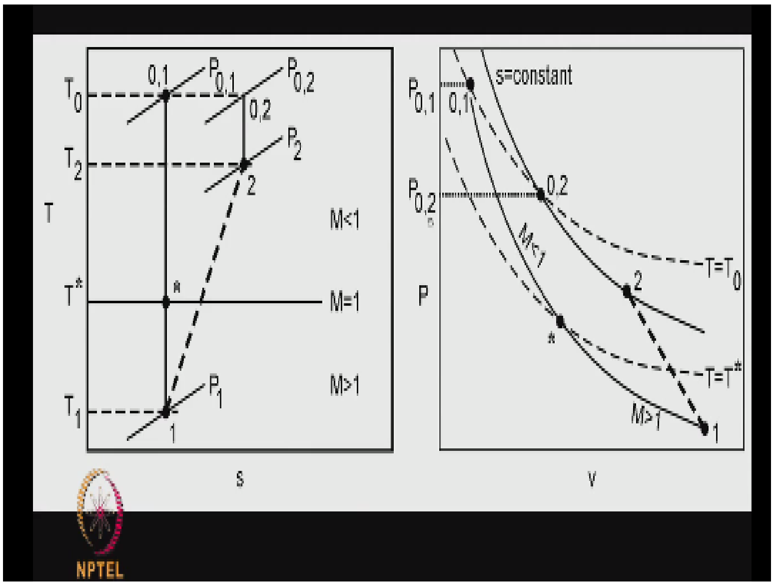

This is the reason why we spend so much time in the beginning trying to draw isobars and isotherms on a TS diagram and isotherms and isentropes on a PV diagram, okay. That is why this is very important because this tells us lots of things about stagnation pressure especially, okay. Let us quickly look at the diagrams before we move on. So, here the same things are illustrated here graphically, this is a much nicer diagram.

So, you can see state1 here, T1 corresponding to state 1 and the isobar P1 corresponding to state 1 and we take s = s1 line all the way up and so we show T0, so this is stagnation state 01, this is the stagnation pressure line P01 and once I know T0, I can calculate T star, so this is the line corresponding to T star and this any state below this line is a supersonic state, any state above this is a subsonic state.

Now, state 2, downstream of the shock is a subsonic state at a higher pressure, higher entropy and higher temperature, so it lies to the right of state 1 over here and I connect these 2 with a straight line like this and again the point of intersection of the line, s = s2, which is a vertical line in these coordinates with the stagnation temperature line gives me state 0, 2 that is what is shown here, okay.

And the isobar, which passes through the state P02 is shown here and you can see that P02 is < P01, okay. The same information is drawn here in a PV diagram, we do the same thing, this is state 1 corresponding to P1, T1 and this solid line is s = s1, so this solid line is s = s1, this is the isentrope corresponding to T = T0 or T01 or T02, so the point of intersection of s =s1 with this isotherm gives me P0.

And this isotherm, which is this one here corresponds to the sonic isotherm T = T star. Now, state 2 is at a higher pressure, lower specific volume and higher entropy than state 1, so state 2 lies here and the 2 states are connected through a dashed line. So, this is the isentrope corresponding to s = s2, so the point of intersection of s = s2 with the stagnation temperature line gives me my state 0 2, which is over here and the corresponding pressure which is P02 is below P01 showing that there is a loss of stagnation pressure, okay.

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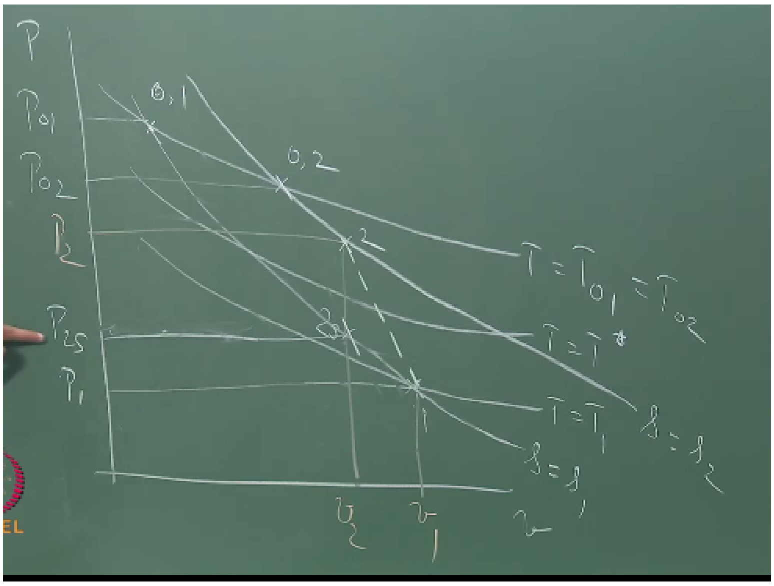

Now, let us look at this diagram a little bit in slightly more detail to get an idea about the effectiveness and the efficiency of the shock wave compression process okay. Now, if I look at state 1 and 2, notice that the change in specific volume, so this is v2, this is v1, so from a given state, this is the change in specific volume and you can also see that this is P2. So, the normal shock compresses the gas from P1 to P2 with this much change in specific volume, correct.

Now, let us say that we have built a device and the volumes are fixed in the device because it has a certain physical size, I cannot keep changing the size right, now in the same device, if I had tried to compress for the same change in specific volume, if I try to compress this in an isentropic manner, what would have been the pressure that I would have achieved, right. So, if I try to compress the gas in an isentropic manner starting from state 1, where would I have ended up for this change in specific volume?

The state would have been along s = s1, which is this line here, so I would have ended up over here, so this would have been my, let us say 2s state, so if I go from here to here, this is where I would have ended up for the same change in specific volume and you can see that the increase in pressure, this line is not very good, so let me just draw a proper line, so this is; so you can see that the resulting pressure that would have; I would have obtained is much < P2.

For the same change in specific volume, the isentropic process gives me a lesser value for P2 when compared to the normal shock compression process, which is why we said in the other class, the normal shock compression process is effective but not efficient, whereas isentropic compression process is efficient but not effective because for the same change in specific volume, it gives me a lesser value of pressure downstream, okay.

Now, this is very important because in practical devices, the specific volume relates to the physical dimensions of the device, right. So, I am very concerned when I design a vehicle, I am very concerned in the kind of sizes that I can have, I cannot have intakes or compressors which are infinitely long, they have to be a finite size. So, within that finite size, I must be able to realize certain pressure rise, so normal shock is very good for that but it comes with the price.

There is a loss of stagnation pressure from P01 to P02, okay. So, in practical situations, how do we make this compromise? We know that normal shock is effective but not efficient, isentropic is efficient but not effective how do we make a compromise? The graph that we looked at before tells us how we can make that compromise, right. So, P02 or P01 is what we are looking at and as you can see from here, when M1 = 2, my P02/P01 is about 0.7.

That means, I am losing about 35 % of my stagnation pressure, so stagnation pressure loss is 30% when M1 = 2 and if I go to Mach number of 3, my P02 or P01 becomes 0.3, which means I am losing about 70% of stagnation pressure, so from M1 = 2 to M1 = 3, the loss of stagnation pressure goes from 30% to 70%. So, normally in practical devices, what we would do is, once the Mach number comes down to 2, we will compress it with normal shock.

Because the loss of stagnation pressure is acceptable for M1 = 2 but we will not do it for M1 =3, so to decelerate the flow from higher Mach number, we use a different method but once we reach a Mach number of 2 or so, we can then quickly go to subsonic state by using a normal shockwave to compress this. So, there the loss of stagnation pressure by a normal shockwave is acceptable, so that is how we draw a compromise in practical devices, okay.

We are going to work out a numerical example next, to demonstrate some of these ideas and work with the real values; numbers give us better feel for what we are talking about, that is what we are going to do next. This worked example is the same as the one that is given in the book, so let me read the example, then we will give the information, transfer the information to the board.

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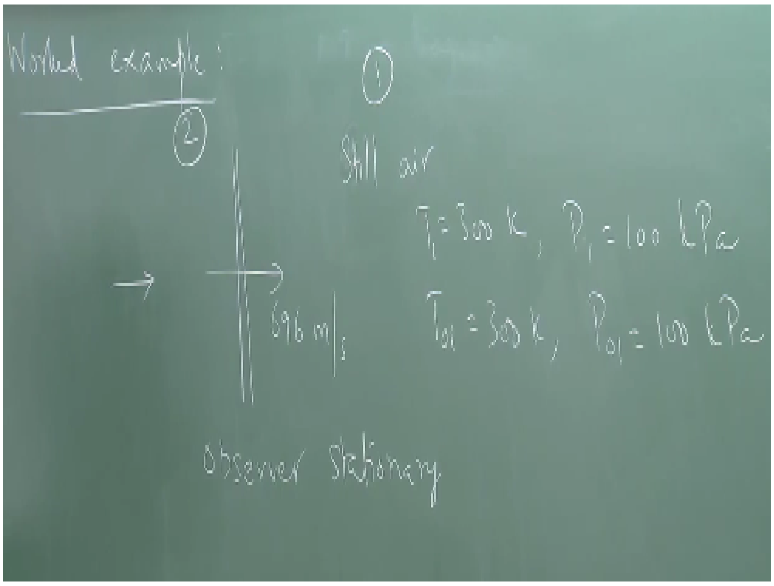

Consider a normal shock wave that moves with the speed of 696 meter per second into still air at 100 kilopascal and 300 Kelvin. Determine the static and stagnation properties I had off and behind the shock wave in the stationary and moving frames of reference? This is the problem statement; let us try to transfer that information to the board, so we have a normal shock wave like this, which is moving into still air.

The speed of the normal shock wave is 696 meter per second, we know that as a result of passage of the shock wave, the flow here is going to move with a certain speed, its pressure would be different, temperature would also be different and the still air is at a static temperature of; so we will label this as 1 as always and label this as 2, so T1 = 300 Kelvin P1 = 100 kPa. Now, what is T01 in this frame of reference, where the observer; remember, in this frame of reference, the observer is stationary.

In other words, I am stationary and I see a normal shock wave moving at 696 meter per second into still air, so the air here is at the static temperature of 300 Kelvin and a pressure of 100 kilopascal, so what is the stagnation pressure in this region? T01 here is going to be 300 Kelvin because it is still air, T01 = T1, so this is 300 Kelvin and P01 = 100 kPa. Now, as we said earlier, we want to move to a frame of reference, where the wave is stationary, right.

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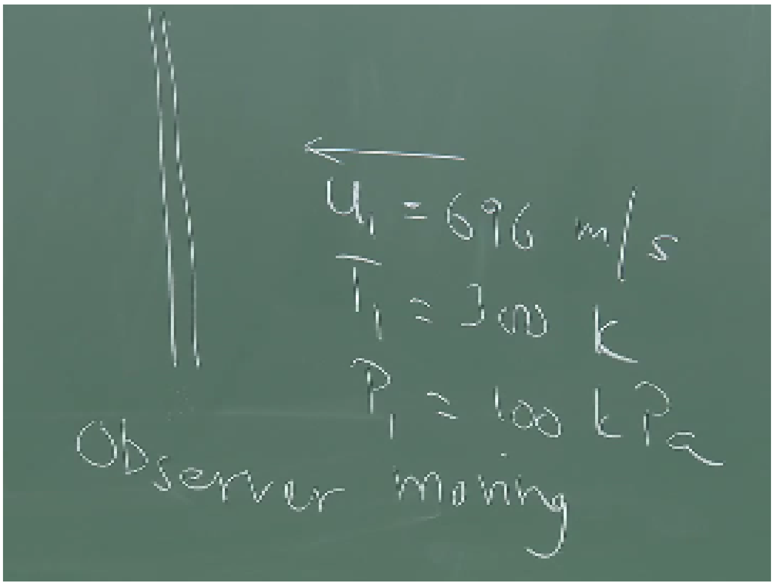

The observer is moving, so we go to that kind of a frame of reference, which looks like this, so here observer; so this is still 1 and this is still 2, so in this case; so this velocity u1 is going to be 696 meter per second, right. So, the wave has become stationary, so the observer perceives the flow to approach with the speed of 696 meter per second, correct. So, initially the wave was moving like this at 696 meter per second.

Now, I have gotten onto the wave, so I am also moving at a speed of 696 meter per second, so the flow seems to approach the wave with the speed of 696 meter per second that is what I have written down here. Now, as we said earlier, static temperature is frame independent whether I measure it in a moving frame or stationary frame, static temperature remains the same, so T1 = 300 Kelvin and P1 is also the same, so P1 is 100 kPa, okay.

Now, this velocity u2 is unknown, so we need to determine all the conditions downstream of the shock wave; the speed, static temperature, static pressure and stagnation pressure and stagnation temperature that is what we are going to do, okay. So, we start the calculation in this frame of reference remember, we are going to do the calculation in this frame of reference, so we first start by looking at M1.

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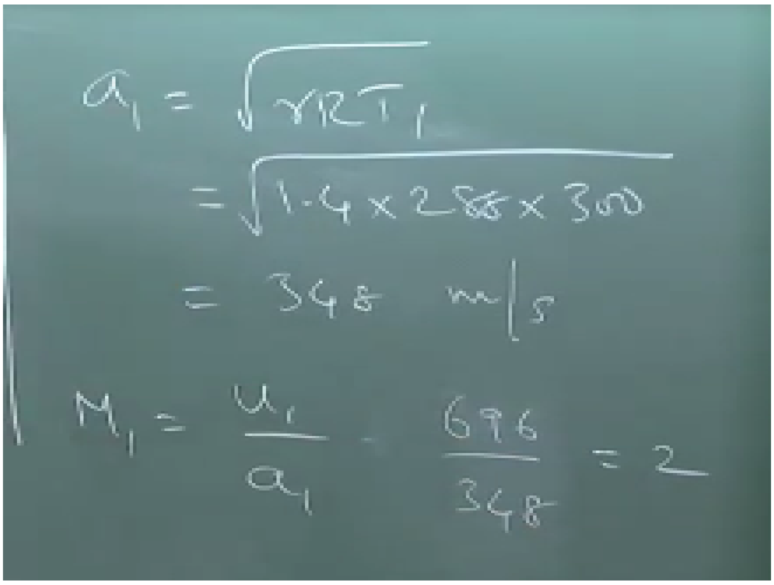


We need M1, once we have M1, right, so if you look at these expressions, once I know M1, I know everything else, correct, so once I know M1, I can calculate M2, once I know M1 and M2, all the other quantities can be calculated, right. So, we start by calculating the speed of sound a1, so I am going to do this; a1 = square root of gamma RT1 and if I substitute the numbers, so this is 1.4 times 288 times 300 Kelvin.

So, this comes out to be 348 meter per second, right, so M1 is going to be u1/a1, so that is 696 divided by 348, which is nothing but 696/348, so M1 = 2, we have a choice to make now, we can calculate M2 and other quantities using this formula by substituting the values or we can make use of gas tables where these things have already been done, so you can go to the table for the given value of M1, I can look up the quantities, okay.

(Refer Slide Time: 41:32)

1.62	6.62511E-01	2.89513E+00	1.40182E+00	2.06526E+00	8.87653E-01
1.64	6.56765E-01	2.97120E+00	1.41578E+00	2.09863E+00	8.79921E-01
1.66	6.51194E-01	3.04820E+00	1.42985E+00	2.13183E+00	8.72014E-01
1.68	6.45789E-01	3.12613E+00	1.44403E+00	2.16486E+00	8.63944E-01
1.70	6.40544E-01	3.20500E+00	1.45833E+00	2.19772E+00	8.55721E-01
1.72	6.35452E-01	3.28480E+00	1.47274E+00	2.23040E+00	8.47356E-01
1.74	6.30508E-01	3.36553E+00	1.48727E+00	2.26289E+00	8.38860E-01
1.76	6.25705E-01	3.44720E+00	1.50192E+00	2.29520E+00	8.30242E-01
1.78	6.21037E-01	3.52980E+00	1.51669E+00	2.32731E+00	8.21513E-01
1.80	6.16501E-01	3.61333E+00	1.53158E+00	2.35922E+00	8.12684E-01
1.82	6.12091E-01	3.69780E+00	1.54659E+00	2.39093E+00	8.03763E-01
1.84	6.07802E-01	3.78320E+00	1.56173E+00	2.42244E+00	7.94761E-01
1.86	6.03629E-01	3.86953E+00	1.57700E+00	2.45373E+00	7.85686E-01
1.88	5.99569E-01	3.95680E+00	1.59239E+00	2.48481E+00	7.76549E-01
1.90	5.95616E-01	4.04500E+00	1.60792E+00	2.51568E+00	7.67357E-01
1.92	5.91769E-01	4.13413E+00	1.62357E+00	2.54633E+00	7.58119E-01
1.94	5.88022E-01	4.22420E+00	1.63935E+00	2.57675E+00	7.48844E-01
1.96	5.84372E-01	4.31520E+00	1.65527E+00	2.60695E+00	7.39540E-01
1.98	5.80816E-01	4.40713E+00	1.67132E+00	2.63692E+00	7.30214E-01
2.00	5.77350E-01	4.50000E+00	1.68750E+00	2.66667E+00	7.20874E-01

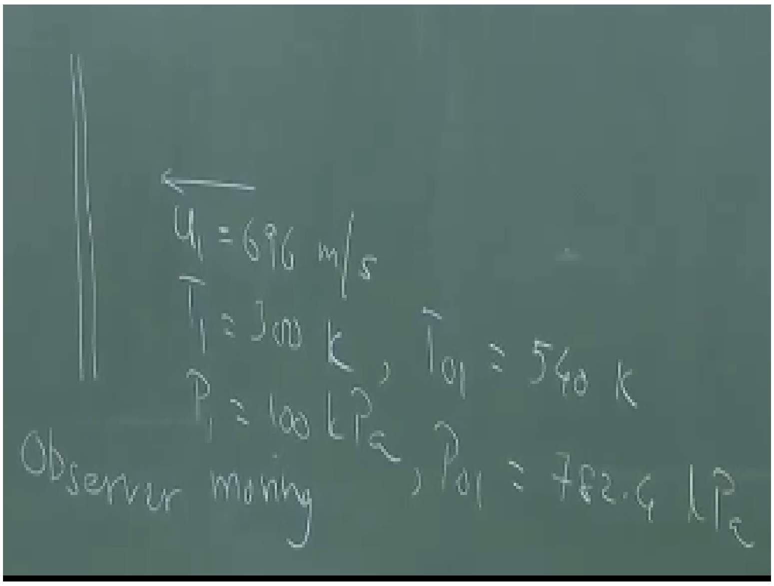
Let us see what that looks like, so here we are looking at normal shock tables, so we can see from the table that for a given value of M1, the calculations in these formulas have already been done, so you get M2, you get P2/P1, T2/T1, rho2/rho1 and P02/P01, so all the quantities are readily available, so we can make use of the table rather than going through the algebra and we can calculate.

(Refer Slide Time: 42:12)

1.76	6.25705E-01	3.44720E+00	1.50192E+00	2.29520E+00	8.30242E-01
1.78	6.21037E-01	3.52980E+00	1.51669E+00	2.32731E+00	8.21513E-01
1.80	6.16501E-01	3.61333E+00	1.53158E+00	2.35922E+00	8.12684E-01
1.82	6.12091E-01	3.69780E+00	1.54659E+00	2.39093E+00	8.03763E-01
1.84	6.07802E-01	3.78320E+00	1.56173E+00	2.42244E+00	7.94761E-01
1.86	6.03629E-01	3.86953E+00	1.57700E+00	2.45373E+00	7.85686E-01
1.88	5.99569E-01	3.95680E+00	1.59239E+00	2.48481E+00	7.76549E-01
1.90	5.95616E-01	4.04500E+00	1.60792E+00	2.51568E+00	7.67357E-01
1.92	5.91769E-01	4.13413E+00	1.62357E+00	2.54633E+00	7.58119E-01
1.94	5.88022E-01	4.22420E+00	1.63935E+00	2.57675E+00	7.48844E-01
1.96	5.84372E-01	4.31520E+00	1.65527E+00	2.60695E+00	7.39540E-01
1.98	5.80816E-01	4.40713E+00	1.67132E+00	2.63692E+00	7.30214E-01
2.00	5.77350E-01	4.50000E+00	1.68750E+00	2.66667E+00	7.20874E-01
2.02	5.73972E-01	4.59380E+00	1.70382E+00	2.69618E+00	7.11527E-01
2.04	5.70679E-01	4.68853E+00	1.72027E+00	2.72546E+00	7.02180E-01
2.06	5.67467E-01	4.78420E+00	1.73686E+00	2.75451E+00	6.92839E-01
2.08	5.64334E-01	4.88080E+00	1.75359E+00	2.78332E+00	6.83512E-01
2.10	5.61277E-01	4.97833E+00	1.77045E+00	2.81190E+00	6.74203E-01
2.12	5.58294E-01	5.07680E+00	1.78745E+00	2.84024E+00	6.64919E-01
2.14	5.55383E-01	5.17620E+00	1.80459E+00	2.86835E+00	6.55666E-01
2.16	5.52541E-01	5.27653E+00	1.82188E+00	2.89621E+00	6.46447E-01
2.18	5.49766E-01	5.37780E+00	1.83930E+00	2.92383E+00	6.37269E-01

Let us see what the value comes out to be, so we look at this and corresponding to M =2, so corresponding to M =2, make it larger; so corresponding to M= 2; M1=2, I have these values here, which i am going to now write down in the board okay, that is what we are planning to do. Now, let me fill in some other information into this diagram before we do that, we have done T1, we have done P1, right; we know M1 now.

(Refer Slide Time: 42:40)



Once we know M1, I can calculate T01, right. T01 is nothing but T1 times 1+gamma – ½ times M1 square, so once I know M1, I can calculate T01 in this frame of reference, which comes out to be 540 kelvin and similarly, P01 in this frame of reference comes out to be; again calculate P01 that comes out to be 782.4 kilopascal.

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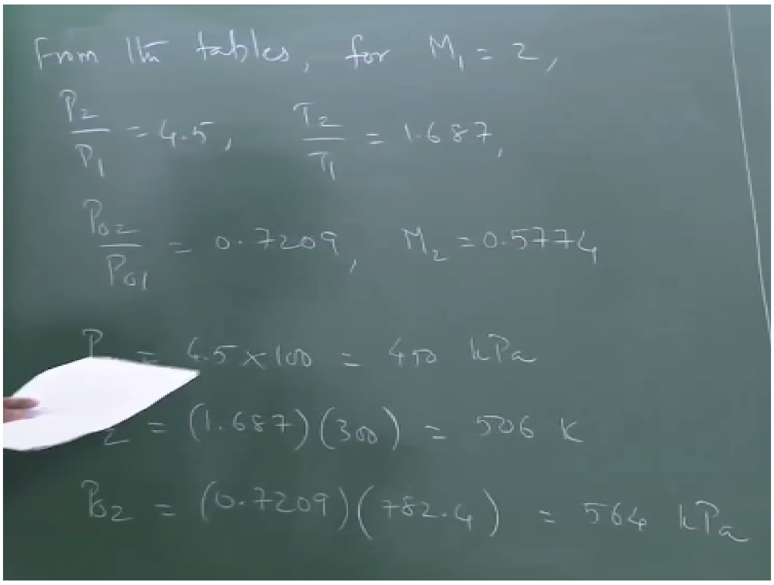
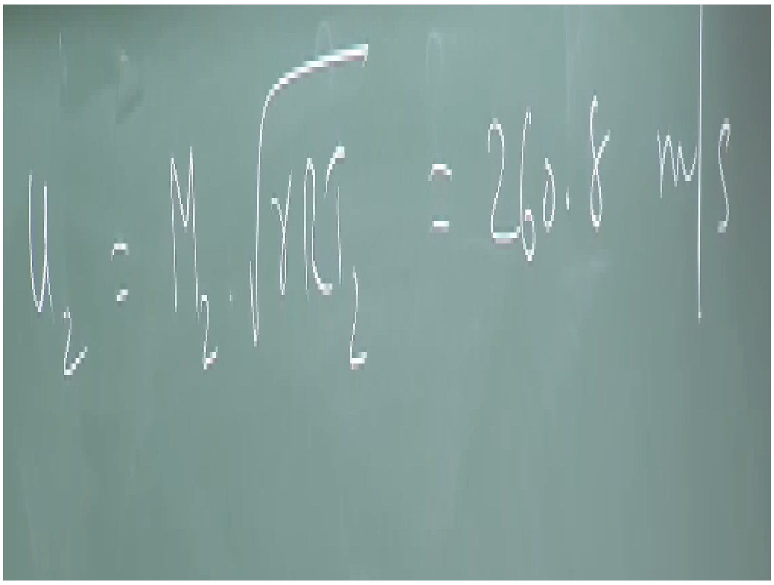


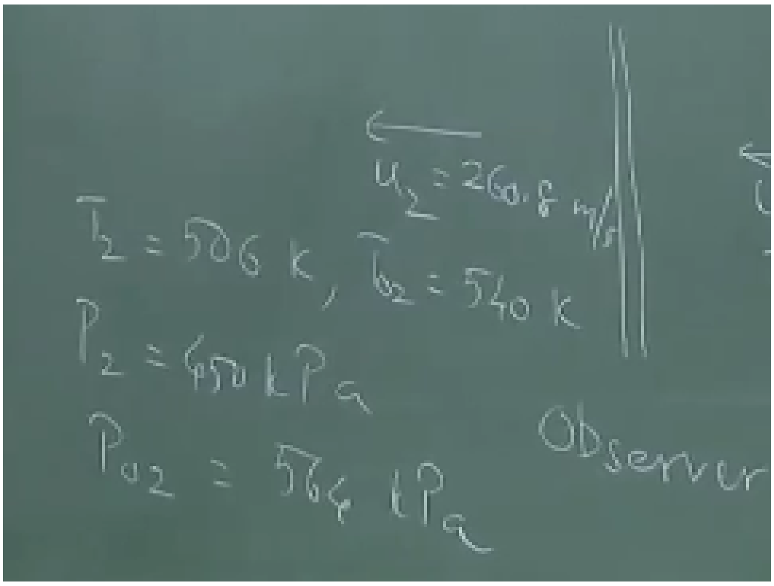
So, from the tables, for M1 = 2, we can retrieve the following quantities, P2/P1= 4.5, so that we can see from the table, so for M1 = 2, we have P2/P1 = 4.5, all the other quantities can be retrieved similarly, T2/T1= 1.687, P02/P01 = 0.7209 and M2 = 0.5774, so we have taken these values from the normal shock tables. So, I can calculate; I know the ratios, so from this, I can calculate the following static pressure P2, P1 is known, so static pressure P2 is going to be 4.5 times 100, that is 450 kilopascal.

(Refer Slide Time: 46:13)



And T2 is going to be 1.687 times 300 Kelvin, which is T1, so this gives me 506 Kelvin and P02 = 0. 7209 times P01 and we know P01 from here, P01= 782. 4, so I can multiply these 2; 782, to get P02 as 564 kilopascal and u2 = M2 times square root of gamma RT2 and if you substitute the values, you get this to be 260.8 meter per second, okay. So, let us now transfer this information to this diagram, here.

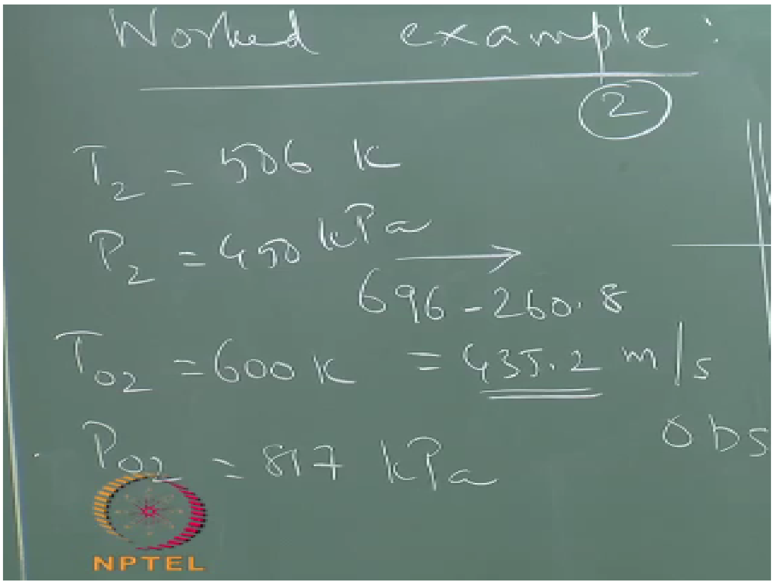
(Refer Slide Time: 46:47)



So, let us write down the quantities, so u2 = 260.8 meter per second, T2 = 506 Kelvin, what about T02? Across a normal shock wave, there is no change in stagnation temperature, so T02 = T01, that is = 540 Kelvin. Now, the static pressure P2 = 450 kilopascal and the stagnation pressure P02 = 564 kilopascal. So, in the moving frame of reference, we have calculated all the quantities that we were asked to calculate; static and stagnation quantities in both frames of reference.

So, moving frame of reference, this picture is complete, notice that P02 is < P01, so there is a loss of stagnation pressure in the moving frame of reference. Now, we are asked to calculate same quantities in the stationary frame of reference also, so now we go from here to here, so we started from here, right we subtracted 696, which is the wave speed, so we got u1 to be 696, so now when we go from here to here, you need to do the same thing.

(Refer Slide Time: 48:35)



So, notice that this velocity is going to be 696 – 260.8, which is = 435. 2 meter per second, right so as a result of passage of the shock wave the air, which was initially still now, begins to move with the speed of 435.2 meter per second, right. So, the air was still and the normal shock passes through this at a speed of 696 meter per second as a result, the air now starts moving at the speed of 435.2 meter per second, okay.

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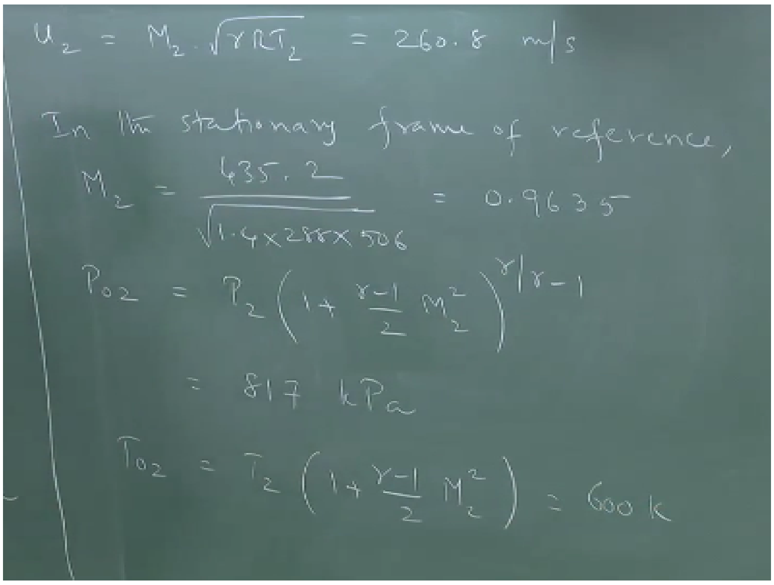

Now, what about the static temperature here, T2? Same, remember static quantities of frame independent, so T2is going to be the same, so 506 Kelvin, P2 is going to be the same, so P2 is going to be 450 kilo Pascal. What about T02 and P02? T02 and P02 is frame dependent okay, so what we do is, we have to use this velocity to calculate T02 and P02, correct. So, in the stationary frame of reference, M2 = 435.2 divided by square root of gamma, which is 1.4 times or 288 times static temperature, which is 506.

So, M2 in the stationary frame of reference comes out to be 0.9635, okay, so P02 = P2 times 1+gamma – 1/2 times M2 square raise to the power gamma/gamma -1, so if you substitute the numbers, you get P02 to be 817 kilo Pascal and similarly, T02 = T2 times 1+gamma – 1/2 times M2 square and if you substitute the numbers, you get this to be 600 Kelvin in this frame of reference. So, T02 in this frame of reference is 600 Kelvin and P02 in this frame of reference is 817 kilo Pascal.

So, you can see that the stagnation quantities are very heavily dependent on the frame of reference that we choose okay. In fact, here P01 in this frame of reference is 100 kilopascal, whereas P02 is 817 kilo Pascal, there is actually an increase of stagnation pressure in this frame of reference, whereas there is a loss of stagnation pressure in this frame of reference, so frame of reference is very, very important when calculating stagnation quantities, static quantities of frame independent.

What we will do in the next class is continue this get some more insights into the normal shock solution before moving on to flow with heat addition or Rayleigh flow.