Gas Dynamics and Propulsion
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Lecture - 22
Oblique Shock Waves

In the previous class, we have gone though worked example.

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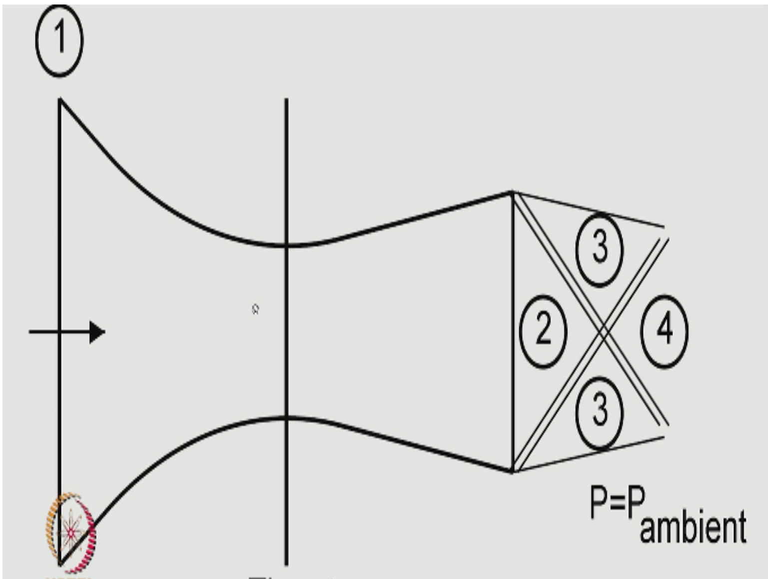

And that worked example is shown here in this figure. We had a convergent, divergent nozzle with the exit to throat area ratio being given and starting from state 1 we evaluated the mach number in the pressure and for the ambient pressure which was given be 100 kappa. We notice that for that ambient pressure the jet was over expanded. So oblique shocks are triggered.

So we evaluated the state properties at 2 then we also evaluated state properties at 3 then we recap this and after doing that we are going to evaluate state properties at 4 and that will terminate this worked examples.

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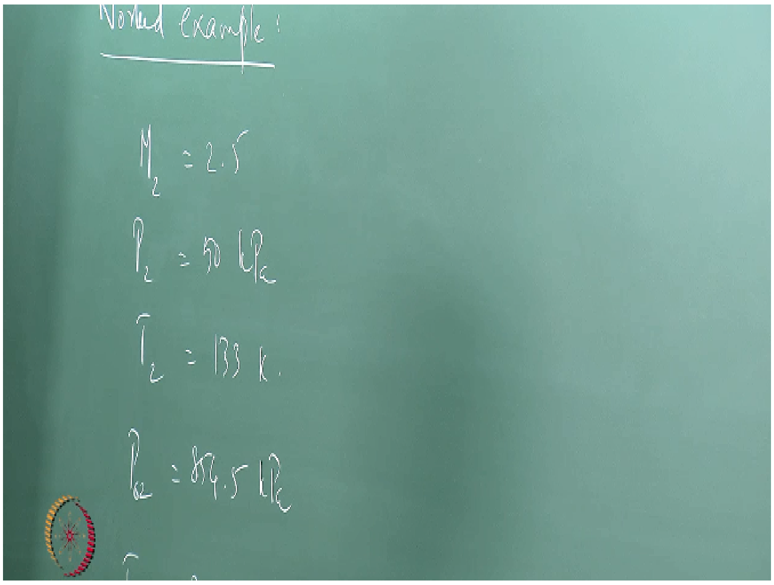

So we are continuing the previous worked example and if you recall from our last lecture we evaluated the following things M2 was calculated to be 2.5, P2 static pressure is 2 was 50 kilopascal and the static temperature T2 was 133 Kelvin and the stagnation pressure P0 2 was 844.5 and the stagnation temperature was 300 Kelvin. So this were the 4 properties that we evaluated at state 2 and for state 3 we calculated the state properties by noting the fact that state 3 is exposed to the ambient pressure.

So the pressure here P3 has to be=100 kilopascal. So that means we knew the pressure ratio and we knew M2. So this was somewhat unusual because if you remember we said theta beta M relation. So given either M or theta or M or beta or theta and beta any of the 2 we could calculate the third one. In this case we are actually seeing that you are given the static pressure ratio and M2 and we are calculating theta and beta so that is what we did.

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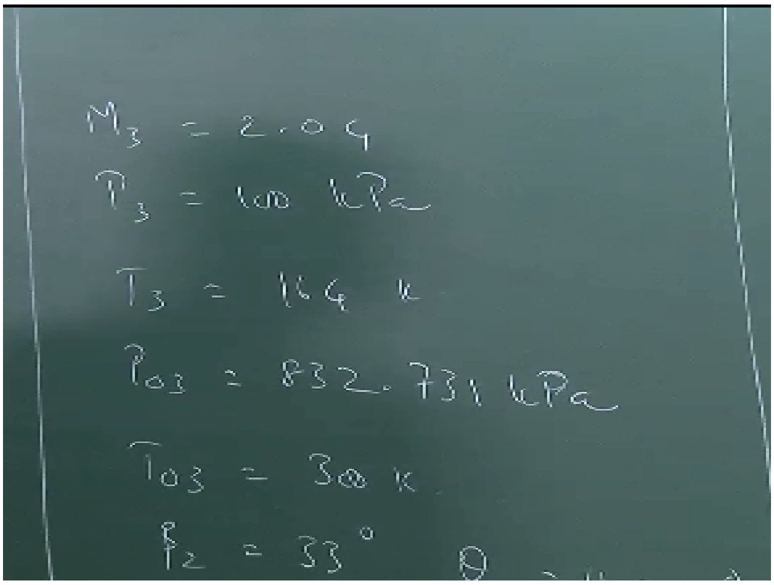

So P3 was known so let us write that P3 was 100 kappa and M2 was known so we calculate M3 to be 2.04 and we calculate T3 to be about 164 Kelvin P03 was evaluated to be 832.731 kilopascal and that T03 of course was 300 Kelvin. And if you recall we calculated beta 2 to be 33 degree and theta 2 the flow deflection angle or the angle through which the jet is deflected was calculated to be 11.2118 degrees.

Now we wish to evaluate state properties at state 4 and before we do that we need to actually understand this angles very clearly.

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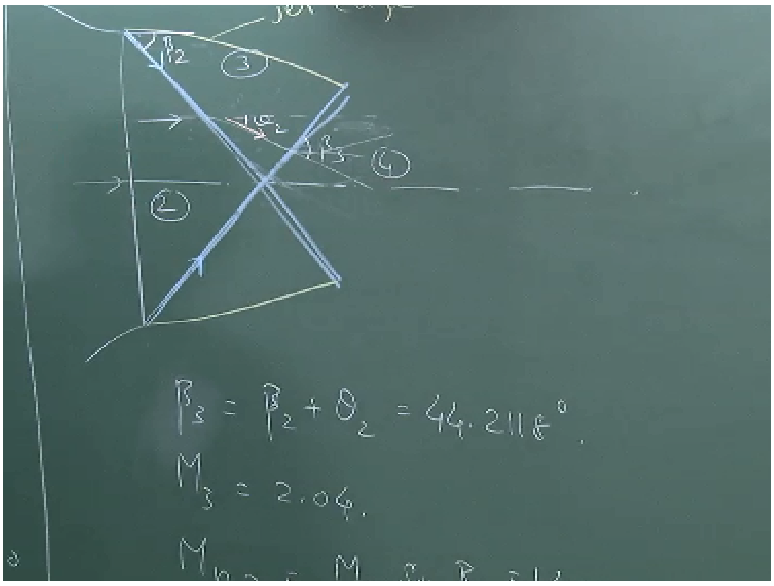

Let us draw an enlarged view of the jet as it comes out. So let us say that this is the exit and the fluid comes out like this. So this is the closer view of the exit from the jet we label this region as 2 and oblique shocks generator from here from the trailing edge of the nozzle and this was labeled as region 2. Now if you look at the flow vector ahead of the shock wave it looks like this.

Remember the direction of the shock waves itself in this case is like this. So after passing through the shock wave the flow is deflected towards the shock wave by an angle of 11.2118 degrees. So that means the velocity vector in region 3 would look something like this. So the velocity vector looks like this in region 3 and this angle is theta we calculate this as theta 2 and we said that the edge of the jet is going to look something like this.

So this is the jet edge. Now notice that angle beta 2 is also measured with respect to the velocity vector ahead of the shock wave. So that means since the velocity vector ahead of the shock wave is like this, this angle is going to be beta 2 which is=33 degrees. So when the flow into region into region 4 which is over here which way is the flow going to be deflected that is the first question.

Remember it is a shock wave so the flow is deflected towards the shock wave so what is the turning angle for the shock wave what is theta for the shock wave. What is the wave angle for region 4? Wave angle is measured with respect to the velocity vector ahead of the shock wave. So the velocity vector ahead of the shock wave is inclined like this. So that means the wave angle has to be measured so this is the wave angle beta 3.

The wave angle is clearly known and in this case we really cannot say that the flow is going to be deflected back to the horizontal. All we can say is that the flow is going to be deflected towards the wave. We know beta 3 clearly so let us write beta 3 and beta 3 is known so beta 3= beta 2+theta 2. So beta 3=the value for this is going to be 44.2118 degrees. Remember M3 is known beta 3 is known.

So we have to calculate theta from this relationship see what theta is going to be whether theta is brought back to the horizontal or not is what we have to see. Are there any questions or doubts is this clear? Go ahead. “Professor - student conversation starts” How come beta 3 we do not know beta 3. “Professor - student conversation ends” We know beta 3 because remember this is the wave angle beta 3. “Professor - student conversation starts” After intersection of 2 shock there will be some other shock. “Professor - student conversation ends”.

No, remember what we said yesterday about intersection of shock wave we said we are going to idealize this and treat this within the framework of our oblique shock theory. All we are saying is if you look at this streamline it passes through the shock wave it is deflected like this and then after passing through this shock wave it is going to be deflected back again like this horizontal or this way towards this.

So beta 3 for this wave is measured with respect to the velocity vector before that wave is like this. So this is beta 3 and if we use the fact that this is beta 2 I can split this into 2 things. So this is my theta 2 and this is my beta 2. So beta 3 = beta2+ theta 2. **“Professor - student conversation starts”** But after intersection direction of shock will be sir something different that one will be beta 2. **“Professor - student conversation ends”**

No, that is what I am saying we are making an idealization that it does not change within the framework of what we are doing we are assuming that the shock wave goes through without any changes. **“Professor - student conversation starts”** Means flow is not parallel to original axis after that shock. **“Professor - student conversation ends”**

Flow need not become parallel to the original axis here. Intersection of shock waves is a much more complex phenomenon which we cannot deal with in our course. So what we are saying is we are neglecting any changes due to intersection of the 2 shock waves by assuming that this is another shockwave this will give us the very good idea of what is going on in such a situation.

So that is an assumption that we are making which I said before. **“Professor - student conversation starts”** If we assume deflection angle as theta 2=theta 3 then we can find out beta 3. **“Professor - student conversation ends”** You do not know theta 2=theta 3 because we know the mach number here and we are saying that there is no change due to the intersection of the shock waves.

So that means I know beta 2 this angle is known this angle with respect to the horizontal is known. So I can calculate beta 3 much more clearly than that is a much more general way of doing this then assuming theta 3 to be horizontal that theta 3 will not be horizontal some other process has to take place. This is much better within the framework that we are talking about this is the much better way of dealing with this problem.

The change in the wave angle due to intersection of the shock wave is also not very large that is well known from doing 2D calculations. So this is a much more accurate way of doing things than assuming theta 3 to be theta 2. “Professor - student conversation starts” P4=P ambient then that theta 2 will be theta 3. “Professor - student conversation ends” No, why are you saying P4 has to be= P ambient.

“Professor - student conversation starts” P4 is P ambient then after all the oblique shocks and if their exit is P4=P. **“Professor - student conversation ends”** No, that is the whole point the thing is why did we say P3 was P ambient here because the fluid was not direct contact with the atmosphere and the jet boundary cannot support a pressure difference. We are not talking about surface tension and so on.

So we cannot have a different pressure there and a different pressure here. Pressure has to be same across the jet boundary. It is a free jet. So it makes perfect physical sense to assume P3 to be P ambient. Here I cannot do this because this fluid first of all is not going to be directly exposed to the ambient their shock wave impinges upon the jet boundary and then it is going to be reflected in some manner.

That means this part of the fluid is isolated from the ambient. So this can be at a different pressure ambient can be of different pressure which is why P4 is not equal to ambient pressure. So the reflection of the shock wave from this type of boundary is something that we will do in the next chapter from a constant pressure boundary, but we can calculate flow properties in region 4 to a reasonable level of accuracy within the framework that we are dealing with.

Turns out to be very close to (()) (11:58) if you do 2D calculation it turns to be very close to 2D (()) 12:01). So I know beta 3 I know M3 remember M3=2.04. So Mn3= M3 sin beta 3. What is that this Mn3 is not the same Mn3 that we calculated before when we were doing calculation for region 3. We got Mn3 from normal shock table for going across this shock wave, but that Mn3 is a different value from this Mn3 because angles have changed now.

So this is the Mn3 that this wave is going to see or the normal mach number approaching this shock wave. So this= 1.4225.

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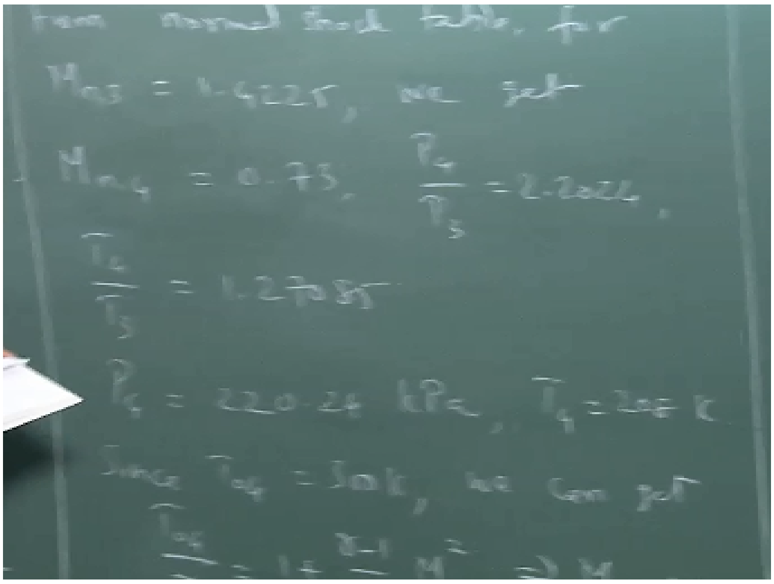

So now you go to the normal shock table from normal shock table for Mn3=1.4225. Remember we are now going to normal shock table with this value of mach number. In the normal shock table this would correspond to M1 in the normal shock table. So for this value of Mn3 we can retrieve the static properties and Mn4. So we get Mn4=0.73 and P4/P3=2.2024 and T4/T3=1.27085 which allows me to calculate P4 as 220.24 kilopascal.

And T4 as 208 Kelvin and since I know T4 and T04 since T04=300 Kelvin and T4 is also known I can get M4 as I can get M4 from the following relationship T04/T4=1+gamma-1/2 times M4 square which gives me M4 as 1.48.

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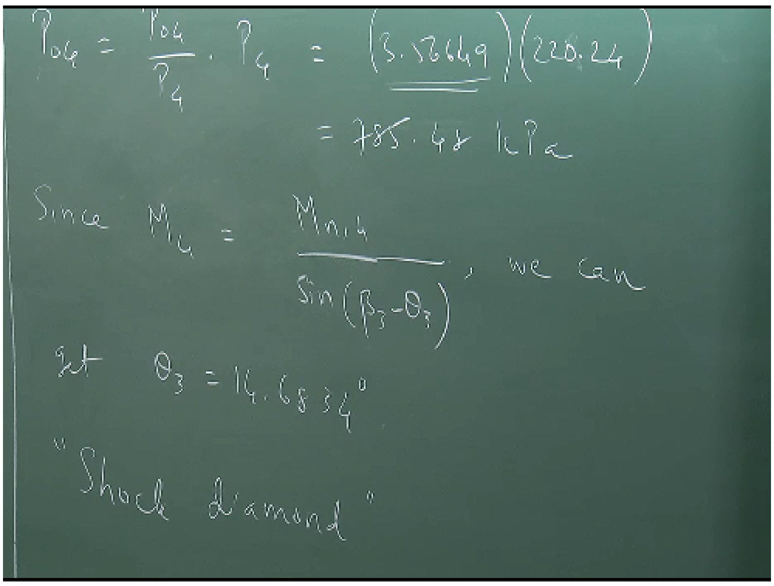

And now I can calculate P04=P04/P4 times P4 and the first number I can get from Isentropic table 3.56649 times P4 is 220.24 kilopascal. So this comes out to be 785.48 kilopascal. So this I have obtained from Isentropic table for a mach number M4=1.48. So stagnation pressure is also known only thing we need to calculate is theta. Since M4=Mn4/sin beta 3-theta 3.

I can get all the quantities are known M4 is known Mn4 is known beta 3 is also known. We can get theta 3 to be 14.6834 degrees. So what this shows is the following. First of all you can see that the pressure is not equal to the ambient pressure that is in fact more than twice the ambient pressure first of all. So what has happened is when we went from region 2 to region 3 we compress it to the correct value then because the shock comes through now it is over compressed.

So what do we have to do to equilibrate with the ambient now. So we have to have expansion facts. So that is something we will discuss in the next chapter. We will continue the solution once more to see, but what happens is you get this alternate compression, expansion, compression expansion for a few nozzle diameters or exit diameter and then eventually the jet becomes equilibrated with the atmosphere.

In fact, this alternate compression expansion is usually called the shock diamond. I will show pictures of the shock diamond later on. So you get this alternating shock waves and expansion fans and so on for several nozzles exit diameters. The other thing that you should notice your suggestion that we should take theta 3 to be=theta 2. Notice that theta 3 here has turned out to be more than theta 2 which means that after the flow goes through this.

Now it is actually deflective this way by 3 degrees. So from this theta to 11 point something degree would have brought me to the horizontal now I have over corrected so I have gone to this side which means I need to do something to bring it back, but it is not the velocity that I want horizontal I want the pressure to be equalized that is the most important consideration and if you actually do a full two-dimensional calculation you will notice the (()) (19:23) you get are almost identical to what this simple theory predicts.

What happening is (()) (19:40) if you go through the central line this shock wave is trying to deflect the flow in this way. The other one tries to deflect the flow this way so that it goes through without any problem, but what can happen in some of these situation is this interaction is little bit complicated. So entropy change for the fluid that goes through this may not exactly be the same the entropy changes for the fluid that goes through these two.

So you will get something called the slip line. So the flow is slightly more complicated then what we have sketched, but the complications are very small and most of the times for engineering application they are really not that relevant. Most of the quantities that we are predicting here will be almost the same is what we get from a much more complicated complication.

In fact, the actual theory what the complex theory would predict is when these 2 shocks impinge about each other like this the theory predicts that one will reflect from that shock and this one will reflect from that shock so that the angles will be slightly different. So this shock does not go through the other one, they do not go through each other. So the first one reflects from that point the second one reflects from that point.

So there can be a slight difference between the two, but the difference is very small and this theory is actually so simple it allows us good estimates of these types of quantities. We will continue this example when we go to the next chapter and talk about expansion facts. So we will take our next worked example which illustrates similar kind of things probably what is most important about this examples are the calculation of the theta and the beta.

Always remember that these angles are calculated with respect to the velocity vector which approaches the shock wave or ahead of the shock wave. So we will do another example which is also something which is used in many applications and then see how these calculations can be done.

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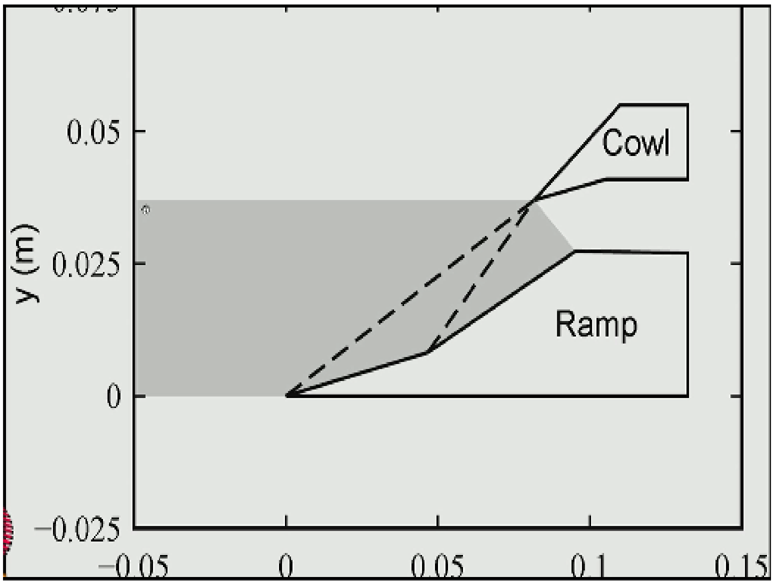

The next worked example looks at something called mixed compression supersonic intake for may be ramp jet engine. So you notice that these are 2 things here one is called the Ramp and the other one is called the Cowl. So the freestream approaches the intake this way and notice that when the freestream comes here that is one deflection of the supersonic flow here so the freestream is at the supersonic mach number.

There is one deflection of the supersonic flow by this part of the ramp which triggers an oblique shock like this and there is the further deflection of the supersonic flow here which decelerate and compresses the flow further. So there is under oblique shock this comes from this corner which further as I said decelerate and compresses and then the flow enters here normally there would be normal shocks which stands just here normally.

But for this particular problem we are not worried about that. So this is called a mixed compression intake because the flow is compressed externally and also compressed internally in a passage here. So we will see details of this later on when we talk about supersonic intakes. For now, this is the mixed compression intake that we are going to work with and let us see what the problem statement says.

This is the geometry and let us see what the problem statement says. The intake shown in the figure is designed for operation at M infinity=3 P3=15 kilopascal and T infinity=135 Kelvin. The Ramp angles or 15 degrees and 30 degrees respectively for the critical mode of operation determined the mass flow rate through the intake cross sectional area the beginning of internal compression and the total pressure recovery for the critical mode of operation.

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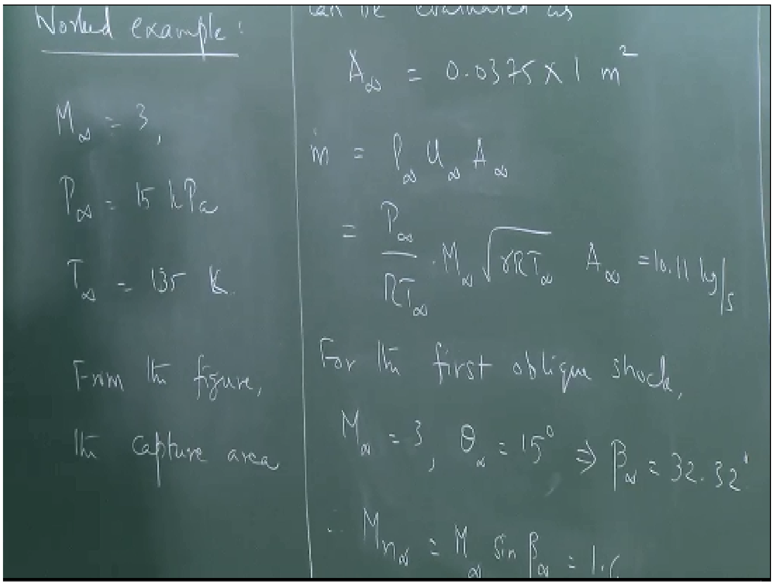

So let us write it down M infinity freestream mach number is given to be 3 P infinity is given to be 15 kilopascal and T infinity is given to be 135 Kelvin. A critical mode of operation refers to a situation when the incidence shocks as shown in this figure the 2 oblique shocks or focus on to the leading edge of the Cowl. So you receive both this oblique shock and this oblique shock focusing on a common point.

So this is referred to as the critical mode of operation so that is what we are looking at now. So that is the critical mode of operation that we are looking at. Now we have also shown the capture area of the freestream tube that enters the intake. Notice that if you take any stream line from here this oblique shock vector has a direction which goes like this. So flow comes like this is deflected towards the shockwave like this then it goes here it is deflected further towards the shockwave like this and then it enters the intake.

What I have sketched here in gray is the capture stream tube. So we can see that the area of the capture stream tube for unit width and the direction normal to the plane of the board I can calculate the area of the free stream captured tube, captured area of the free stream. So let us go ahead and calculate it. So from the given dimension from the figure the capture area can be evaluated as A infinity=0.0375 assuming unit width in the normal direction. So it is 0.0375 times 1.

So captured mass flow rate M dot can be evaluated as rho infinity U infinity times A infinity and if I write this as P infinity/R times T infinity and this as M infinity times square root of gamma RT infinity and this as A infinity. I know all the quantities P infinity T infinity M infinity A infinity everything is known I can evaluate this as 10.11 kilogram per second for unit width normal to the board.

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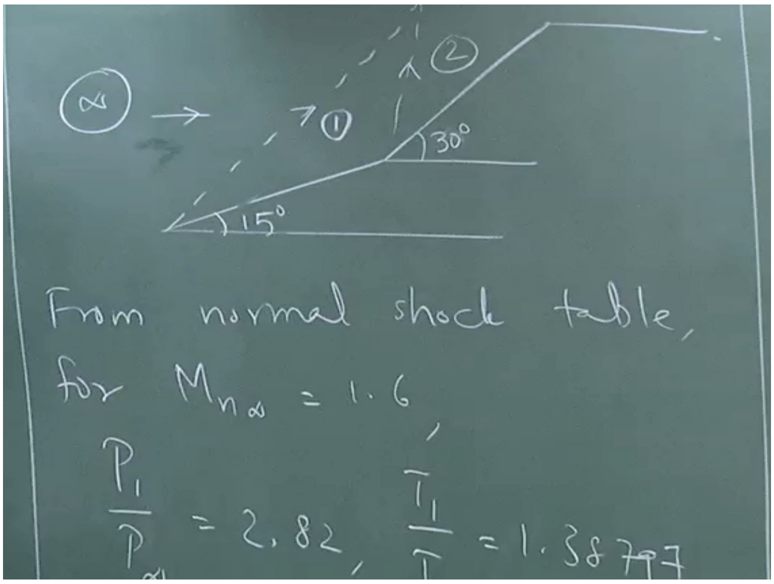

So let us make a sketch of the flow situation. So we have this is the first shock this is the second shock. So we are labeling this region as infinity. So this is how the velocity vector looks like ahead of this shock wave the direction in this shock wave is like this, the direction in this shock wave is like this. So we call this region 1 and we call this region 2. So this is the cowl which starts from there.

And the angles are given to be 15 degrees here for this theta and another 30 degrees here. So for the first oblique shock M infinity=3 theta infinity=15 degrees and from the oblique shock table this tells me that beta infinity=32.32 degrees from the theta beta M relation or from the oblique shock table we get this to be 32.32 degrees. Therefore, Mn infinity=M infinity sin beta infinity which comes out to be 1.6.

So we now go to the normal shock table with this value of M1. So from normal shock table for Mn infinity=1.6. Remember this becomes M1 when I go to the normal shock table so we retrieve the values from the normal shock table. So we get P1/P infinity to be 2.82 T1/T infinity to be 1.38797 and we get Mn 1 to be 0.668437. So the flow upon passing through this shock wave is deflected through an angle of 15 degrees towards the shock wave.

And we have obtained these quantities.

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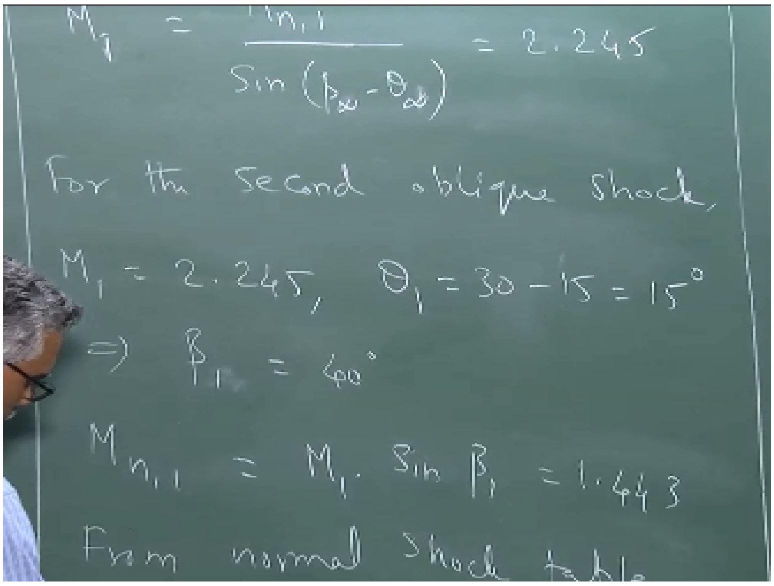

So once I know theta and beta I can calculate for example M1=Mn1/sin beta 1-theta1 and this I can calculate as 2.245. So the mach number went from M infinity 3 to M1=2.245. So for the second oblique shock wave the mach number approaching the shock wave M1 is 2.245 and what about the flow deflection angle through this shock wave. Remember this velocity vector is like this and the ramp angle is 30 degrees.

So as you can see from here the flow deflection angle what is the flow deflection angle going to be for this case 15 degrees from here it is deflected further this way. It is already deflected 15 degrees so that means it is going to be deflected another 15 degrees so that means theta 1=15 degrees, but let me explicit the choice of number is little bit unfortunate so let me be explicit and say that this is 30-15=15 degrees and this angle been let say 45 then this would have been 45-15=30.

It seems that I did not choice this number wisely so it is just coming out to be 15 again and is just a coincidence. So we know these values so from the theta beta M relationship I can get beta 2 for this shock wave. We already have beta 1 oh we already had theta 1 also or theta infinity and theta 1 we have beta infinity so that means this angle is going to be I am sorry this is beta infinity-theta infinity please change there. So now we have beta 1.

So beta 1 comes out to be 40 degrees. Now Mn1 for the second oblique shock wave Mn1=M1 times sin beta 1 and so this comes out to be 1.443. So when you do this calculation on your own you must be very careful about the notation. Remember Mn1 for the second shock wave is this whereas Mn1 from the previous one was calculated from the normal shock table. The two are not the same.

This Mn1 we obtained from the normal shock table post the first oblique shock. This is Mn1 pre the second oblique shock which means it has to be calculated like this. So when you do these kinds of calculations you must be very, very clear about the terminology, the notation and how you calculate these quantities. I do not wish to use a different notation for this that will be very confusing I do not want to do that.

So you must understand how we are calculating these numbers that is why I am writing all these things explicitly. So now from normal shock table for Mn1=1.443 so now this becomes M1.

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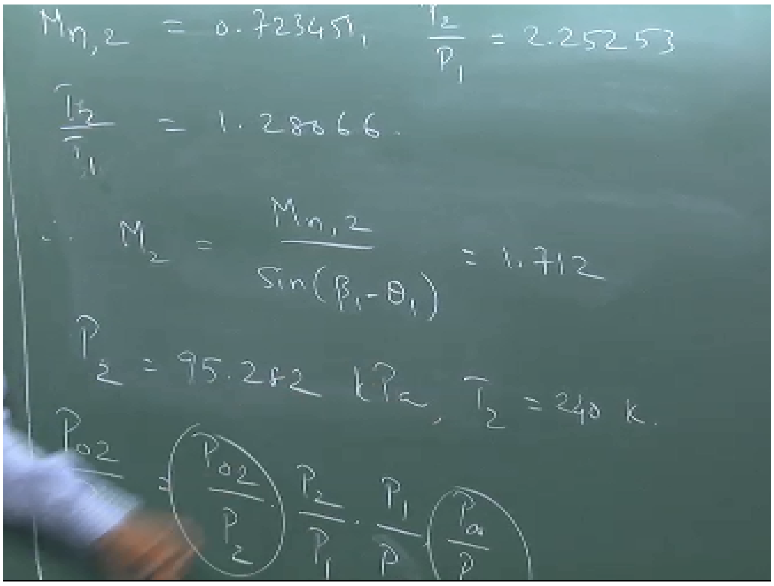

When I go into the normal shock table I get Mn2 to be 0.723451 P2/P1 to be 2.25253 and T2/T1 to be 1.28066. So we have retrieved the static quantities and the normal component of mach number downstream on this oblique shock wave from the normal shock table. Therefore, M2=Mn2/sin beta 1-theta 1 and this can be evaluated as 1.712. So the mach number went from freestream value of 3 to a value of 2.245 after the first oblique shock wave and then to 1.712 after the second oblique shock wave.

Now as we can see now we are ready to compress this using a normal shock the mach number as dropped below one which is why usually you will have a terminal shock standing at the entrance to the intake which will complete this compression process. 1.7 is low enough that I can compress with normal shock. It is not only affective, but also efficient as such low mach number below 2 to 1.7 is okay so I can do that.

So that is the intent of all the supersonic intakes to decelerate from freestream value to a value below 2 through a series of oblique shock waves and I get my P3 to be now I have P2/P1 so I can evaluate P2 as 95.282 kilopascal and my T2 to be 240 Kelvin. We are also asked to calculate the pressure recovery at the end of the internal compression process. So P02/P0 infinity=P02/P2 times P2/P1.

So we are looking for P02/P0 infinity so we do P02/P2 times P2/P1 times P01/ and one more did I do this correctly P2/P1 times let me do it like this now this is P1/P infinity I am sorry P infinity/P0 infinity. So I can substitute these values we have all this values and calculate this quantity. This can be obtained from Isentropic table. This can also be obtained from Isentropic table.

These two values we have already calculated using normal shock table so the internal pressure recovery the stagnation pressure recovery can be calculated usually it will be a number which is around 80% or so for this or maybe slightly less than that. One more thing that we are asked is to calculate the area cross sectional area at the end of the external compression process.

This we calculate by looking at the mass flow rate and equating the mass flow rate at this section to the free stream capture mass flow rate. So let us go ahead and finish the example.
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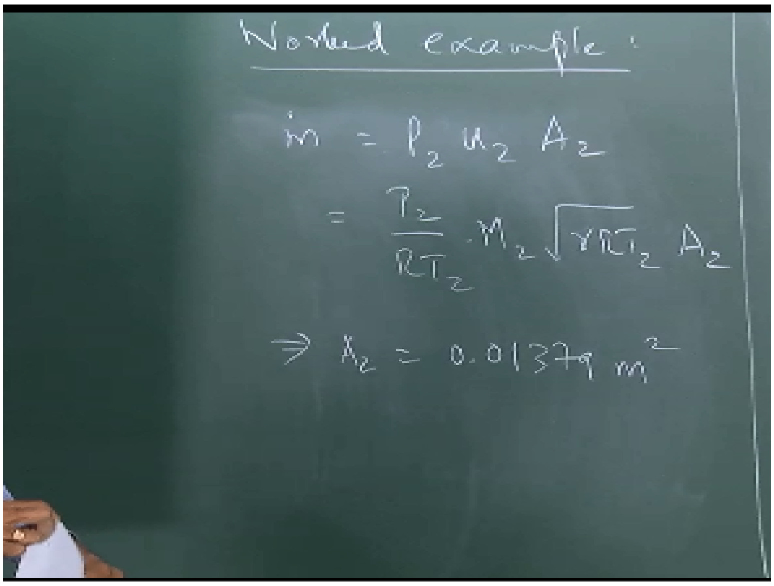

So m dot at the entry to the internal compression is rho 2 times u 2 times A2 and rho 2 once again can be written as P2/rT2 and this can be written as M 2 times square root of gamma rT2 times A2. M dot is the same as the capture mass flow rate. So M dot=10 kg per second. So from this I know all the values I can calculate my area to be 0.01379 meter square. So that completes the second worked example.

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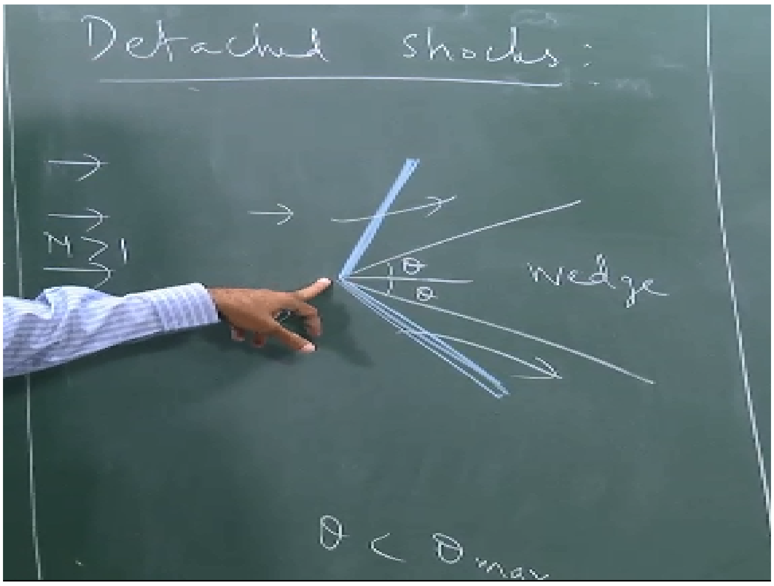

So we move on to the next topic which is Detached Shocks. So all the solutions that we have looked at so far are attached shock solution. In reality as we said earlier when theta flow deflection angle is greater than theta max for that particular mach number the shock becomes detached. So obtaining a solution for this case is very complex and because the flow is much more complicated than what we are assuming.

But we can still draw some useful inferences based on the theory that we have discussed so far. So let us look at flow over wedge. So let us say that we have a wedge with a semi vertex angle theta like this. So this is also theta and let us say they are supersonic flow approaches the wedge. If the value of theta that we have shown here if this is less than theta max corresponding the freestream mach number, then we have two oblique shocks.

One on the top surface and another one on the bottom surface which reflects the flow like this and then the flow flows along the wedge this is a wedge. Remember the reason why I keep emphasizing this is this is not a cone although it looks like a cut section of a cone. A cone has 3 dimensional. So three dimensional effects are much more in a cone whereas the wedge is the two dimensional object which just goes like this.

So what we are seeing here is flow over a wedge and not a cone. Although there are similarities, but they are really different. Now what happens when I increase this theta to a value above theta max corresponding to this mach number that is when the shock becomes detached and in the case of body like this is usually refer to as a bluff body. So keeping a bluff body in a supersonic flow many times will cause the shock to be detached.

And it stands ahead of the bluff body.

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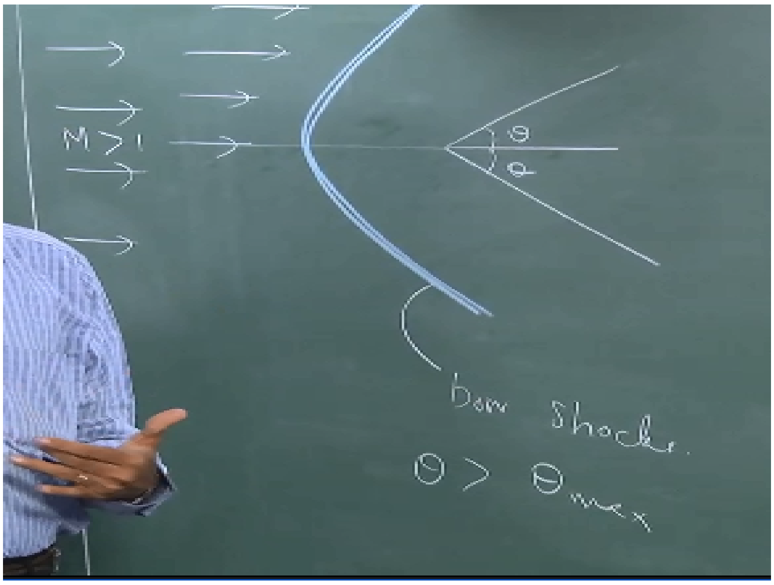

So let us say that theta now is like this greater than theta max corresponding to the freestream mach number which remains the same. So in this case the oblique shock becomes detached and it stands in front of the bluff body like this. So this is the detached shock and the standoff stand meaning the distance between the apex of the bluff body and the shock wave depends upon the angle how much more then theta max this is.

If it is only slightly more than theta max, then the standoff distance will be very small. As it increases the standoff distance also increases. This is extremely important in many applications especially in reentry flows the space shuttle or any other craft reenters the atmosphere there is usually a very strong bow shock which stands in front of the vehicles and creates a lot of problems.

Of course the space shuttle will be blunt object and the mach number at which it is travelling so you have something like this. We can get some idea about the nature of the flow behind the shock wave and the bow shock itself from the theory that we have discussed so far. These are going to be only qualitative inferences not quantitative calculations. Again infer certain things qualitatively based on what we have discussed so far.

If you look at the flow so the flow is approaching like this. Notice that for the flow along the central line what is the flow deflection angle just across the shock wave 0. Now far away from the object where the freestream does not even know that there is such an object what is the flow deflection angle 0. So the flow deflection angle is 0 here and we can easily infer that it reaches a maximum somewhere here.

It increases reaches a maximum and then again decreases to 0 far away from the object that is theta. And then let us look at the strength of the shock wave at each location like this. Here we can infer that the strength is going to be the highest here and then far away from here where the freestream does not even know that there is an object. There is no loss of stagnation pressure just proceeds as it is which means that the strength of the shock wave far away as I move away along this far away from here the strength of the shock wave is 0.

That means it has become almost a acoustic wave Isentropic compression wave. So the strength increases monotonically from a maximum here and becomes 0 or loss of stagnation pressure is a maximum here and become 0 as I approach this end. So the flow deflection angle 0 reaches a maximum again become 0 reduces and the loss of stagnation pressure maximum decreases monotonically it is 0.

So this suggest to me that what I am seeing here is the following.

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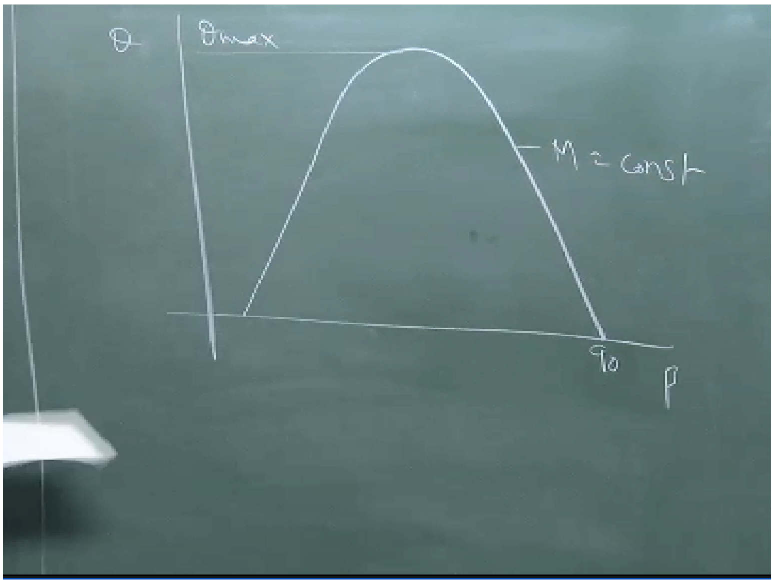

If I draw the theta beta M curve corresponding to this mach number remember this is for an attached shock solution this is a detached shock wave. So the inferences for that reason are going to be only qualitative what this suggest to me. There are two things that we have said and we can actually infer from here to here based on the two things that we have discussed so far. We will pick it up in the next class and then continue from there.