Gas Dynamics and Propulsion
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Lecture - 34
Calculations for Thrust and Fuel Consumption

So, in the last class we looked at calculating the exit static state from the nozzle. What we need to do is to calculate both the specific thrusts are the thrust produced by the engine as well as the fuel consumption for the engine. So the exit static state from the nozzle will allow us to calculate the thrust produced by the engine. What we are going to do in this class is to evaluate the fuel consumption for the engine.

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Specific Thrust & TSFC Calculation

Energy balance for the combustor gives

$$\dot{m}_H C_p T_{03} + \dot{m}_f H_{fuel} = \left(\dot{m}_H + \dot{m}_f\right) C_{pg} T_{04}$$

$$\dot{m}_{f,ideal} = \frac{\dot{m}_H \left(C_{pg} T_{04} - C_p T_{03}\right)}{\left(H_{fuel} - C_{pg} T_{04}\right)}$$

$$\dot{m}_{f,actual} = \frac{\dot{m}_{f,ideal}}{\eta_{comb}}$$

So the fuel consumption for the engine can be obtained by doing an energy balance for the combustor. So, if you will get the combustor at steady state operation so you have mH dot, amount of air coming in at a stagnation temperature T03 and we have a certain amount of fuel mf dot which comes in with negligible velocity, but with calorific value given by this amount. So this is the rate at which energy is being brought into the combustor by the fuel.

This is what happens for the fuel and the exit stream has mass flow rate mH dot+mf dot*cp gas*T04. This is the exit stagnation temperature. Remember T03 we have already calculated from the given overall pressure ratio for the combustor and T04 is actually prescribed as the maximum allowable temperature in the cycle. So the turbine entry temperature is given to us and once again mH dot+mf dot is usually not mf dot is negligible when we add to mH dot.

So we keep that in mind, but notice that here we cannot ignore mf dot here because H fuel is a very large number. If you remember H fuel is 45,000 kilo joule per kilogram. So it is a large number so that multiplied by mf dot will still be a very big number so we cannot neglect this. If you neglect this, then this equation will become meaningless and also there is an indirect check of this equation.

So the ideal amount of fuel flow rate that I need to provide then comes from this equation. Notice that here I have neglected mf dot in comparison with mH dot. This I am allowed to do, because mf dot is usually a much smaller number compared to mH dot that I am allowed to do. So, I can rewrite this equation for the mass flow rate of fuel as something like this.

So this is the amount of fuel that I need to burn to take a stream of air from a stagnation temperature T03 to a stagnation temperature T04 under ideal conditions, but as we discussed earlier when I burn a certain amount of fuel in a combustor there are going to be losses. Some of the heat goes towards increasing the temperature of the combustor components, some is lost to the ambient air, so there are lot of losses.

So, there are lot of inefficiencies. So I account further by saying that this is the amount of fuel I need to burn under ideal circumstances so in the actual circumstance the amount of fuel that I burn is going to be the ideal value divided by the efficiency of the combustor. So this is going to be more than the ideal value understandable. So this gives me the actual mass flow rate through the combustor.

So, given value for eta combustor I can calculate this, I know this, I can calculate mf dot actual. So this allows as to calculate the thrust and the mass flow rate from which I can calculate the specific thrust at the thrust specific fuel consumption. So we are now ready to proceed with our calculation for a particular example.

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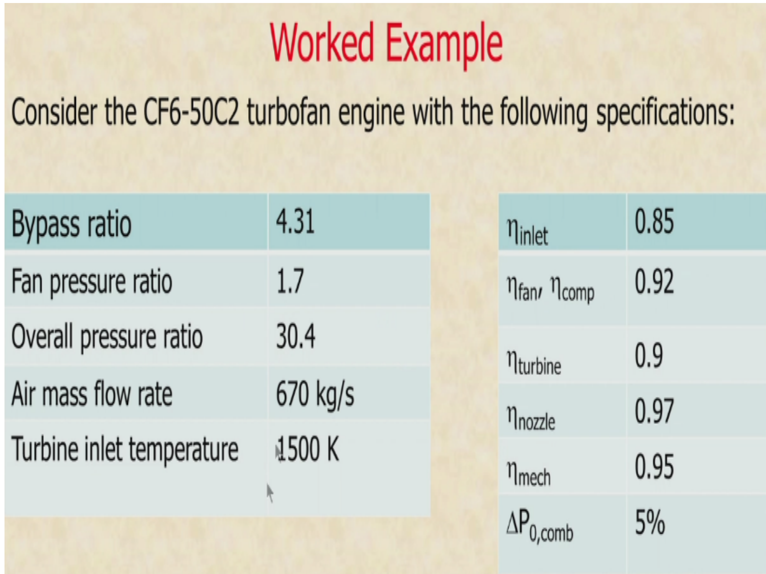

So, for the worked example I have chosen the following engine. The CF6-50C2 turbofan engine is the mid size engine and manufactured by GE/SNECMA and it has the following specifications: Bypass ratio 4.31, fan pressure ratio is given 1.7, overall pressure ratio is given, air mass flow rate, and the turbine entry temperature is also given. The component efficiencies for the inlet, for the fan and the compressors, turbine, the nozzle, and the mechanical efficiency is given.

There is 5% loss of stagnation pressure as shown here. So the properties as we listed earlier the properties also need to be given. We have already discussed that so for air we will use a certain value for gamma and CP and for the combustion gases we will use a certain value for gamma and CP. So I will let you write these numbers and then we will go to the next slide where the property values are given. So here the performance, the matrix and the efficiencies are given here.

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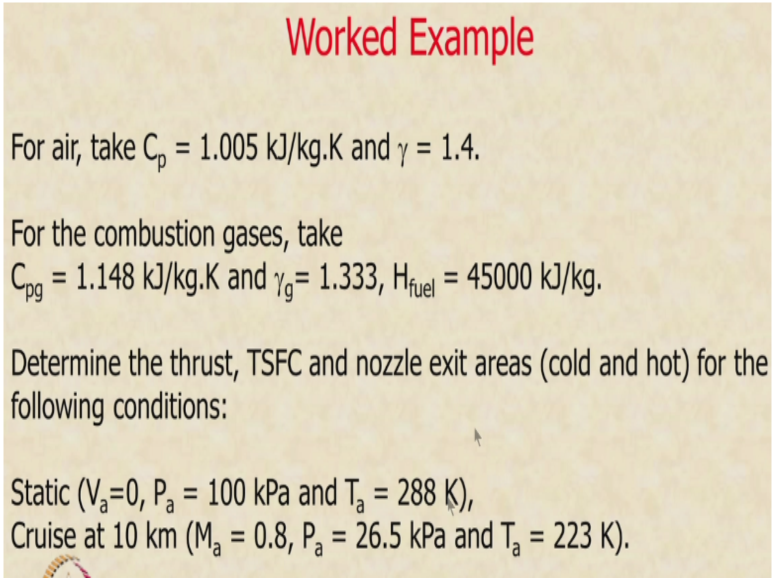

So the properties are given here for air. Cp and gamma are given, combustion gases, calorific value of fuel is also given here. We are asked to determine the thrust, the thrust specific fuel consumption and the nozzle exit areas both the cold and the hot nozzle for the following conditions. 2 conditions are considered one is the static thrust meaning the engine is on the (()) (06:00) and VA = 0, Pa = 100 kPa.

Ambient condition, Ta = ambient temperature 25 degree Celsius. I am sorry this corresponds to 15 degree Celsius correct yes, relatively cold day, and then we are also asked to calculate the thrust TSFC and nozzle areas for cruise at 10 kilometer altitude which corresponds to running conditions of 0.8 for the Mach number, ambient pressure 26.5 kilo Pascal and ambient temperature 223 Kelvin.

So we are asked to calculate this for both these conditions. Notice that the nozzle that we use in an air craft engine like this CF6 engine the nozzle area is fixed. It is not a variable area nozzle. It is not an after burner nozzle. The nozzle area is fixed. So you must keep in mind that the nozzle exit areas will be the same for static and cruise condition. It is not going to change, but there is an important factor that we must use during our calculations.

So now we start the step procedure, we start from the static state a. We evaluate T01, P01, T02, P02, T03, P03, T04, P04, and then T05, P05, T06, P06. Once we have that then based on T06 and P06. We can calculate PeH and VeH for the hot nozzle. Now based on P02 and T02 we can calculate PeC and VeC for the cold nozzle then we can do the thrust calculation that is going to be our strategy.

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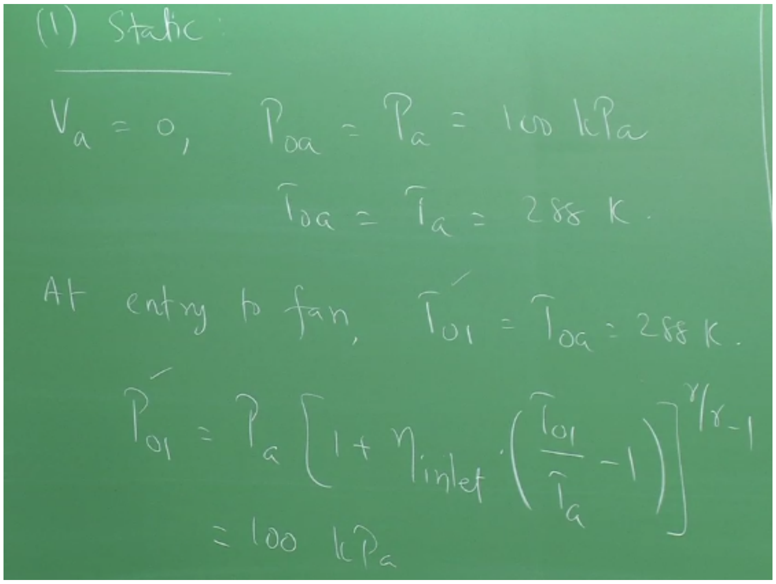

So, let us first do the calculation for static conditions. So it is given that Va = 0 so that means P0a = Pa = 100 kPa and T0a = Ta = 288 Kelvin. So this is the free stream condition so at entry to fan this is the inlet. So at entry to fan, since there is no work or heat addition or extraction in the inlet we have T01 = T0a = 288 Kelvin and P01 at the end of inlet was = Pa*1+eta inlet*T01/Ta - 1 raise to the power gamma - 1.

So eta inlet is known. All the other quantities are know so if you plug in the numbers. The fact that T01 = Toa and the fact that T0a = Ta makes this term 0 because we are looking at static conditions T01 is = T0a and Toa is = Ta so this goes to 0 so in fact this stagnation pressure P01 also comes out to be 100 kPa. This makes sense. Because the air is not moving so the inlet cannot take a static air and increase its pressure.

The inlet is designed to take the air that is moving and convert some other momentum into static pressure. In this case the air is not at all moving so there is no question of taking the air and increasing its static pressure so this number make sense. So our definition for the isentropic efficiency of the inlet seems to be consistent it works fine. So that P01 and T01. So we know the entry conditions at the inlet to the fan. So we know P01 and T01. Next, we have to determine P02 and T02.

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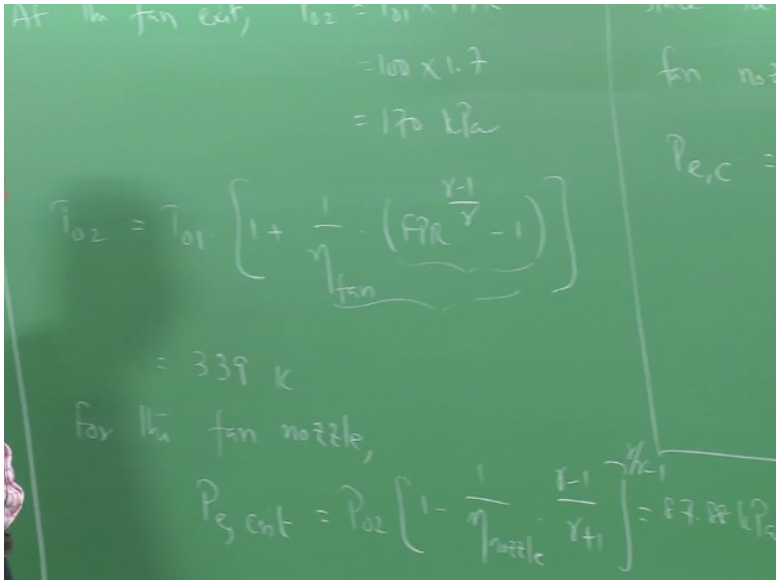

So at the fan exit, P02 = P01*the fan pressure ratio. This was how the fan pressure ratio was defined so P02 = P01*fan pressure ratio and if I substitute the value for fan pressure ratio let me just go back here a little bit so the fan pressure ratio is given to be 1.7 so that means P02 = 100 kilopascal*1.7 so this is 170 kilo Pascal. So we know P02. Now, we can evaluate T02 from the expression for isentropic efficiency for fan and the isentropic relationship itself.

So T02 = T01*1+1/eta fan*fan pressure ratio raise to the power. You may recall that this quantity inside remember comes from this is nothing by T02s and then we use the isentropic efficiency of the fan to calculate T02. So we had P02 so that means once I have P02 I know P02s. Once I know P02s I can actually calculate T02s, state 02s and 02 are on the same isotope. So once I know P02 is I can calculate T02s.

Once I known T02s I can use the definition of the isentropic efficiency of the fan to calculate my T02 that is what I have done here. So from this we can also calculate P02s then calculate T02s and then calculate T02 that is also okay. So if you substitute the numbers here then you should get T02 to be 339 Kelvin. Would you prefer to do these calculations that way rather than doing these expressions is that better?

The P02s, T02s is better or this is okay. It is alright. This seems to compression too many of the things into this that is the only thing but in an actual calculation it is probably better to do this using the P02s expression that we have written down earlier. So now we have P02 and T02. So let us go ahead and complete the fan nozzle also.

For the fan nozzle once I know P02 I can calculate the critical pressure for the fan nozzle. So for the fan nozzle PE critical = the upstream stagnation pressure P02*1- 1/eta nozzle*gamma - 1/gamma+1 and if you substitute the numbers you get this to be 87.88 kilo Pascal that is Pe critical for the fan nozzle. The ambient pressure is given to be.

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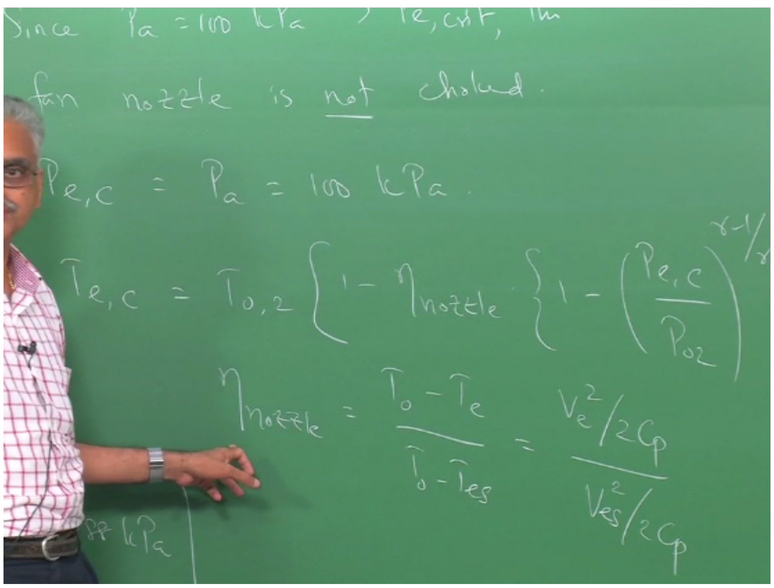

Since Pa = 100 kilo Pascal for this case which is greater than the Pe critical. This means that the core nozzle is not choked. Therefore, the exit static pressure of the core nozzle Pec = ambient pressure and that is = 100 kilo Pascal. **"Professor - student conversation starts"** Yes, Pe critical expression having the power of gamma/gamma+1 so this has gamma/gamma – 1, yes thank you.

So please include that also. So there is an exponent here gamma/gamma -1. **"Professor - student conversation ends"** So I know the exit static pressure. I need to evaluate the exit velocity or the exit static temperature any of those things would do. So let us calculate the exit static temperature. Remember once I know the exit static pressure in the fan nozzle Pe, Pes = Pe because they both lie on the same isobar.

Then I know the stagnation quantities at the inlet I know T0 at the inlet to the nozzle, I know P0 at the inlet to the nozzle. Now I know the static pressure Pes, which means I can evaluate Tes using the isentropic relationship. So once I have Tes then I can evaluate Te at the exit to the nozzle. So let us now we go about doing this. So Tec = T02*1 - eta nozzle*1 - Pec/P02 raise to the power gamma - 1/gamma that is what we have done here.

So once I know the exit static pressure we outline the calculation procedure once I know the exit static pressure you know Pes I can evaluate Tes. Once I evaluate Tes I can evaluate Ves square over 2 CP and then I can evaluate Ves or I can also evaluate this static temperature and then go from there. Remember the expression for nozzle efficiency itself the expression for nozzle efficiency we wrote this down as T0 - Te divided by T0 - Tes.

This was what we wrote down the expression as and if you use the definition for stagnation temperature you can see that this can written as Ve square over 2 Cp divided by Ve square/2Cp. so once I know my Pes I know T0. I know P0. I can either evaluate Tes from this expression or I can directly evaluate Ve from this expression. Ve square and then Ve from this expression both are okay that is what I have done here. So if I go ahead and substitute the numbers I get the static temperature Tec to be.

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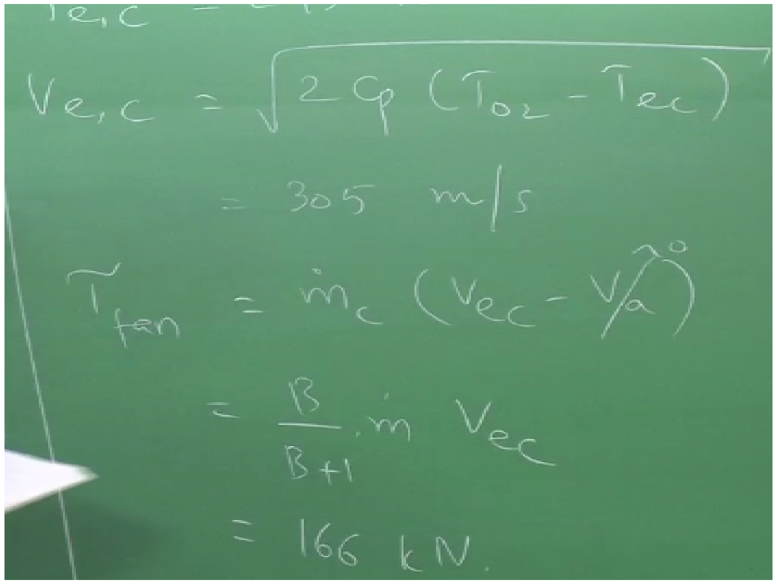

The static temperature at the exit of the core nozzle to be 293 Kelvin and so the velocity at the exit Vec comes out to be square root of this quantity and that works out to approximately 305 meter per second. So now I can calculate the fan thrust. Remember the fan nozzle is not choked, So Pec = Pa which means the pressure thrust is absent we have only the momentum thrust so the thrust from the fan comes out to be m dot C*Vec - Va and Va itself is = 0.

Because the engine is on a static thrust stand and m dot C is nothing but b/b+1*m dot that is the amount of air that goes through the fan so this *vec. So I known B, I known m dot, the total mass flow rate, so I can calculate the fan thrust to be 166 kilo Newtons, So, this takes care of the fan stream completely Now we need to do the calculations for the core engine stream. Let us do the next.

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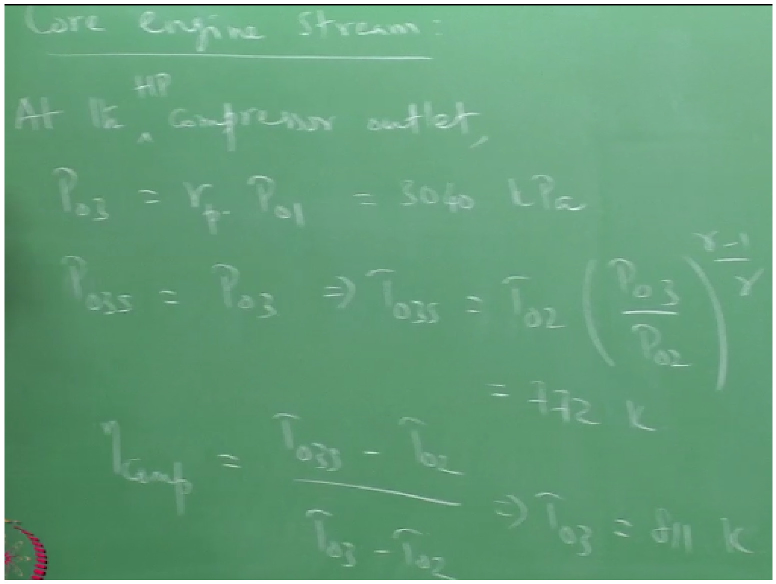

So for the core engine stream, so I start with the compressor outlet. So this is the HP compressor if you remember so this is the hp compressor so P03 = the pressure ratio rp*P01. Remember the pressure ratio itself is defined as P03/P01. So I can calculate this to be 30. P01 is 100 ka Pa so I can calculate this to 3040 kilo Pascal. So once I know P03 I know P03s and I can calculate T03s let us go ahead and do it that way.

So please calculate so P03 as = P03, so which means that T03s is going to be what is this going to be? We use the isentropic relationship to calculate T03s so calculate please calculate and tell me what T03s is? So as shown here To3s is going to To2*Po3s divided by Po2 to the power gamma - 1/gamma. So if you substitute the numbers what you get for T03s is 772 Kelvin.

Now the efficiency of the compressor if you recall, the efficiency of the compressor is given as T03s - T02 divided by T03 - T02. Now I know efficiency of compressor is given T03s is known T02 is known. so I can calculate T03 from this expression so if I go ahead and do that what do I get for T03? The efficiency of the compressor is given to be 0.92. I have 811 Kelvin that is close enough. Let us go to the next one.

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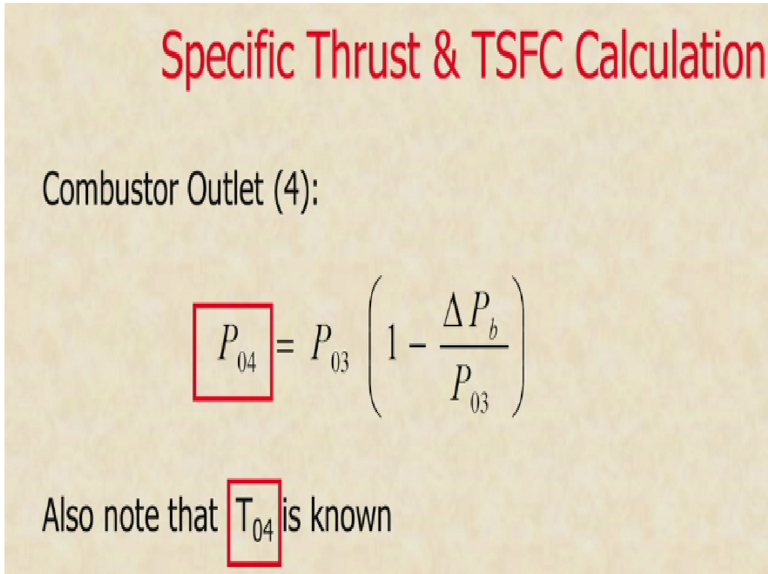

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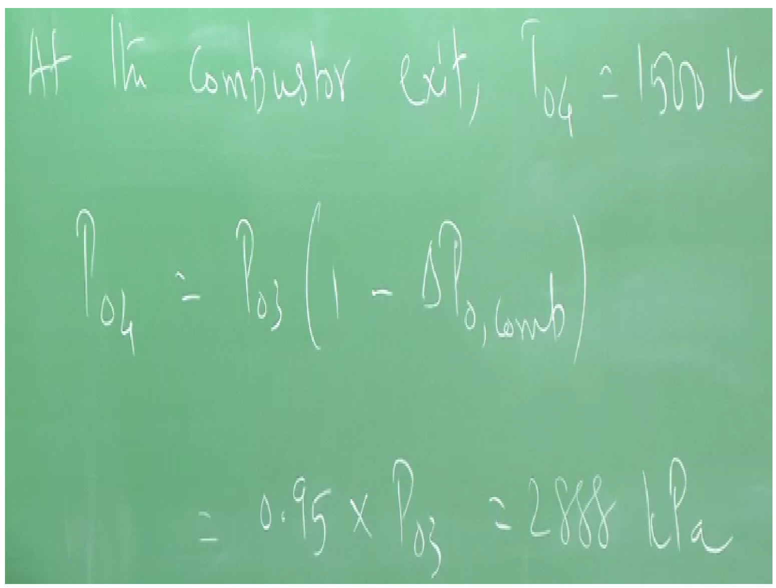

At the combustor exit, T04 the maximum temperature is given so T04 is given to be 1500 Kelvin and P04 can be evaluated like this. P04 is = P03*1 - delta P0 the combustor. So there is a pressure loss in the combustor which is given and this is given to be 5% so that means this is 0.95*P03 which works out to 2888 kilo Pascal. So, now we know P04 and T04. We will go to the next component which is the HP turbine.

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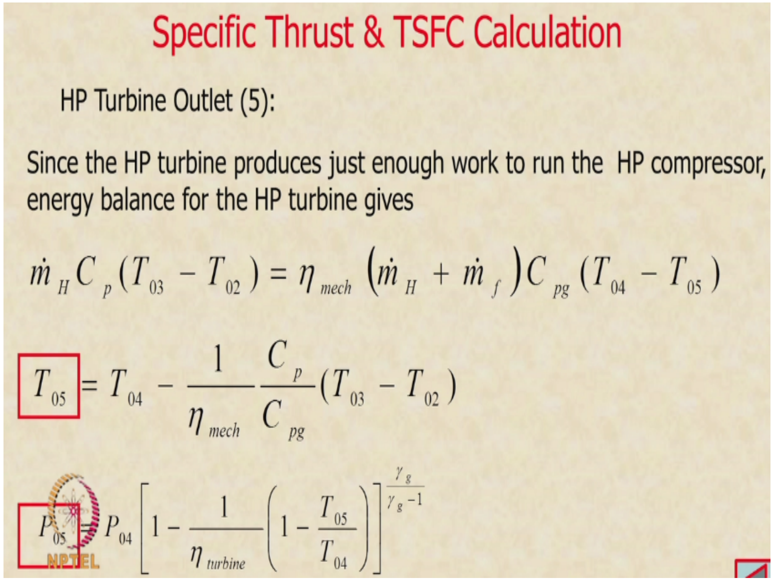

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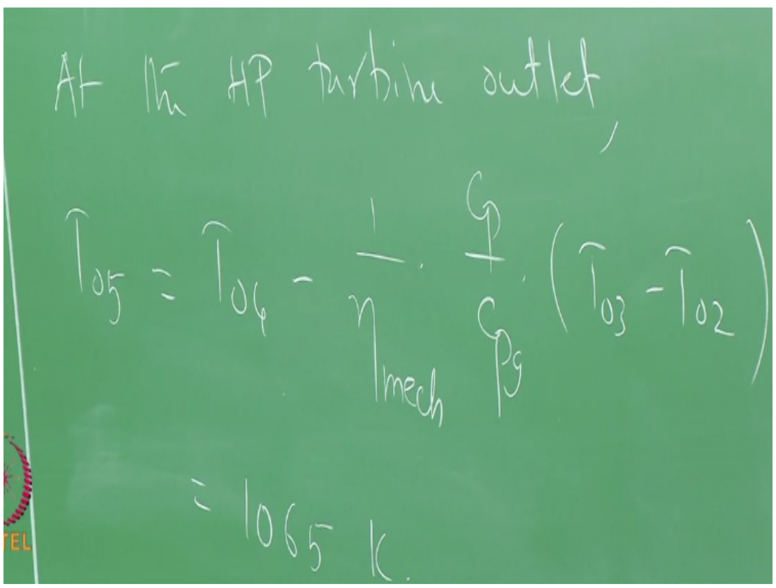

At the HP turbine outlet, we obtain this expression by doing an energy balance for the HP turbine and the HP compressor. So the HP turbine produces just you know for to run the HP compressor and based on the energy balance we obtained that the exit temperature at the exit of the HP turbine. HP turbine = T05 = T04 - 1/eta mechanical*Cp/Cpg*T03 - T02. So I know all the quantities on the right side. If I substitute the values, I get T05 to be 1065 Kelvin. So once I have T05 I can use the definition of the efficiency of the Turbine.

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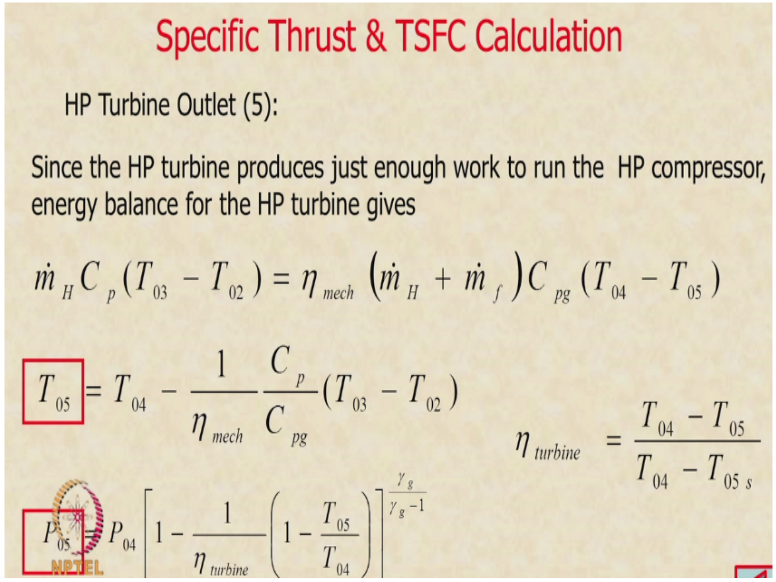

The efficiency of the turbine is defined like this.

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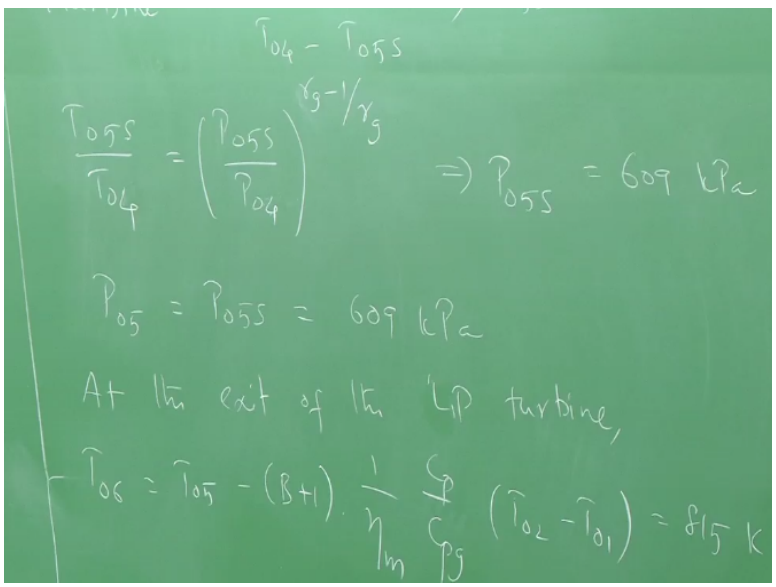

Eta turbine = T04 - T05 divided by T04 - T05s. So I can calculate T05s from this expression. I know eta turbine also. Eta turbine is given to be 0.9 so I can calculate T05s from this can you calculate this and let me what it comes out to be? To5s is 1017 Kelvin. That is T05s. So once I know T05s I can calculate P05s using the isentropic relationship. So if you remember T05s divided by T05 = raise to the power gamma g - 1/gamma g.

So I can evaluate P05s from this equation. I am sorry this is T05s divided by T04, P05s divided by P04. So I can calculate P05s from this equation. Can you calculate and let me what it is? 609

kilo Pascal and you know that P05 = P05s so this is going to be = 609 kilo Pascal. So we have T05 and P05. Both 05 and 05s lie on the same isobar based on our definition of the isentropic efficiency for the turbine. So that is the high pressure turbine. We now go to the low pressure turbine. So at the exit of the low pressure turbine we have the following equation.

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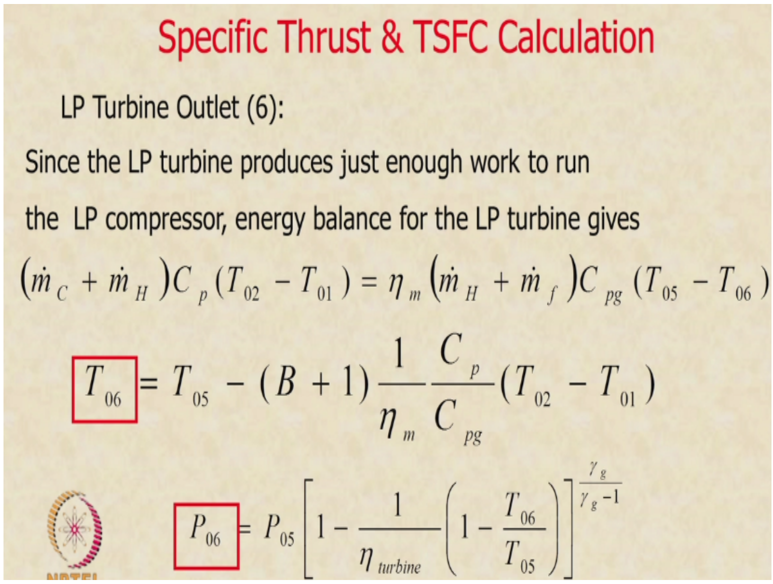

The low pressure turbine produces enough work to run the fan which means that T06 is given as T05 - remember the amount of mass flow rate through the fan is m dot through the entire fan is m dot c+m dot H. So that is why we have the B+1 term here. The fan handles the entire amount of mass flow rate. M dot H+m dot C and if you take that into account then this is what we get for the expression for T06.

And if you substitute the numbers this comes out to be 815 Kelvin. So I have T06 and from the expression for the isentropic efficiency of the turbine.

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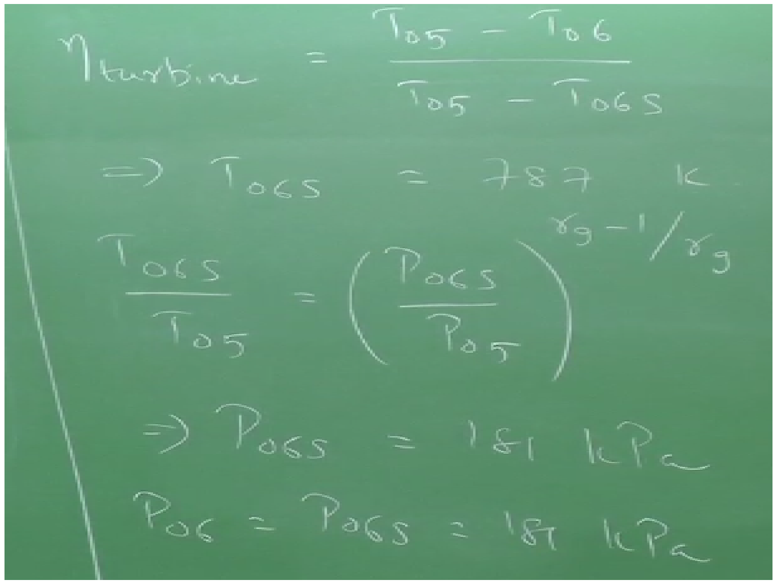

I can write the following T05 - T06 divided by T05 - T06s. So, I known T05, I know T06. Efficiency of the turbine is also given so I can evaluate T06s from this expression. So I get T06s to be 787 Kelvin. So once I know T06s I can use the isentropic relationship between 06s and 05 and write the following. T06s divided by T05 = P06s divided by P05 raise to the power gamma g -1 divided by gamma g.

So from this I can calculate my P06s. Everything is known in this so I can calculate P06s. What do you get for P06s I have 181 kilo Pascal and since P06 = P06s we have now completed this state also. So this is 181 kilo Pascal. So we have T06 and P06. So now we need to move on to the hot nozzle. We have completed the calculation for the other components. Let us do the hot nozzle.

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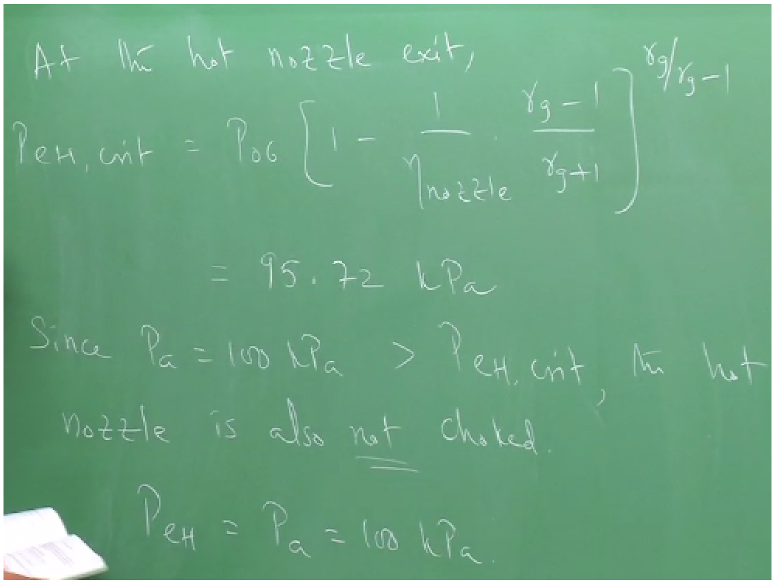

So at the hot nozzle exit since I have P06 I can immediately evaluate the critical pressure. So at the hot nozzle exit the critical pressure PeH critical we derive this expression before this is = P06*1 - 1/eta nozzle*gamma g - 1/gamma g+1 raise to the power gamma g/gamma g -1. So if I substitute the values for this value of P06 the critical pressure comes out to be 95.72 kilo Pascal and the ambient pressure is 100 kilo Pascal.

Remember the engine is on a static thrust and so since the ambient pressure Pa = 100 kilo Pascal > PeH critical for the hot nozzle. The hot nozzle is also the core nozzle was also not choked and for operation at sea level static pressure you can see that the hot nozzle is also not choked therefore this is also not chocked therefore Peg = Pa = 100 kilo Pascal. So what do we do next.

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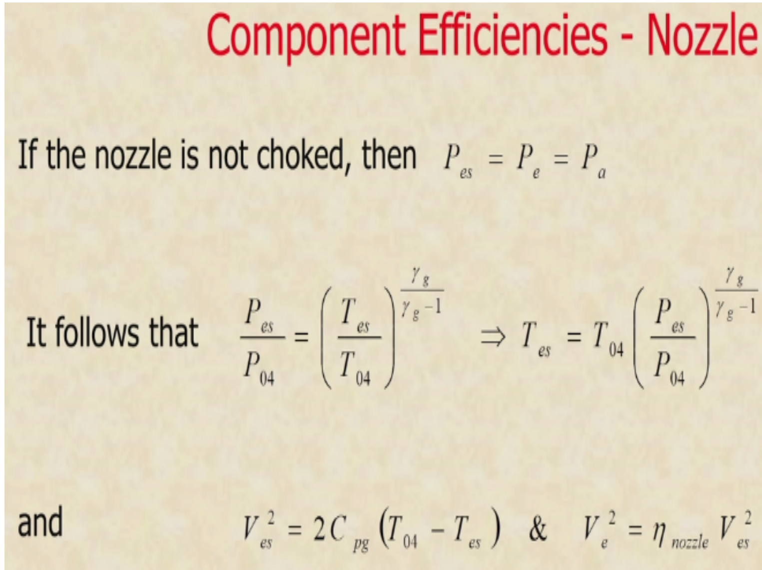

So as you can see from here. The hot nozzle is not choked then Pes - Pe = ambient pressure. So once I have Pes I can calculate Tes from this expression.

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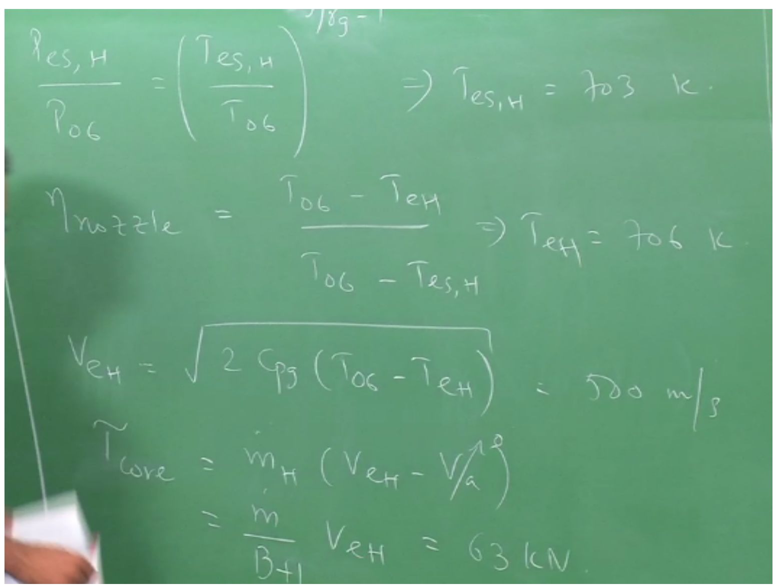

So I can write Pes for the hot nozzle divided by P06 = Tes for the hot nozzle divided by T06 to the power gamma g/gamma - 1. From this I can calculate my Tes for the hot nozzle. So what you get if you do this calculation. gamma g/gamma g -1 and what do you get for Tes, H this is correct is it 703. So, the efficiency of the nozzle if you remember is T06 - Tes divided by T06 – Tes, H so from this I can calculate my Tes.

What do you get for Tes? 706. That is what I also have. So this is 706 Kelvin. So once I have this I can calculate the velocity. I am sorry this is TeH. Please change this. This is TeH so I can calculate my TeH to be 706 Kelvin. So VeH can be calculated as 2*cpg*T06 - TeH and if you substitute numbers for this what do you get for VeH 500 meter per second. So the thrust produced by the core range in.

Remember here also the pressure thrust is 0 because the nozzle is not chocked so Pe = P ambient that means the pressure thrust is 0 so the thrust produced by the core engine is going to be m dot H*VeH - Va and Va = 0 and because that the engine is on a thrust and static thrust and so Va is 0 and m dot H itself is nothing by m dot/B+1*VeH.

So if you substitute the numbers we get the thrust produced by the core engine to be 63 kilo Newton. So the net thrust produced by the engine which is the sum of the thrust produced by the fan and the core engine. So the net thrust produced by the engine which is the sum of the thrust produced by the fan and the core engine comes out to be like this.

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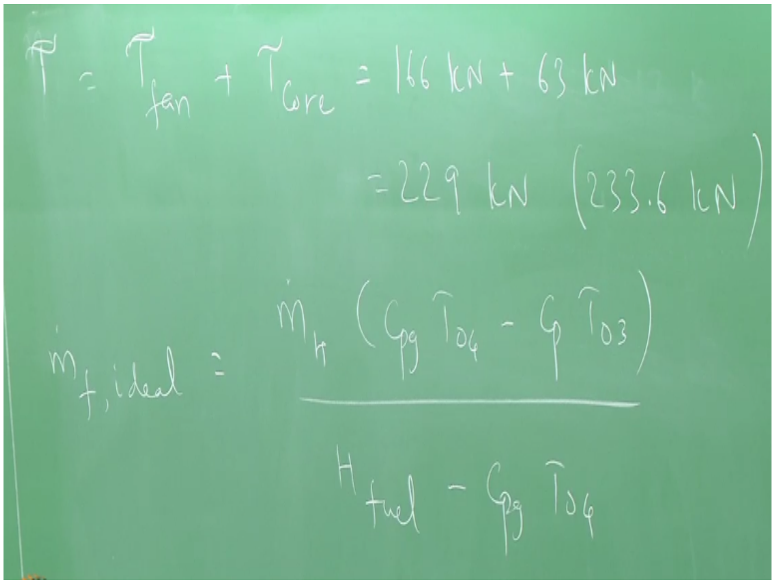

So total thrust T = Thrust produced by the fan + thrust produced by the core engine and this comes out to be 166 kilo Newton+63 kilo Newton which is = 229 kilo Newton which encouragingly compares extremely well with the value reported by the manufacture. So the manufacture quotes static thrust value for this engine to be 233.6 kilo Newton.

So this is from the manufactures website. So you can see that the calculation seems to be alright. You are getting reasonably good numbers and so now we can calculate the mass flow rate of fuel and if you remember the mass flow rate of fuel the expression for that.

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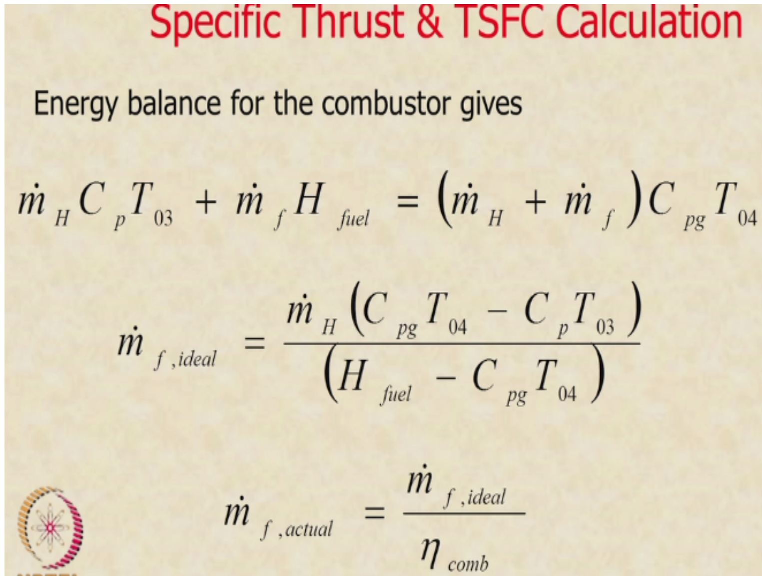

So this is the mass flow rate for fuel so mf dot ideal = mH dot*Cpg T04 - Cp*T03 divided by H fuel - Cpg*T04. So please substitute the numbers and let me what you get for this 45000 kilo joules per kilo gram that is the enthalpy of the fuel. So, we will continue it in the next call we will continue in the next class.