Gas Dynamics and Propulsion
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Lecture - 35
Calculations for Thrust and Fuel Consumption

So in the last class, we derived the final or we calculated the value for the total thrust under static condition.

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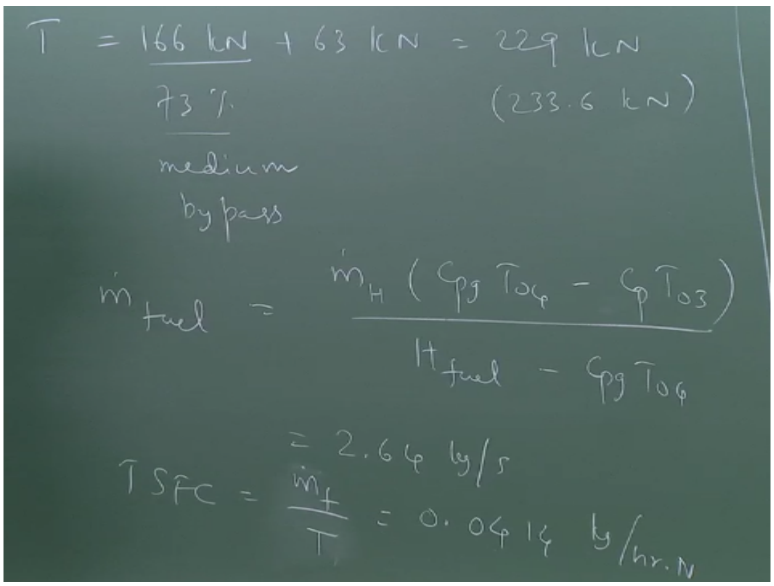

So, we said that the total thrust was composed of 166 kilo Newton of thrust from the fan and 63 kilo Newton of thrust from the core engine which worked out to 229 kilo Newton of thrust and we compared with this value with the value quoted by the manufacturer which was about 233.6 kilo Newton. So the comparison appears to be reasonably good, so we come to the conclusion that the calculation process is probably okay.

One point that you should notice is that the fan contributes to about 73% of the overall thrust which tells you that this is medium to high bypass ratio turbo fan engine. So medium to high bypass will have numbers in the range from 75 to 80 so this tells you that it is a medium bypass ratio engine. The next thing that we wish to calculate is the mass flow rate of fuel so let us go ahead and calculate the m dot fuel based on an energy balance for the combustor.

So we did energy balance for the combustor and we showed that the mass flow rate of fuel is given by this expression m dot H*Cpg T04-Cp*T03 divided by the calorific value of the fuel-Cpg*T04, and if you go ahead and substitute the numbers that we have calculated earlier then we get this value to be 2.64 kg per second as we suspected earlier this mass flow rate of 2.64 kg per second is much less compared with the total amount of mass flow rate that goes through the engine.

Remember the air mass flow rate is around 600 kg per second or so. So this is indeed much less than that. So we were correct in neglecting m dot fuel when we add the mass flow rate of air to this so it is indeed a small number and if I calculate the thrust specific fuel consumption TSFC, TSFC is nothing but the total thrust divided by m dot fuel. In this particular problem, the combustion efficiency is not given.

So the ideal value that we calculate using this formula for mass flow rate of fuel also happens to be the actual value since combustion efficiency is not given. So T divided by m dot f comes out to be 0.0414 in units of kg per hour Newton is what we calculated for this and the manufacturer quotes value of.

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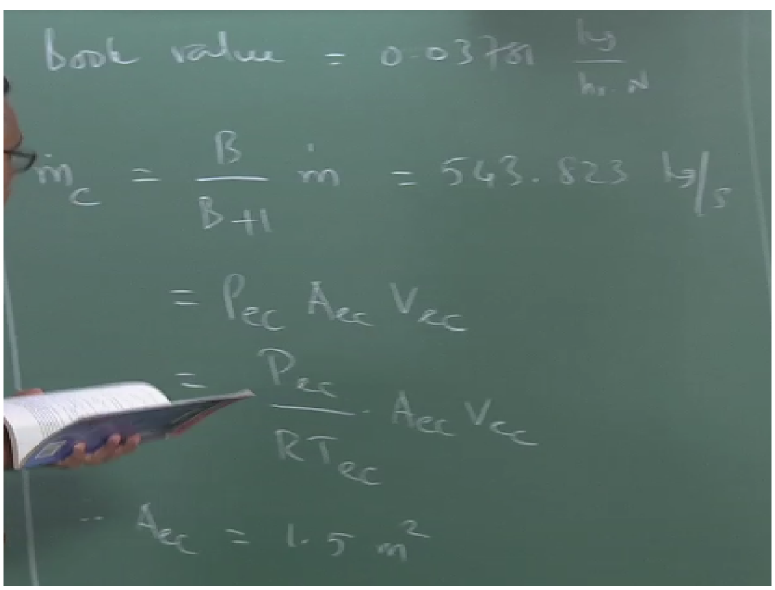

So, book value for TSFC 0.03781 kg per hour Newton. It seems to be reasonably okay for the kind of assumptions that we have made. This seems to be reasonably alright. It is the other way around. So this is m dot f divided by T thank you. So the comparison seems to be alright. We are also asked to calculate in addition to thrust and TSFC we are also asked to calculate the area of the cold nozzle and the hot nozzle.

So let us go ahead and do that so the mass flow rate through the cold nozzle m dot c is nothing but B over b+1*the total mass flow rate and if you substitute the numbers this comes out to be about 543.823 kg per second and we can also use the following expression m dot c is also going to be = rho e at the exit for the cold nozzle*Aec*Vec right. M dot is also = rho av and in this expression I can write this rho as p/RT*Aec*Vec.

So I know all the quantities in this expression except Aec and I can calculate the cold nozzle area to be if you substitute the numbers you get the cold nozzle area to be 1.5-meter square. Remember the cold nozzle is an annular nozzle. So this is the area of the annular nozzle and in the same manner I can calculate the mass flow rate through the hot nozzle.

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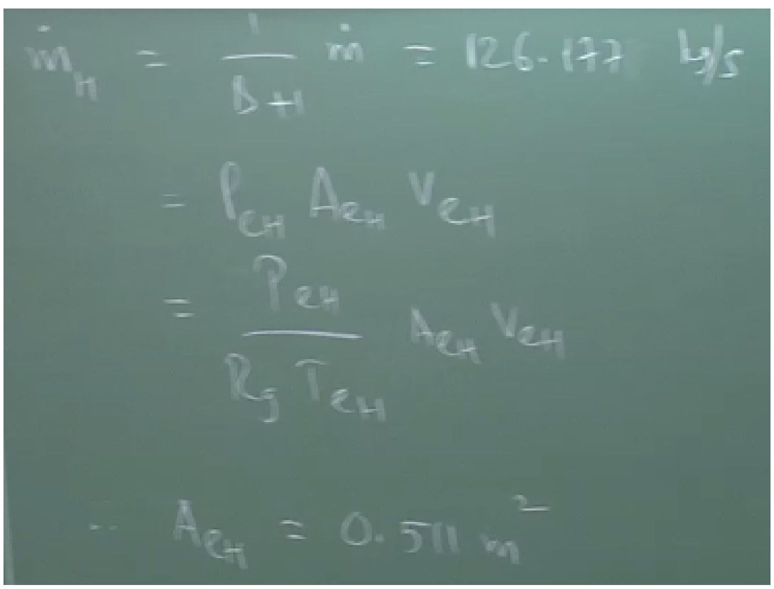

So m dot H is going to be = 1 over B+1*m dot. So the mass flow rate through the hot nozzle is about 126.177 kg per second and this is also = rho eH*AeH*VeH or this is nothing but PeH divided by Rg*TeH. Remember these are combustion gases. So we use Rg here and R here for the cold air, here this is cold air and this is combustion gases so we use Rg for this*AeH*VeH and so this gives me AeH to be the hot nozzle area to be 0.511-meter square.

Now since the calculated value of thrust and the TSFC agree reasonably well with the manufacturer quotes we believe that these areas are correct. This is what the actual values are so we have faith in these areas and we will assume that these are correct. This is important as you place a role later on in what we are going to do. So this completes the calculation procedure for static conditions.

What we will do next is calculate the same quantities for cruise at the given altitude that is what we are going to do. We have been asked to do the same thing for cruising at an altitude of 10 kilometers.

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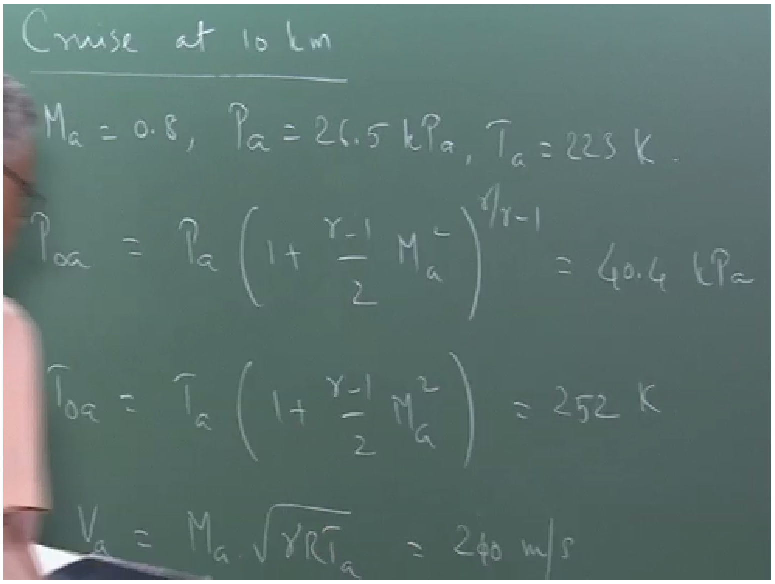

And it is given that Ma = 0.8, the ambient pressure at this altitude is given to be 26.5 kilo Pascal and the ambient temperature is given to be 223 Kelvin. Now we know from experience that when the engine or when the air craft cruises at this kind of an altitude the thrust that the engine will produce or thrust the engine is required to produce is much less than the take of thrust. Remember the static thrust or the take off thrust which are almost the same is quite high. So the same engine will produce much less thrust when it is actually cruising.

The throttle settings will be changed mass flow rate of fuel will change and other things may also change, but in the problems statement none of these things are given. We are not given we are only given that the altitudes cruises at this kind of an altitude. I am sorry the aircraft cruises at this kind of an altitude. So we have to figure out how the conditions have to change. We will do that to some extent as we go along.

For now, we will simply use the same values and go through the calculations to see if there is anything wrong with the predictions. That is our strategy now. We will start with this and let us go through in the same manner. So I can calculate P0a to be Pa*1+gamma-1/2*Ma square raise to the power gamma/gamma-1 and if I substitute the numbers I get this to be 40.4 kilo Pascal and T0a can also be calculated in the same manner.

T0a = Ta*1+gamma-1/2*ma square and this comes out to be 252 Kelvin and the free stream speed Va can be calculated as Ma*square root of gamma RTa and this comes out to be 240 meter per second. So starting from this free stream conditions, we will go through each and every state as we did before. So we go to the end of the inlet, end of the fan, end of the fan nozzle which completes the fan stream then we will go to the compressor core stream and then finish the calculation.

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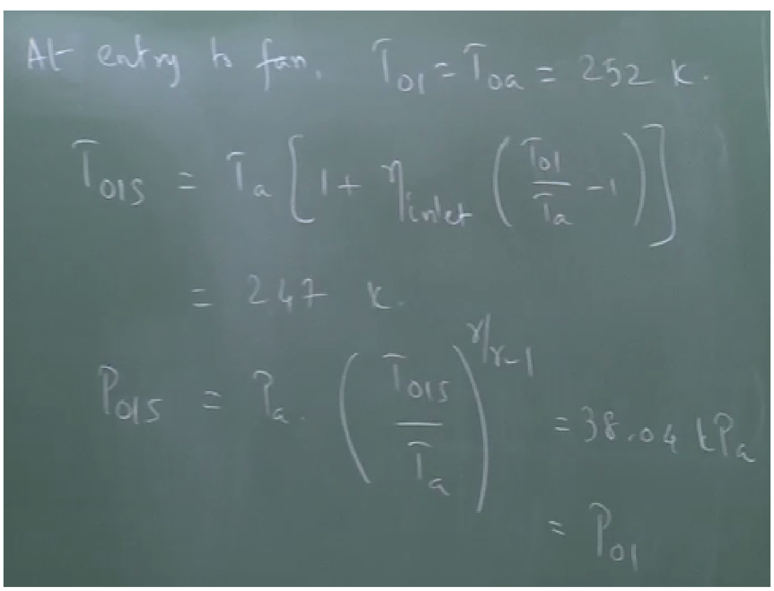

So at the end of the inlet or at entry to fan since there is no energy exchange, heat or work interaction in the inlet T01 = T0a = 252 Kelvin. So I need to calculate P01 also remember we want T01 and P01 so I calculate P01 in the manner that we did before. So I calculate my T01s from the definition of the isentropic efficiency of the inlet. So T01s = Ta*1+eta inlet*To1 divided by Ta-1.

This is the definition of the isentropic efficiency for the inlet. So I substitute the numbers I get my T01 is to be 247 Kelvin. Once I have T01 is I can use the isentropic relationship to calculate P01s. Po1s = Pa*T01s divided by Ta to the power gamma/gamma-1. So if I substitute the numbers, I get this to be 38.04 kilo Pascal and if you remember from our earlier lecture P01 = P01s because both of them lie on the same isobar. That is how we define the efficiency so this is = P01. So now I have T01 and P01.

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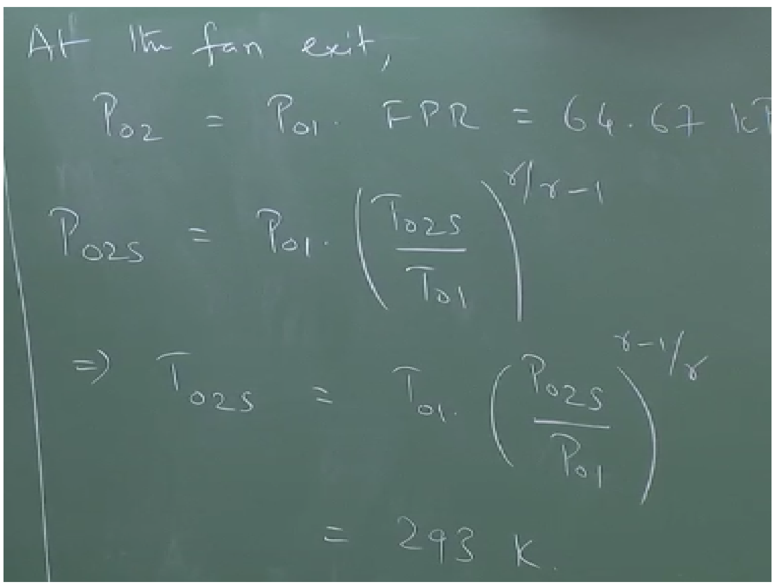

Now at the fan exit we move to the next state. At the fan exit P02 = P01*fan pressure ratio and fan pressure ratio is given to be 1.7 so we change P02 is 64.67 kilo Pascal. So I have P02 now. I have to calculate T02. Earlier I had the stagnation temperature and I needed to calculate the corresponding stagnation pressure. Now I have the stagnation pressure and I need to calculate the corresponding stagnation temperature which is what we are going to do next.

So remember P02s = P02. So I use that fact and do the following. Remember P02s = P01*T02s divided by T01 raise to the power gamma over gamma-1 which tells me that I can calculate my T0s as T01*P02s divided by P01 to the power gamma-1/gamma and P02s = P02 and both the states lie on the same isobar so I can substitute the numbers and If I do that I get T02 is to be 293

Kelvin. So once I have T02s by using the definition of the isentropic efficiency of the fan I can calculate T02.

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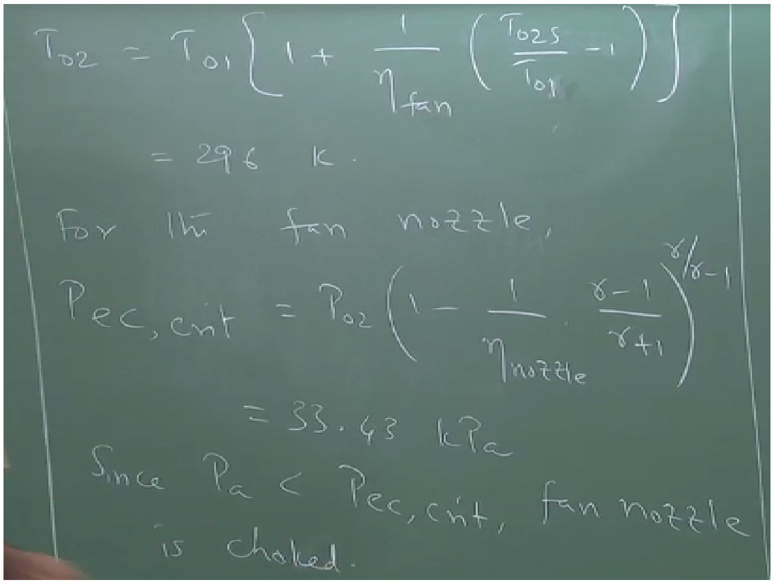

So T02 is going to be 15:38 = To1*1+1/eta fan*T02s/To1-1. So this is the definition of the isentropic efficiency of the fan and if you substitute the numbers I get T02 to be 296 Kelvin so now we have P02 and T02. So the next component in the fan stream is going to be the fan nozzle. So let us go ahead and complete the fan stream. For the fan nozzle since I know P02 I can calculate the critical pressure P for the cold nozzle.

Critical pressure = P02*1-1/eta nozzle*gamma-1/gamma+1 raise to the power gamma/gamma-1 and if you substitute the values we get the critical pressure to be 33.43 kilo Pascal and contrary to the static condition where the ambient pressure was 100 kilo Pascal. Now the ambient pressure is 26.5 kilo Pascal and the critical pressure is 33.43 kilo Pascal which tells me that the fan nozzle is going to be choked now in the previous case it was not choked now fan nozzle is choked since Pa < Pec critical, fan nozzle is choked.

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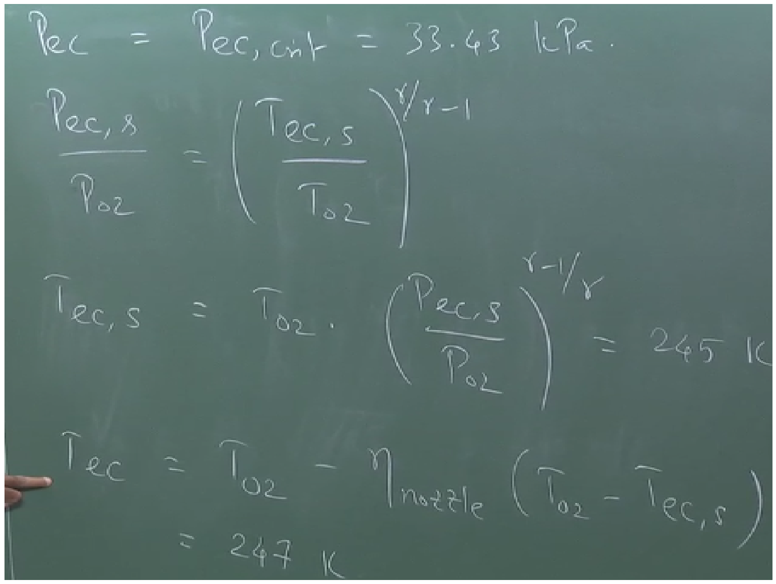

So that means that the exit pressure Pec = Pec critical since the nozzle is choked and that is = 33.43 kilo Pascal. So I know the exit static pressure. I need to calculate the exit velocity. I calculate the exit velocity by making use of the fact that Pec isentropic = Pec both of those states lie on the same isobar so which means that I can do the following. I can use the following relationship P02 let me arrive it like this.

Pec for an isentropic process divided by P02 = Tec, s divided by T02 raise to the power gamma/gamma-1. So from this I can calculate Tec, s. So, Tec, s = T02*Pec, s divided by P02 raise to the power gamma-1/gamma and this value is = Pec. So remember both this and Pec lie on the same isobar. So I can substitute the value and get this temperature to be = 245 Kelvin.

Now the actual exit static pressure Tec can be calculated from the definition of the isentropic efficiency of the nozzle in this way so this = T02-eta nozzle*T02-Tec, s. So this gives me the actual static temperature at the exit of the cold nozzle. Substitute the numbers we get this to be 247 K. So I have the static pressure I have the exit static temperature from which I can calculate the exit velocity. Vec can be calculated very easily. Vec = let us do that.

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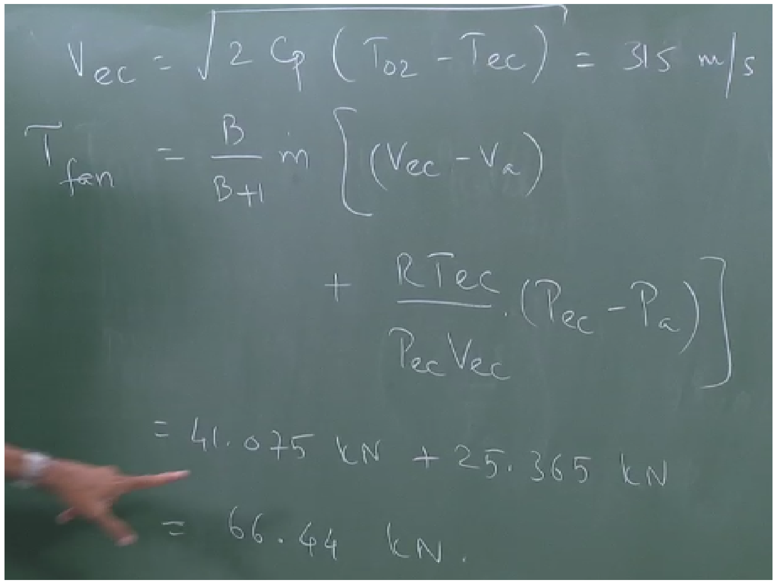

Vec = square root of 2 Cp*T02-Tec and this comes out to be 315 meter per second. Working with this number also gives you an idea of what values or magnitudes these numbers have what kind of velocities do you have at the exit of the nozzle, fan nozzle and the exit of the core engine nozzle and so on. Now the thrust from the fan stream can be written as so this is the momentum thrust plus the pressure thrust which is nothing but so the second term is the pressure thrust.

There was an area term which normally multiplies the pressure thrust. I have taken out the mass flow rate. I have divided by m dot c. So that area divided by m dot c is what gives rise to this expression here. M dot c is nothing but rho ec*Aec*Vec so that is what I have used here to write this expression. This is the momentum thrust, this is the pressure thrust and if you substitute the numbers we get this to be 41.075 kilo Newton+25.365 kilo Newton which adds up to 66.44 kilo Newton.

Notice that we have not touched or changed this value of M dot. We are still assuming M dot to be whatever was given for sea level static condition. Remember m dot was given to be 670 kg per second. We are using the same value here and as I said earlier it is very likely that when the engine is cruising at 10 kilometers altitude the mass flow rate is likely to be less than what it takes in under sea level conditions.

So now the mass flow rate is less due to 2 reasons number 1 the density of the air itself changes at the higher altitude and number 2 the throttle setting may also change to produce a certain amount of thrust. So we will see what changes need to be made. For now, we will use this and then go ahead with this. Now notice that the momentum thrust is about roughly 2 times the pressure thrust in this case.

Under sea level condition, this was completely 0 only this momentum thrust was present. So that completes the fan stream. We now move on to the core engine stream.

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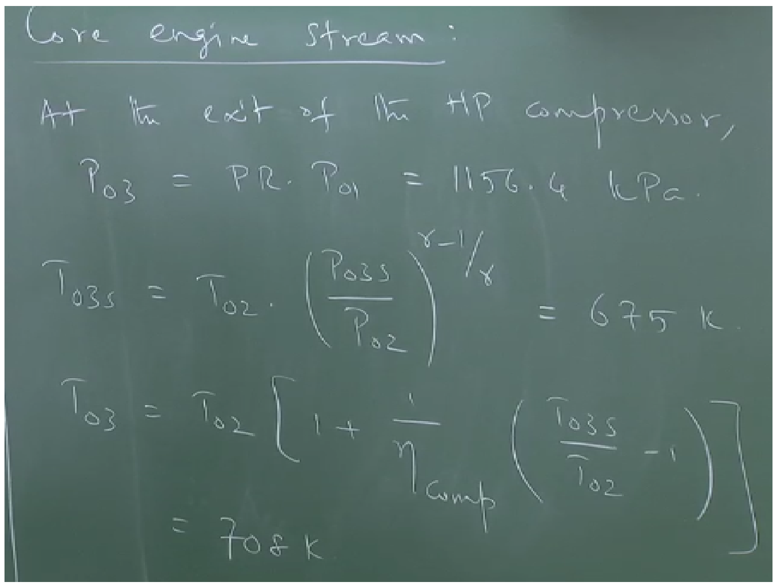

So as you said earlier part of the air and amount = m dot H goes through the fan and then it goes through the core engine. So we are going to track the m dot H amount of here which now goes through the core engine. So we start from the fan outlet and then we go to the exit of the high pressure compressor. So at the exit of the HP compressor P03 = pressure ratio PR*P01 so if you plug in the numbers you get this to be 1156.4 kilo Pascal.

So I know P03. I need to calculate T03 now. So what we do? same procedure as earlier. We evaluate since I know P03 I know P03s so I evaluate T03s. So T03s = T02*P03s divided by P02 raise to the power gamma-1/gamma and if I substitute the values P03s = P03 and if I substitute the values I get this to be 675 Kelvin. Now since I know T03s I can use the definition of the isentropic efficiency of the turbine to calculate my T03 and so T03=T02*1+1/eta compressor * T03s divided by T02 – 1.

And if I plug in the numbers I get this to be 708 Kelvin. So this is consistent with what we were telling earlier that the temperature at the end of the compression process is around 700 Kelvin or so. So this confirms what we are saying earlier. so the air at the end of the compression process comes out at a temperature of around 700 Kelvin then we add fuel, burn fuel, to raise the temperature to some value around 1500 or 1600 Kelvin so that is how much energy is being added in the combustor and that is what we are going to look at next.

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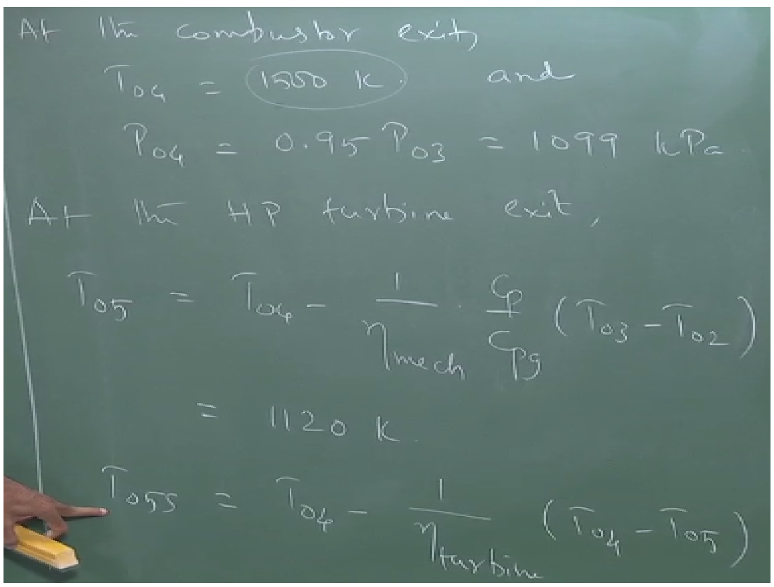

So at the combustor exit T04 = 1500 Kelvin and P04 = it is given that there is a 5% loss of stagnation pressure in the combustor so P04 is 0.95*P03 and this comes out to be 1099 kilo Pascal. Once again just like the total mass flow rate we are not going to change this quantity also for now. but when we are cruising at an altitude of 10 kilometers for in order to produce less thrust the engine will take in less amount of air plus the amount of fuel burned will also be less,

Little bit less than what it is going to be for takeoff condition, but we do not know that value. You have not been given that value in the problem description. So we will proceed with this and then try to correct this or rectify this later on. So let us proceed. We will remember that this value most likely needs to be changed and the m dot value also most likely needs to be changed. So this value also most likely needs to be changed.

We will keep that in mind and proceed. So the next component is the high pressure turbine. So at the HP turbine exit, from an energy balance remember the HP turbine produces the amount of work that is required to run the HP compressor. So T05 can be calculated based on an energy balance so T05 = T04-1/the mechanical efficiency*Cp/Cpg*T03-T02, and if you substitute the numbers we get this to be 1120 Kelvin.

So I have T05 now. I need to evaluate P05. So to do this what I do is I calculate my T05s using the definition of the isentropic efficiency of the turbine so I can calculate T05s = T04-1/eta turbine*T04-T05. So this is the definition of the isentropic efficiency of the turbine so I can calculate T05s.

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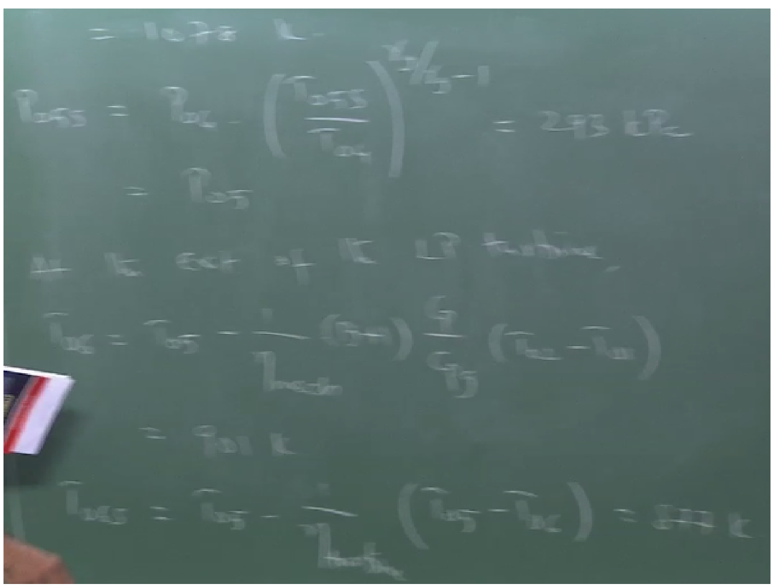

So if substitute the numbers you get T05s to be 1078 Kelvin and once I have T05s I can actually calculate P05s using the isentropic relationship. So P05s = P04*T05s divided by T04 raise to the power. Now we are using gamma g please bear that in mind these are combustion gases so this is gamma g/gamma g -1 and gamma g itself was given to be 1.333. So we substitute these numbers and we get P05s to be 293 kilo Pascal and this is also = P05.

So now we have both T05 and P05. The next component is the low pressure turbine. Let us go ahead and do that. So at the exit at the LP turbine so the LP turbine produces enough power to run the fan so energy balance for this gives me T06 = T05-1/eta mechanical*B+1*Cp/Cpg*T02-T01. Notice that the expression for the exit stagnation temperature of the LP turbine is little bit different from the expression for this exit temperature for the HP turbine mainly.

Because there is no mass flow rate here, but there is a B+1 here. Mainly because the HP compressor handles an amount of air = m dot H. The HP turbine also handles an amount of air = m dot H. So that m dot cancels out whereas in this case, the LP turbine handles an amount of air = m dot H whereas the fan handles an amount of air = m dot c+m dot H which is why we are getting this additional B+1 here.

So you substitute the numbers and we get this to be 901 Kelvin and once I have T06 I need to evaluate T06 and we will use the same procedure. So we calculate T06s may be I can write it over here so T06s looks like this. So T06s = T05-1/eta turbine*T05-T06. So this is the definition of the isentropic efficiency of the LP turbine and if we substitute the numbers you get this to be 877 Kelvin. So once I have T06s I can calculate P06s from the isentropic relationship.

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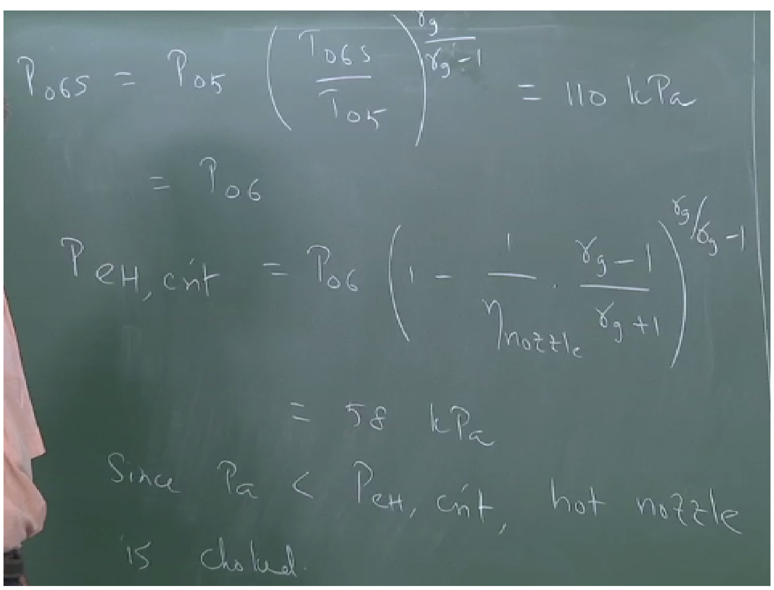

So P06s = P05*T06s divided by T05 raise to the power gamma g divided by gamma g-1 and this value comes out to be 110 kilo Pascal. So I have now this is also = P06. So I have T06 and P06.

The next component is the hot nozzle. So what we do is we first evaluate the critical pressure. So PeH for the hot nozzle exit pressure hot nozzle critical value is given by.

P06*1- 1/eta nozzle*gamma g-1 divided by gamma g+1 raise to the power gamma g/gamma g – 1 and if you plug in the numbers we get this to be 58 kilo Pascal and the ambient pressure is let us see. The ambient pressure is given to be 26.5 kilo Pascal in this case so since Pa < Pe H critical hot nozzle is choked, which means that the exit static pressure is = the critical pressure.

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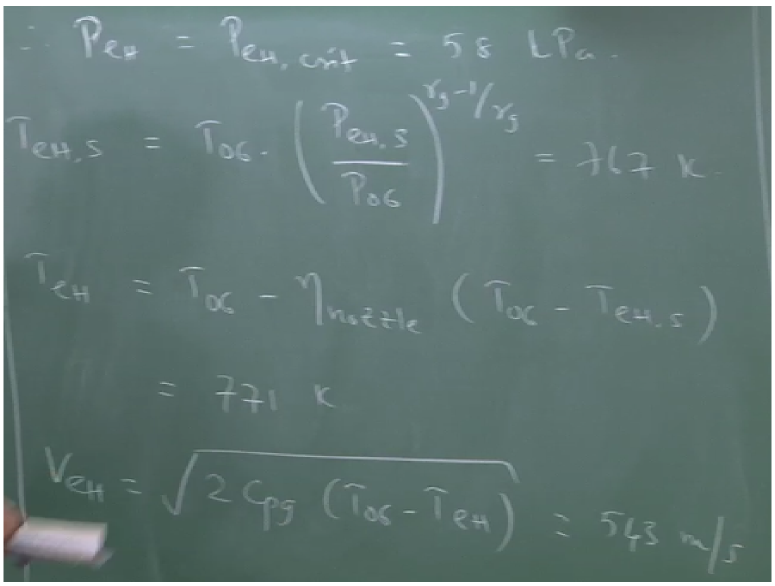

Therefore, the exit static pressure PeH = PeH critical which is = 58 kilo Pascal. So I have the static pressure. Now I can calculate the I have to calculate the static temperature so what I do is I calculate TeH,s. So TeH for isentropic process is going to be T06*PeH isentropic process divided by P06 raise to the power gamma g-1/gamma g at I know PeH for the isentropic process because this is = Pe H.

Both the state points lie on the same isobar. I know this so I can calculate TeH, s which comes out to be 767 Kelvin. Once I have this I can evaluate TeH by using the definition of the isentropic efficiency of the nozzle so Te, H is = T06-eta nozzle*T06-TeH, s. so the exit static temperature we substitute the numbers comes out to be 771 Kelvin and now I can calculate the exit velocity VeH is = square root of 2*Cpg*T06-TeH and this comes out to be 543 meter per second. So now I can go ahead and calculate the thrust from the core engine.

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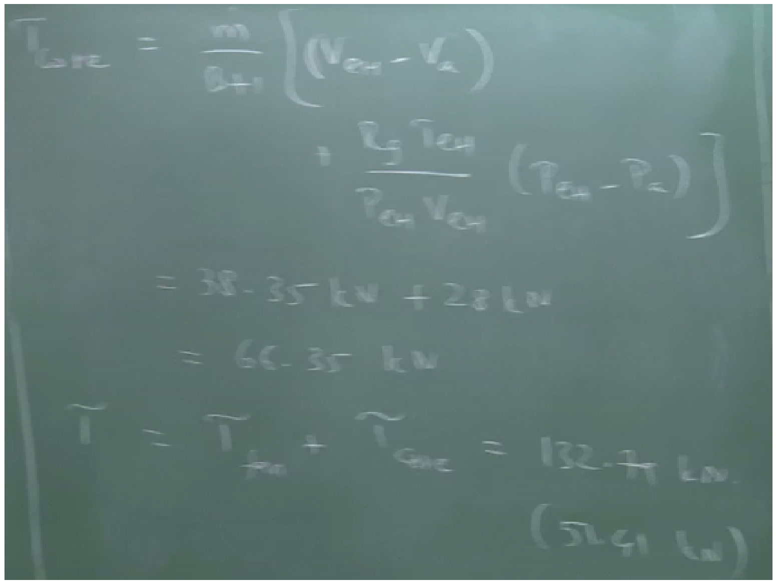

So T from the core engine is = m dot/ B+1*VeH-Va which is the momentum thrust +Rg*TeH divided by PeH*VeH*PeH-Pa which is the pressure thrust and once again I am not going to touch this mass flow rate I will use the same value as before and if I do that I get this to be 38.3 kilo Newton+28 kilo Newton which gives a total value of 66.35 kilo Newton from the core engine.

So the total thrust T is = fan thrust + core engine thrust which works out to the problem is the fan thrust if you check back your numbers you will see that the fan thrust was about 66. something. So the core engine thrust is also 66. something. Now they appear to be the same okay, but the overall thrust itself reduces to about 132.79 kilo Newton. So the thrust has gone down from 229 kilo Newton to 132.79 kilo Newton. Let us calculate the mass flow rate of fuel and then we will compare our numbers with the book values.

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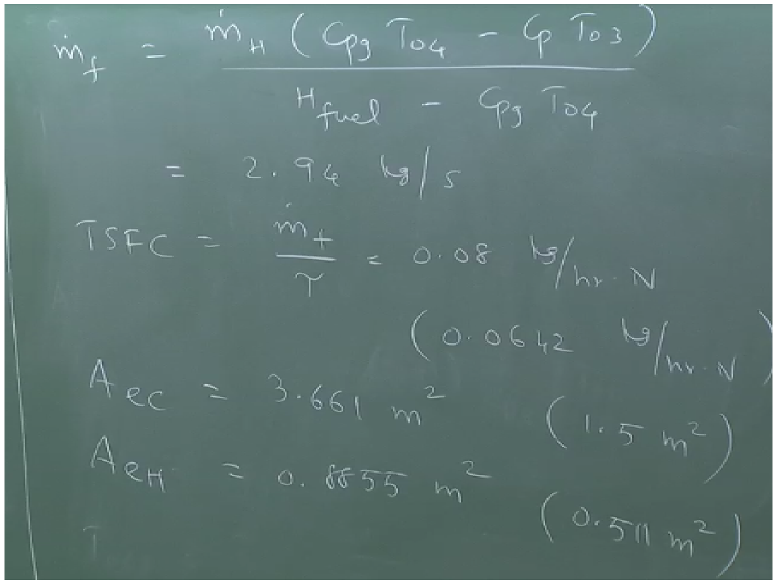

So the mass flow rate of fuel m dot f can be evaluated as m dot H*Cpg T04-CP*T03 divided by H fuel-Cp*T04 and if I substitute the numbers I get this to be 2.94 kg per second and the TSFC works out to be m dot f/total thrust works out to be about 0.08 kilogram per hour Newton. So now we have to see what we are seeing at the end of the tunnel a sunlight or the head light of the oncoming train that we do not know.

That we will know only when we compare these values with the actual book values now the quoted value by the manufacturer for cruise thrust at this altitude is about 51.41 kilo Newton and the TSFC is the manufacturer quotes is about 0.0642 kg per hour Newton. So it appears that this not light at the end of the tunnel, but the head light of an oncoming train. So we need to fix do something.

We need to do something to make this compare better. What do we do? We can figure out what to do if we if you proceed in the same way as before. What if I calculate the area of the core engine nozzle and hot nozzle for these conditions So if I calculate the area of the cold nozzle and the hot nozzle for this conditions I get this to be 3.661 meter square and the hot nozzle area to be 0.8855 meter square and if you remember earlier.

We calculated the same values to be 1.5 meter square and this was calculated to be 0.511-meter square. So now we can see what the problem is. We had kept m dot and T04 the same. If you do that and go ahead with the calculation, we can see that there is a poor agreement with the value quoted by the manufacturer and the reason for that lies in this to pass the same mass flow rate and have the same T04 we need these kinds of areas for the cold and the hot nozzle.

But remember this is the turbo fan engine with fixed nozzle. It does not have adjustable area nozzle which means that I need to redo my mass flow rate so that the nozzle area will come out to be same. So the same nozzle must pass that mass flow rate so let us start with the fan nozzle we can simply scale down the mass flow rate to agree with this remember our exit velocity and other things that we calculated for example if you look at VeH, mass flow rate was not involved.

In any in these kinds of expression so the VeH is not going to change everything was taken into account properly everything was a ratio. So if I scale down the mass flow rate by a factor of half Vec will remain the same and I should be able to get a better agreement with manufacturer quoted values so that I can do for 1 nozzle then what do I do the other nozzle that is the question that we must answer next. Let us see what happens with this. So let us do this part and then we will pick it up in the next class.

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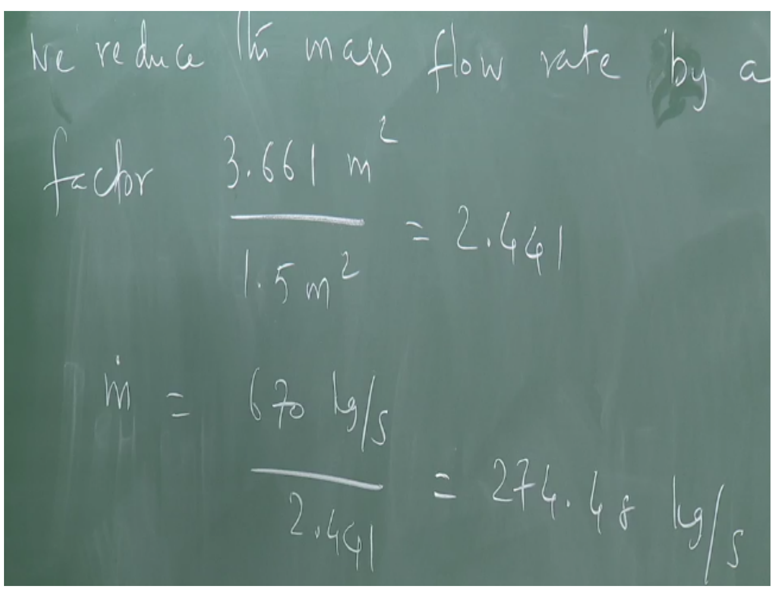

So we reduce the mass flow rate by the factor 3.661-meter square divided by 1.5 meter square which gives me about 2.441 which means m dot = 670 kg per second divided by 2.441 so the mass flow rate at this altitude should be 274.48 kg per second. So this directly tells me how much the mass flow rate should be changed.

So now if I do this and if I recalculate my Aec for corresponding to this mass flow rate and the same Vec, Vec is not going to change if I do that then I will notice that Aec comes out to be 1.5-meter square perfect what happens to AeH and how do we fix that is what we are going to discuss in the next class.