Gas Dynamics and Propulsion
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Lecture – 32
Thermodynamic Analysis of the Engine

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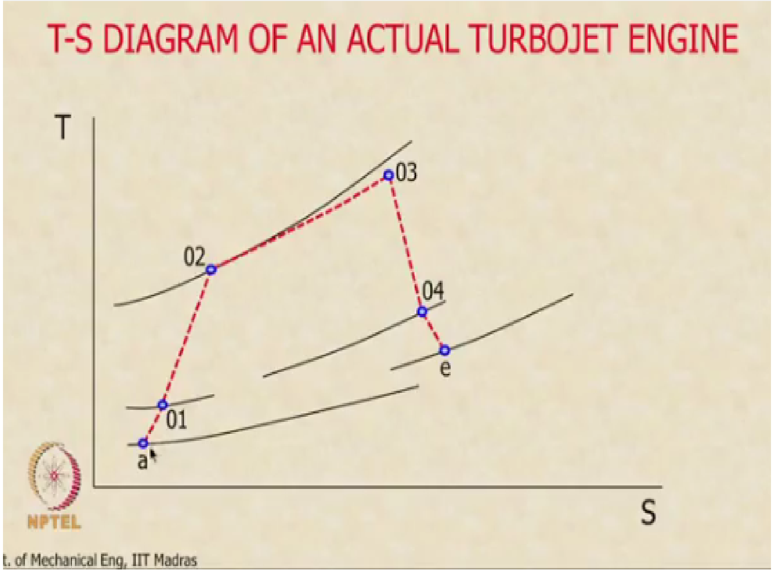
T-S DIAGRAM OF AN ACTUAL TURBOJET ENGINE

So, in the last class we said that the next step in our analysis would be to take into account irreversibilities and losses associated with irreversibilities in our analysis. So, what we have sketched here is the basic turbojet cycle. In the earlier version of this cycle, all the processes were ideal, so we went from a to 01 along a vertical line like this and from 01 to 02 along a vertical line and then there was a constant pressure heat addition process and so on.

Now, you can see what irreversibility is due to the process and the state points, so you can see that there is an increase in entropy in going from a to; state a to state 01 and so consequently, the state point 01 shifts to the right and once again, when we go from stagnation state 01 to 02 again due to irreversibilities, we can see that there is a shift to the right and this is going to cause, is it going to be a loss of stagnation pressure associated with this.

Although, this is a compressor and we are putting in work to increase the stagnation pressure as a result of the irreversibility, some of the work that we put in is definitely going to be lost, okay and then in going from 02 to 03, we can see that the heat addition process also results in loss of stagnation pressure and also there is a loss of static pressure as you can see from here only. So, this is due to the mixing which is an irreversible process and other chemical reactions and other things.

So, there are definitely going to be losses associated with this and 03 to 04 is the expansion process in the turbine, which once again because irreversibilities, you can see that the state point shifts to the right and whatever work it could have produced right, if it is at the expansion be an isentropic, we would have just gone down here, so whatever work we could have obtained for the turbine is now going to be; whatever we are getting now is going to be less than whatever we could have obtained earlier, right.

So, because of the irreversibility, then here you see expansion in the nozzle and similar to what we discussed for the other components, the expansion process; for example, will give only slightly less velocity then, what we would have obtained had the process been isentropic, okay. So, what we need to do now is come up with a way to include the effects of these irreversibilities in our calculation.

So, for example when the state changes from a to 01, 01 to 02 and so on, we need to have a way by which we can take this into account, so we are not going to actually try to evaluate the amount of losses due to this irreversibilities because that is not a measurable quantity, okay you know only from experience that you know there is so much loss of pressure, so we do not know exactly what the mechanisms are which caused that loss of pressure.

So, what we will do is, we will define an appropriate efficiency for each one of this process and then assume the process to be less efficient than the ideal process. So, if the process is ideal, the efficiency is 100% and that would be an isentropic process. So, what we would say is the process is non-ideal, so let us say the efficiency is only 90%, then how do we calculate the downstream states, if the efficiency were 90%.

So, that is the way in which we will proceed, we will not try to justify why the efficiency is 90% or why it is 95%, that is a value that we can take from practice, then it appears to be 90% efficient, we will accept it as it is, without asking any questions about how it became 90%. So, we start; so what we are going to do now is, we will take each leg of this simple cycle and define efficiency and then see how the downstream states can be calculated based on this.

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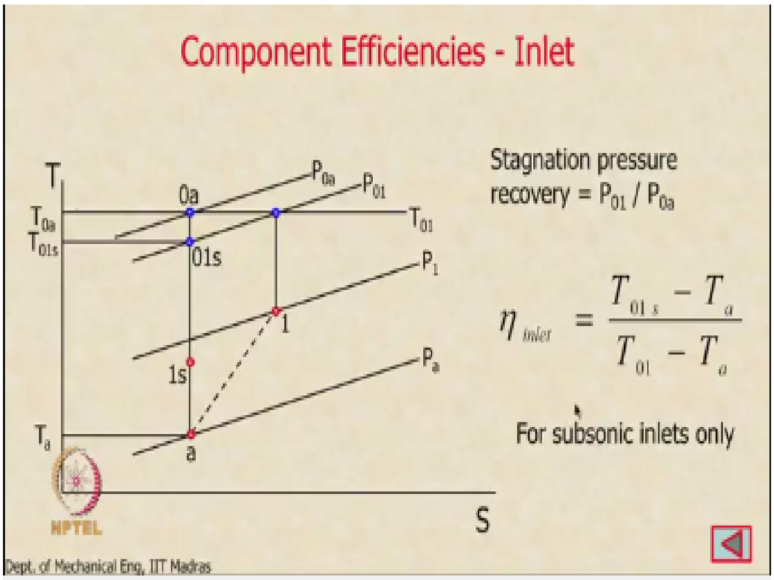

Component Efficiencies - Inlet

Stagnation pressure recovery = P_{01} / P_{0a}

$$\eta_{inlet} = \frac{T_{01s} - T_a}{T_{01} - T_a}$$

For subsonic inlets only

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So, the basic idea is to keep track of this stagnation stage 01, 02, 03, 04 and then this static state e, at the exit that is the basic idea, okay and this is the free stream state, a. So, let us start with the first component, which is the inlet okay, so the inlet as you can see has a static state corresponding to this, so this is the inlet; this is the free stream stagnation, I am sorry; this is a free stream static temperature.

And this is the free stream static pressure and this is the inlet state, so the flow approaches the engine at this static state, this is a static state, okay. Now, the corresponding stagnation state for this can be depicted like this, so this is 0a that is this stagnation state corresponding to the free stream static state and you can see that the stagnation temperature is T0 and the stagnation pressure is P0a, okay.

Starting from this static state, the flow is going to be decelerated at the inlet and there is going to be an increase in static pressure, right that is what we are looking at. So, you can see that starting from this static state, the flow undergoes a deceleration process, so that there is an increase in static pressure, right, notice that the velocity component which was equal to this, so this is nothing but va square/ 2P, it is starting from there.

Now, you can see that the velocity has reduced remember, stagnation temperature remains the same because there is no work or heat interaction in the inlet, so stagnation temperature remains the same, so you can see that static pressure has increased but v square/ 2cP has reduced in magnitude and so we go from here to here and I will sketch this using a dashed line to illustrate that there is irreversibility associated with the process.

And you can see that this is the stagnation state 01, this is the stagnation state 01, this is P01, notice that T01 = T0a because there is no heat or work interaction during the process, one performance metric for evaluating this particular component, which is the intake is to use this stagnation pressure recovery, P01/P0a is a suitable metric right, so there is a loss of stagnation pressure because of the irreversibility as you can see from here.

The stagnation pressure line shifts from this, which is P0a to this, so the state point stagnation state moves to the right and that moving to the right is a result of this process moving to the right, so due to increased entropy and irreversibility. So, P01/P0a p will be = 1 if the compression process were isentropic, otherwise it is something less than 1. So, this is a metric which is used routinely and it can be used.

There is also a slightly different metric, which is used for subsonic inlets. This is a metric that is typically used for supersonic intakes and for subsonic intakes; there is a metric which is used, which goes like this. The idea is, we know that the stagnation state changes from P0a to P01, right, the stagnation temperature remains the same, so the stagnation state goes from here to here.

So, we are willing to concede that this change in stagnation state is inevitable, it is always going to happen but what we try to say is the following; had this change in stagnation state taken place as a result of an isentropic process, what would the stagnation temperature have been? Okay, so you need to listen to this carefully remember, the stagnation state here goes from here to here because the stagnation temperature remains constant in the actual device.

What we are going to do is come up with the definition for the efficiency of the device by arguing giving like this. What if we say that we go from P0a to P01 in an isentropic process, if you had done that what would the change in stagnation temperature been? Right, so this is what the stagnation state would have come to. Remember, this is along the same isobar right, so the P01s = P01, the stagnation pressure here is the same as this state.

But notice that because we are constraining it to be isentropic, it has to lie along this vertical line and not along this horizontal line, which means that the stagnation temperature is this. Since in our steady flow energy equation, we have the stagnation temperature appearing in our steady flow energy equation, we try to relate the efficiency of the devices to changes in stagnation temperature.

Because that we can relate to either heat or work interaction right, our energy equation does not have stagnation pressure, it has only stagnation temperature, so that is the rationale for relating these 2 changes in stagnation temperature, okay. The corresponding static state would have been equal to this. So, in the actual inlet, we go from this static state to this static state and due to the irreversibilities, there is a loss of stagnation pressure from here to here.

But because there is no heat or work interaction, stagnation temperature remains the same, there is a scenario in the actual intake. Now, in order to quantify this, we are proposing a metric which states the following; had the compression process be an isentropic, what would the stagnation temperature be at the end of such a compression process, while the stagnation pressure is = P01, okay.

“Professor - student conversation starts” Yes, one is the static state at the end of such an isentropic compression process. It is not = P1; It is not =P1, what we are saying is 1s is the static state, which would correspond to a stagnation state 01s, so we are coming back from the 01s state, we are saying P01s is such that is =; P01s = P01 corresponding static state would have been this, okay.

So, the efficiency can be defined like this, for subsonic inlets it is better to define the efficiency in this way, so we say that the efficiency is T01s - Ta divided by T0a or T01 – Ta. Yeah! Go ahead, why there is loss of stagnation pressure if we assume it to be this one; isentropic, why this loss? Yes, I am assuming it to be an isentropic process only until I reach stagnation pressure P01s.

I am not assuming it is an isentropic process all the way, if I do that, then the efficiency is 100%, remember there is a loss of stagnation pressure, correct in the actual intake, there is a loss of stagnation pressure, we go from P0a T0a to P01 T01 with T01 being = T0a. What I am going to do is, I am going to convert this into an equivalent changes in stagnation temperature okay, so we go from P0a T0a to P01s and T01s, okay along an isentropic path. “Professor - student conversation ends”

So, if you look at this diagram, you can see that we go from 0a to 01s and 01s is such that P01s = P01, okay and so the efficiency is defined like this. Notice that, if the entire process, is state1 were to be over here, then this will have 100% efficiency otherwise, we have a certain value for the efficiency, okay. So, this is used for subsonic inlets only, for supersonic inlet, we will use either this or a slightly different metric, okay.

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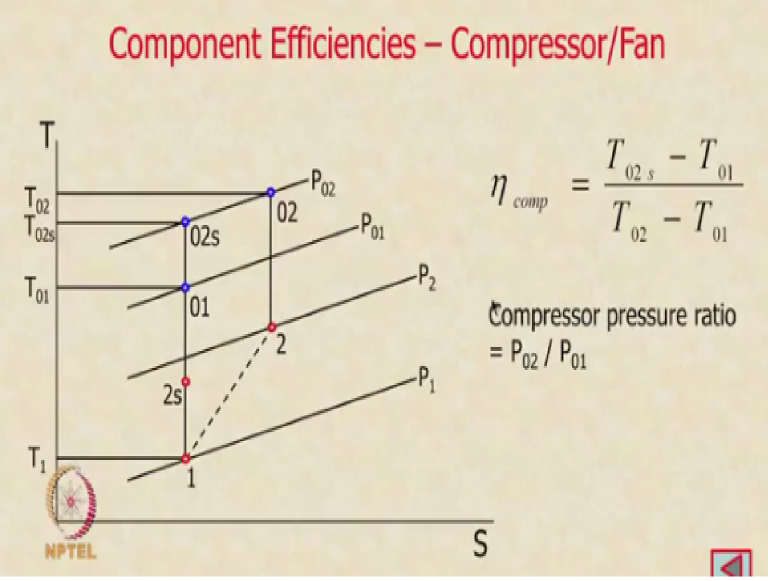

Component Efficiencies – Compressor/Fan

$$\eta_{comp} = \frac{T_{02s} - T_{01}}{T_{02} - T_{01}}$$

Compressor pressure ratio $= P_{02} / P_{01}$

So, we will take a look at supersonic inlets, when we go to ramjet engines. For now, we will use this definition for this, okay. So, we now go to the next component, which is the compressor or the fan, both you know can be handled in the same framework, so we look at the compressor and so as you can see from here, here we go from static state 1, which is at the end of the inlet and at entry to the compressor, the static temperature is T1, static pressure is P1 here.

So, we go from there, the corresponding stagnation state is 01, so you can see that T01 and P01 are like this. Now, in the compressor, we are going to add work such that the pressure ratio becomes let us say, 20, 30 or 40, so we are adding work in the compressor, right. So, there is going to be a change in stagnation pressure right, but how much of this change in stagnation pressure is due to an increase in (()) (13:16) input of what.

And how much is the loss due to irreversibility that is what we need to look at, right. So, the compression process takes the static state from 1 to 2 like this, right and because we have put in work, the stagnation pressure corresponding to this state will be higher than P01, right, so this is stagnation state 02, right and it is going to be higher than P01. So, P02/P01 is going to be equal to the pressure ratio across the compressor 20, 30 or 40, whatever we specify for the particular engine.

Now, we are going to quantify the efficiency of this device in the same manner as we did before, so you can clearly see that the stagnation state changes from 01 to 02, right we want to relate this change to a; we want to make sure that we can relate this change in stagnation temperature for an isentropic process. So, what we are going to say is, if the stagnation pressure changes from P01 to P02 in an isentropic process, what would the stagnation temperature have been, right?

That is what we are going to see, so if you look at this, as I said earlier compressor pressure ratio is P02/P01, that will be 20, 30 or 40 and so you can see that this is the stagnation state that we are looking for, had the increase in P01, I am sorry; I had the increase in stagnation pressure from P01 to P02, had it taken place in an isentropic manner, this would have been my end state T02s, right.

Then, you can see that the amount of work that I am putting in which is related to the change in the stagnation temperature T02s - T01 as you can see is < T02 – T01, right which means in the actual process due to irreversibilities, I have put in more work to accomplish the same change in stagnation pressure, I am now putting in more work that is what this definition allows me to do, right.

So, T02s - T01 is the work that I would have put in in an isentropic process for the same pressure ratio, instead of that I am now putting in T02s - T01 in the actual compressor to accomplish the same pressure ratio, so the efficiency of the compressor can be defined like this, T02s - T01divided by T02 - T01. Notice that I have kept the pressure ratio the same; P02s = P02, so the pressure ratio for this isentropic process is the same as the pressure ratio for this irreversible process.

“Professor - student conversation starts” Go ahead. Sir, in the inlet, there was no work condition; there was no work condition. Why we are going for the same kind of definition for this case also and there was no energy transfer but here there is a transfer? Yeah! But there was a change in stagnation pressure there due to irreversibilities correct, so we are trying to convert that into a change in stagnation temperature.

Remember, what we said in the first lecture, when we talked about stagnation pressure, any loss of stagnation pressure is tantamount to a loss of work, correct, which is why I am relating changes in stagnation pressure to changes in stagnation temperature because our energy equation, if you remember our energy equation contains T + u square/ 2cp, which is the stagnation temperature.

So, now right hand side can contain w dot, which will be a rate at which I am adding work, so any change in stagnation temperature, I am able to relate to either a heat input or a work input, which is why changes in stagnation pressure are being related to changes in stagnation temperature. In the previous case, there was no work or heat interaction, so there was no change in stagnation temperature but there was a loss of stagnation pressure which I translated to a change in stagnation temperature.

To illustrate the fact that; that is loss of work, equivalent to a loss of work, Here, I am actually putting in work and for the amount of work that I am putting in, I am able to increase the stagnation pressure from P01 to P02, if this process had been isentropic, I could have realized the same change in stagnation pressure with the less amount of work, as you can see from here okay, I am sorry; as you can see from the difference between these 2.

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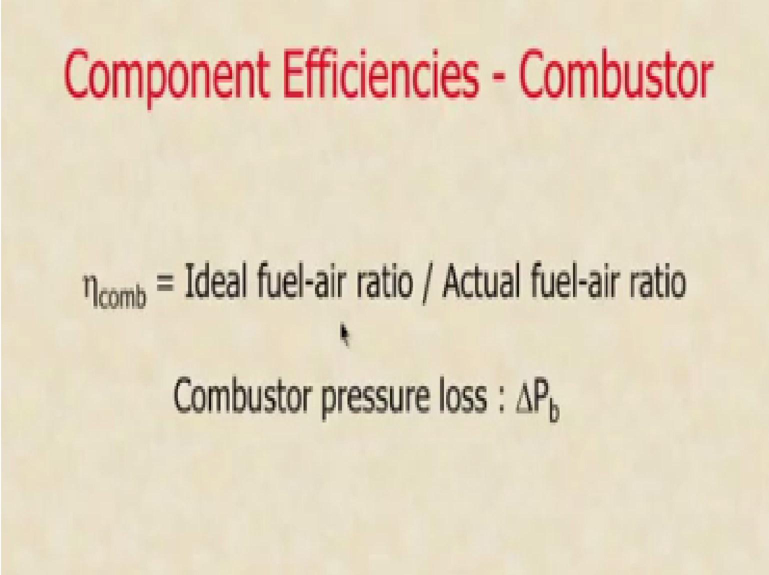

Component Efficiencies - Combustor

η_{comb} = Ideal fuel-air ratio / Actual fuel-air ratio

Combustor pressure loss : ΔP_b

So, that is why we are relating everything to an equivalent change in stagnation temperature okay, right. **“Professor - student conversation ends”**. So, you can see that this is the efficiency that we are going to use for the compressor. Combustor; since, we are not looking too much into the actual heat release mechanism or the process in the combustor and we are not interested in doing that also.

Because as we showed earlier, the processes in the combustor essentially incompressible, so we are not worried too much about it, what we will do is, define 2 matrix for the combustor performance; one is the loss of stagnation pressure, okay. So, I go into the combustor with a certain stagnation pressure as we saw let say, P02, how much am I losing in the combustor, as a result of mixing and combustion and so on.

So, I will specify; usually, a percentage is specified, we may say that there is a 10% loss of stagnation pressure in the combustor and so when we come out of the combustor, we will say exit; at the exit of the combustor, the pressure is 0.9 times the pressure at the inlet. So, I know this stagnation pressure at the exit and now, if I also think about the combustor itself, it is quite possible that I you know; I put in a certain amount of fuel into the combustor.

So, I know that based on the calorific value of the fuel, there should be a certain change in stagnation temperature, right there should be a certain release of heat but in the actual combustor, lot of the energy goes towards; you know, heating the various combustor parts and so on, they are also cooling, so there are many heat losses in the combustor. So, I may put in 1kg of fuel and burn 1kg of fuel in the combustor.

But I may not be able to realize the calorific value contained in 1kg of fuel, as in increase in the stagnation temperature of the fluid, right. So, I define my combustor efficiency that way, I say that what is the amount of fuel that I need to put in for realizing a certain change in stagnation temperature, that is the ideal fuel air ratio. So, I know that my turbine entry temperature cannot be above this value, so I calculate.

So, given my inlet condition to the combustor and my exit temperature, what is the amount of fuel that I need to burn to go from here to there under ideal circumstances? And under real circumstances, I need to put in more because I am losing some of the heat to heating of the components, to cooling and so on, so the actual value will be more. So, that I define to be; may combustion efficiency.

We will assume some typical values for combustion efficiency, 90 % is typically a good value, which means that if you want to realize a certain change in stagnation temperature, I need to put in more than the ideal amount of fuel to realize the change in stagnation temperature, okay and that is how we are going to characterize the operation of the combustor, we do not want to get into details beyond that.

"Professor - student conversation starts" Yes, stagnation pressure is changing because of 2 parameters; one is heat is added and the other is irreversibility. Correct, correct, we are considering both of them, this 10% loss is inclusive of both that is obtained from practice. We can actually take both these effects into account remember, the flow in the combustor is almost on the incompressible limit, okay.

So, the loss of stagnation pressure due to heat addition, which is typically something that you would see in a compressible flow will be largely absent in the actual combustor. The loss of stagnation pressure that you see in the combustor is due to mixing; it is a lot of mixing so that is what causes the loss, so the loss of stagnation pressure due to heat addition will be somewhat minimal in this case due to reduced compressibility effects, okay.

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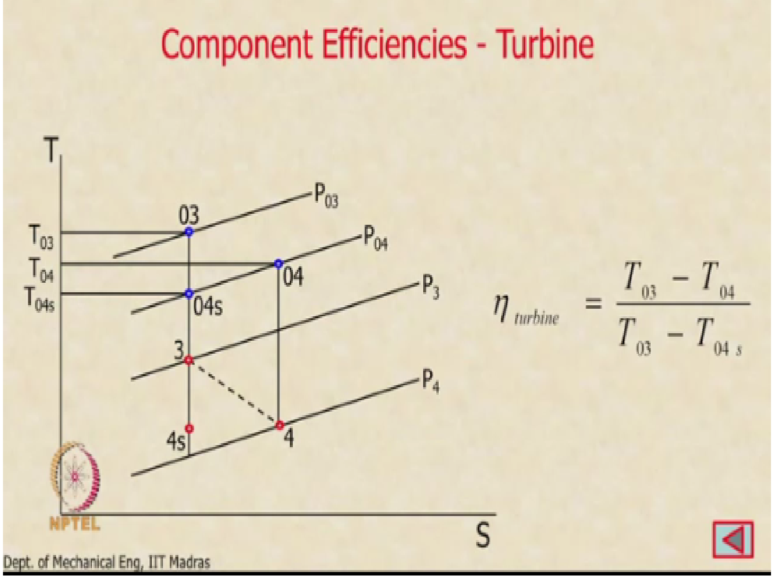

Component Efficiencies - Turbine

$$\eta_{turbine} = \frac{T_{03} - T_{04}}{T_{03} - T_{04s}}$$

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So, this 10%, which should be more than able to include all the other things and these are numbers as I said, these are numbers derived from practical situations; practically, typically in these combustor what would you see, so that is what is used here okay, without going into the details of how this value came about. **"Professor - student conversation ends"**. Next component that we are going to look at is the turbine.

And TS diagram of the process in the turbine looks like this, so we start from static states 3, which is at a pressure P3 and the corresponding stagnation state is 03, with stagnation temperature T03 and stagnation pressure P03, so now we have an expansion process as you can see, the expansion process shifts to the right, so the downstream state from the turbine is over here at a higher entropy and this is a static pressure T4.

Now, corresponding stagnation state is 04 with stagnation temperature 04 and stagnation pressure is 04; P04, so what has happened in this process is that the stagnation pressure has decreased from P03 to P04, which means some amount of work has been taken out remember, decrease of stagnation pressure can be due to work extraction or irreversibility, so what we are trying to quantify now is how much of this is work extraction, how much of this is irreversibility.

And we quantify that by saying, if the same change in stagnation pressure had it taken place isentropically, what would have been the change in stagnation temperature, which would then tell me how much work I would have obtained, right. So, if I do that, the corresponding static state would have been 4s and the corresponding stagnation state would be for 04s, so you can see that if the process had been isentropic, then the amount of work that I would have obtained would be T03 – T04s, which is more than what I am getting now, which is T03 – T04, okay.

So, this nicely quantifies the loss due to the irreversibility that is the usefulness of this particular calculation. Remember, all these other components, the compressibility effect is important except for the combustor we showed in our first lecture and compressibility effect is important in all the other components, which is why we are dealing with stagnation quantities rather than static quantities in all these definitions, okay.

Your probably, first or second year thermodynamics course, would have probably defined these efficiencies using static states rather than stagnation states. It is oaky for a basic thermodynamics course, but we are doing a propulsion course, so we need to make sure that the numbers and the models and views or as realistic as possible okay, so the efficiency can now be defined as T03 – T04, which is the lesser amount of work that I am getting now compared to T03 – T04s.

So, the common thread in all these definitions of efficiency is that there is a change in this stagnation state, right what if the change in the stagnation pressure had been achieved using an isentropic process, then what would have been the change in this stagnation temperature so and we can relate that to work or what basically we are not worried about heat, so we are talking about work in all these cases.

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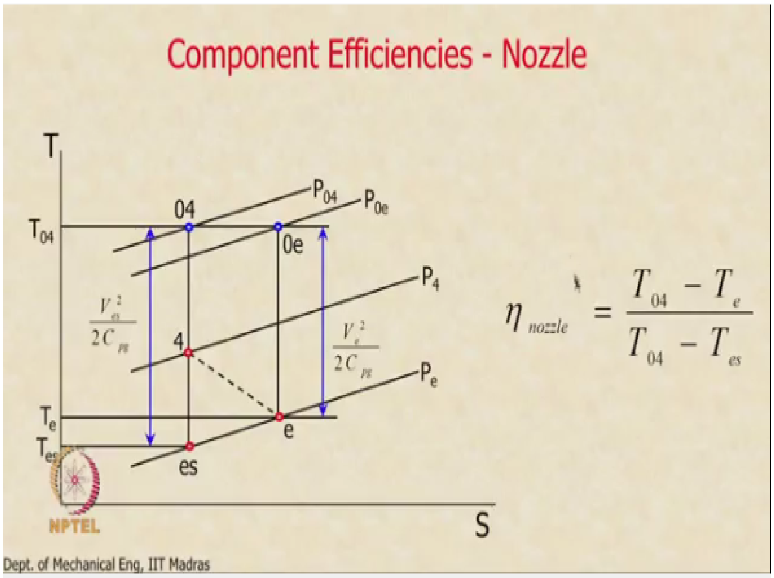

Component Efficiencies - Nozzle

$$\eta_{nozzle} = \frac{T_{04} - T_e}{T_{04} - T_{es}}$$

We can relate that to work through our energy equation, so that is a common thread in all these definitions the last component is the nozzle and again the process in the nozzle looks like this, so we start from state 4, which is at static pressure P4 and the corresponding stagnation state is 04; T04 and P04 and the expansion takes place from 4 to an exit state e, to a static pressure Pe, remember in the nozzle what we need to think about is the exit static pressure.

Because the thrust is determined by the exit static pressure, which in turn determines the velocity as well as the pressure thrust, okay so that is what we must think about in this case. Remember, in the inlet also we use the static quantities a and then, we related that to the stagnation quantity, so in the nozzle also we do the same thing. In the compressor and the turbine, we relate stagnation quantities at inlet and at exit, okay.

Whereas, for the inlet and the nozzle, we relate these stagnation quantities to the appropriate static quantity either at the inlet or at the exit, in this case it is at the exit, okay. Now, as a result of this, what is the stagnation state corresponding to e? That is T0e, the stagnation pressure of P0e, what is that? This is = Ve square/ 2Cpg, where this additional subscript g denotes the fact that we are dealing with combustion gases and not air.

So, the Cp for that is going to be different from Cp for air, we need to take that into account. Cp for this will be higher actually at a higher temperature although, we are; strictly speaking, not abandoning the calorically perfect assumption but this is still something that we can easily we are assuming the Cp to be constant for this entire gas downstream of the combustor, so downstream of the combustor, we assume a different value for Cp, different value for gamma.

Upstream of the combustor; we assume the value as same as that of air that is something that we can easily accommodate, it is not a problem and the results will be more realistic, okay. So, now what are we going to do, we can see that there is a change in stagnation pressure from P04 to P0e, had this change taking place isentropically, what would the exit velocity of the nozzle would have been? What would the exit velocity from the nozzle have been, right?

So, the exit state remember, we want to go to the exit state, es because we are going to expand against the same pressure; exit pressure, so the corresponding state would have been this, okay. The change in stagnation pressure here is accommodated in this device using by comparing these 2 velocities okay. Notice that in this case, we are not going to go down here and say that had the process being isentropic, what would the velocity have been.

What we are saying is, if I expand from this stagnation state to this static pressure, remember the static pressure has to be the same in both cases for the comparison to be fair for this device, so we are saying this is my ve square / 2Cpg, this is the velocity that I would have obtained, if I had expanded isentropically to this static pressure, okay. In the real case, this is the velocity that I am getting okay, so the efficiency of the nozzle can be defined like this; T04 - Te divided by T04 – Tes.

Because the conversion of the enthalpy to kinetic energy is what we are trying to do in this particular device, okay. Similarly, if you look at the efficiency expression for the inlet, you would notice that you get a similar kind of thing, velocity square at the exit of the inlet divided by velocity square at the free stream right because the inlet is also supposed to recover the momentum of the fluid as pressure that is why that efficiency is defined in that way.

This is also defined in the same way because of the objective that we are trying to accomplish in this device. So, the efficiency metric should take that into account, what all we trying to accomplish with this device, right. If it is a work addition or work extraction device, then we need to track the changes in stagnation pressure, if it is a device where we are trying to convert momentum into enthalpy or enthalpy into momentum of the fluid, then we need to make sure that the definition of the efficiencies reflects that fact, that is what we have done here, okay.

So, this is the way in which we take the irreversibilities and losses into account. When we do the calculation, we will assume certain typical values for all these efficiencies, these are the so called isentropic efficiencies, we will assume certain typical values and do our calculations okay. Now, with the nozzle there is one more complication okay, what is the complication? The complication is we have said that this is exit pressure Pe.

Now, this exit pressure Pe is going to be different depending upon whether the nozzle is choked or not. If it is choked, then the exit pressure is equal to the ambient pressure, if it is not choked, then the exit pressure is = the critical pressure M = 1, so it is equal to the critical pressure. So, how do we evaluate the critical pressure for a nozzle in which there are irreversibilities. So, previously the nozzle process was isentropic, now it is irreversible.

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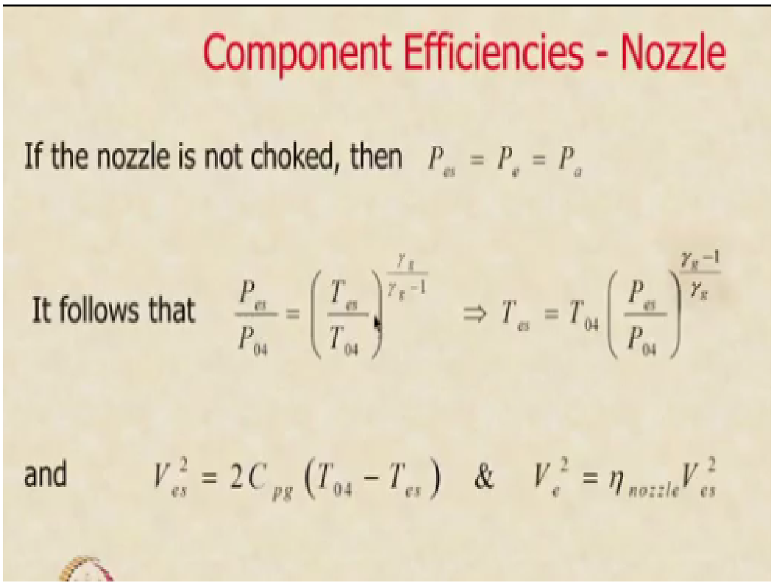

Component Efficiencies - Nozzle

If the nozzle is not choked, then $P_{es} = P_e = P_a$

It follows that $\frac{P_{es}}{P_{04}} = \left(\frac{T_{es}}{T_{04}}\right)^{\frac{\gamma_g}{\gamma_g - 1}} \Rightarrow T_{es} = T_{04}\left(\frac{P_{es}}{P_{04}}\right)^{\frac{\gamma_g - 1}{\gamma_g}}$

and $V_{es}^2 = 2C_{pg}\left(T_{04} - T_{es}\right)$ & $V_e^2 = \eta_{nozzle} V_{es}^2$

So, we need to make sure that we know how to do that, how do we evaluate this Pe, right that is the extra complication with the nozzle, let us see how we do that. So, we take this situation one at a time, the nozzle is not choked that is a simple case, then Pes = Pe = Pa, remember the state es and state e lie on the same isobar, right; state es and state e lie on the same isobar, so Pes = Pe = Pa, correct.

And so, then I can write the following Pes /P04 both of these states lie on the same isentrope, correct; es and 04 lie on the same isentrope, which you see the earlier figure, we can see that very easily, so I can use this relationship to evaluate my Tes, T04 is known, Pes is now known, P04 is also known, so I can evaluate my exit static temperature this way. So, when you want to evaluate the thrust from this engine, what quantities do we require?

We are going to show that next but we have already talked about thrust and other things. There are only 2 quantities that we need at the exit, those are velocity and exit static pressure. So, now in this case, I know the exit static pressure and I need to calculate the velocity ve, right, so I can calculate the velocity corresponding to that state because T04 = Tes + Ves square/ 2 Cpg, so I can calculate Ves square based on this relationship.

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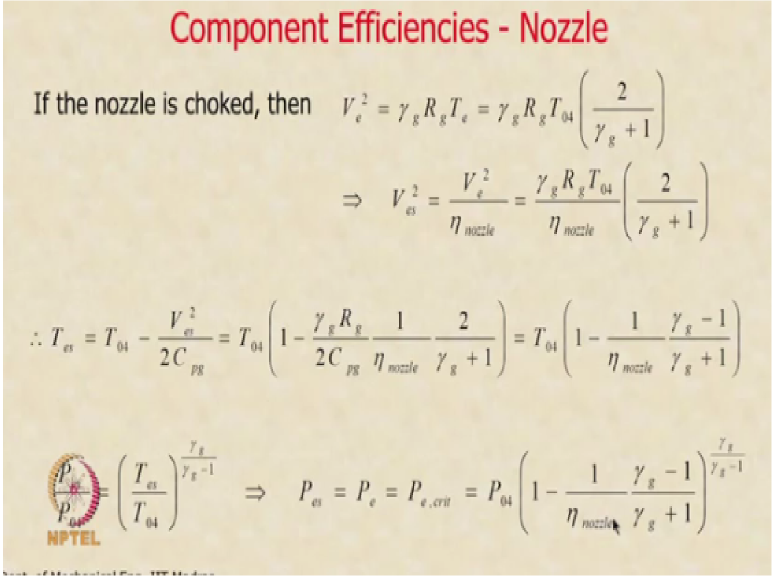

Component Efficiencies - Nozzle

If the nozzle is choked, then $V_e^2 = \gamma_g R_g T_e = \gamma_g R_g T_{04}\left(\frac{2}{\gamma_g+1}\right)$

$$\Rightarrow V_{es}^2 = \frac{V_e^2}{\eta_{nozzle}} = \frac{\gamma_g R_g T_{04}}{\eta_{nozzle}}\left(\frac{2}{\gamma_g+1}\right)$$

$$\therefore T_{es} = T_{04} - \frac{V_{es}^2}{2C_{pg}} = T_{04}\left(1 - \frac{\gamma_g R_g}{2C_{pg}}\frac{1}{\eta_{nozzle}}\frac{2}{\gamma_g+1}\right) = T_{04}\left(1 - \frac{1}{\eta_{nozzle}}\frac{\gamma_g-1}{\gamma_g+1}\right)$$

$$\frac{P_{es}}{P_{04}} = \left(\frac{T_{es}}{T_{04}}\right)^{\frac{\gamma_g}{\gamma_g-1}} \Rightarrow P_{es} = P_e = P_{e,crit} = P_{04}\left(1 - \frac{1}{\eta_{nozzle}}\frac{\gamma_g-1}{\gamma_g+1}\right)^{\frac{\gamma_g}{\gamma_g-1}}$$

Once, I have this, I can calculate the exit velocity using the definition of the nozzle efficiency okay, so now I have both the quantities that I wanted, the exit pressure; exit static pressure and the exit velocity, this is for the case when the nozzle is not choked, okay. If the nozzle is choked, then the exit velocity we know is equal to the speed of sound, right. So, Ve square = gamma RT, right.

And if I write this in terms of my stagnation temperature, remember the stagnation temperature does not change in the nozzle, only the stagnation pressure decreases due to irreversibilities, stagnation temperature does not change, so I can rewrite this in terms of that and after using the fact that Mach number is = 1, right. So, I can write it like this, correct T04/Te = gamma g + 1/2 because the Mach number is = 1 at this point.

So, I can write it like this, so I have this relationship. So, once I have this, I can calculate Ves square from the definition of the nozzle efficiency, so which makes this expression look like this. So, I get this expression for Ves square, remember from our earlier figure Ves square is always greater than Ve square okay, so this definition has a slight theoretical difficulty, which we must actually overlook.

Because we have no choice remember, in this case we have said that the nozzle is choked, so we have used that fact here, the actual nozzle is choked right, so this is equal to the speed of sound. Now, I am calculating Ves square, which is more than this value but it is still a convergent nozzle, so it is theoretically from a gas dynamic perspective, it is not possible to have a speed more than the speed of sound in a converging nozzle.

But remember, this definition does not take gas dynamic aspects into account, it is only a thermodynamic definition; isentropic efficiency is only a thermodynamic definition, okay. It assumes that you will do whatever it takes to accomplish this velocity Ves square, so in the theoretical definition, we may say okay, maybe it is a convergent divergent nozzle but somehow you will accomplish that Ves square, okay.

So, that is something that you must keep in mind, to reconcile this with the gas dynamic aspects, what we are saying is this may be a slightly different nozzle that the velocity is Ves square, which is slightly more than this okay because many times when you do the calculation, you realize this and then you are startled, should this be equal to the speed of sound or should this be equal to the speed of sound.

Remember, the actual nozzle is choked, so correct representation of what you see in practice is to keep this equal to speed of sound and then assume that in this process remember, this is an imaginary process, right. This imaginary process, I can accomplish in many different ways starting from the same stagnation and static state, I can make this expand in a CD nozzle such that I accomplish this velocity, keeping other things also the same that is possible to do, okay.

So, once I know this velocity, I can calculate my static temperature again from the definition of the stagnation temperature, right; Tes is T04 - Ves square/ 2Cpg, so this finally gives me Tes like this, remember what am I looking for? I am looking for 2 things; exit velocity and exit static pressure. I already have the exit velocity, I am looking for exit static pressure, so what I am going to do is, do the same as what I did in the previous case relate Pes and P04 and Tes and T04 using an isentropic relationship, right.

So, that is what this does, Pes/P04 = Tes/T04, right so, in this relationship I know Tes, I know T04, I know P04, so i can evaluate my Pes and remember, state point es and state point e lie on the same isobar, so Pes = Pe, which in this case is also equal to the critical pressure, so this is what the relationship looks like. So, this is the expression for critical pressure in a nozzle where there are irreversibilities.

If you set eta nozzle = 1, meaning there are no irreversibilities, then you recover the expression that we derived earlier for an isentropic process. So, the only change is this goes inside now, so at the end of this exercise, now I know Ve square and also Pe, so we have to look at 2 situations, we have to first of all determine whether the nozzle is choked or not. In the actual calculation, what we will do is given P04 eta nozzle and the other quantities; we will determine the critical pressure.

Then, compare the critical pressure with the ambient pressure to see whether the nozzle is choked or not. Remember, the flow in the nozzle can only be under expanded not over expanded, which means the ambient pressure can only be less than the critical pressure not more than the critical pressure, so we evaluate this and then determine whether the nozzle is choked or not.

Once we make the determination, we can then go through and determine the exit velocity, all right we will demonstrate this as we go through the calculations, so that takes care of irreversibilities in each one of the components in their device. So, when we add for example, something like a fan or other turbines, we can use the same definitions to deal with that okay, so we have demonstrated this for a turbojet engine.

But the definitions do not change, if you use it for a turbofan engine also, okay that is what we will do because the addition of a fan; fan is like a low pressure compressor, so there is no problem in using this. The fan nozzle is a cold nozzle again gamma g will be replaced by gamma in a fan nozzle that is the only change that we are going to see, so we can use this for a turbofan engine also.

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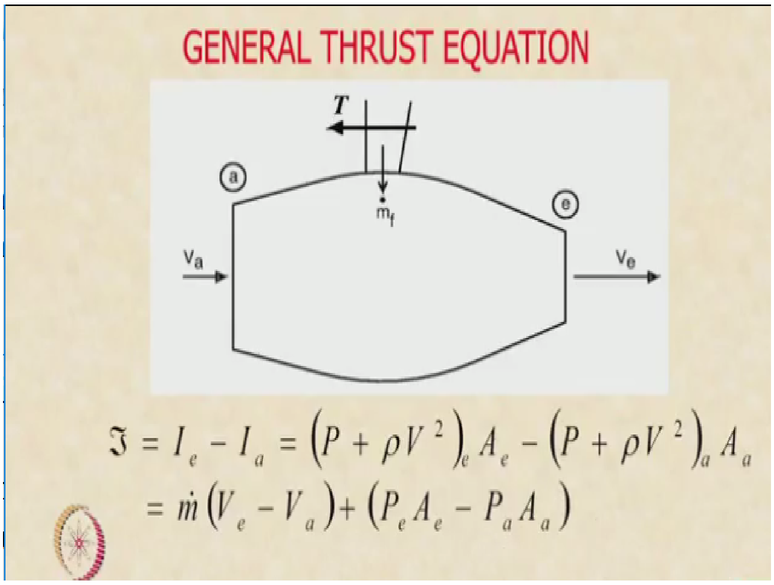

GENERAL THRUST EQUATION

$$\Im = I_e - I_a = \left(P + \rho V^2\right)_e A_e - \left(P + \rho V^2\right)_a A_a$$

$$= \dot{m}\left(V_e - V_a\right) + \left(P_e A_e - P_a A_a\right)$$

The next part that we need to do is develop expressions for thrust, we want to do thrust calculations and thrust specific fuel consumption calculation, so we start with that. So, we first derive a general thrust equation okay, so we look at an engine, so this is an engine, right air approaches the engine with velocity Va and air leaves the engine with velocity Ve, okay. So, we are putting in a certain amount of fuel flow rate and the engine produces a certain amount of thrust which is what is measured or felt by the frame in which the engine is mounted, okay.

So, we label this state; entry state as, a and we label the exit state as, e. So, if you apply impulse function between these 2 states, I can calculate the thrust, it is not a problem, so thrust is equal to impulse function at the exit minus impulse function at the inlet, right. So, if I substitute for this, I get the following; so, impulse function is nothing but P + rho V square times A, right, so this is the impulse function and the inlet, this is the impulse function at the exit.

Remember, the mass flow rate through the engine, if I do not take this into account; m.f is usually much smaller compared to the air flow rate, we know that much, right. So, mass flow rate at any station is going to be rho times A times V, right, so the mass flow rate at inlet is going to be rho a times Va times this and mass flow rate at exit is also rho e times Ve times this. So, if i make use of that, I get the following.

I can write this in terms of mass flow rates like this okay. Usually, the mass flow rate that goes through the engine in this direction, I have mass flow rate is usually of the order of about 500, 600 or even more, 700 or 1000 kilogram per second, whereas the fuel flow rate is of the order of probably few grams per second, kilograms per hour grams per second approximately, okay so, we can easily ignore this and use this expression.

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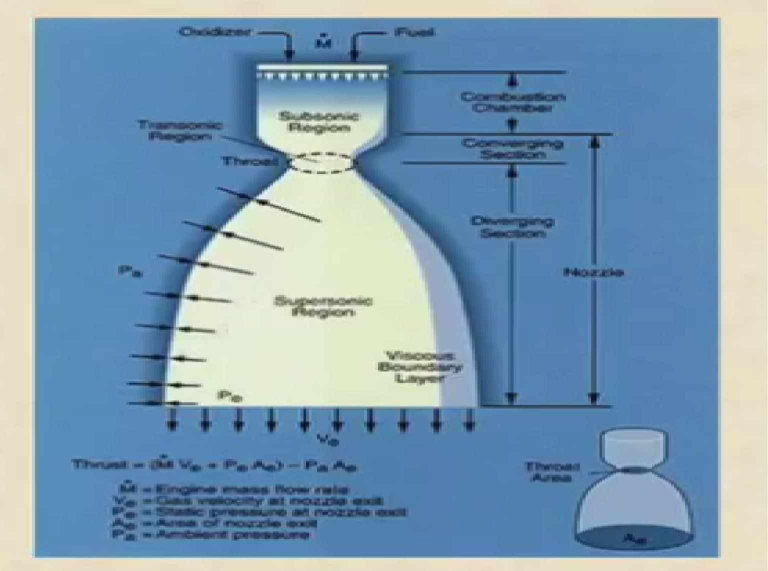

Now, you must keep in mind what we said earlier right, if you remember in our earlier discussion on thrust, then where we said that thrust can also be evaluated by integrating the pressure force on the metal surfaces and we showed this graphic earlier, we can see that the net force on the metal surface is going to be the internal pressure minus the ambient pressure, which is constant.

So, you can see that the ambient pressure vector remains constant here, so the net force in this is going to be this pressure minus the ambient pressure and I do this throughout okay, which means that at every point here, I need to measure my actual pressure with respect to the ambient pressure. I have to subtract the ambient pressure from this pressure throughout, right in order to calculate the net force on the metal surface, right.

So, we are calculating thrust which means that this pressure must be measured with respect to the ambient pressure; local ambient pressure; remember, local ambient pressure keeps changing and the engine takes off the ambient pressures sea level static, when it is cruising at 30, 000 feet it is less than that, so we subtract the ambient pressure from these 2 and if you do that, what do you get? This becomes Pe - Pa and this becomes 0, exactly.

So, this term disappears, so we end up with this equation, which is the general thrust equation for a turbojet engine; single stream engine where we have assumed only one stream here, okay.

So, there is only one stream that enters and leaves okay, so you must be very clear about why this term disappears and that has to do with the fact that net force on the engine is always the pressure force after subtracting the ambient pressure that is the net force, okay.

Any questions, what we will do in the next class is develop this expression, look at this expression closely, analyse it, draw some inferences and then write a similar expression for a turbofan engine also. Remember, turbofan engine has 2 streams; one is the bypass stream or cold stream, the other one is the core engine stream or hot stream. So, we will develop an expression for the turbofan engine also and then we will keep everything ready.

In the next module, we will try to do thrust calculations, evaluate the thrust for actual engine using these expressions.