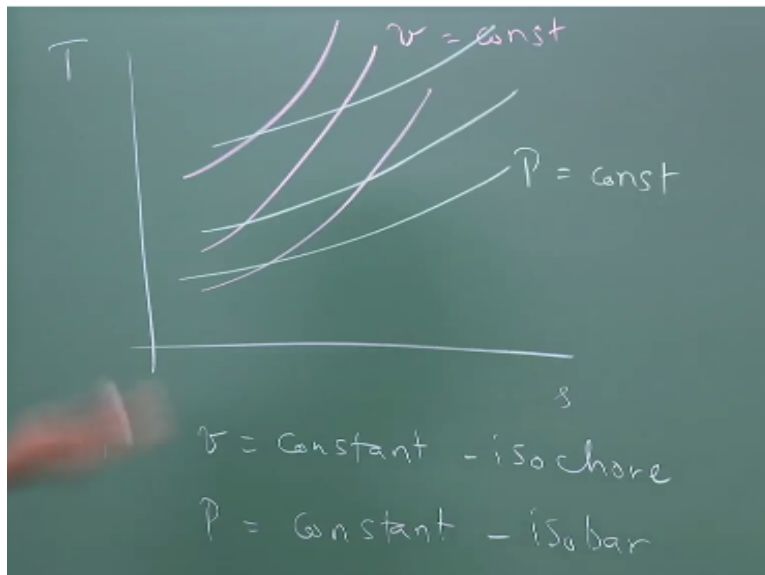


Gas Dynamics and Propulsion
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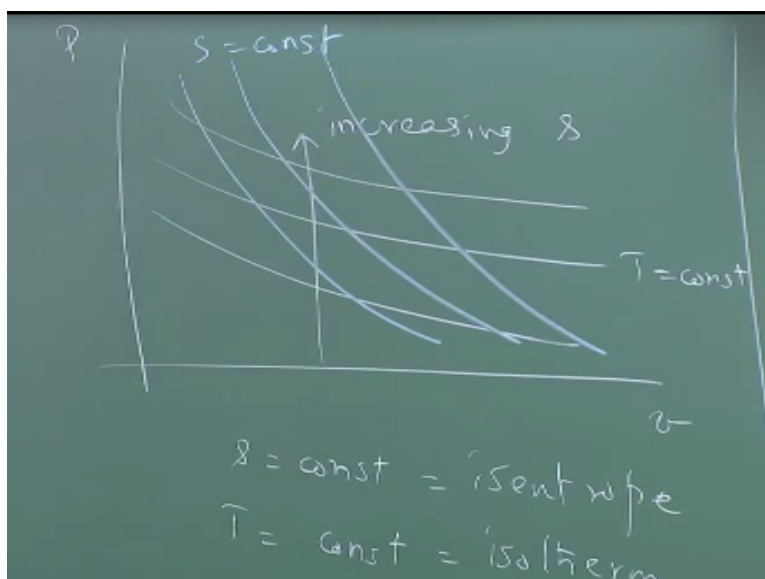
Lecture – 05
Fundamental Ideas / Normal Shock Waves

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In the previous class, we looked at depicting state points on a TS diagram and we showed that constant volume lines; we depicted both constant volume line which is an isochore and we depicted the constant pressure line in this diagram. So, the constant volume line; we showed that constant volume lines steeper than constant pressure lines, so these are $v = \text{const}$; and the constant pressure lines are less steep than the constant volume lines.

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And what we are going to start with today is to depict say, same states on a PV diagram; on a PV diagram then we would like to show just like this, we would like to show; $s = \text{constant}$, which is an isentrope line on this diagram and also an isotherm which is at $T = \text{constant}$ on this diagram. So, that is what we start out with and if you remember the equations that we wrote down for change in entropy, let us start with that equation.

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$$\text{If } dv = 0, \text{ then } ds = C_v \frac{dP}{P}$$

$$\text{If } s = \text{const}, \text{ then } ds = 0,$$

$$C_v \frac{dP}{P} + C_p \frac{dv}{v} = 0$$

$$\frac{dP}{dv} = -\gamma \frac{P}{v}$$

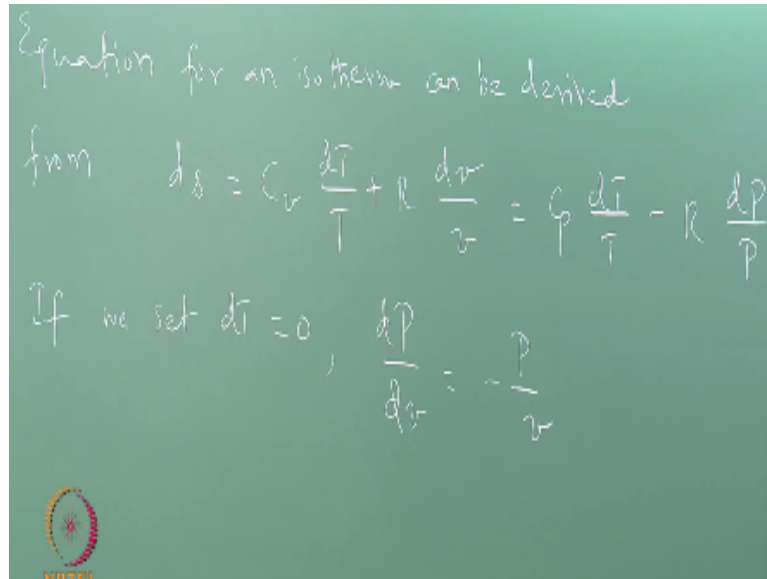
And if you remember, this equation looks like this; $ds = c_v \text{ times } dP / P + c_p \text{ times } dv/v$ and remember, I am trying to see, what ds ; $s = \text{constant}$ line looks like on this diagram and if you; if you look at this, if I travel along the vertical direction, then you can see that, if I let us say travel along this vertical direction, then $dv = 0$ and you can see that $ds = c_v \text{ times } dP / P$ and this shows that as the pressure increases; as I travel along this line, pressure increases.

And so entropy also increases along this direction, so increasing yes along this direction and in order to obtain the equation for an isentrope or an $s = \text{constant}$ line, I simply said if, $s = \text{constant}$, then $ds = 0$, just like what we did earlier and this gives me then, $c_v \text{ times } dP/P + c_p \text{ times } dv/v = 0$ and if I rearrange this, I easily can show $dp/dv = - \gamma \text{ times } p/v$. So, this equation tells me that the slope of $s = \text{constant}$ line is negative, right.

This tells me that the slope of the $s = \text{constant}$ line is negative, so contrary to what we; what we showed earlier where the slope was positive, now these lines are going to run like this, so that is one information that we get from here. So, this is the equation that describes the $s = \text{constant}$ line and they can also see that the lines become steeper as P increases or as v decreases.

So, when I am over here, where the P is low and the v is high, the slope is less and as I move towards this side because the slope is negative, I moved this way along the line and when I come to this part, where the P is high and the v is small, then the slope is higher; larger and negative, so the lines become steeper here and they are shallower over here for a $s = \text{constant}$ line. I can do the same thing for $T = \text{constant}$ line that is the next thing that we want to do.

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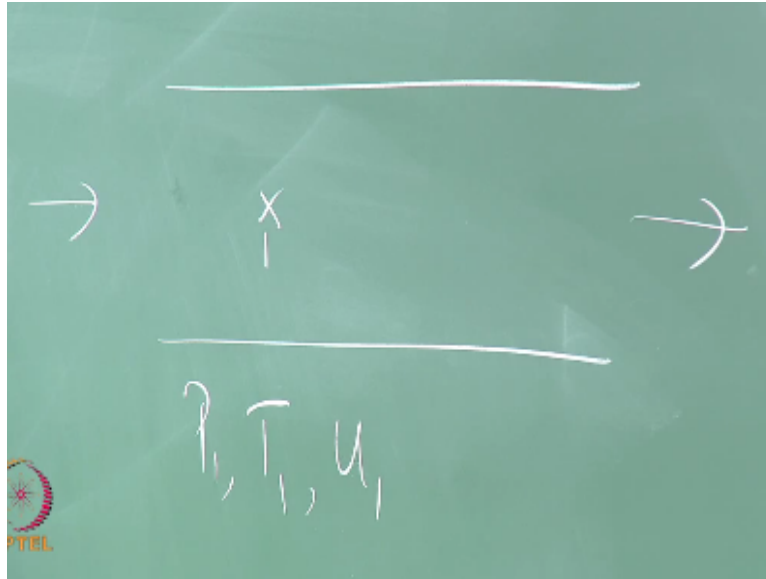


Equation for an isotherm can be derived
 from $ds = C_v \frac{dT}{T} + R \frac{dv}{v} = C_p \frac{dT}{T} - R \frac{dp}{p}$
 If we set $dT = 0$, $\frac{dp}{dv} = -\frac{p}{v}$

We want to draw an isotherm in the same diagram, so the equation for an isotherm can be derived very similarly from the same entropy equation, we know that $ds = c_v dT / T + R dv$, which is also $= c_p \text{ times } dT / T - R \text{ times } dp / p$, so if I take the last 2 equalities and set $dT = 0$ to obtain the equation of an isotherm, we get the following. We get $dP / dv = - P/v$ and once again the inferences from this; is that the slope of the isotherms is also negative and just like the isentrope, the lines are shallow in this part of the PV diagram, when P is small and v is large.

And as I go towards this side, again the lines become steeper. Now, most importantly when I compared the slope of an isotherm with the slope of an isentrope, I can see that isentropes are steeper than isotherms okay. Now, we are in a position to draw the true line, so we know that isentropes are steeper than, which means that if this are $T = \text{constant}$ lines and this will be; these are $s = \text{constant}$ lines.

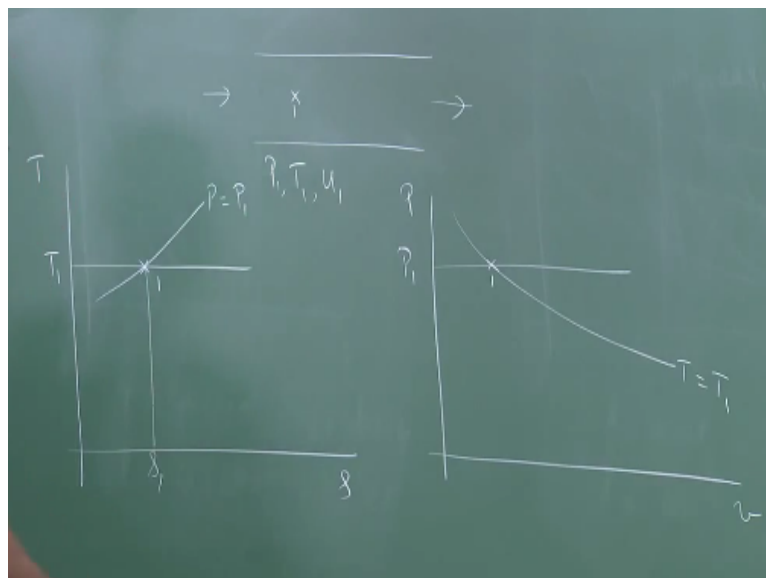
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So, the next task is to take a particular state point in a flow field and depict the state point on this diagram on the TS diagram and the PV diagram that is what we want to do next, so that we can sketch processes, once we draw a states, we can then sketch processes on the TS and PV diagram that is what we are going to do next. If you remember what we want to depict on this state diagram is a straight point you know, one dimensional flow field.

Let us say that this is the one dimensional flow field that we are looking at and let us say that this is state point one and the information that is available in at a state point one is nothing but P_1 , the static pressure, T_1 the static temperature and u_1 the velocity, this is the information that is available and we want to show this on a TS and a PV diagram that is what we are going to do next.

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So, let us say that say this is a TS diagram and this is a PV diagram, so I start by drawing let us say, the isotherm corresponding to $T = T_1$, so let us say that this is the isotherm corresponding to $T = T_1$ and then I draw the isobar corresponding to $P = P_1$, we know what these isobars looks like, so I am just going to approximately sketch this; like this, so this is the isobar corresponding to $P = P_1$ and so the state point lies at the point of intersection of these 2. So, this is state point one.

So, we have now shown p_1 and T_1 and this is state point one, right and this is s_1 , if required we can calculate s_1 but we usually do not need to calculate the absolute value of the entropy. Now, we are still to indicate this on the TS diagram, we will do that next. Let us let us display the same information on a PV diagram, so $P = P_1$ is known, so I draw the isobar corresponding to $P = P_1$.

And now I draw the isotherm corresponding to $T = T_1$, I know that that looks like this, right. So, this is the isotherm corresponding to $T = T_1$, so this is state point one, right. Notice that I have drawn the same things here, I have drawn the isotherm corresponding to $T = T_1$ here and I have drawn the isobar corresponding to $T = T_1$ here and I have drawn the isobar, I am sorry; isobar corresponding to $P = P_1$ and isobar corresponding to $P = P_1$, okay.

This was the reason why we spend so much time to see how an isotherm and an isentrope looks like on this diagram and an isobar and isochore looks like on this diagram. So, T and s , we should be able to denote P and v ; P and v , we should be able to denote T and s that was the objective of that exercise. So, now I have this, now I need to figure out how to show the velocity information on this diagram, but this is a thermodynamic state diagram okay.

The energy equation that we wrote down gives us a clue as to how we can depict velocity also in this diagram, right.

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$$dh + d\left(\frac{u^2}{2}\right) = 0$$

If you remember, we wrote down the energy equation as the $dh + d$ of u square/ $2 = 0$ and if i make use of the fact that $h = c_p$ times t for an ideal gas, I can rewrite this equation as follows.

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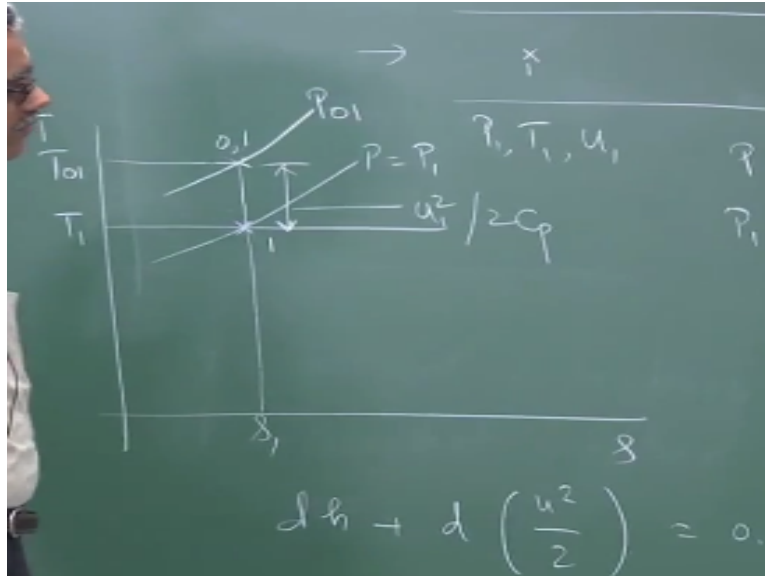
$$d\left(T + \frac{u^2}{2c_p}\right) = 0$$

units of temperature

I can write this as, $T + u$ square / $2c_p = 0$, in the absence of any heat addition, right, I can write this, so the quantity u square/ $2c_p$ has units of temperature, same as T , right. So, this has; this quantity has units of; so this gives me an idea as to how I can illustrate velocity on a TS or a PV diagram, so I simply calculate u square/ $2c_p$ and I add that to the static temperature, right.

So, all I do here is I go up to here, T_1 is known and I add u_1 square/ $2c_p$, so if I do it like this, let us say this is then, this is u_1 square / $2c_p$.

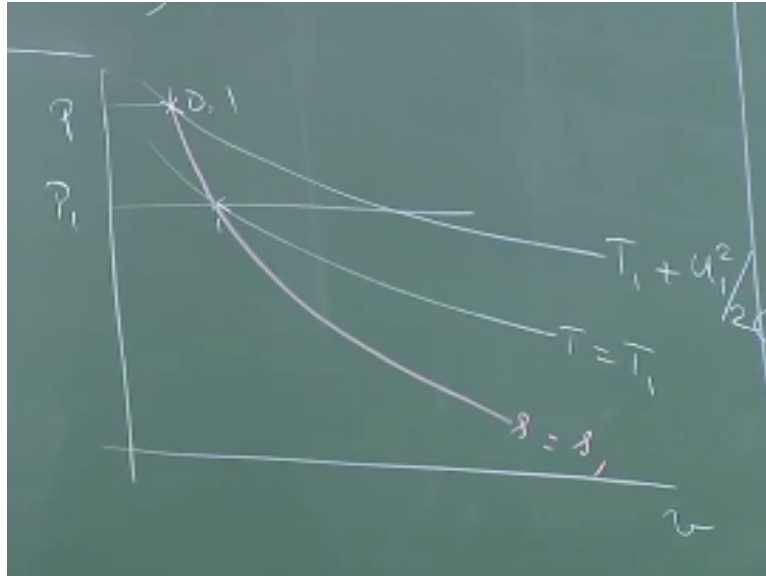
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So, now I have illustrated T_1 , I have illustrated velocity and pressure, so the information that we wanted here; P_1 , T_1 , u_1 is now shown here, so this is P_1 , this is T_1 and this is $u_1^2 / 2c_p$ and what is this quantity =, $T_1 + u_1^2 / 2c_p$, this is nothing but the stagnation temperature of the flow, right. So, if I draw a horizontal line like this, this would be T_{01} , right and if I draw the isobar that passes through this point, right. If I draw the isobar that passes through this point, that would be P_{01} .

So, you can see that, now we have also indicated the stagnation state in this diagram, so this point here is the stagnation state corresponding to the static state 1, so I can denote that as 0, 1, so we have captured these 3, in addition stagnation information also in this diagram. We need to do the same thing in this diagram, right. So, let us see how we do that. Now, as you can see from here, when I went from state 1 to state 0, 1, I do so in an isentropic process, right.

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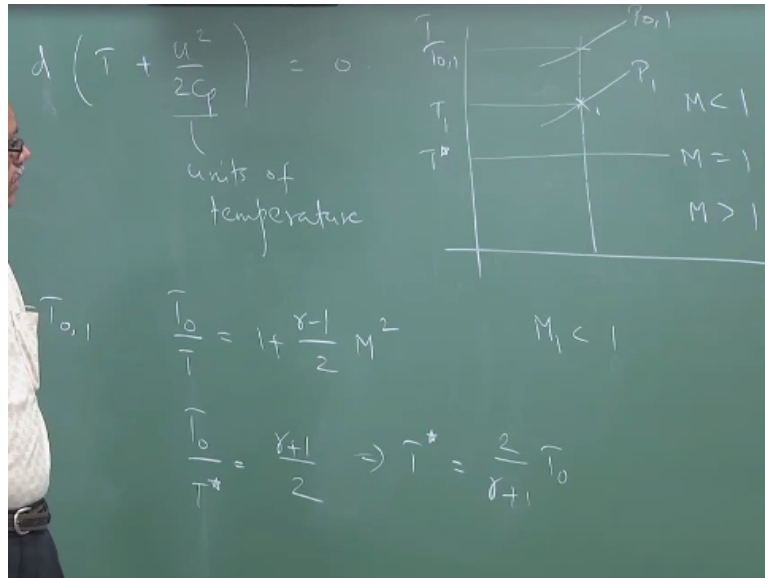
So, when I want to do the same thing here, what I do is, I look for the isentrope, which passes through state point 1, let us say that this is the isentrope, which passes through state point 1, so this is $s = s_1$, same as this isentrope and this is $s = s_1$, I have drawn the same isentrope there, so what I do is, I take this is this and now I look at; I am adding $u_1^2 / 2c_p$ to this static temperature value.

So, that means I look for the isotherm, which is like this, right, so this isotherm corresponds to $T_1 + u_1^2 / 2c_p$, right, that is this isotherm. So, stagnation state 0, 1 would then come right here; 0, 1 and this is nothing but $T_{0,1}$, right, this is $T_{0,1}$ and this pressure would be $p_{0,1}$, right. Notice that I am traveling along the same isentrope to go from 1 to 0, 1, just like what I did here.

I travelled along the same, I am sorry; I travelled along the same isentrope to go from 1 to 0, 1, I am doing the same thing here and travelling along the same isentrope to go from here to there. So, now I have illustrated now all the information that I wanted to illustrate. Now, we become somewhat greedy when we see whether we can in depict even more information on this diagram.

By more, what I mean is, is it possible for me to distinguish whether a particular state is subsonic, sonic or supersonic that would be extremely useful information to have. How do we illustrate that kind of information on this diagram, is what we are going to look at? okay.

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I am going to draw that in a separate diagram because otherwise, this will become very cluttered okay, so that part alone we will draw in a separate diagram. Let us take a look and see what that looks like, let us say that this is my TS diagram and here is my state 1, for the sake of argument we will assume that state 1 is subsonic, we will also show what it looks like, if state 1 happens to be a supersonic state.

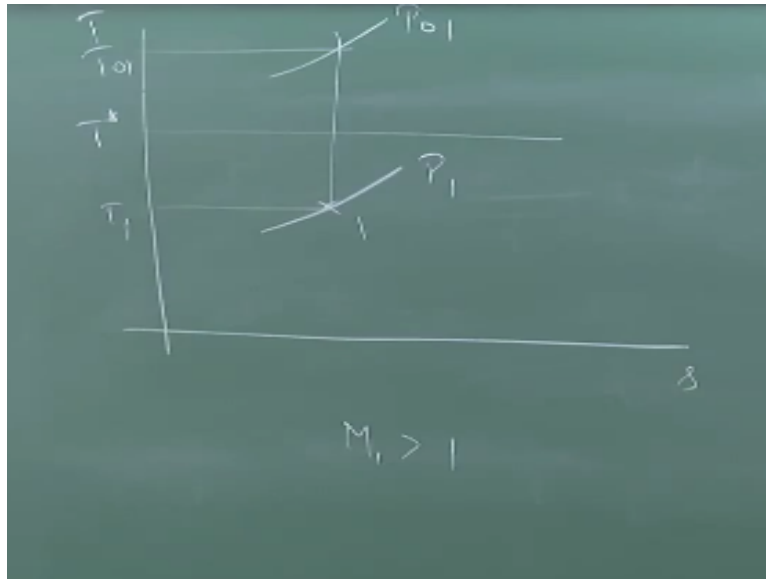
So, let us start by saying that state 1 is a subsonic state, so I have T_1 , right; this is my T_1 , this is my p_1 , this is my $T_{0,1}$, this is my $p_{0,1}$, I have indicated all that here and this is $= u_1^2 / 2c_p$, that is what we showed in the previous diagram. Now, if you remember we wrote down the definition for stagnation temperature as follows; $T_{0,1}$ or T_0 / T in a general case $= 1 + \gamma - 1 / 2$ times M square, correct.

Now, if I know T_0 , I can evaluate T^* from this, right. What is M corresponding to T^* ; the sonic state, $M = 1$, so I substitute $M = 1$ and this expression gives me $T_0 / T^* = \gamma + 1 / 2$, right. So, once I know T_0 , I can evaluate T^* as $2 / \gamma + 1$ times T_0 , now this I can show here, right. I can show T^* on this and since we assume that $M_1 < 1$, T^* is going to fall like this, this is $T = T^*$.

So, once I know T_0 , I can show T^* . Once I show T^* , I immediately know whether the state is subsonic or supersonic, right. Subsonic States will lie above this and supersonic states will lie below this, okay. So, let us show that also though, so this is the $M = 1$ line, this; any state here is a subsonic state and any state here is a supersonic state, right. So, now in

addition to P, T and u, we have also shown the Mach number related information on this diagram, okay.

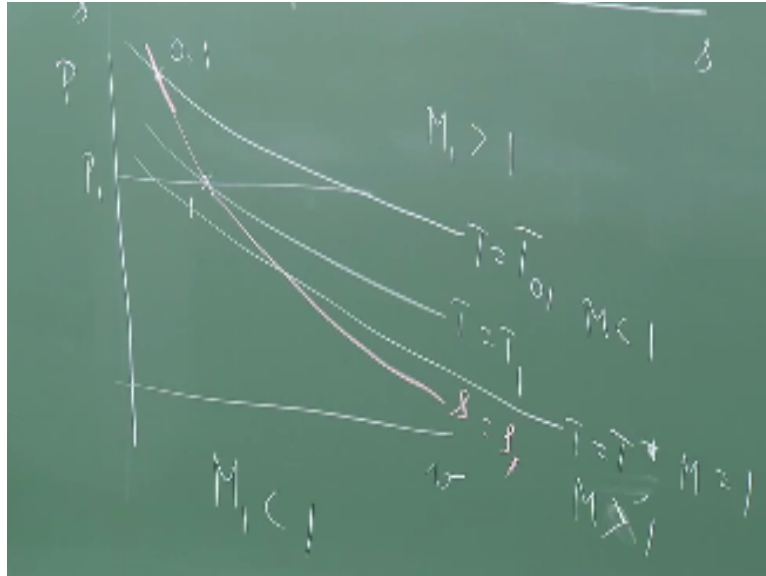
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So, it is an extremely powerful way of illustrating the process and states for the type of flows that we are going to encounter. Now, had the initial state being a supersonic state, right, how would this have looked? I would have gone through the same process but my diagram would be slightly different, if M_1 ; so this is my TS diagram, if my initial state had been a supersonic state and I would have depicted state 1 like this, so this is T_1 and this is P_1 , so this is T_{01} and this is P_{01} .

Now, when I calculate T^* for this case, my T^* would have come right there, my T^* would have come right there, so that state 1 is a supersonic state. The procedure is the same right, so the advantage of depicting this kind of information is that, if T^* or if T_0 is constant for the flow, then I can draw that line continuously and show all subsequent states in the same flow field. It is an extremely powerful way of illustrating the flow field, okay.

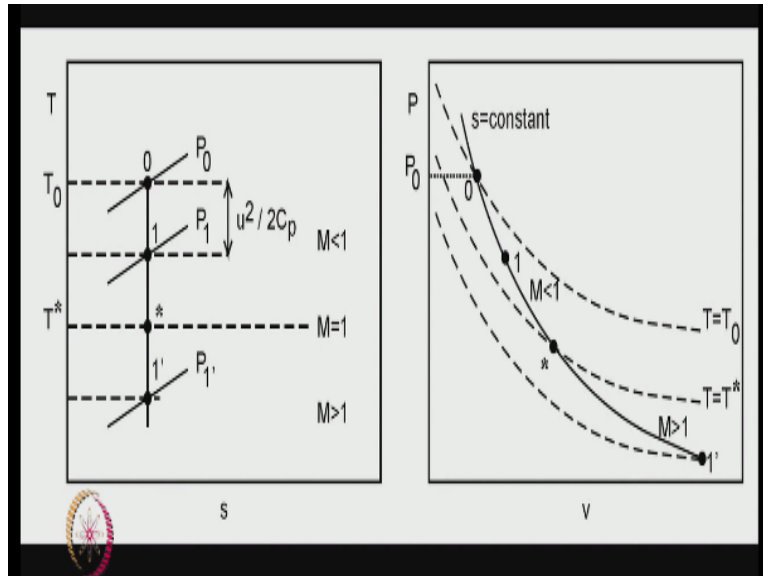
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And that is something that we will use throughout in the course of the lecture. Now, let us try to show this information on a PV diagram just for the sake of completeness. So, if you remember our PV diagram, I am going to redraw the PV diagram, so this is; let us say p_1 and this let us say is T_1 , so that state 1 falls here and let us say that this is my isentrope corresponding to $s = s_1$, so I travel along this isentrope up to this point, that is the stagnation state 0, 1.

So, this is the isotherm corresponding to $T = T_{01}$ and now if M_1 happens to be a subsonic state, then my isotherm corresponding to $T = T^*$ will fall like this, right. So, this would be my isotherm corresponding to $T = T^*$, so states that lie about $T = T^*$ or subsonic states, states that lie below $T = T^*$ or supersonic state, so any state that lies here M is < 1 , this corresponds to $M = 1$ and any state that lies below s , I am sorry; $M > 1$, okay.

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This is what is illustrated in the diagram; let us take a quick look at the diagram. So, you can see that state 1 lies, here $P = P_1$ and T^* lies here and notice that we have added $u^2 / 2C_p$ to go from state 1 to the stagnation state, T_0 is the stagnation temperature, P_0 is the stagnation pressure. So, any state that lies above a T^* will be a subsonic state, $M < 1$ and any state that lies below the T^* isotherm is a supersonic state as you can see from here on a TS diagram.

Same thing is illustrated on a PV diagram here; we have done the same thing. We start with state 1 and it travel along this $s = \text{constant}$ line, $s = s_1$ line to reach the stagnation state and this is the stagnation isotherm corresponding to the stagnation temperature and notice that this is the isotherm corresponding to $T = T^*$, so that subsonic states lie above this isotherm along this $s = s_1$ line and supersonic states lie below this isotherm along this $s = s_1$ provided the flow is isentropic, otherwise we can show the process exactly and then go from there okay.

Are there any questions? Okay, in that case let us move on to the next chapter. The next chapter is going to deal with normal shockwaves. We have already looked at a wave solution to the governing equations that we wrote down for one dimensional flow and that particular wave solution use the fact that the change across the wave was an isentropic process. If you remember, we said that represented the propagation of an acoustic wave.

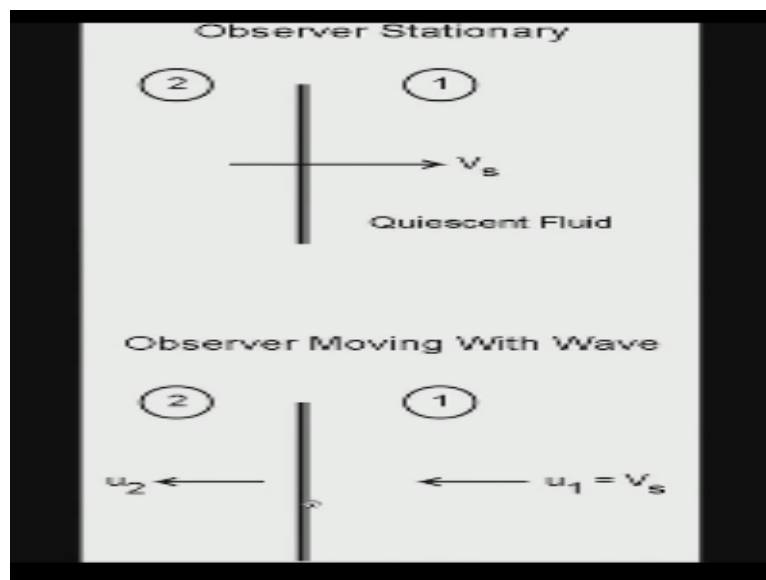
So, any change across the wave is infinitesimally small and the process itself is an isentropic process and we showed that such a wave would travel through the medium with the speed

equal to the speed of sound that is what we showed in the earlier case. What we are going to look at next is another wave solution, where we relax the requirement that the process should be an isentropic process.

Now, such waves are seen as a result of; let us say an explosion or a blast and so on, it is a very strong waves which emanate, when there is a sudden release of energy in a compressible flow as in an explosion or a blast wave and that is the solution that we are going to look at because these waves are so strong, we cannot imagine them to be isentropic, it has to be a non-isentropic process.

And since there is no; as the wave propagates, there is no heat addition, the entropy has to increase in such a flow.

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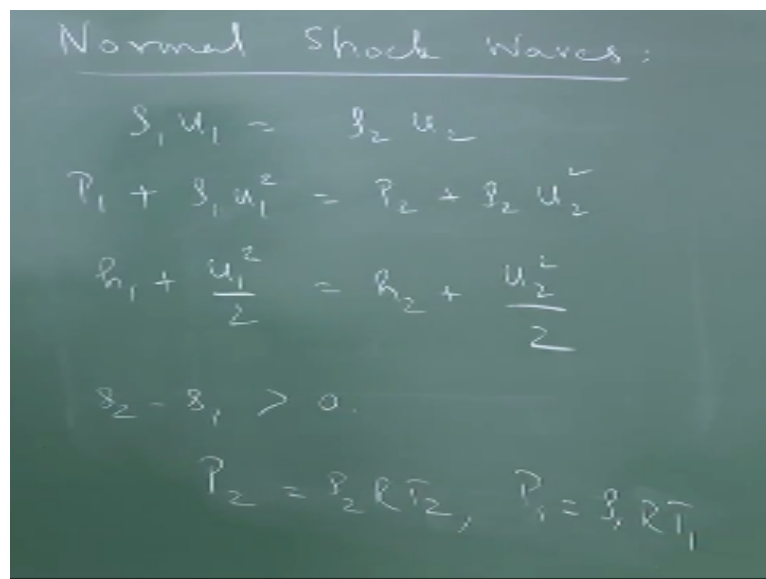
Because of the irreversibility, okay, so that is what we are going to look. The scenario is illustrated in the diagram that we have shown here. Notice, that this diagram is very similar to what we what we did earlier, so in a stationary frame of reference, where the observer is standing like this and the explosion wave or the blast wave goes like this, the picture on the top is what this observer would see.

So, the wave propagates into a quiescent fluid and the flow behind this would have actually acquired some velocity which is not known, which has to be determined. Now, just like what we did before, if the observer also has the wave moves like this, if the observer also starts moving along with the wave, then you see the flow approaching you with the speed equal to

the speed of the wave and you see the flow behind you residing with the different speed, right.

That is the frame of reference that we are going to use, as we have shown here, right. So, the flow seems to approach the wave with the speed u_1 which is equal to the speed of the shock wave itself as denoted by v_s , in this frame of reference and the flow downstream of the wave recedes with the speed or velocity = u_2 and the objective is to calculate the given; the state of the fluid at 1, what is the state of the fluid at 2.

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Normal Shock Waves:

$$\rho_1 u_1 = \rho_2 u_2$$
$$P_1 + \rho_1 u_1^2 = P_2 + \rho_2 u_2^2$$
$$h_1 + \frac{u_1^2}{2} = h_2 + \frac{u_2^2}{2}$$
$$\rho_2 - \rho_1 > 0.$$
$$P_2 = \rho_2 R T_2, \quad P_1 = \rho_1 R T_1$$

In other words, what are the velocity of the fluid, pressure, temperature and so on at state 2 given the information in state 1, that is what we are looking for and we use the same governing equation as before, if you remember the continuity equation looks like this and the momentum equation, the energy equation and if you remember the change in entropy, we are saying that the change in entropy must be positive.

Because this represents an irreversible process, so if you look at this equation in addition to this, we also know that the gas obeys perfect gas equation of state, so which means that $P_2 = \rho_2 R T_2$ and $P_1 = \rho_1 R T_1$, so given these quantities, we are trying to given u_1 , P_1 and T_1 , we are trying to determine the corresponding quantities on the other side, so that is the exercise that we are going to go through.

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Given u_1, P_1, T_1 determine u_2, P_2, T_2

Continuity eqn can be written as $\frac{P_2}{P_1} = \sqrt{\frac{T_2}{T_1}} \cdot \frac{M_2}{M_1}$

$u = M\sqrt{\gamma RT}$, $P = \rho RT$

Momentum eqn can be written as $\frac{P_2}{P_1} = \frac{1 + \gamma M_1^2}{1 + \gamma M_2^2}$

Energy eqn can be written as $\frac{T_2}{T_1} = \frac{1 + \frac{\gamma-1}{2} M_1^2}{1 + \frac{\gamma-1}{2} M_2^2}$

So, we are given u_1, P_1, T_1 determine u_2, P_2 and T_2 , notice that u_2, P_2 and T_2 are determined in a reference frame, where the observers moving along with the wave, so if you want that in a stationary reference frame, then you have to go back to the stationary reference frame, we will illustrate that through a worked example okay. For now, we will just try to determine u_2, P_2 and T_2 , so we have 3 unknowns and 3 equations, so we must be able to solve this to obtain a solution to this.

So, we start by writing the continuity equation, if I use the definition of density, then I can write this as $P_2/P_1 = M_2/M_1$, where M is the Mach number of the flow, so here we have used the fact that $u = M$ times square root of γRT and of course, $P = \rho RT$, right. So, we have used these 2 relationships here to come up with this equation. Now, the momentum equation can be written like this, $P_2/P_1 = 1 + \gamma M_1^2 / 1 + \gamma M_2^2$.

The energy equation can be simplified and written like this, $T_2/T_1 = 1 + \gamma - 1/2$ times M_1^2 square divided by $1 + \gamma - 1/2$ times M_2^2 square and if I combine these 2 equation, notice that this as P_2/P_1 and T_2/T_1 , this has P_2/P_1 , this as T_2/T_1 , so I can eliminate P_2/P_1 and T_2/T_1 from here using the remaining 2, then I will be left with the relationship, which involves only M_1 and M_2 , right, that is what I am going to do next.

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Eliminate $\frac{P_2}{P_1}$ and $\frac{T_2}{T_1}$ from these equations,

$$\frac{1 + \frac{\gamma-1}{2} M_1^2}{1 + \frac{\gamma-1}{2} M_2^2} = \frac{M_2^2}{M_1^2} \left(\frac{1 + \gamma M_1^2}{1 + \gamma M_2^2} \right)^2$$

Quadratic in M_2^2 .

Only meaningful solution is

$$M_2^2 = \frac{2 + (\gamma-1)M_1^2}{2\gamma M_1^2 - (\gamma-1)}$$

So, if I eliminate P_2/P_1 and T_2/T_1 from these equations, I end up with the following relation, this is the equation that I end up with. Notice that, since I know u_1 and T_1 , M_1 is known, so I have a single equation, where M_2 is the only unknown, right. So, I have managed to write everything in terms of one equation with one unknown, which is this; this equation looks formidable but actually, is not.

If you look at this equation, this is actually a quadratic in M_2 square, so it is not a very, very difficult equation to solve. So, if it is a quadratic in M_2 square, how many solutions will it be going to have? 4 solutions, right, it is going to have 4 solutions and the only meaningful solution from this equation looks like this, is M_2 square = $2 + \gamma - 1$ times M_1 square divided by $2\gamma M_1$ square - $\gamma - 1$.

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Rearranging, we get

$$M_2^2 = 1 - \frac{\gamma+1}{2\gamma} \cdot \frac{M_1^2 - 1}{M_1^2 - 1 + \frac{\gamma+1}{2\gamma}}$$

If $M_1 > 1$, then $M_2 < 1$ - compressive

$M_1 < 1$, then $M_2 > 1$ - expansive

$\beta_2 > \beta_1$ allowed

$\beta_2 < \beta_1$ Not allowed

So, the quadratic for M_2 square gives two solutions and this is the only solution, which is meaningful. Now, this itself, if I take square root of this, this itself will give me 2 solutions, in fact, if I rewrite this equation; slightly rearrange this equation, I can get the following M_2 square = $1 - \gamma + \frac{1}{2}\gamma$ times M_1 square - 1 divided by M_1 square - 1 +; so, notice that $\gamma + \frac{1}{2}\gamma$ okay, if you take this quantity $\gamma + \frac{1}{2}\gamma$ is going to be < 1, right.

So, if I look at this expression and I can see that if M_1 is > 1, then M_2 is going to be < 1, right and vice versa. If M_1 is < 1, then M_2 is > 1, right. So, if M_1 is > 1, then I get a solution for which M_2 is < 1 and if M_1 is < 1, then I get a solution for which M_2 is > 1. The first solution as you can obviously see is a compressive solution, the Mach number has decreased across the wave and it is a compressive solution.

You can in fact, show that this is a compressive solution by looking at P_2/P_1 also and this is an expansive solution meaning; as the wave passes through the flow in the first case, the pressure and temperature are increased; static pressure and static temperature are increased, whereas in the second case, as the wave passes through static pressure and static temperature are decreased, so it is an expansion process.

The first case corresponds to a compression process but remember, we actually have 4 equations, we have only used the first 3 equations to obtain these solutions right, we have used the first 3 equations to obtain this solution, we still have not looked at the fourth equation, remember this also must be satisfied, right. So, if you apply this condition, then it turns out that for the first solution; for the first solution, s_2 is > s_1 , so that is allowed.

For the second solution, s_2 is actually < s_1 , so this is not allowed by second law of thermodynamics. With a little bit of algebra, we can easily show that s_2 is > s_1 for this case and s_2 is < s_1 for this case, this is shown in the textbook, so I suggest that you look at the textbook and see how this comes out but the gist of it is that, this solution; the compressive solution is the only physically allowed solution.

Although, mathematically you may have several solutions, this is the only one which is physically seen. So, you cannot have an expansion process across which entropy decreases that is what this one says but later on, when we go and study Prandtl Meyer waves, you will

see that a Prandtl Meyer wave can be both an expansion wave as well as a compression wave, that is permitted because Prandtl Meyer wave is an isentropic wave.

There is no change in entropy across the waves and it represents an infinitesimal compression that is permitted, what this talks about is a finite compression or finite expansion. Finite compression is permitted but finite expansion is not permitted, so we cannot have these types of expansion waves, okay. So, we have derived a solution for M_2 in terms of M_1 , once we have that, other quantities can be derived very easily.

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$$M_1 > 1, \text{ and } M_2 < 1$$

$$\frac{T_2}{T_1} > 1 \text{ and } \frac{P_2}{P_1} < 1$$

$$\frac{P_2}{P_1} > \frac{T_2}{T_1} \Rightarrow \frac{\rho_2}{\rho_1} > 1$$

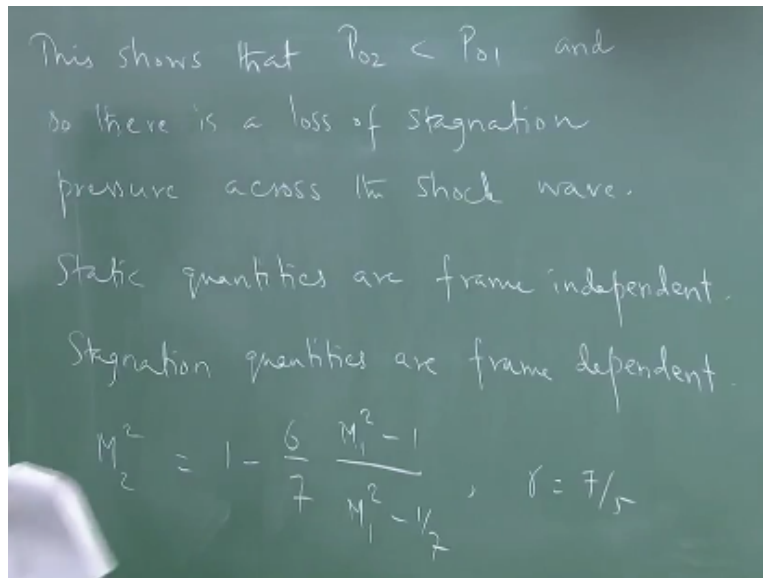
$$\rho_2 - \rho_1 = -R \cdot \ln \left(\frac{P_{02}}{P_{01}} \right)$$
 Since $T_{02} = T_{01}$

Let us take a look at that next, okay. So, we have derived the solution for M_2 and we showed that the only permitted solution is $M_1 > 1$, supersonic and M_2 is subsonic, this is the only solution that is permitted and for this solution, if you look at the earlier expressions that we wrote down, you can easily show that $T_2/T_1 > 1$ and $P_2/P_1 < 1$, it is not very difficult to; I am sorry, P_2/P_1 is also > 1 .

For the solution that we have written down both the temperature; static temperature and the static pressure increase across the shock wave that can be shown very easily but what about the density? Remember, density is P divided by RT , so if the P increases and the T also increases, we need to see, what happens to the density? So, in fact it is also possible to show from the expressions that we wrote down that P_2/P_1 is actually $> T_2/T_1$ and this implies that $\rho_2/\rho_1 > 1$.

So, the density increases across the shock wave as well just like static pressure and static temperature, the static density also increases across the shock wave and based on the expression that we wrote down earlier, if you remember, we wrote down an expression for entropy change earlier, it is $2 - s_1$ because T_02/T_01 remains the same. I can write this as; I am sorry, since T_02 remains the same, so you can see that the increase in entropy is equal to this.

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And this shows that P_{02} must be $< P_{01}$, right, so there is a loss of stagnation pressure across the shock wave. So, this shows that P_{02} is $< P_{01}$ and so, across the shock wave; now please, bear in mind that we are still working in a reference frame that the observer moves with the shock wave but stagnation quantities are frame dependent, static quantities are not frame dependent, stagnation quantities are frame dependent right.

So, if I measure the static pressure using a static pressure probe right, it does not matter whether as an observer, I am stationary and I measure the pressure or whether, I am moving and I measure the pressure, I get the same static pressure reading. Similarly, if I measure the static temperature, if you ignore convection effects, right, if you measure the static temperature it does not matter, whether I am standing and measuring the static temperature or whether I am moving along with the wave and measuring the static temperature.

So, static quantities are frame independent whereas, stagnation quantities are not frame independent that is a very important point, so you must bear that in mind. Let us write it down now, so static quantities are frame independent whereas, stagnation quantities are not;

are frame dependent, so the greater the velocity with which the observer moves, right; the greater the dependence on the frame for these stagnation quantities.

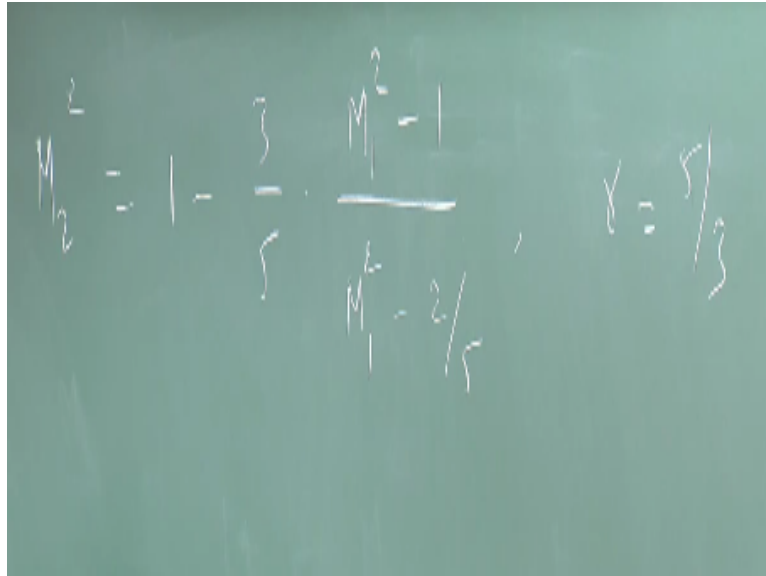
So, when we say that there is a loss of stagnation pressure across the shock wave that is only for the reference frame in which the observer is moving along with the wave, what this will be in the observer stationary reference frame is going to be different and that is something that we will calculate, okay. Now, before we proceed further, let us also find out; we have so far only said that it is a shock wave, why is it called a normal shock wave, right?

It is called a normal shock wave because there is no change in flow direction. If you look at this figure, you can see that there; there is no change in the flow direction, the flow approaches normal to the wave and it also departs normal to the wave. In other words, there is no deflection of the flow as it passes through the wave, there is no change in the direction of the flow as it passes through the wave.

Later on, when we look at oblique shock waves, you will see that there is going to be a change in the direction of the flow after it passes through the shock or in other words, the flow is deflected either towards itself or in some cases, away from itself in these types of cases. So, this will be oblique waves, here it is normal because the flow direction remains normal to the wave before and after the flow, okay.

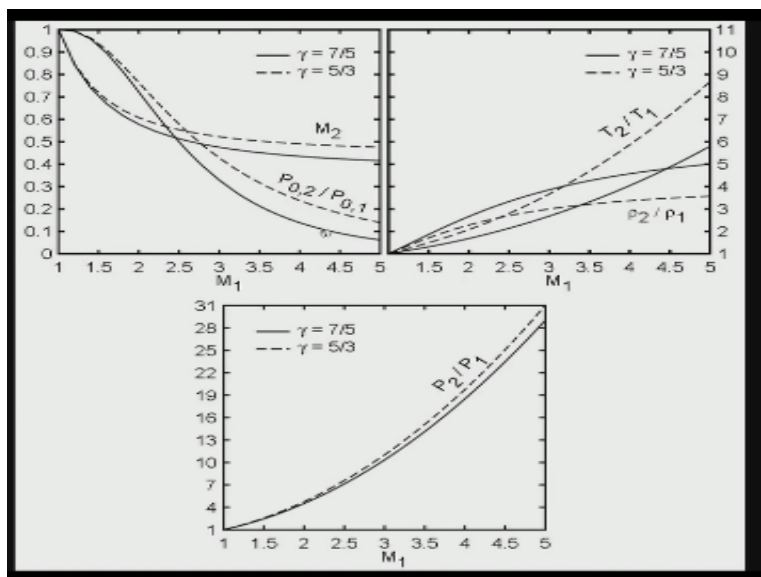
Normal here does not refer to the usual language word normal, right. Normal here means normal to the direction of the wave, so the flow remains normal to the direction of the shock wave both before and after that is why we use the word normal in this case. So, what we are going to do next is to try to draw some inferences on the shock wave solution. Before, we do that, let us just take a quick look, we have derived the solution already.

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And let us just take a quick look at the Mach number M_2 square for some of these cases, notice that M_2 square can be written as $1 - \frac{6}{7} M_1$ square - 1 divided by M_1 square - $\frac{1}{7}$ for diatomic gases for which gamma is $\frac{7}{5}$ and for monatomic gases M_2 square = $1 - \frac{3}{5} M_1$ square - 1 divided by M_1 square - $\frac{2}{5}$, so this is for monatomic gases for which the gamma is $\frac{5}{3}$ rds.

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So, we can actually look at this figure and see how the various quantities depend upon the gamma, so here we have illustrated quantities like Mach number downstream of the shock wave M_2 , stagnation pressure downstream of the shock wave here, ratio of temperatures across the shock wave ratio of densities and P_2/P_1 for the different cases, you notice that for P_2/P_1 , there is another significant difference as Mach number increases in these cases, right.

P_2/P_1 , in fact we can see approaches values like 25 and so on across the normal shock wave right. So, if you remember our discussion earlier on the aircraft engines, right, what kind of pressure ratios are we talking about across the compressor? What 30 to? 30 to 40, so you can see that when my flight Mach number approaches 5, I can actually achieve those kinds of pressure ratios by decelerating the flow properly, right.

So, my P_2 is $> P_1$, T_2 is $> T_1$, u_2 is $< u_1$ and that is what; that is what has happened here, so this tells you how or why at higher flight Mach numbers, I can do away with the compressor and if I do this properly, I can actually compress the flow to the required pressure ratio just by decelerating it, that is what this; that is what this graph is telling you okay.

But you do notice significant differences, when you know; when you are looking at T_2/T_1 right, you can see that T_2/T_1 tends to values like 9 or so for Mach number 5 and when the temperature; static temperature reaches these types of values, the temperature is already very high, we may not be able to jet a fuel and burn it because the gases will begin to ionize and dissociate and they will dissociate.

And then start ionizing the static temperature is too high at the end of the compression process that is what, this is trying to show you and you can also look at the loss of stagnation pressure for example, if you want a pressure ratio of 25, which is what you are seeing here. Let us say at a Mach number m_1 of 4.5, so corresponding to M_1 of 4.5, you can see that the loss of stagnation pressure is P_02/P_01 is 0.1 in this case for $M_1 = 4.5$.

That means, you have lost 90% of the stagnation pressure, which is a tremendous loss of work, so this kind of compression is very effective but not very efficient, right. Isentropic compression process is very efficient but not very effective whereas, this one just across the shock wave, the shock wave is a discontinuity across the shock wave, we are able to achieve a compression ratio of 10, 15 or 20.

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Normal shock compression is effective
but not efficient

Isentropic compression process is efficient
but not effective.

If you tried the same thing with an isentropic compression process, it would not be possible over such a short distance, it will take a much longer deceleration, much gradual deceleration and so on, which is why normal shock compression is said to be is effective but not efficient due to the; it is not efficient because of the loss of stagnation pressure whereas, isentropic compression process is efficient but not effective. So, depending upon the application at hand, we need to use one or the other.