Gas Dynamics and Propulsion
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Lecture - 23
Oblique Shock Waves/Prandtl Meyer Waves

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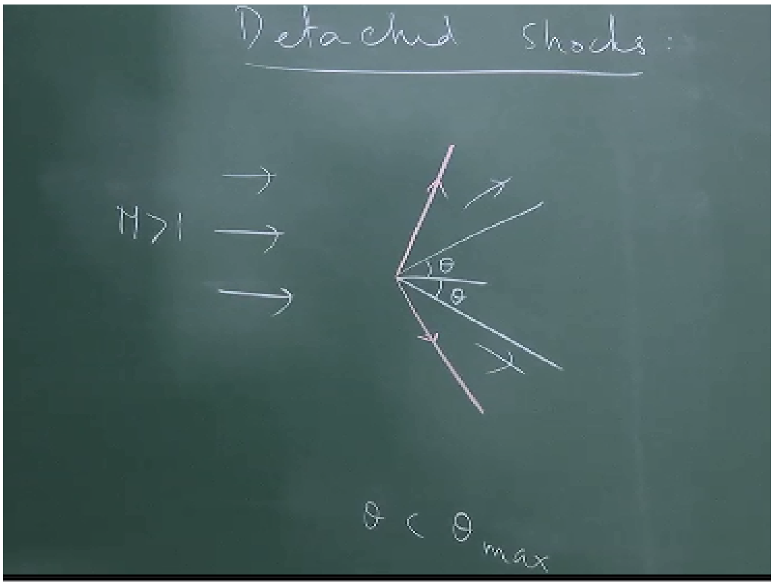

In the previous class we started our discussion of the structure of Detached Shocks and we looked at 2 situations both involving flow over wedge. In the first case, the semi vertex angle of the wedge theta was < theta max corresponding to the freestream mach number. So supersonic flow approaches the wedge and in this case we said that the semi vertex angle theta is < theta max corresponding to this mach number.

And in this case we saw that there is going to be an oblique shock which is attached to the vertex of the wedge and the flow is then deflected through this. So this is the direction of the shock wave and the flow then passes through and goes like this. In the second case, we said that the semi vertex angle is more than or > theta max corresponding to this mach number.

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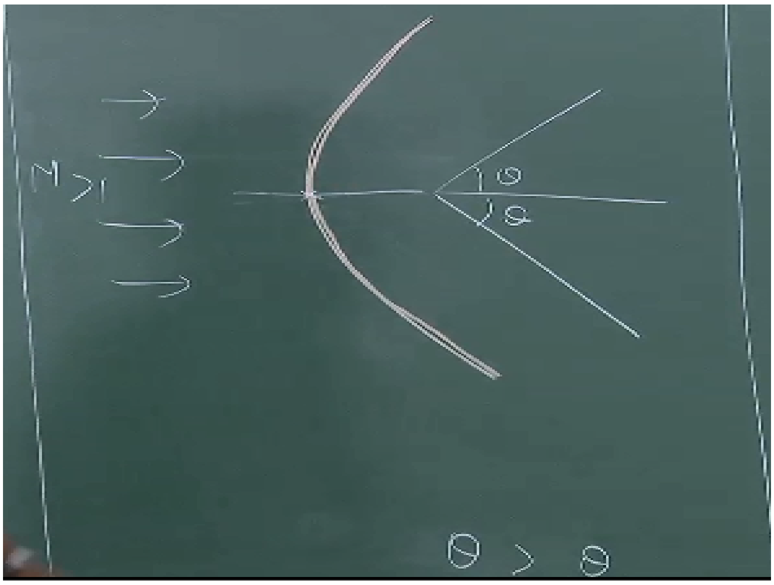

So we had a situation where we were looking at something like this. So the semi vertex angle theta is now > theta max corresponding to the freestream mach number and in this case we had said that was going to be detached shock like this and we had also qualitatively inferred that the strength of the shock is the maximum here and becomes weaker because far away the freestream does not even know the presence of the wedge.

So that means that far away the freestream just goes through without any problem. So this should be very strong here and the strength should decrease as we go outwards eventually the strength will become vanishingly small or this will become a like an acoustic wave that was the one inference that we do. The other thing that we said was far away from the wedge the flow deflection angle was 0, the flow does not even know that the wedge is there.

And at this point also the flow deflection angle is 0, but the flow deflection angle is non 0 in this location. So we inferred from this that the flow deflection angle starts at 0 increases probably reaches a maximum and then again decreases to become 0 as we move outwards.

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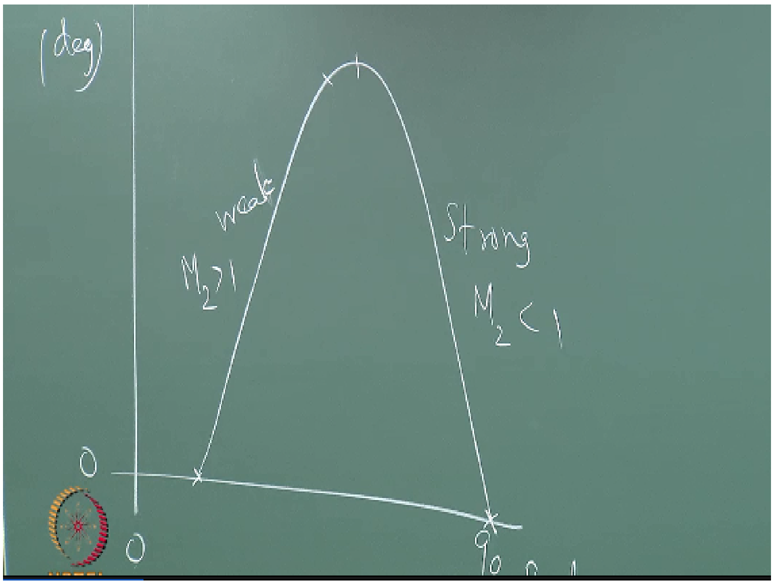

So based on this we can correlate this with for example the theta beta M curve corresponding to this mach number. So let us say that this was our beta in degrees and this was our theta again in degrees. Now if I draw the theta beta M curve corresponding to the freestream mach number that curve should look something like this. So this is theta=0 and this is beta=0 and this is beta=90.

So this is the theta beta M curve corresponding to this mach number. Remember the important point here is this is the theta beta M corresponding to this mach number and represents the locals of all possible solutions for a attached shock waves, but here the shock wave is detached. So whatever we are going to say is going to be only qualitative not quantitative.

But by enlarge you know the qualitative inferences seem to be accurate when you look at solutions obtained using a very complex CFD calculation for example. So inferences are qualitatively consistent and accurate which is why we are going through this exercise. So if you look at this curve if I start from here the flow deflection angle here is 0 and then as I go along this the flow deflection angle increases reaches a maximum then when I come down the flow defection angle decreases and becomes a 0 here.

And that was exactly what we said was happening along this curve what we said should happen along this curve that the flow defection was 0 here and 0 there so reaches a maximum somewhere in between. Let us see where this is consistent with the other inference which we said about the strength of the shock wave or loss of stagnation pressure and again you noticed that the loss of stagnation pressure or the strength is very high at this point then as I move along this.

This is the strong shock branch it becomes weaker and continuous to become weaker. Once I move into the weak shock solution and then becomes the weakest infinitesimally weak when I reach the mach wave solution or the Isentropic wave solution. So this is the strong shock branch and this is the weak shock branch. So with regard to the strength of the shock wave, strength of the shock wave decreases continuously from strongest here to weakest here.

Flow deflection angle is also consistent with what we had inferred. So we can say that from here this point of the shock wave represents a normal shock solution and as I move here is going to be some part which is part of the strong shock and there is going to be some part which is going to be the weak shock solution. This is the qualitative inference this is how we expect this to be.

So you can see that we are actually able to see a strong shock now. Previously we said that you know attached strong shocks are seldom seen probably never seen, but detached strong shocks can be seen and or seen in many real life applications. As I said when a missile or a space vehicle reenters the atmosphere the bow shock ahead of this will have a strong shock portion and a weak shock portions, but this has other implications.

If you remember we said that for the strong shock solution M2 is always < 1 and for the weak shock solution, we said that M2 is almost always > 1 almost always except for a small portion here. M2 here is almost always > 1.

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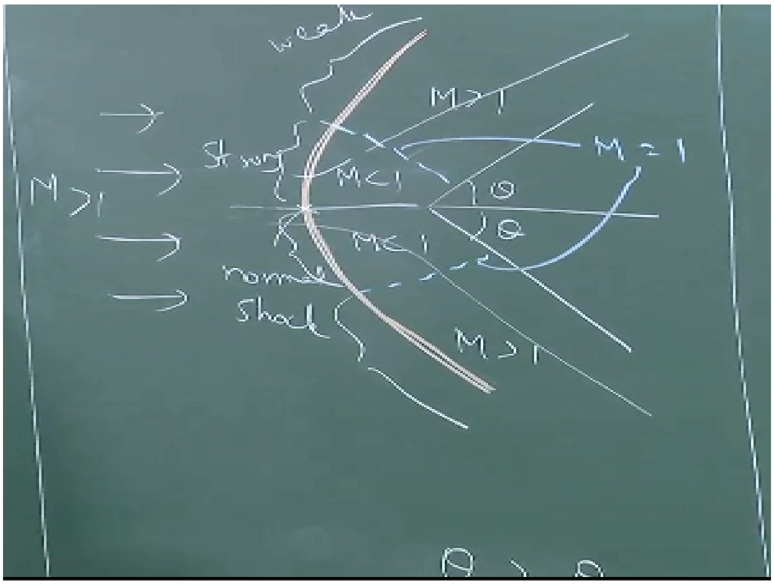

So what that means is that the flow when it comes across this part the M is going to be < 1 here and the flow when it comes across the weak shock portion the M is > 1 which means that there is a sonic line in between these two. So I can draw a similar thing on this side also. So we will have a strong shock here and weak shock here. So M is < 1 here and M is > 1 here which means that we are going to have something like M=1 sonic line. So this represents M=1.

So the pressure increases across this static pressure always increases, but then the flow is actually accelerating when it goes across a surface of the wedge in this case. So subsonic flow accelerates and attains a supersonic velocity eventually it attains the freestream if you terminate this in some fashion.

We really cannot go into that part because we need to know how this is terminator. So that is a much more complicated thing, but you can see for now that the flow deflection has to be very large here and the flow deflection than gradually eases out and it becomes parallel to the surface of the wedge so same thing here, but what is interesting is that the flow actually accelerates when it goes across like this.

Across the surface of the cone that is something that you probably would not have expected to begin with. So but these inferences these are only qualitative inferences and this are actually reasonably accurate when you do a (()) (08:39) dissimilation of this particular flow and because this occurs in many applications you can get very good idea about pressures rise across the wave and other flow features just based on this.

The important point is you see this kind of strong shock solution only in this type of situations not attached strong shock solutions. Any questions?

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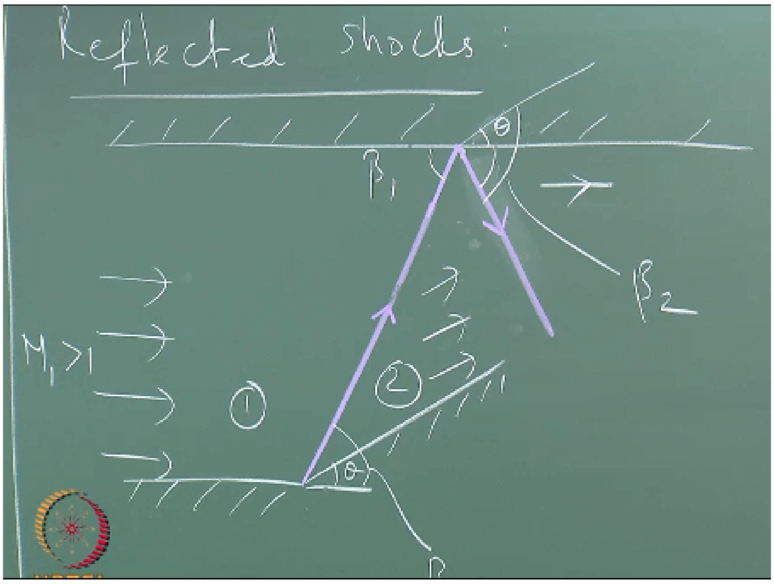

So the next topic that we are going to look at is Reflected Shocks what happens when an oblique shock wave impinges upon a surface. So far we are talking about the corner like this let say at certain angle theta. So we said supersonic flow let say M1>1 approaches this type of the corner and we had an oblique shock wave that was generated at the corner and then the flow went like this.

So let us call this 2 and let us call this 1 before this and we looked at the flow and how the shock wave is deflected and so on and this angle was denoted as this angle in our usual notation is denoted as beta 1. What happens when this kind of an oblique shock is triggered inside a duct? In other words, let say I have a wall like this. So we have a duct in which we have this corner and what happens when this shock goes all the way.

So remember there is also a direction to this shock wave and let us indicate that explicitly. So this is the direction the question is some of the questions are easy to ask, but the answers are not obvious. The question is how does this shock wave come back, does it come back if it comes back how does it come back? Is it like a reflection from a mirror, but the angles are same or not those are the things that we need to discuss.

Let us see what happens let us say that I do not know how the shock comes back. I do not know whether it comes back as a shock whether it comes back as some other wave. Remember there are many wave solutions that are possible. Let us say that it comes back as something, but I know for sure that the direction is going to be this on this wave. Let us say this is an unknown wave, but the direction is known and I also know that the flow after it passes through this what is the direction of the flow?

The flow has to be parallel to the top wall you know that for sure. So the flow direction has to be this. So the flow goes into the wave like this and then comes out like this so that means it is deflected towards the wave. So that tells me that this is going to be an oblique shock wave. So now I can remove this and say that this is going to be reflected as a shock wave so that answers my first question. It is reflected and it is reflected as a shock wave.

You may wonder why this is so obvious it has to be come back as a shock wave. In the next chapter when we look at reflection from a constant pressure boundary like a jet boundary. You will notice that it does not come back as a shock wave it comes back as something else. So how it comes back is determined the fluid dynamics of the problem that we are looking at. So in this case because the flow has to straightened out parallel to the wall it has to reflect as a compression wave because we need to deflect the flow towards the wave.

So now the question is what about the angle. So we know that this angle is beta 1. What about this angle what is this angle equal to, is this equal to beta 1 or is it something else that is the next question that we must answer and remember in oblique shock wave the angles are always measured with respect to the velocity vector ahead of the shock wave. So if I look at this, this is the velocity vector ahead of this shock wave.

So that means this angle is theta 1 and what is the wave angle now. The wave angle is theta+ this angle. So this is the new wave angle beta 2 and M2 <M1.

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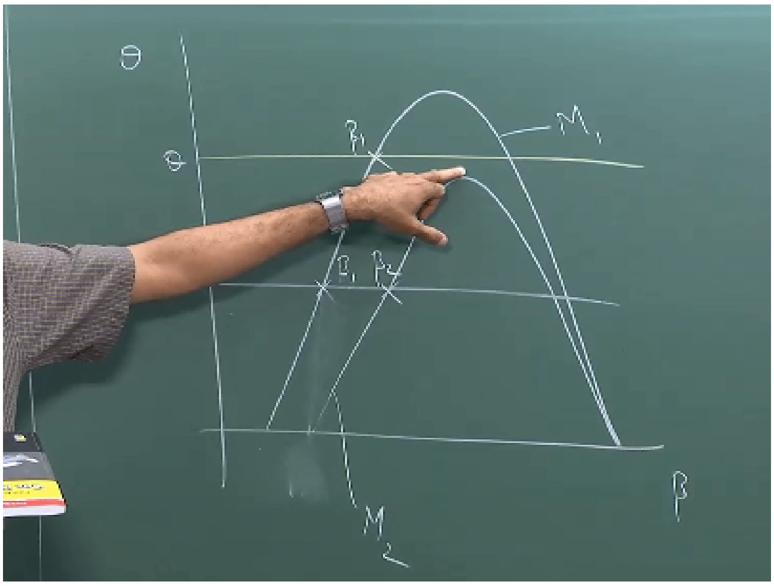

So if I go to my theta beta M diagram let us say that this is the curve corresponding to M1. The curve corresponding to M2. M2<M1 so the curve corresponding to M2 is going to look like this. What is the flow deflection angle for the second shock wave same as the previous one? So if I draw horizontal line let us say this is my flow deflection angle theta in this particular problem the previous situation at this as my beta 1.

So this is a little bit confusing so let me write it like this. So let me call this beta 1. So I want the same deflection angle. but now the mach number has reduced which means that my deflection angle is now going to be higher. So I am going to pick up this solution for the same angle. This will be my beta 2 which tells me that this reflection is not necessarily going to be at the same angle that angle is determined by M2 and the fact that theta is a constant.

So if everything is okay then this shock is reflected back this way I can actually calculate beta 2 from my theta beta M table there are no issues. So this is what is called a regular reflection. So we have an oblique shock it is reflected back as a regular oblique shock wave with the different wave angle, but there can be also be some difficulties with this reflection that is also another type of reflection.

Notice that the way the theta beta M curves are and the way the theta beta M curves are because we are operating at let say this is the deflection angle and this was my initial wave angle. Now this is my new wave angle. M2 decrease so everything was okay. What happens if my initial flow deflection angle is like this? This is my initial flow deflection angle theta. So I increase this angle it is still < theta max for this mach number.

So this would be my initial beta 1, but the mach number decreases $M_2 < M_1$. So you see that regular reflection is not possible in this case because there is no solution we do not get a beta 2. So regular reflection is not possible in this case.

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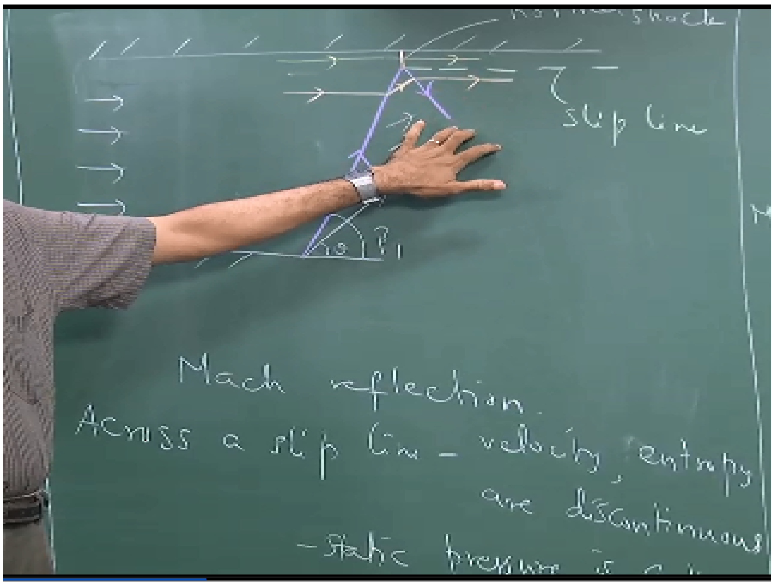

So what happens in this case is that we get a flow which looks like this. So the first oblique shock can be generated without any difficulty. This is theta and this is beta 1. So the first shock wave can be generated without any difficulty and the arrow on this looks like this and the flow is like this, but then that curve tells me that regular reflection is not possible. So what happens in this case is that you generate a normal shock here close to the wall.

And then the oblique shock comes back like this. So this is a normal shock. This type of reflections is known as a Mach reflection. So an attached oblique shock solution is not possible in these cases. So we have a normal shock wave then the flow comes back like this, but when you have these types of situations notice that if you take a streamline which goes through there is no flow turning here.

There is no flow turning across the normal shock wave so if you look at the flow which goes through like that and another one which goes through like this. So this is the streamline which goes through like this deflected this way and then deflected back again this way. So this is the other stream line. If you look at the entropy change across this one, the entropy change is going to be higher because it goes through a normal shock.

But if you look at this one the entropy change is going to be less because it has gone through two oblique shocks which means that in these types of situation you get new structure called a slip line. A slip line is a discontinuity in the flow field. Normally velocity is discontinuous entropy is discontinuous, but pressure is continuous across the slip line. So across the slip line velocity entropy or discontinuous. Normally static pressures is continuous and temperature too/

"Professor - student conversation starts" How come static temperature can be seen because normal shock will have a higher temperature. **"Professor - student conversation ends"** No static temperature normally in real flow they are going to have other mechanism which will cause the temperature to equilibrate, but from gas dynamic solution static temperature will also be discontinuous yeah that is correct.

That is determined by where you are operating in this curve. How far away you are from this theta max the further away you are the greater the extent of the normal shock will be because if you operate if this theta is such that you know let say you are just over here then still attach shock solution are possible. Let say you increase it slightly yes just barely so that means you are now you are beginning to get the normal shock portion.

So the further away you are from the theta max corresponding to M2 the greater the normal shock portion. That is a good question. Normally what is done is if you want further reflections then you do something if you want to terminate the shock how do we terminate the shock. You can terminate the reflections if you do not like them you can do the following. You adjust this angle in such a way that after this the flow is completely straightened out.

And the shock is terminated. It is possible to do that. So in some cases you may want several reflected shocks to achieve further compression before you go to the next components. So depending upon what you want you can either terminate the oblique shock at some point or you can also have multiple reflections. So if you want to terminate then this theta can have only a certain value so you can determine that from this combination and then do that.

One of the exercise problem asked you to do that. **"Professor - student conversation starts"** Reflected shock exactly come at the point where we turn it. **"Professor - student conversation ends"** Yes so the reflected shock comes and impinges at this corner. Remember if you look at this corner we are going to see this in the next chapter what type of a corner is this. Generally, if you look at the flow.

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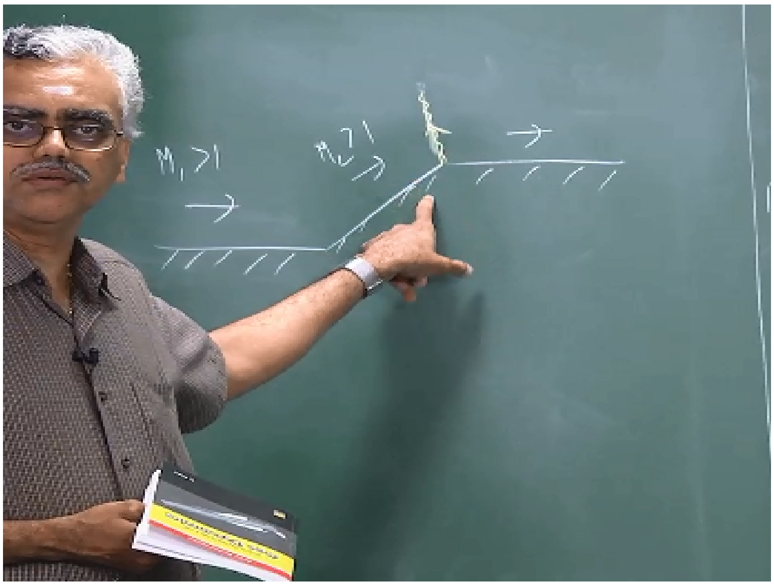

Let me draw this separately so that it is not confusing. We have to look at the corner in isolation. So if you look at this so I had a flow which came like this I have a flow which comes like this still supersonic. So M is still > 1 M1 is > 1 so the flow has to go around this corner now which means that I am going to generate some wave. I do not know what that wave is. I did not assume anything about that wave.

I am going to generate some wave from this corner and add some angle. Let us say some wave like this, but I know for sure that the direction of the wave let me change the color I do not want to have the same color as the other one let us use yellow. So I am generating some wave here of an unknown type, but with a direction like this. So after passing through the wave the flow has to be like that is our intent.

We do not want any reflection and we wanted to be parallel to the surface. How are we turning the flow in this situation? We are turning the flow away from the wave. So whatever the wave is it has to be an expansion wave. So that is what happens if you look at this corner in isolation.

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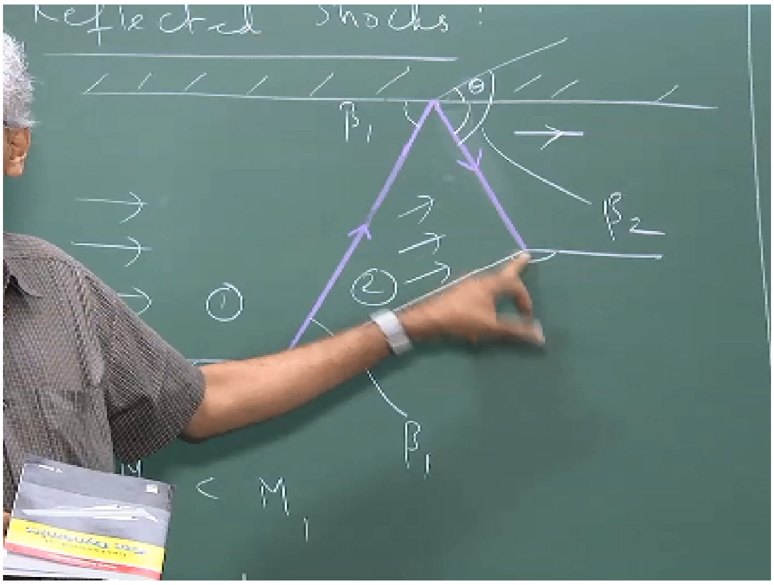

Now you look at the corner in combination with this one. So you are generating an expansion fan here and an oblique shock is also incident upon this corner so the two cancel each other and we get a flow free of reflections. That kind of design has to be done very carefully it is always done in intakes in fact we will go back and revisit one of our worked examples you will see that definitely happens in practical intakes.

And this will have lot of implications later on such reflections will have lot of implications later on when we look at for example fan blade we are going to look at propulsion that is our objective. So when we look at fan blades we will notice that you know we are operating with the transonic fan blade which means part of the fan blade has to dissimulate and compress a supersonic flow in the blade passage.

So that means the passage has to be designed properly and it is very difficult to avoid such expansion corner. A mach number increases across the expansion corner which means I have to bring it down again which means loss of stagnation pressure. So the design is very tricky in these types of situations, but a knowledge of gas dynamics is very, very important. So people have designed very clever blade profile for fans where you have controlled diffusion.

So that you avoid these types of problems such sections are called controlled diffusion air foil CDA and we will look at that when we talk about fan blades because gas dynamics we remove. We started the lecture by saying compressibility effect or significant in inlet fan compression, turbine and nozzle that is why we are going through in such details. Everyone of this concept that you learn here will be very useful later on when we look at propulsion.

So technically that completes the discussion of this chapter what I wish to do now is to continue the worked example that we looked at in the previous classes. I was kind of unhappy with the way we left it.

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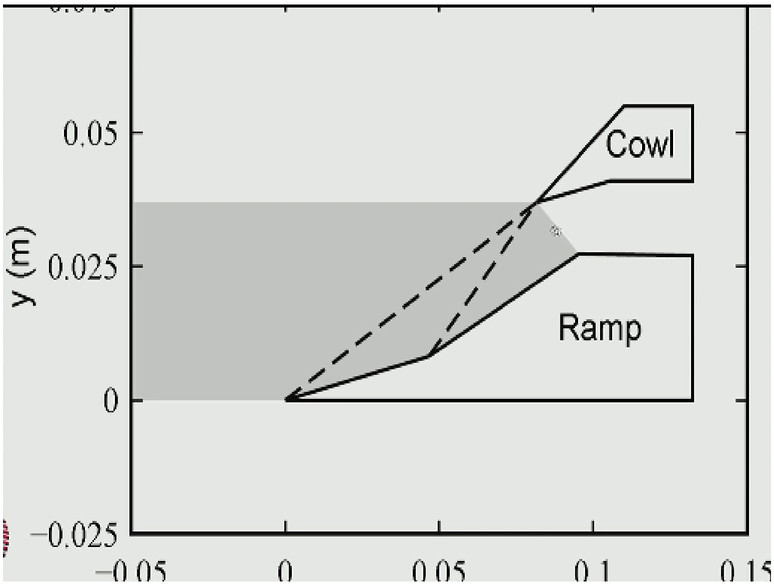

So I am going to revisit this worked example and we said that normally there will be terminal normal shock here before we complete this compression process. Let us go ahead and do that continue the worked example with a normal shock here and then look at the stagnation pressure recovery in the intake as the result of this. So that will just take a few minutes let us go ahead and do that.

One of the important things that you should learn from this chapter is the theta beta M curve. You should be able to draw the curve completely and know every aspect of the curve that is an extremely important curve in real applications because the possibility of solutions, detached shock, attached shocks and other things variation and so on. All of them are tied to that single curve.

So you should know the behavior of the curve, should be able to reproduce the curve at any time.

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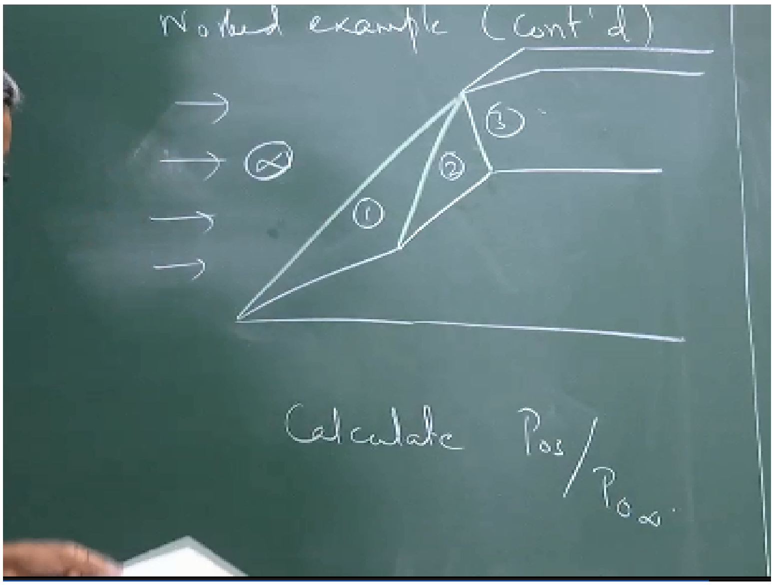

So let us continue our worked example. So what we are doing is the following. So this was the intake that we looked at in the previous class. Now I have added a normal shock at the end of the compression process so that the flow become sub sonic as it goes inside the intake and we want to look at P03/P0 infinity to see how affective the shock wave compression process is.

Once again notice that in this case also there is an expansion corner here so the flow will actually try to accelerate around this expansion corner that is something that you have to deal with because we are trying to decelerate the flow, but there is going to be local acceleration around this so we try to design things in such a way that this is minimized. And we are still operating with the quasi-one dimensional or slightly better than quasi-one-dimensional assumption.

So this is the best that this theory can provide. So let us go ahead and have a terminal normal shock and calculate P03. So we used to calculate P03/P0 infinity and let me just quickly recap whatever we had calculated in our previous class.

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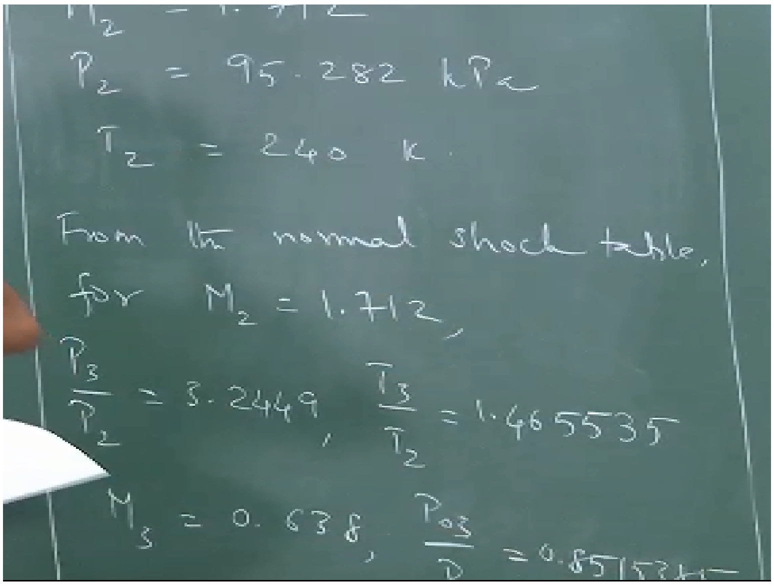

So if you remember we had calculated our M2 so was 1.712 if you remember and P2 was 95.282 kilopascal and the T2 was 240 Kelvin this is static temperature. So now we are going to have a normal shock at 1.712. We go to the normal shock table from the normal shock table for M2=1.712 so this will go into the normal shock table as M1. So we go to the normal shock table with this value of mach number and we retrieve the following quantities.

We have P3/P2 static pressure ratio 3.2449 T3/T2 1.465535 and M3=0.638 and in this case because it is a normal shock wave I can also retrieve P03/P02 0.8515385. So we can see that the mach number finally becomes subsonic after the normal shock wave.

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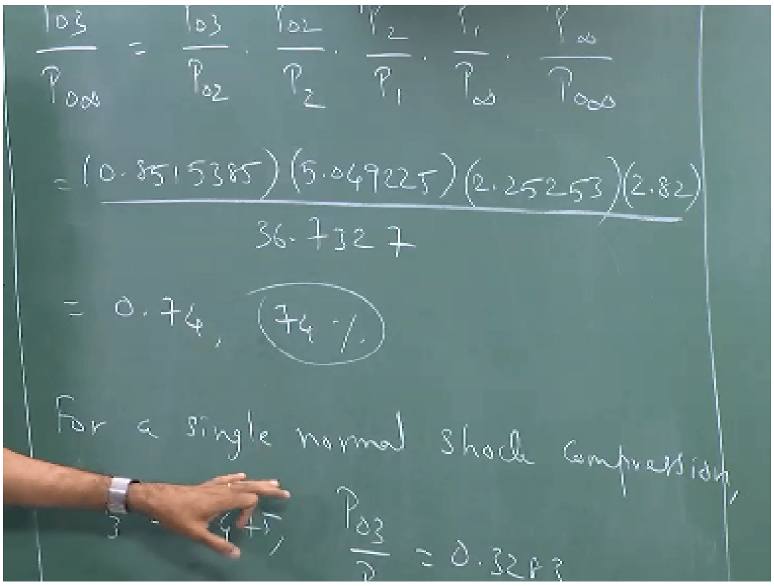

So P03/P0 infinity can be written as P03/P02 times P02/P2 times P2/P1 times P1/P infinity times P infinity/P0 infinity. We have calculated all these values during our earlier calculation.

So let me just substitute the numbers 0.8515385 times 5.049225 times 2.25253 times 2.82/ P0 infinity/P infinity we will go into the denominator as 36.7327. So this quantity is P0 infinity/P infinity.

So if you calculate this comes out to be 0.74 or 74%. And remember M infinity was=3. So M infinity=3 in this case. So from M infinity of 3 we have decelerated the floor to a mach number of 0.638 and in the process we have lost about 26% of stagnation pressure this is called the stagnation pressure recovery 74% we have lost 26%.

The same compression if we are done using a single normal shock we can check that also if you do the same compression using a single normal shock at mach 3 for a single normal shock compression we get the final mach number M3 to be 0.475 and P03/P0 infinity comes out to be 0.3283 approximately. So we can see the tremendous difference that the oblique shock compression makes.

So here we are losing only 26% of the stagnation pressure here we are losing nearly 68% of the stagnation pressure. So you can see how much more better this is if we can do it in a stable manner this intakes are not without that problem and that is something that we will discuss in the later part of the course when we talk about supersonic intakes, but this demonstrates very clearly how much better the oblique shock wave compression process is.

And for this reason alone I wanted to continue the example and finish it off today. So now we go to the next chapter.

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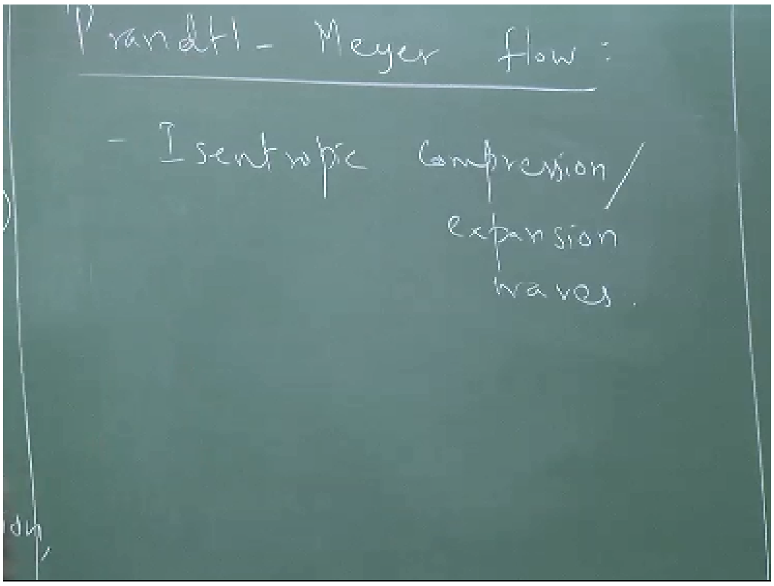

This is the last chapter we will discuss in gas dynamics Prandtl–Meyer Flow. So here we are going to talk about oblique expansion as well as compression waves, but waves which are Isentropic. So we are going to look at Isentropic compression/expansion waves.

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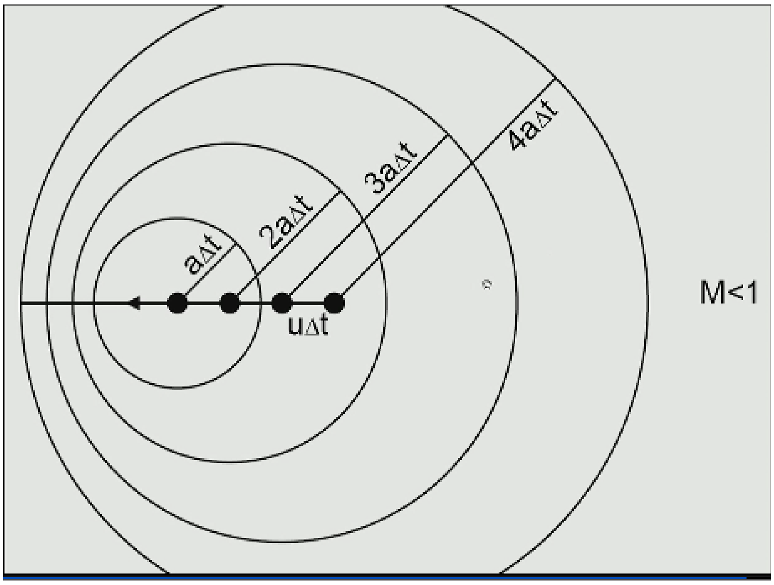

And to see how this waves were generated we start our discussion by looking at the following figure. So here we are looking at point disturbance okay this is a very small object almost like a point. So a point disturbance is travelling from right to left with the speed=u and I have sketched the location of this object at different instants of time as it passes through like this. So for every delta T increment in time the object goes from here to here.

So I have shown the object in increments of delta T. The object is travelling continuously, but I have shown this only in increments of delta T like this. So when this point disturbance starts moving the moment it start moving it starts generating these acoustic disturbances which travel outward as you can see with speed=speed of sound. So this was spherical wave front in 3D and this move out with speed=the speed of sounds.

And I have also shown this wave fronts at different instance of time. So for example if you are here then the wave front that this generator would have travelled A delta T in a time delta T and the wave front that was generated by this would have travelled to a delta T in the same time and so on. So you see this waves expanding outward, but a new wave being generated at every instant as the disturbances passed through undistributed fluid.

So this is how it happens so any observer who is located somewhere the observer can be located anywhere in this space. So any observer who is located here will actually hear the disturbance before he sees the disturbance. In some case you get to know of the disturbance even before the source arrives to the person. For example, a person standing here would have heard the disturbance long before that source actually rise at his location because the source is moving with the speed which is < the speed of sound.

So I can see that the source is moving at the speed which is less than the speed of sound. So the disturbances seen by the observer before the source of the disturbance actually arise. If I increase the speed of the source to a speed equal to the speed of sound, then you can see what happens. So the source as you can see the point disturbance is always just at the wave front of the wave that it generated delta T or any of the previous instances.

So the source is always at the front here and moving along with the wave front as it is generating new waves. but it is also keeping up with the waves. So in this sense an observer who is standing let say somewhere here gets to know about the disturbance and the source for the disturbance at the same time both arrive at the same time both the disturbance and the source that makes the disturbance arrive at the same time.

That is when the speed with which the disturbance moves through the still air= speed of sound. One very important point that you should always keep in mind here is that this type of scenario is possible only when the source of disturbance is a point disturbance. It is not an object of finite dimensions it is infinitesimally small point for example if a bullet moves through the air you will not get the same situation because it is a finite sized object.

If it is a finite sized object, then we are talking about oblique shock waves. This is an infinitesimal point object so we get this scenario only in this case so this is a very special case.

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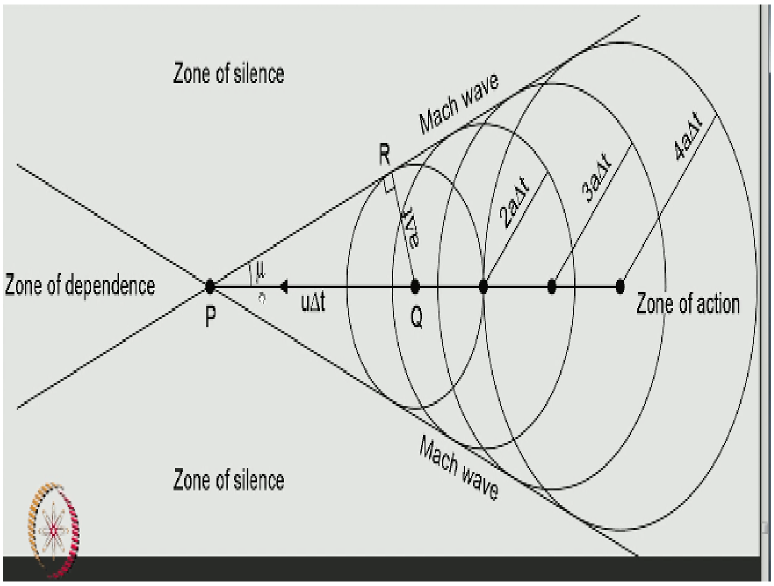

Now what if I increase the speed of the disturbance even more. Let say that I make it travel with supersonic speed then this is the situation I have. Let me rotate this wheel so that we can see this better. Now you can see that the disturbance is beginning to travel in such a way that the observer any observer who is located along the axis of this sees the source of disturbance passing by before information about the disturbance itself arise there.

So the observer as you can see from here observers who is located here will see the disturbance passing by, but any information about the disturbance that it has created will be received only after the source has passed by which means that the region in which the disturbance is going to be felt as a result of passage of this is confined to this cone shaped region.

So you see a cone here. So all the information about the disturbance is contained here anyone outside this cone will not hear anything, will not know that something has passed. The observer will come to know about the disturbance only after the observer get into the zone into the cone. In other words, the cone is going like this so when the cone passes over the observer that is when he actually knows about the disturbance, but the disturbance itself has long horn.

So that is why this is called the zone of action. Any information about the disturbance to this cone shaped surface in 3d flow. Nothing is known about nobody here will know anything about the source. This region is a region so the source itself is affected or the source itself can see only disturbances which are confined to this zone of dependence. So the disturbances are confined to this zone of dependence then the source can perceive or see only these disturbances.

If there is a disturbance that takes place here the source will be (()) (41:37) to these kinds of disturbances. In other words, if I want to influence the source in some way I need to make sure that I do it within this region and not outside this region. So I get distinct regions where information is available where information is not available and regions which allow me control and other regions in which I do not have any control.

So that happens when the source begins to travel with the speed which is greater than the speed of sound. So the source is always ahead of the waves that it produces. So in such a situation as you can see from here the surface of the cone is like a wave front. In addition to these wave front at each one of these surface of this spherical wave front is also a wave front. In addition to this wave front we also have the surface of the cone itself acting as a wave front as you can see from here and that is given the name mach wave.

So that is a discontinuous change in flow properties across the surface of the cone. So that itself actually behaves as if it is as wave, but because we are talking about sound waves and acoustic disturbances the changes across such waves are infinitesimally small and the process are Isentropic which is why we said that we are going to look at Isentropic compression and expansion waves.

So the process across this mach wave notice that the process across the mach wave can be a compression or an expansion process. In this particular case we talked about the source starting from subsonic speed and then accelerating to supersonic speed what about the source which decelerates from a supersonic speed to a subsonic speed. So the flow will be exactly the opposite and one case you get expansion and in other case you get compression both being Isentropic.

So we are going to look at Isentropic compression expansion solutions the solutions are called Prandtl–Meyer Solution the wave itself is called a mach wave. So we will pick this up in the next class and then discuss this in great detail.