Gas Dynamics and Propulsion
Dr. Babu Viswanathan
Department of Mechanical Engineering
Indian Institute of Technology – Madras

Lecture – 33
Thermodynamic Analysis of the Engine

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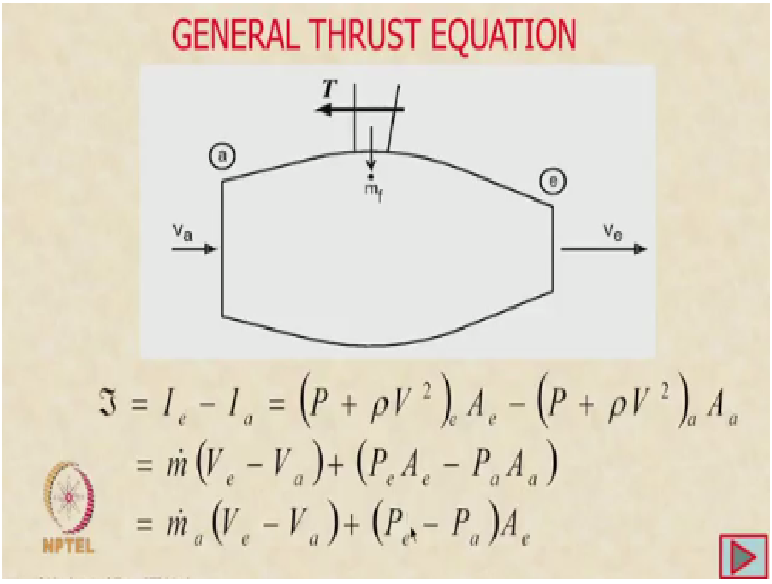

In the previous class, we looked at the general thrust equation and we looked at an aircraft engine like this, where the air enters with the velocity Va and leaves with the velocity Ve and we applied the impulse function at section e and section a and calculated the thrust to be Ie – Ia, where I is the impulse function which is nothing but P + rho u square times a, so if we expand this then, we had the expression for thrust like this.

So, P + rho V square times a is the impulse function, the subscript e denotes that this is being evaluated at the exit section and this is being evaluated at the inlet section and then when we make use of the fact that rho times V times a = the mass flow rate at that section, I can simplify this; rearrange the terms and then simplify this to look like this, so we have a momentum term here or change in velocity here, multiplied by the mass flow rate and changes in pressure.

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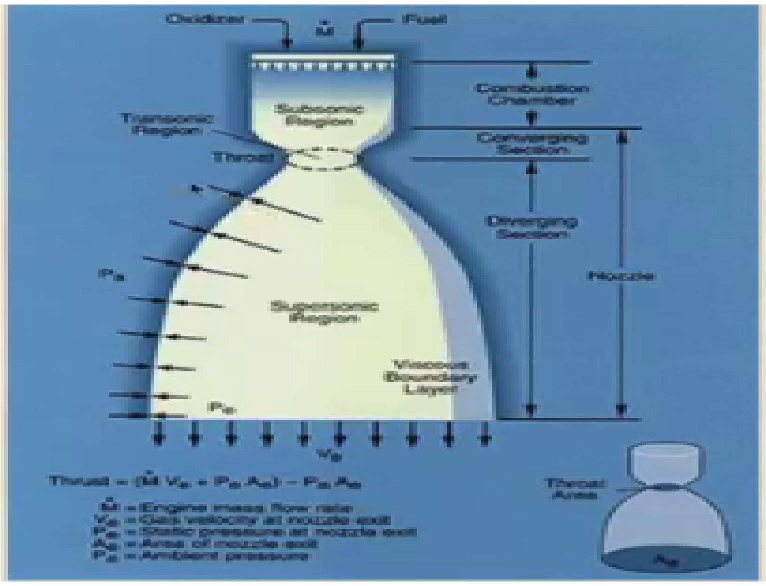

And we argued based on our considerations of forces on pressure force on solid surfaces that the net pressure force on any solid surface, which is what eventually translates into thrust, has to be evaluated after subtracting the ambient pressure from the local pressure that is what we argued. So, keeping this in mind, the pressure term in this equation or the pressure; static pressure in this equation has to be evaluated with the reference to the local ambient pressure, right.

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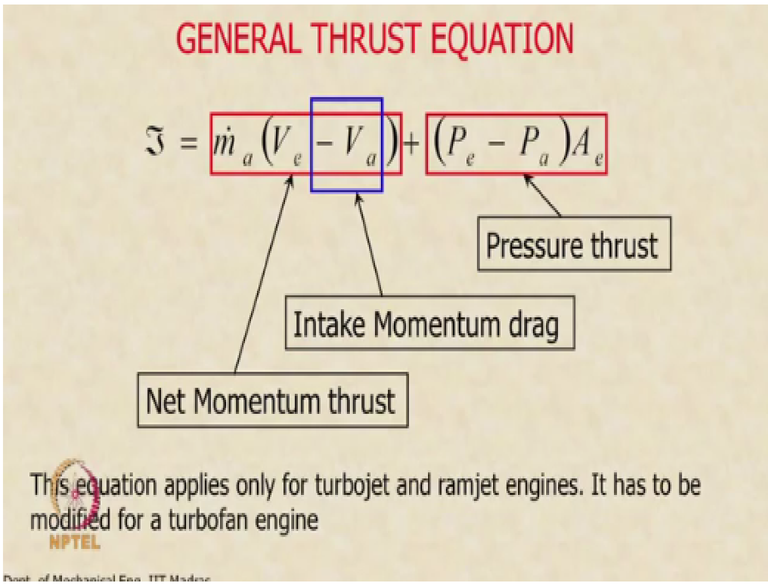

So, local ambient pressure in this case is Pa, so we can then rearrange this or rewrite this equation like this, where this Pe is now, you can see the Pe is the evaluated with respect to Pa, so we have Pe – Pa here and this goes to 0 because Pe – Pa is nothing but 0, so this was where we left over last lecture and so to summarize, this is our thrust equation and let us take a closer look at the thrust equation.

So, you can see that there are basically 2 terms here; the first one involves changes in velocity and the second one looks at the pressure forces and the exit plane, right so and if I look at this term, so the first group of term is usually called the net momentum thrust, so this is the change of momentum of the air flow rate or the air that flows through the engine m dot a is the mass flow rate of air through the engine.

So, this times the change in velocity of the air stream as it goes through gives rise to the net momentum thrust, okay and the second term is the pressure thrust, if the exit pressure is above the ambient pressure, then the difference translates into a force on the exit plane and this is nothing but the pressure thrust. If the flow is correctly expanded at the exit, then Pe becomes = Pa and of course, the pressure thrust becomes 0.

Now, in this case the Ve will also be different, so the situation when Pe is more than Pa will give rise to a certain value of Ve, which is less when compared to the case when Pe = Pa remember, we are talking about a convergent nozzle here; the propulsion nozzle is a convergent nozzle, so which means that Pe cannot be below Pa; Pe can either be = or > Pa, we cannot have over expanded flow in a converging nozzle.

Now, this term itself with the negative sign is usually classified as the intake momentum drag, this is m dot a times Va, okay so that is a negative term which; so this actually acts as a drag term, so this difference is what contributes to the net positive thrust from the engine, this is something that we have to live with, there is nothing that we can do about this, so this drag has to be lived with.

So, under cruise conditions, Va can be quite large, so we must make sure that Ve is even larger than Va to make sure that we get enough thrust from the engine, okay. Now, this equation applies only to a single stream and we have looked at single stream and we evaluated Ie and Ia for a single stream, so which means it is applicable only for a turbojet and a ramjet engine, both of which have only single stream.

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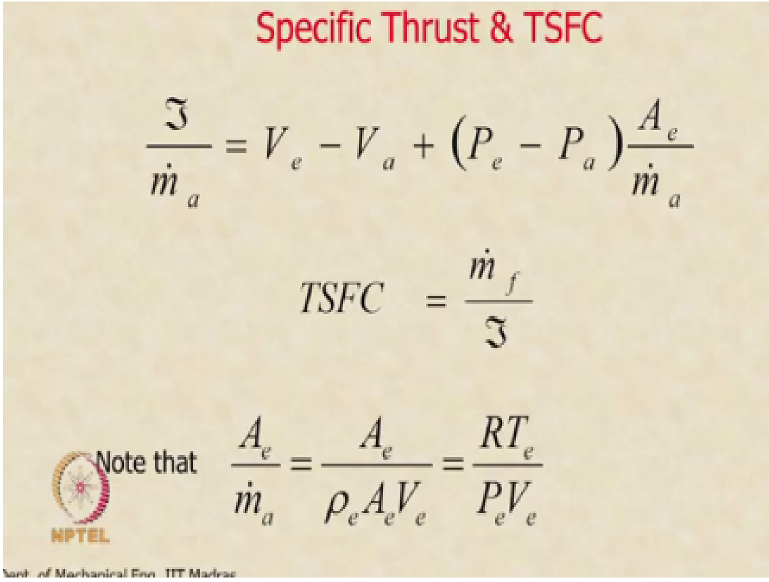

Now, a turbofan engine has 2 streams, as we already saw; one is the fan stream or the cold stream, another one is the core engine stream or the hot stream. So, we have to modify the expression slightly for a turbofan engine. Now, let us take a look at this; if you want to evaluate specific thrust, right, specific thrust is thrust produced per unit mass flow rate of air that flows through the engine, okay.

Now, why would we want to calculate specific thrust, okay? Specific thrust actually is very useful as a comparison metric, let us say that you have 2 engines; one engine takes in let us say, 500 kg per second of air and produces a certain amount of thrust, another engine takes in, let us say 600 kg per second of air and produces slightly more thrust, okay. Now, how do we say which one is better, we need a figure of merit.

Is this increase in thrust entirely due to the increase in mass flow rate or is the second engine really better than the first engine and so we need a figure of merit that will allow us to make this kind of comparison, so the correct figure of merit in these cases is this specific thrust. So, how much thrust does the engine produce for every kilogram per second of air that it takes in, so you give both engines 1kg per second of air.

If the first engine produces more thrust then, obviously that is better, right. I can make the engine smaller for a given amount of thrust if this is superior then the engine size will become smaller, right so, the specific thrust allows us to make these kinds of comparisons, an engine may be producing more thrust simply because it is larger but that is of no use to us, remember the most important metric is thrust per unit weight of engine.

So, the larger the engine is the heavier it is going to be, so we want more thrust, at the same time we do not want the weight to increase too much, so the correct metric in these cases is the specific thrust, which is what is given here thrust per unit mass flow rate of air that flows through the engine. So, if a given engine produces 1 kilo Newton of thrust using a certain amount of air, another engine requires more air for producing the same thrust that means the first one is preferable.

Because it is going to be smaller since m dot a is smaller; the cross sectional area will go down and it is going to be smaller, so it is to be preferred, so that is the reason why we calculate thrust; a specific thrust for these kinds of problems, thrust; absolute thrust is important but specific thrust allows us to compare engines depending upon their performance. Now, thrust specific fuel consumption is the amount of fuel or mass flow rate of fuel through the engine divided by the thrust that the engine produces, okay.

The mass flow rate of fuel through the engine can be calculated based upon the increase in stagnation temperature that we desire. Remember, we said that the turbine entry temperature has to have a certain value, so based on that, we can actually calculate the mass flow rate of fuel and the mass flow rate of fuel per unit thrust is what we are interested in. This is also very important as a performance metric, so we will calculate both these things.

The value of m dot f itself is of interest because m dot f itself, you know as we said earlier will determine the size of the fuel tank that is required, right. For example, if a certain engine consumes certain amount of fuel per hour, let us say so many kilograms per hour, right. Let us say, 2 kilograms per hour, then if I look at the amount of fuel that I have to carry, let us say you know, we are talking about a flight from London Heathrow to Chennai that is about 11-hour flight.

So, if the engine; each engine is going to burn 2 kilograms of fuel per hour, right that means it is-- we are assuming the specific gravity of fuel to be approximately 1000, same as water; 1000 kilogram per meter cube. In reality, it will be slightly less but 1000 is a good number for us to work with, so that means we are going to consume; each engine is going to consume 2000 litres of fuel per hour.

So, for a 10-hour flight that would work out to how much; 20, 000 litres of fuel and if it is 4 engine aircraft that is going to work out to be something like 80 to 100,000 litres, in reality, it will be more than that, okay. So, we are talking about tanks, which must be capable of holding about 100 to 150,000 or more litres of fuel, so the absolute value of fuel flow rate is important because that determines the duration of the flight, right.

If you want the flight to last so many hours, you need to carry so much of fuel, right, so this value is important. The thrust specific fuel consumption is also important because once again, it allows us to compare different engines for example, we just know we said that, if one engine takes in 1kg per second of air produces a certain amount of thrust, another engine takes in 1kg per second of air produces let us say, the same amount of thrust.

Then, we cannot immediately conclude that both are the same, okay, the specific thrust is same. What we are going to do next is, how much fuel does the first engine consume to produce the same amount of thrust and how much fuel does the second engine required to produce the same amount of thrust, so the fuel efficiency is also important, so thrust specific fuel consumption is also a good performance metric that we will use, okay.

So, both these are important performance metrics and we will calculate both these quantities. Notice that this expression has exit velocity, which is something that we have to calculate, exit static pressure also, the area usually is not known but the advantage is this quantity Ae/ m dot a can be expressed like this; Ae/ rho times A times V at the exit, right. The m dot is Rho times A times V and if I write rho as P/ RT.

Then, you can see that this expression Ae/m dot a involves can be written, I am sorry; can be written entirely in terms of exit static quantities, velocity static pressure and static temperature. So, if I know Ve and Pe, I can also evaluate this quantity, so what we are saying is any thrust calculation must be able to calculate the starting from the intake or free stream, we must be able to calculate the exit static pressure or the nozzle exit and the velocity at the nozzle exit.

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General Thrust Equation – Turbofan Engine

$$\Im = \dot{m}_H\left(V_{e,H} - V_a\right) + A_H\left(P_{e,H} - P_a\right) + \dot{m}_C\left(V_{e,C} - V_a\right) + A_C\left(P_{e,C} - P_a\right)$$

$$\dot{m} = \dot{m}_H + \dot{m}_C \qquad B = \frac{\dot{m}_C}{\dot{m}_H}$$

Once we have that, other quantities can be calculated that is what the calculation procedure will involve, okay. Now, as I said earlier, the previous expression is applicable only for a single stream engine, now the turbofan engine as you know is a 2 stream engine; there is a hot stream indicated with the subscript H and there is a cold stream indicated with the subscript C. So, the total thrust; the net thrust is the sum of these 2 terms.

So, the term here or the quantity here is the net thrust from the hot stream, okay and the term here is the net thrust from the cold stream, right. So, this is the thrust from the cold stream, this is a thrust from the hot stream and remember, the ratio m dot C/ m dot H is the bypass ratio okay, remember the bypass ratio is defined as the amount of air for every kg per second of air that goes through the core engine, how much air goes through the bypass stream, okay.

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Specific Thrust & TSFC

$$\frac{\Im}{\dot{m}} = \frac{1}{B+1}\left(V_{e,H} - V_a\right) + \frac{B}{B+1}\left(V_{e,C} - V_a\right) + \frac{A_{e,H}}{\dot{m}}\left(P_{e,H} - P_a\right) + \frac{A_{e,C}}{\dot{m}}\left(P_{e,C} - P_a\right)$$

$$TSFC = \frac{\dot{m}_f}{\Im}$$

So, that means m dot C divided by m dot H, the total mass flow rate through the engine itself is the sum of these 2; m dot H + m dot C, right and the specific thrust can be evaluated by dividing this thrust by m dot and that is the net mass flow rate through the engine, so thrust; this divided by m dot gives me the specific thrust, as you can see from here and I have written everything in terms of the bypass ratio, okay.

So, the specific thrust for the turbofan engine can be calculated using this expression and we can also calculate the specific thrust for the turbojet and ram jet engines also with the simplified expression, in fact, if you said be B = 0, in this, you recover the expression for turbojet and the thrust specific fuel consumption for the turbofan engine again is nothing but fuel flow rate, mass flow rate of fuel divided by the net thrust that the engine develops, okay, all right.

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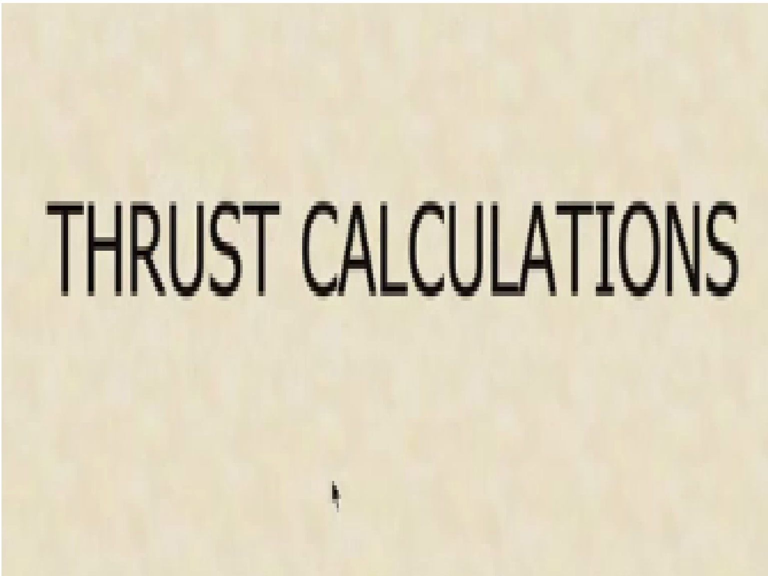

So, in this case also you can see that I need to evaluate Ve, at the exit and Pe at the exit; other things can be calculated, right. It is not a problem, all right. So, that is what we are going to look at next, how do we do thrust calculations for realistic engines using the concepts that we have described, so far. We have the general thrust equation and the general thrust equation says that I need to evaluate Ve and Pe to calculate the thrust.

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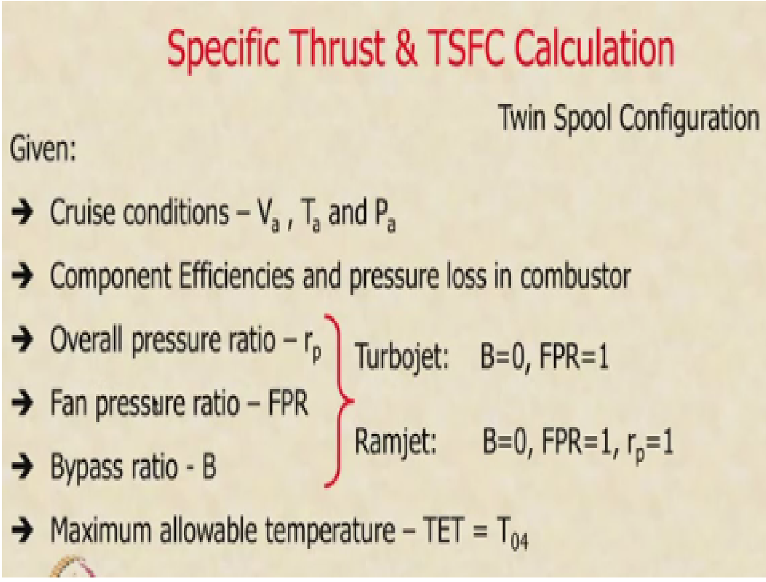

And I also have written down expressions for the efficiency of each component; the isentropic efficiency for each component, so that allows me to go from one component to another and evaluate one state property to another state property, so I can just walk through the cycle like that okay, all right. So, let us see what we are going to do, we are going to do thrust calculation for a twin spool engine, we are going to assume 2 spools.

There is no loss of generality, if you add one more spool, the calculation can easily handle that also, so there is no loss of generality in assuming a twin spool configuration, it is easier. So, these are the conditions that are given to us; these are the operating conditions that are given to us. The cruise conditions are given either cruise Mach number or cruise velocity and static temperature and static pressure.

So, once you are given the altitude and the cruise Mach number, these quantities can be evaluated; the static pressure and static temperature and velocity can be evaluated. We also assume that component efficiencies for the different components like inlet, fan, compressor, combustor, turbine, nozzle they are all available that will be given to us and pressure loss in the combustor is also usually given.

If for example, a particular value is not given, then we assume the behaviour of that particular component to be same as ideal behaviour, if component efficiency is not given, we assume it to be 100%. If pressure loss in the compressor is not given, we assume that there is no pressure loss in the combustor; we assume ideal behaviour in the absence of any other information like okay.

So, the overall pressure ratio of the engine denoted by rp is also given, this may be given as 30 or 40 or whatever depending upon the particular rating for the engine, fan pressure ratio is also given okay and bypass ratio B is also given and maximum allowable temperature, which is nothing but the turbine entry temperature T04 is also given, so these are quantities which will be given, okay.

Notice that the calculation procedure although, we are doing it for a turbofan engine it is actually a very general procedure because if I set the bypass ratio to 0, I recover the turbojet equation, if I said B = 0 and the fan pressure ratio to be 1 that corresponds to a turbojet engine okay, if I said B = 0, fan pressure ratio = 1, in addition if I also said the compressor pressure ratio to be 1, then I recover a ramjet because ramjet has no compressor, no turbine.

So, if I set rp = 1, then I recover the ramjet, so the calculation procedure that we are going to develop is a very, very general procedure, which can be used for all these engines; turbojet turbofan, ramjet and also an afterburning turbojet, okay, it is very general, so we will go ahead with this. So, these are operating conditions for the engine that will be given to us; operating conditions and other parameters like the efficiencies.

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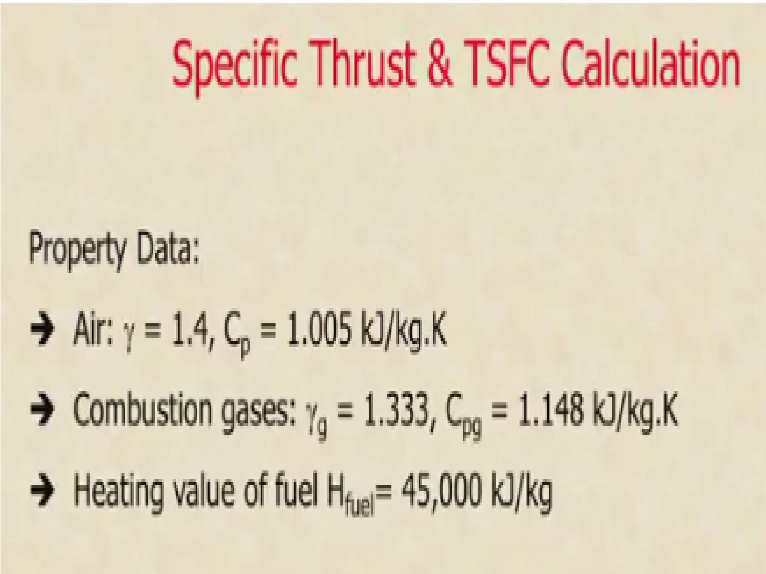

In addition, property data is required okay, for air, we will take gamma to be 1.4 and Cp to be 1.005 kilo Joule per kilogram Kelvin and for the combustion gases, so we will assume that the composition of the working fluid is that of a combustion gas downstream of the combustor, for all components downstream of the combustor, we will assume gamma to be 1.333 and Cp to be this.

Notice that, the Cp value here is higher than the Cp value for the cold air, Cp increases with temperature, so we kind of try to take that into account here okay, but still we assume the air here to be calorically perfect and we assume the combustion gases to be calorically perfect, we just have 2 different gases in 2 different parts of the engine with different properties that is okay and we will also assume reasonably that the calorific value of the fuel is 45,000 kilo Joule per kilogram, this is true for any typical hydrocarbon fuel, okay.

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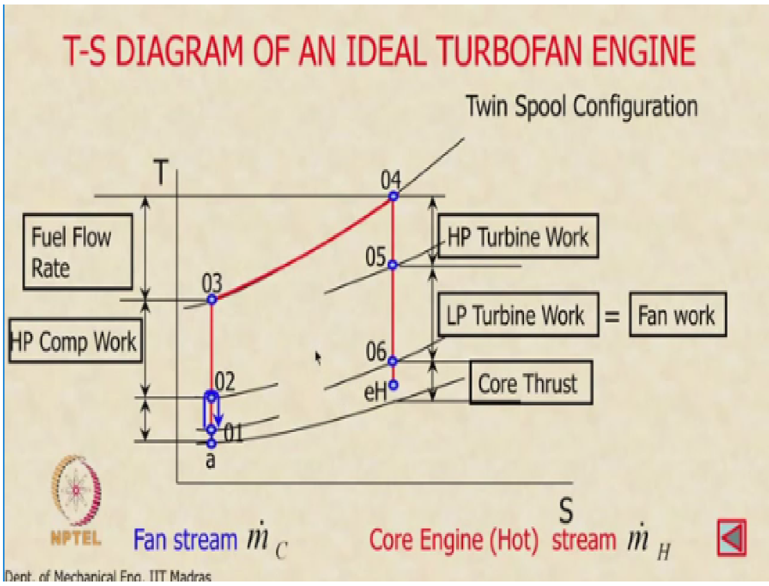

So, these are reasonable things, so we will assume this throughout okay and we will do the thrust calculation based on this assumption and let us just quickly revisit the twins spool cycle just to give you an idea. Remember, the free stream state is denoted by a, 01 is the state at the end of the intake, 02 is the state at the end of the fan, 03 is the state at the end of the compressor, 04 is the state at the end of the combustor.

05 at the end of the high pressure turbine, I am sorry high pressure turbine and 06 is at the end of the low pressure turbine, eH is at the end of the hot nozzle, in addition right; in addition, there is a fan stream, so the fan stream goes from 01 to 02 and then from 02, it goes to the fan nozzle exit state at the exit of the fan nozzle is denoted as eC, so the subscript H denotes hot, subscript C will denote cold, okay.

So, what we will do is starting from the free stream, we will walk through this process diagram and we will calculate at each state point, we will calculate the stagnation temperature and stagnation pressure, so starting from static state a, we calculate P01and T01, P02, T02, P03, T03 and so on until we come here and then at the exit here, we need to calculate VeH and PeH. Similarly, for the fan nozzle, we need to calculate VeC and PeC that is the objective of this exercise, okay.

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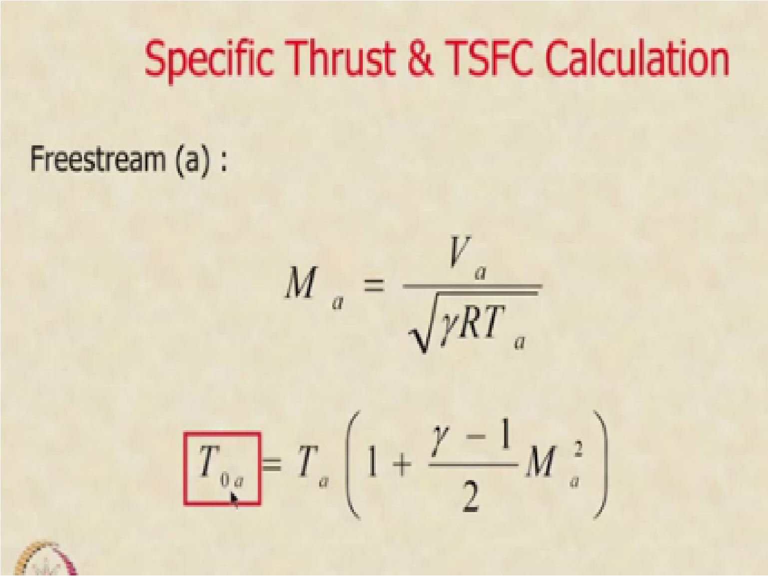

And also mass flow rate of fuel, right, those are the things that we are going to calculate, so let us go back and continue okay, so free stream state, let us assume that the velocity is given, static temperature and static pressure are given that was what we wrote down in the previous slide, so Va, Pa and Ta are known, so from Va and Ta, I can calculate the free stream Mach number, okay.

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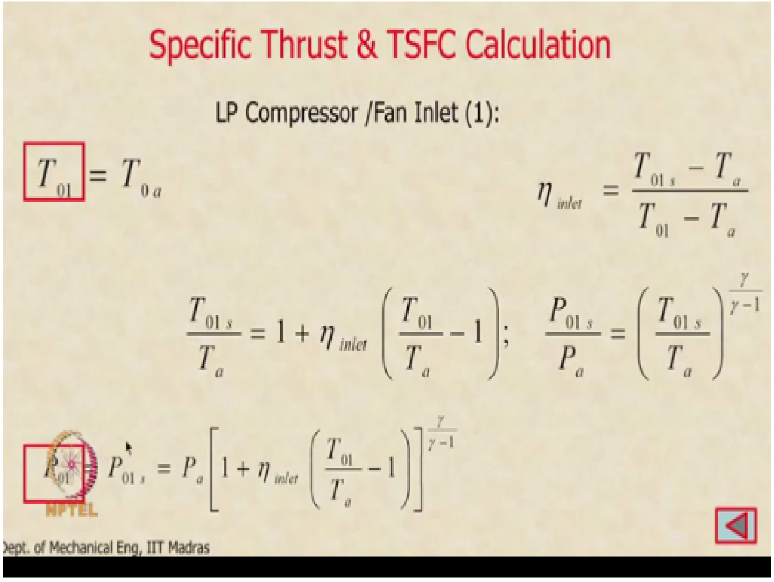

I substitute that into this expression here, I can evaluate the stagnation temperature of the free stream T0a, right. Next, we go to 1, remember this is the exit of the inlet, right, so there is no energy addition; energy addition or removal, there is no work exchange in the intake, so stagnation temperature remains constant, so T01 = T01, okay. Remember, what do we have to calculate for this state; T01 and P01.

So, I know T01 already, as I have indicated with this box, now we need to calculate P01, we calculate P01 by using the definition of the isentropic efficiency of the inlet, right. If you remember this was how the efficiency of the isentropic efficiency the inlet was defined, correct so we assume that eta inlet is known that is given to us. Now, I know T01, I know Ta in this expression, I do not know T01s, correct.

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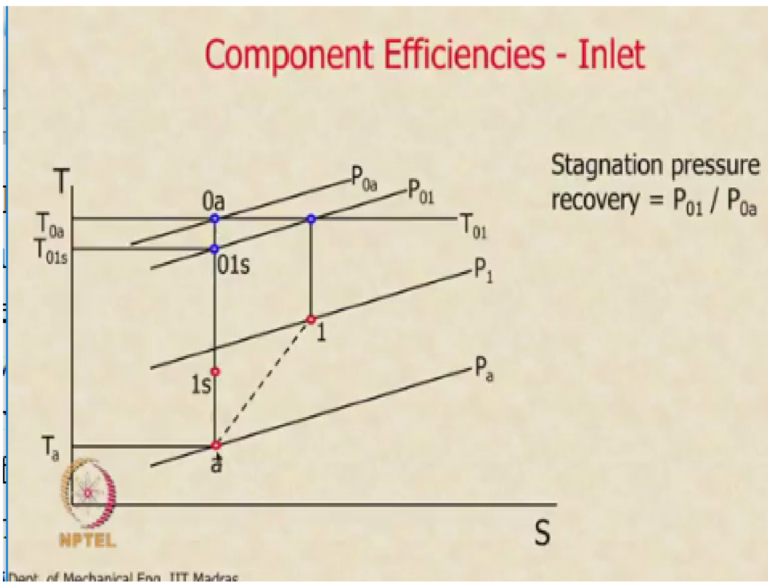

I can evaluate T01s from this expression because I know all the other quantities, right. Once, I know T01s right, so I can calculate T01s this way from this expression, correct; T01s/ Ta is nothing but 1 + eta inlet times T01 / Ta – 1, once I know this, notice that I can use the fact that 01s and a lie along the same isentrope, right. If you recall; right, 01s, right and a, lie on the same isentrope, right.

So, P01s/ Pa, remember P01s itself is = P01, so if I evaluate P01s, then I know P01 that is what we are going to do, I know T01s, I know Pa, I know Ta, so I can evaluate P01s using the fact that they lie on the same isentrope, right so that is what I am going to do here. So, P01s/Pa = T01s/Ta to the power gamma or gamma – 1, correct. So, now I know P01s also, so P01 = P01s, which itself is = this from this expression, correct, is that clear.

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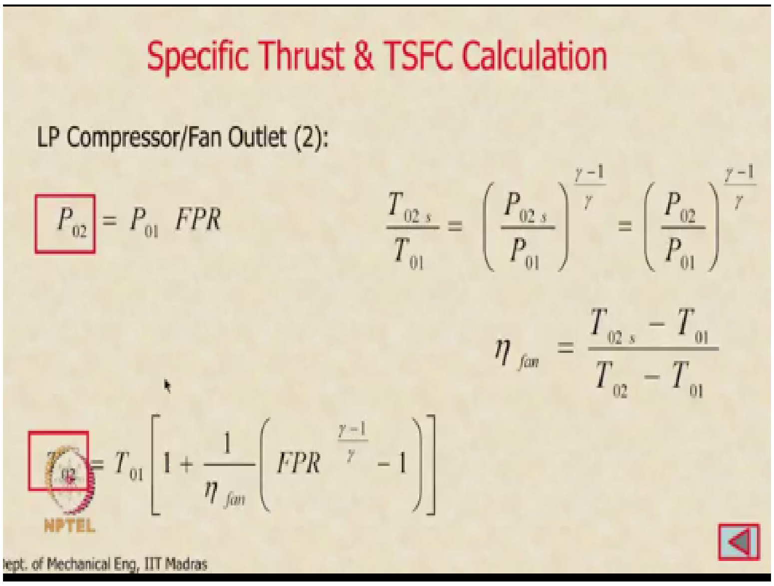

So, now I have T01 and P01, right, are there any questions? Okay, alright, so we go to the next state, which is state2, this is at the exit of a fan, right. Now, in this case P02/P01 is given to us because the fan pressure ratio itself is given to us, right, so P02 is nothing but P01 times the fan pressure ratio because there is a definition of the fan pressure ratio, I know fan pressure ratio so I can calculate P02.

So, I have put it inside a red box, right, so we know that. What do we need to evaluate next? T02, okay, in the case of the inlet, T0 was known, we calculated P0 using the isentropic efficiency and the isentropic relation, now P0 is known, so we are going to do the exact opposite, right, the isentropic relationship and the efficiency definition to calculate T0 okay. Let us see how we do that, right.

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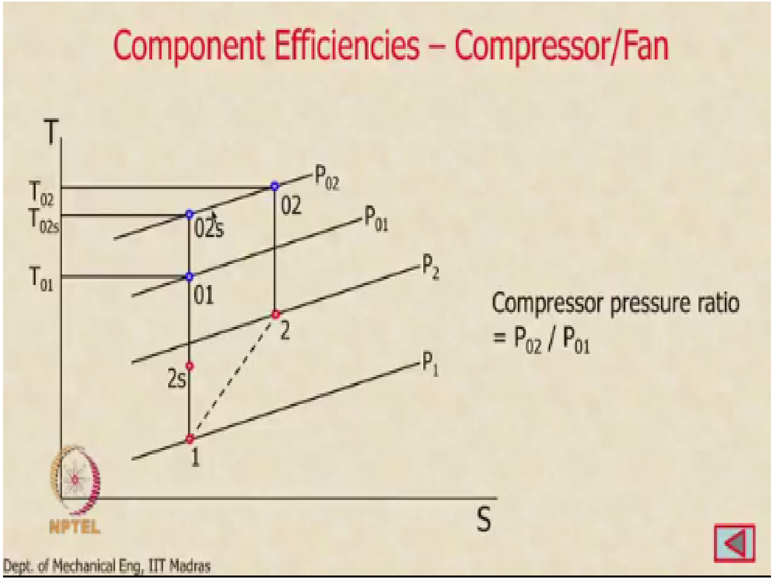

So, the isentropic relation tells me this, T02s/T01= P02s/P01 to the power gamma – 1/ gamma right, remember, so you can see that T02s and if I raise this T02 or this lie on the same isentrope, right, I am going to say T02s/T01, so you can see that these states lie on the same isentrope; T02s/ T01; T02s is also = P02, correct; this lie on the same isobar, right. So, I can go back to that expression to that this thing.

So, you can see that because they lie on the same isentrope, I can write this expression and P02s = P02, so I can write this. Do you follow that logic? Is that clear, I am able to write the first equality because 02s and 01 lie on the same isentrope and I am able to write P02s = P02 because these 2 states lie on the same isobar, so in this expression P02/P01 is known that is equal to the fan pressure ratio, T01 is known, so I can evaluate T02s from this, right.

I can evaluate T02s, notice that the isentropic efficiency of the fan is defined this way, so once I know T02s from this expression, the eta fan is given to us, T01 is known, T02s is known, so I can evaluate T02 from this expression, okay. So, we are now going the opposite way, given the stagnation pressure, we use the isentropic relationship to calculate the other quantity, then we use the isentropic efficiency expression to calculate the stagnation temperature, right.

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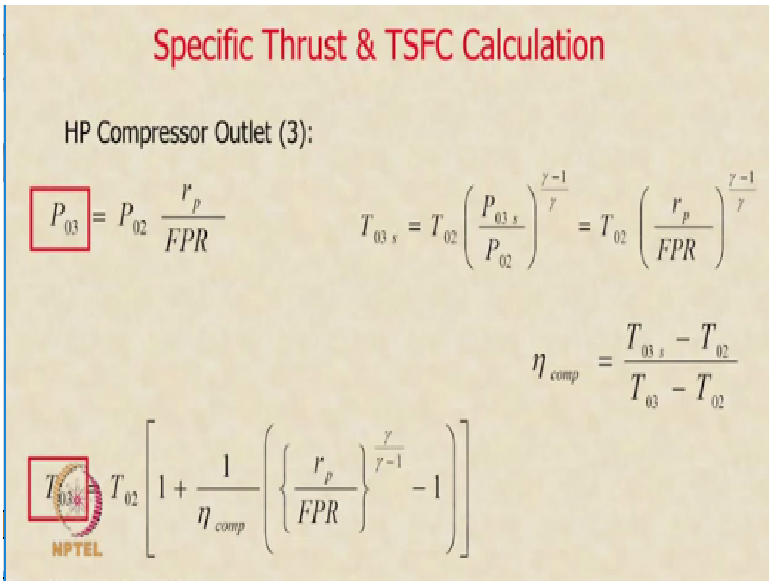

So, that stagnation temperature T02 = T01 and we make use of the T02s value to evaluate it, okay. So, now I have P02 and T02, okay. So, if this is given, then we go this way to calculate that, as we did in the previous case. If this is given, then we go through this way to calculate this, okay. Next stage HP compressor outlet, remember the overall pressure ratio of the engine is given.

So, the overall pressure ratio is nothing but P03/P01, okay, so P03 itself will then be = P03/P02 times P02/P01 times P01, right, so I get P03 to be = this, okay, so this is straightforward. So, now I have this stagnation pressure, I need to calculate the stagnation temperature, so I do this the same way. So, I use this fact and I use the isentropic relationship T03s = T02 times P03s divided by P02 raised to the power gamma -1/ gamma, right and P03s itself is = P03, right.

So, I have substituted for that here P03/P02 from here is = rp / fan pressure ratio FPR right, so P03s = P03 and P03/P02 from this expression is = rp / fan pressure ratio that is what I have done here, so I have T03s compressor efficiency is written like this, I know T03s, I know T02, so T03 can be evaluated because eat compressor is also known to us, right. So, you can see that T03 = T02 + 1/ efficiency of compressor times this quantity.

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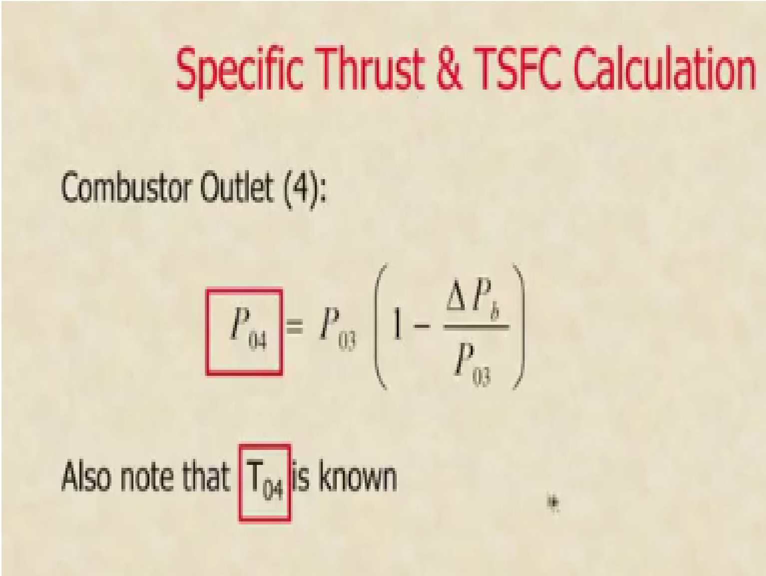

So, now I have indicated this also in the red box, so that now we are done with state 3; we know P03 and T03, okay because the next component is the combustor, now again, we need P04 and T04. T04 is given, that is the turbine entry temperature that is given to us, so there is nothing that we need to do for this, P04 is calculated, if the pressure loss in the combustor is given; normally, that is given as a percentage.

We may say there is a 10 % loss of stagnation pressure in the combustor, so then I can convert it into an expression like this, right, so this is the loss of stagnation pressure right, so this ratio will be = 0.1, so the stagnation pressure at the exit of the combustor is 0.9 times the stagnation pressure at the entry to the combustor. So, now I know T04 and P04, so combustor is relatively straight forward.

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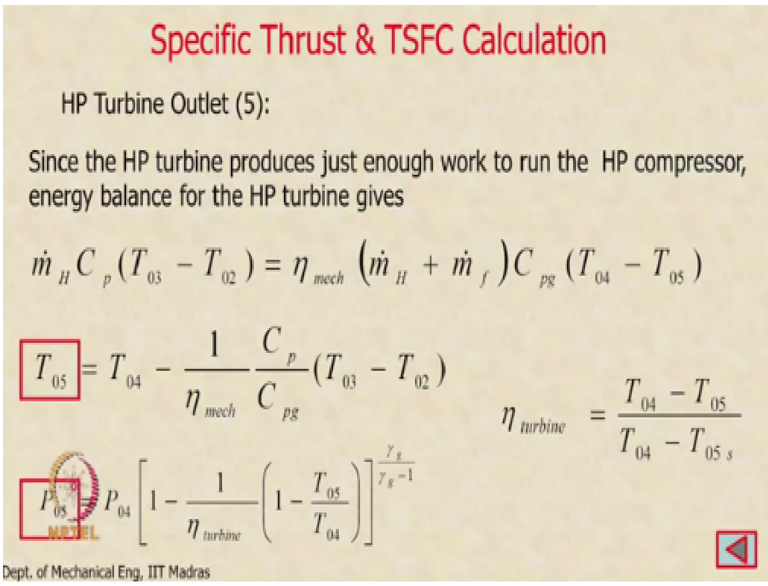

In fact, later on knowing the change in stagnation temperature across the combustor, I know T04, I know T03, I can calculate the mass flow rate of fuel based on this change in stagnation temperature that also we will do later on. The next component is the high pressure turbine outlet right that is state 5, so we need T05 and P05. Now, things get a little bit different remember, the HP turbine produces just enough work to run the HP compressor right, as indicated here.

The HP turbine produces just enough work to run the HP compressor, so if you do an energy balance for the turbine right, so the energy balance for the turbine reads like this, m dot H + m dot f that is the net amount of mass that is going through the high pressure turbine, correct. The change in stagnation temperature across high pressure turbine is T04 – T05, okay. Normally, there could be a loss of power due to friction.

You know, we take the high pressure turbine we are connecting it to the compressor, so this may produce 1 kilo watt of power but we may lose some amount of this in the transmission friction and so on, so this may get only 0.9 but I know that this wants so much power, so if you take the friction into account, how much power should the turbine produce, that is what this expression says, right.

So, the power required by the HP compressor is given by this expression m dot H times Cp times this to produce this much power, I need to produce this much work from the turbine, okay. Strictly speaking, the m dot f must be accounted for here but in general, the m dot f is usually much much smaller than m dot H, so we can safely whenever, we are adding these 2 types of terms, we can safely ignore this okay.

So, we can safely ignore this mH dot + mf dot is almost the same as mH dot, so we can neglect this, cancel the mH dot on both sides and we get the T05, using this expression, okay. What is that this component is unlike the compressor that we looked at okay, in the compressor, the pressure ratios were given; the fan pressure ratio was given, which means that I can calculate the exit stagnation state using the given fan pressure ratio.

For the turbine, which is going to run the compressor, I have to calculate the exit stagnation state by using the fact that it needs to produce so much work okay, the pressure ratio across the turbine is not given. However, I know that it needs to produce so much work to run the compressor, the pressure ratio across the compressor is given right, so I used that to calculate this stagnation state at the exit of the compressor.

For the turbine, I use the fact that the work produced is known to calculate the exit state of the turbine that is what I have done here, so as you can see from here. Notice that again, the gas that goes through the turbine has different properties when compared to the air that goes through the compressor, so this is Cpg and this is Cp, okay. So, now I know T05, how do I calculate P05? By using the same trick as before, right.

I know T0, I calculate P0 by using the isentropic relationship and then the definition of the efficiency or efficiency and then isentropic relationship, right. So, I use the efficiency definition and then the isentropic relationship to calculate the value for P05. So, what I do is, with the T05, this is the efficiency expression for the turbine, right. So, in this expression I know eta turbine, I know T04, I know T05, so I can calculate T05s.

Once I know T05s, I can calculate P05s; correct using the isentropic relationship, once I know P05s that is = P05 because they are on the same isobar that is what we are doing here, all right. So, the calculations can be very involved, so you have to be very methodical in doing these calculations. In the next class, we will try to work with numbers to demonstrate this calculation procedure; it is going to be quite involved, all right.

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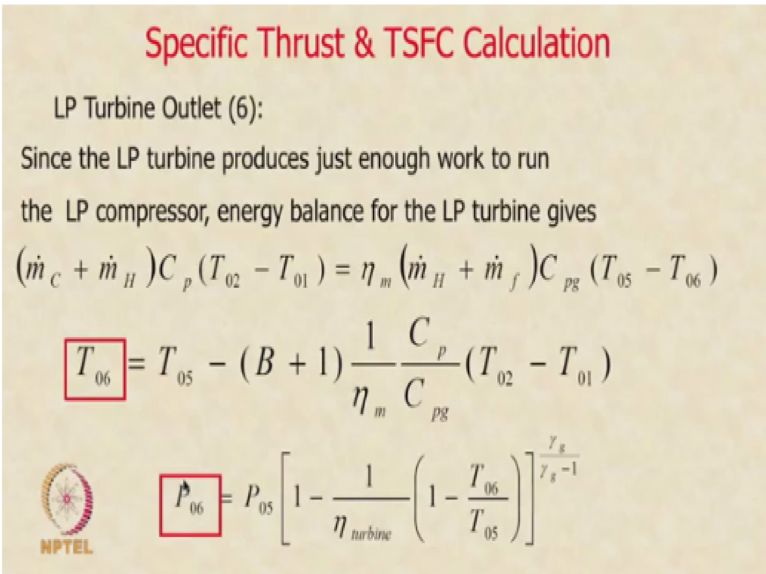

Now, same procedure with the LP turbine outlet with a slight difference, now the LP turbine produces just enough work to run the LP compressor, right. Remember, this is 2 spool engine, so LP compressor means, fan plus whatever stages we have in the LP compressor but notice now, that the mass flow rate through the compressor is not m dot H but it is m dot C + m dot H, right, that is the first stage.

So, the entire mass flow rate goes through the fan right, so the amount of work required to run the LP compressor or the fan is going to be the total mass flow rate times Cp times change in stagnation temperature okay, by LP compressor what we mean here is the fan. Once again, the amount of air that goes through the LP turbine however, is only m dot H + m dot f, okay with m dot f being small, I can neglect this in comparison to this and I can simplify this expression okay.

But this is a very important difference from the previous case, the HP turbine runs the HP compressor but the HP compressor is part of the core engine, which means only m dot H kg per second of air goes through the HP turbine or the HP compressor, right. The LP compressor or the fan is located outside the core engine, which means that total mass flow rate is going to go through that, so I can evaluate T06 from this after making use of the fact that m dot f is much smaller than m dot H.

So, I have been able to write this in terms of the bypass ratio, so T06, the stagnation temperature at the exit of the LP turbine can be calculated. Once I know this, I can use the definition of the isentropic efficiency of the turbine and the isentropic relation itself to calculate the stagnation pressure at the exit of the LP turbine, okay. So, let us see how that is done, so that is what we have done here. What is that?

We use the definition of the isentropic efficiency of the turbine to calculate this quantity and then we use the isentropic relation to calculate P06s, once we have P06s, then I can calculate P06 because they are both = each other that is what I have done to calculate this, we have put both these inside the red box, so now we are done with state 6 also. So, what follows from here is the nozzle.

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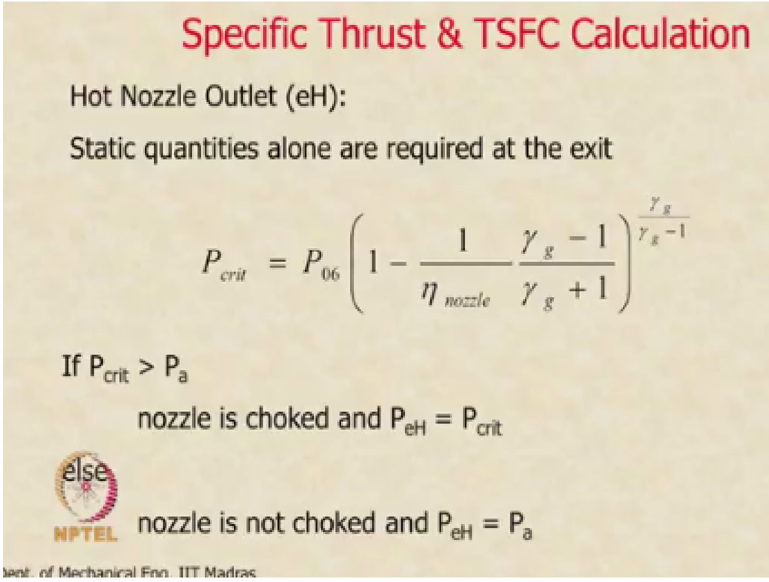

And remember, the nozzle; analysis for the nozzle has to be done carefully because there are 2 possible states; one the flow is choked, another one the flow is not choked okay, so that is what we need to take into account when we do the calculation for the nozzle, okay. Let us see how this is done, are there any questions so far? So, let us see how the calculation for the nozzle is done, so we will first do the calculation for the hot nozzle, okay.

So, the outlet or the hot nozzle; the state at the outlet of the hot nozzle is denoted by eH; e for exit and H for hot, okay and remember, we need only the static pressure, static temperature and the velocity at the exit of the nozzle not the stagnation quantities okay. If you recall, we developed an expression for the critical pressure of a nozzle, which has irreversibilities, we derived this expression in one of the earlier classes, right.

So, if the nozzle has irreversibilities, then the critical pressure is related to the inlet stagnation pressure using this expression okay, in fact if you said eta nozzle = 1 and you will recognize the term inside the bracket to be nothing but 2/gamma g + 1, which is what we have in a isentropic case, right. So, in the case, when there are irreversibilities in the nozzle this is what the critical pressure is, so we evaluate the critical pressure.

Remember, we know P06, right; we know P06, I know eta nozzle and I know other quantities, so once I have P06, I can evaluate critical pressure using this relationship, so that is what I would do next. Once I evaluate P06, I evaluate the critical pressure, then I compare the critical pressure with the ambient pressure. Remember, we are given the ambient pressure and the ambient temperature, right, so I know the ambient pressure, so I compared the critical pressure with the ambient pressure.

If the critical pressure is for the given value of P06, if the critical pressure is > the ambient pressure, then I know for sure that the nozzle is choked and the exit pressure PeH is nothing but the critical pressure itself okay, so one as soon as, I have P06, I do this evaluation and then I make this determination. If P critical is more than P ambient, then the nozzle is choked and the exit static pressure is the critical pressure, otherwise the nozzle is not choked.

And the exit static pressure is equal to the ambient pressure okay, so this allows me to evaluate the exit static pressure immediately, what I need now is the exit velocity. Once I have the exit velocity, remember there is no energy addition or removal in the nozzle, which means the stagnation temperature at the exit; nozzle exit is the same at the inlet, there is no change in stagnation temperature.

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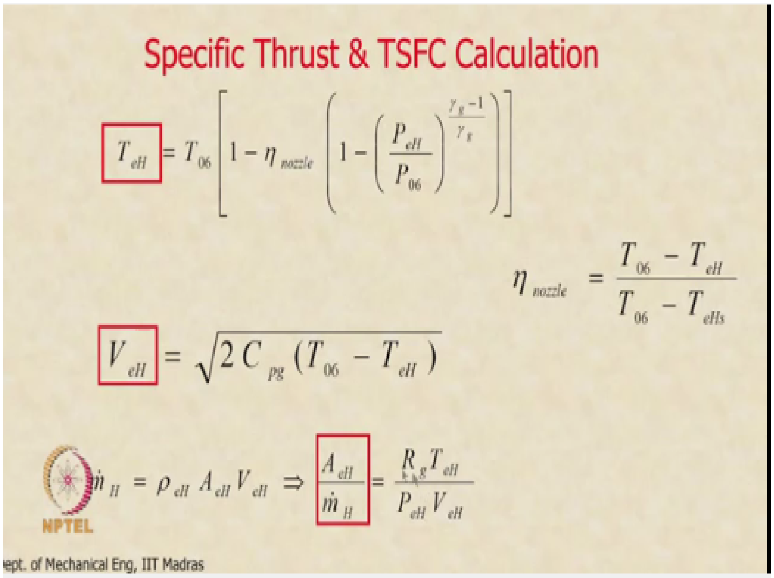

So, if I have V, then I can evaluate the static temperature or if I have the static temperature, I can evaluate the velocity at the exit okay, so I need only 2 quantities at the exit right, oh! Is this procedure clear? Okay, so how do we calculate; how do we calculate the static temperature, we derived this expression in our earlier lecture that if the; once the exit pressure is known, I can write the; exit static pressure is known, I can write the exit static temperature using this relationship, right.

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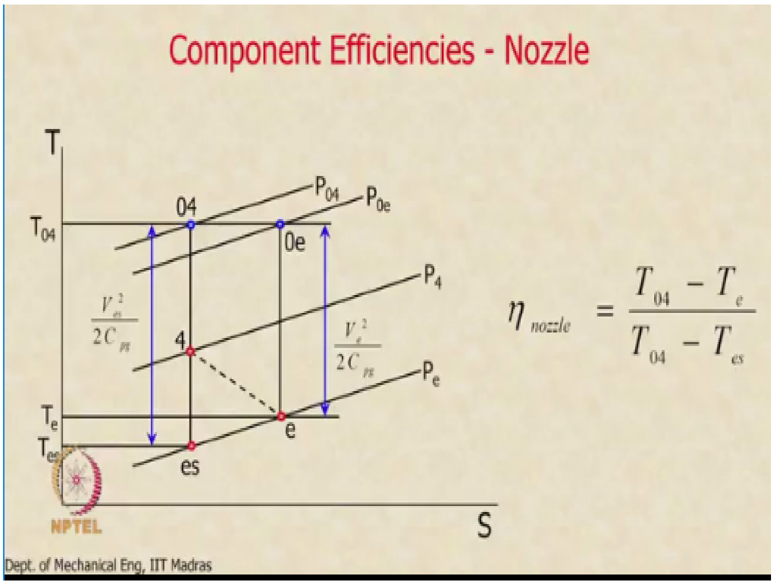

This was what we derived in our earlier discussion involving the nozzle, right. Let us quickly go back and take a look at that, so if you remember this was what we did earlier right and in fact if you do this okay, so if you look at the nozzle itself, you can see that the operating conditions for the nozzle. So, 4 here is the inlet to the nozzle and eE is the exit of the nozzle and if you remember this is Ve square/ 2Cpg.

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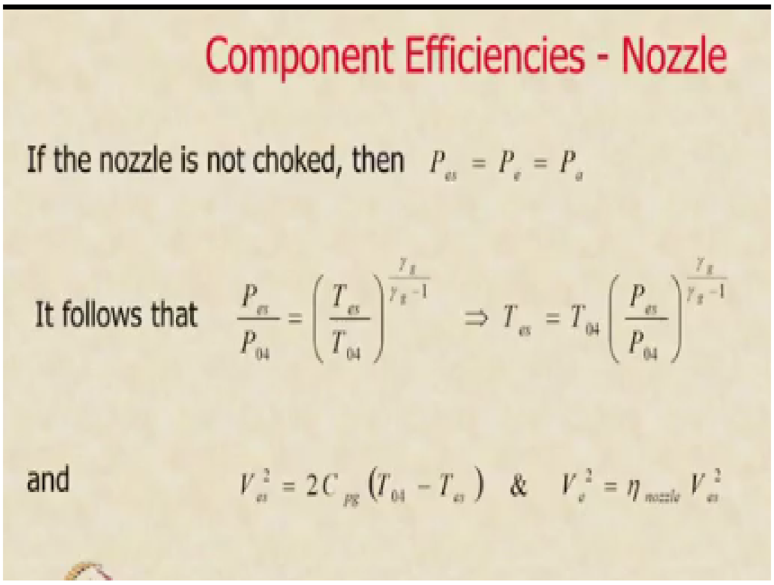

And this is a static State es which is at the same pressure as which is at the same pressure as Pe, right and this is Ves square/ 2Cpg, right and then we define the isentropic efficiency of the nozzle this way, T04 - Te divided by T04 – Tes; Tes has to be evaluated to complete this expression, so if you remember, if the nozzle is not choked then we said Pes = Pe = Pa, correct and then, so Pes/P04 both this lie on the same isentrope.

So, I can use this relationship right, from which I calculated my Tes. Once I know Tes ,I can calculate Te from the definition of the; or once I know Tes, I can calculate Ves square right and then I can calculate Ve from the definitional nozzle efficiency, right. So, once I know the exit pressure; once I know the exit pressure, I know Pes and once I know Pes, I know P 04, I can calculate the exit velocity, in this case when the nozzle is not choked.

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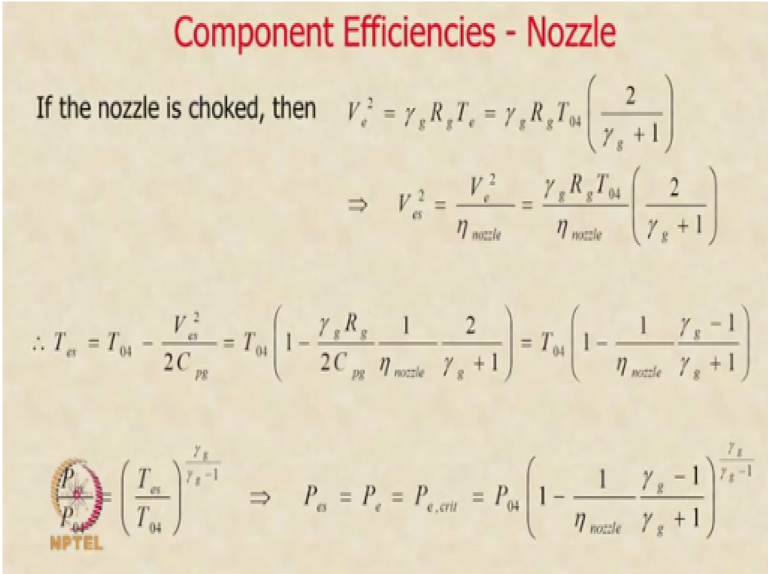

Similarly, when the nozzle is choked right, then this is the process that we went through and remember, the Ve is now equal to the speed of sound and we went through this right, so Tes can be evaluated and once again, Pes is also know, Pes = Pe same as before. Once I know the exit pressure, I can calculate the other quantities okay. For example, if I know Tes, right or I am sorry; if I know the exit pressure, then I know Pes, I know P04, I know T04.

I can calculate Tes, correct and then once I know Tes, I can calculate Ves square/ 2Cpg. Once I know Ves square/ 2Cpg, I can calculate Ves square/ 2Cpg okay, so this is the expression that we are using now to calculate because I know the exit pressure in both cases right, Pe = Pes, right, once I know Pe, I know Pes, then I can use this relationship to calculate Tes. Once I know Tes, I know Ves square/ 2Cpg and then I know Ve square/ 2Cpg.

So, once I know the exit pressure; exit static pressure I can calculate the exit static temperature and VeH can be evaluated, that is not a problem, right. Notice that here we are evaluating the exit static temperature by using the factor PeHs = PeH, both are on the same isobar, that is what I am using here and once I have this, I substitute the value for TeHs here to evaluate TeH, okay, so what I am doing here is once I know the exit static pressure, I know PeHs.

Once I know PeHs, I can evaluate TeHs, right, as soon as I know TeHs, I can evaluate TeH from this expression that is what I have written here, Is it clear? **"Professor – student conversation starts"** Go ahead, because the TeH is required here; the TeH is required here, notice that the numerator here is nothing but VeH square/ 2 Cpg, right, the denominator is VeHs squared/ 2Cpg, so I need this and then I can get this.

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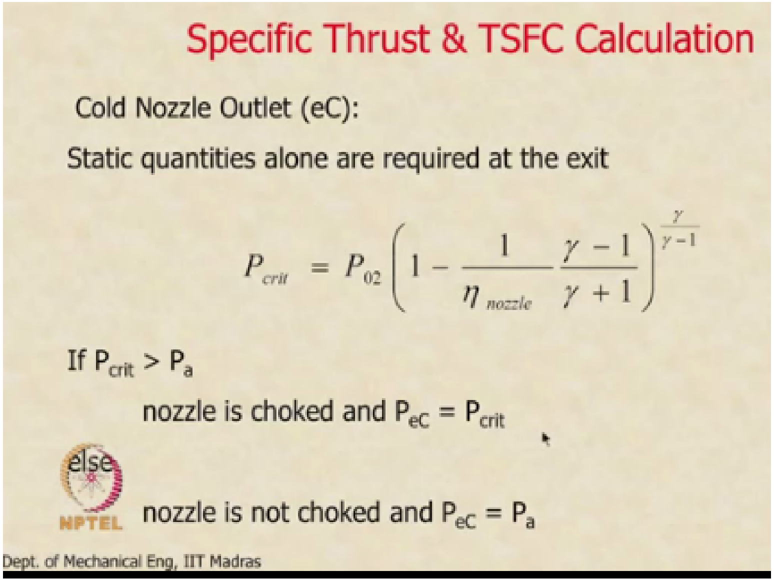

Once I get this, I can get this and notice that this term AeH/ m dot H can also be calculated this way, okay. **"Professor – student conversation ends"** Any questions, all right, so this is how we do the calculation for the hot nozzle. Now, for the cold nozzle, we do the same thing, once I have P02, I can evaluate the critical pressure; I evaluate the critical pressure compare that with the ambient pressure.

And when if the critical pressure is more than the ambient pressure, then the nozzle is choked and I state that the exit static pressure is equal to the critical pressure okay, otherwise the nozzle is not choked and the exit static pressure is equal to the ambient pressure, so same as before. The in fact, the procedure is in the hot and the cold nozzle are the same as before okay, so we do the same thing here.

So, we use the definition of the nozzle efficiency, right, so once I have the exit static pressure, I know; once I know PeC, I know PeCs, once I know PeCs, I can evaluate TeCs, once I know TeCs, I can evaluate TeC, using this expression. Once I know TeC, I can calculate VeC using this and again this ratio can be calculated like this okay. Any questions, so, what we will do in the next class is continue this and finalize the procedure, then go through a numerical example and calculate some of the values, okay.