Gas Dynamics and Propulsion
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Lecture – 21
Oblique Shock Waves

In the last class, we looked at the derivation of the theta, beta, M curve and what we are going to do today is look at the behavior of the curve. We are not going to try to solve the equation because it is a very complex nonlinear equation. But, we are going to look at the behavior of the curve for different values of the parameters and remember, as the name indicates the 3 parameters here.

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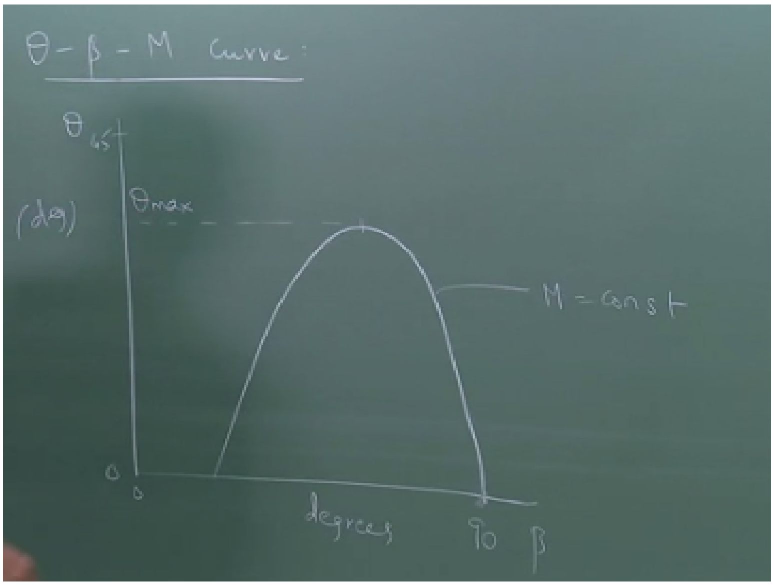

The Mach number, ahead of the shock wave, the wave angle and the flow deflection angle are the 3 parameters that we are seeing in this equation. So, if I draw let us say that I take this axis to be the wave angle and denote quantities in degrees. So, this is 0 and let us say this is 90 and let us say plot theta along this axis and again in degrees, say this is 0 and I can go up to let us say 45, no more than that, that is sufficient.

So, if I take a constant value of M, so for a given value of M, if I go to my theta, beta, M equation and then, look at how theta varies against beta for a given value of M, the curve would look something like this. This is how the curve looks and so this is for M = constant. Now, let us look at the main features of the curve as we vary theta, as we look at the variation of theta against beta.

The first thing that you notice is that there is a value theta max for the given Mach number. So, for the given Mach number, what this say is that if the flow deflection angle is greater than theta max, then the solution for the theta, beta, M equation is not possible, okay. So, what this actually is telling me is the following. If you recall, we sketch the flow situation in the previous class.

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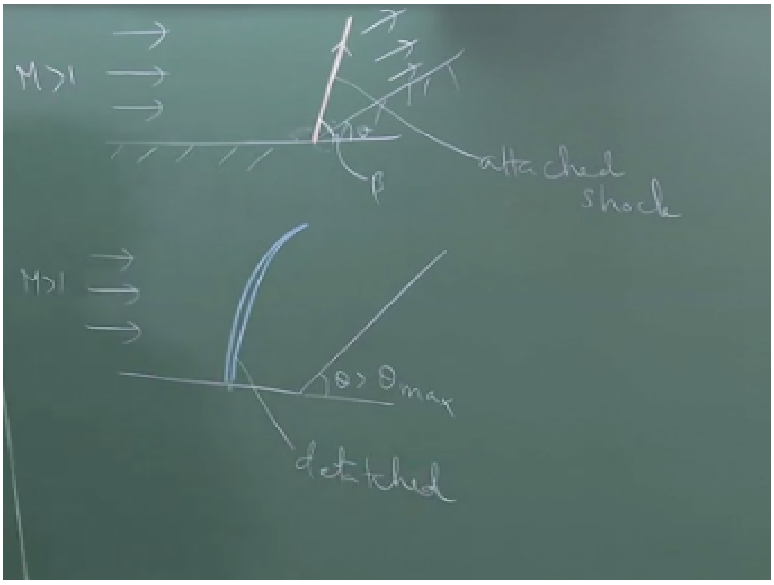

So, we said that this is a compression corner, where the flow is turned through an angle theta and this was our theta and when I have a supersonic flow at a given Mach number approaching this corner, an oblique shock was generated from the corner and the flow was then deflected like this and this was beta. Now, what this curve is trying to tell me is that for a given value of Mach number, remember we are keeping Mach number constant along the entire curve.

So, as I keep Mach number constant and I keep increasing the flow deflection angle and as I keep increasing the flow deflection angle, I keep moving like this. So, what I am doing is, I have this flow deflection angle. I keep increasing it. When I increase it beyond a certain value theta max, then there is no solution that is possible. So, what this is trying to tell me is this. Let us say that this is my, this theta.

Let us say is > theta max corresponding to this Mach number, then this curve is telling me that an attached shock wave solution. Notice that this is an attached shock; this is attached to the corner. So, this is attached to the corner, which is actually generating the shock wave.

Remember, we also gave a direction to this. So, the shock wave is generated from this corner and travels in this direction. So, this is an attached shock.

Now in this case, when the theta becomes larger than theta max, then an attached shock wave solution is not possible and the shock wave becomes detached and looks something like this. So, this is detached. So, the shock wave detaches from the corner and then moves like this. So, if I keep M constant and then vary theta, attached shock wave solutions are possible only up to a certain value of theta.

Above this for this Mach number, solutions are not possible, okay that is the first point that we noticed from this. The second point is for a given value of theta, so this is a given value of theta; notice that 2 solutions are possible, one on this branch and another one on this branch. The left branch is usually called the weak shock wave solution and the right branch is called the strong shock solution.

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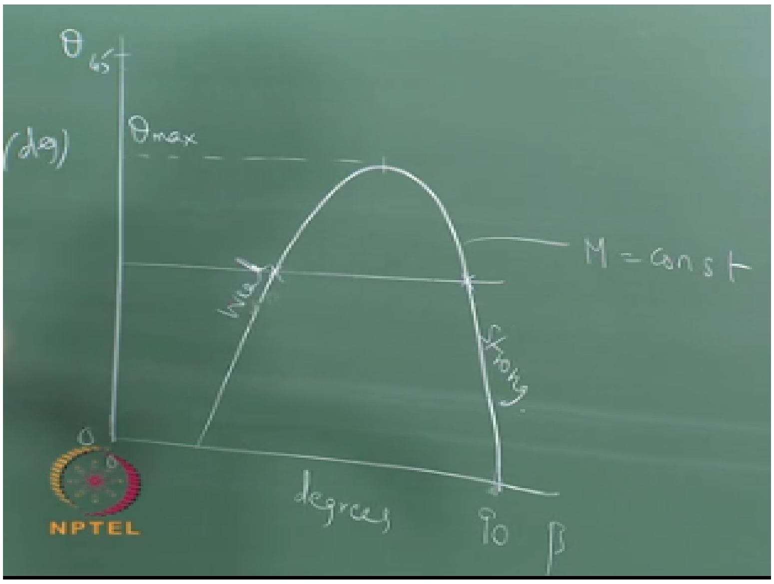

So, notice that the beta corresponding to the strong shock solution is larger than the beta corresponding to the weak shock solution. So, what this is telling me is the following. For a given value of theta, if this is my theta and for a given value of M, the weak shock solution has a smaller value of wave angle compare to the strong shock solution. So, the weak shock solution would look something like this, right.

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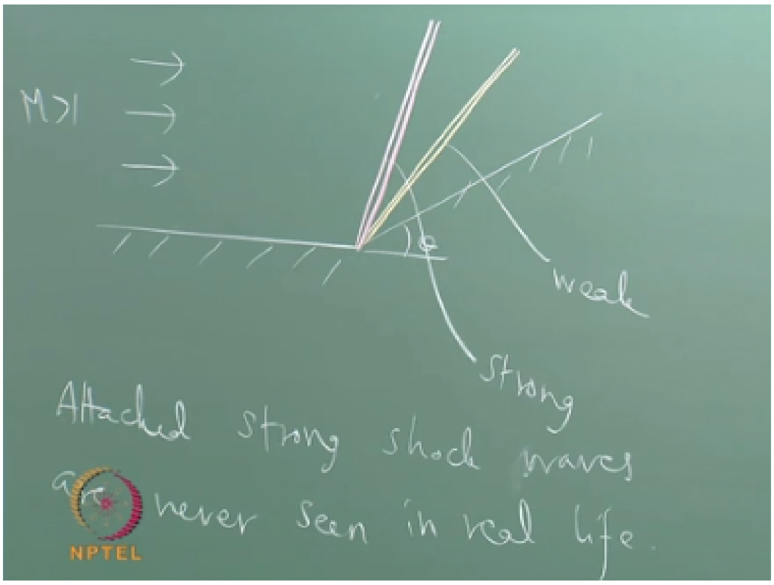

So, this is the weak shock solution and the strong shock solution would look something like this, so that is the strong shock solution. So, the wave angle corresponding to the strong shock solution is larger than the wave angle corresponding to the weak shock solution, which is what we are seeing here. So the wave angle corresponding to the weak shock solution < the wave angle corresponding to the strong shock solution, although 2 solutions are possible.

In reality, strong shock solutions are not seen at least in the attached situations, okay. So, attached strong shock waves are never seen in real life. I said attached strong shock solutions, but when a shock wave detaches itself like this, right, then part of the shock wave, we will discuss the structure of this shock wave later on, part of the shock wave is actually strong and the part of the shock wave is actually weak.

So, detached shock waves can have strong shock in some part and weak shock in other parts. But, an attached shock solution like this is never seen in real life. We always get only the attached weak oblique shock, okay that is the second point that is important about this curve. Now, the third point is the following: For the strong shock solution, M2 is always < 1, remember M1 is > 1 for both sides, right. M1 is greater than 1 for both sides.

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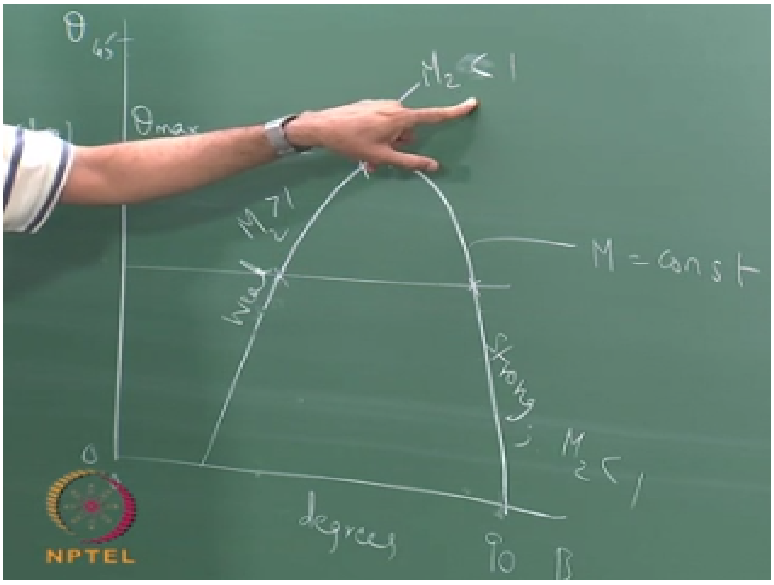

For the strong shock solution, M2 is always < 1. Notice that, I am saying here M2 is always < 1 and for the weak shock solution, M2 is > 1 except very close to the maximum value. So, for this part of the weak shock solution, M2 is < 1. So for the most part, M2 is greater than 1 for the weak shock solution except when the deflection angle is close to the theta max when M2 becomes less than 1, okay.

So let us write that down, for the strong shock, M2 is always < 1. For the weak shock, M2 is > 1 except when theta is close to theta max. But in both cases, Mn2 is always < 1 for both cases. So, this is for one value of Mach number. Now, curve at a lower Mach number, same curve corresponding to a lower Mach number would look something like this and a curve with a higher Mach number would look something like this, okay.

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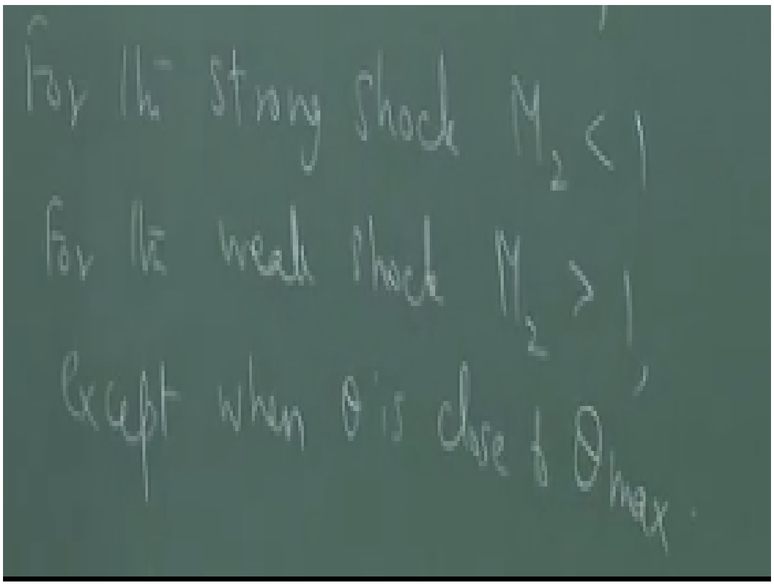

So, this theta max keeps shifting to the left, okay. So, let us look at the figure and see what this curve looks like. So, we can see the theta, beta, M curve, sketch for different values of M, so you notice that the theta max shift slightly to the left and then behaves in this manner, okay, right. Slightly to the left and then goes like this. Initially it starts from here, it shifts like this and then, it straightens out like this, okay.

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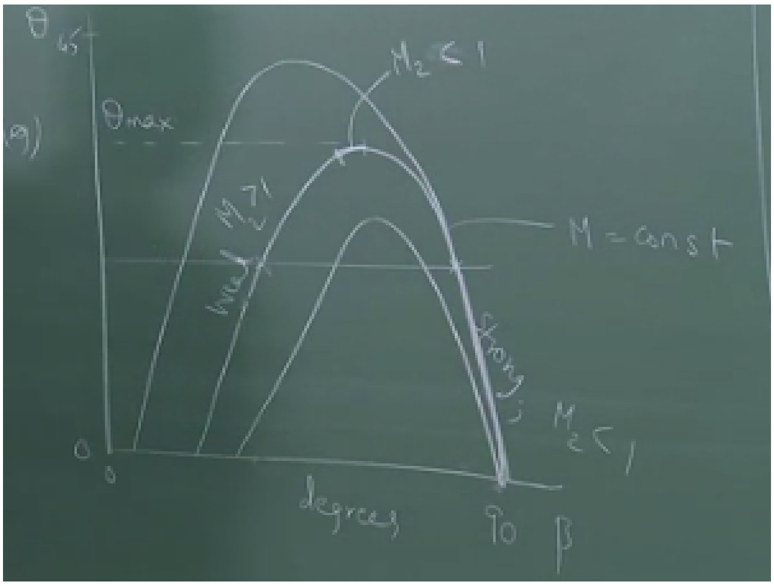

You see the strong shock branch, you see the weak shock branch, then this dash line here, this chain line gives the locus of stage for which M2 is < 1 for the weak shock solutions. So for this value of m, the weak shock solution produces M2 < 1 in this small portion of this curve and so on, okay. So, that is what this curve looks like for different values of M. Notices that as M approaches 1, the shock becomes infinitesimally weak.

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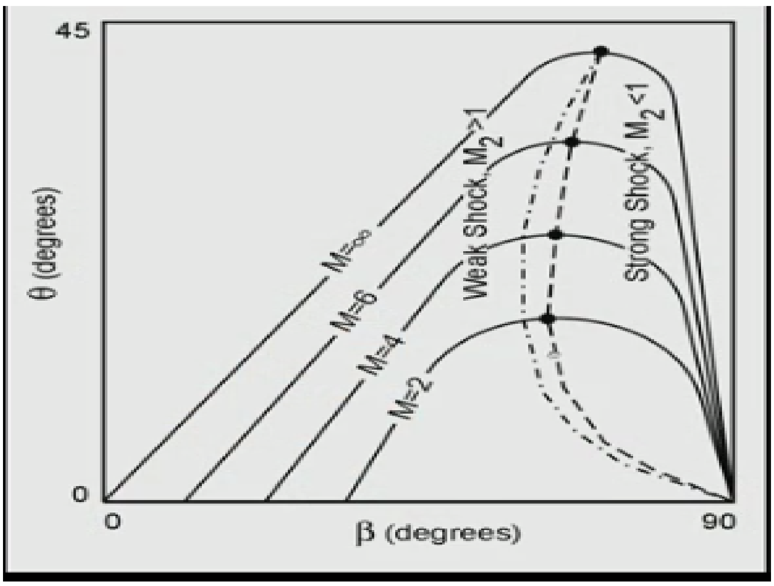

So, the shock keeps collapsing like this, right. So this curve keeps collapsing like this and for M = 1, you will not get anything, it becomes an acoustic wave. Remember, theta for an acoustic wave is 0. There is no deflection of the flow and the flow velocity is also normal to the wave. So, beta is 90 for the acoustic wave and also for the normal shock solution, right.

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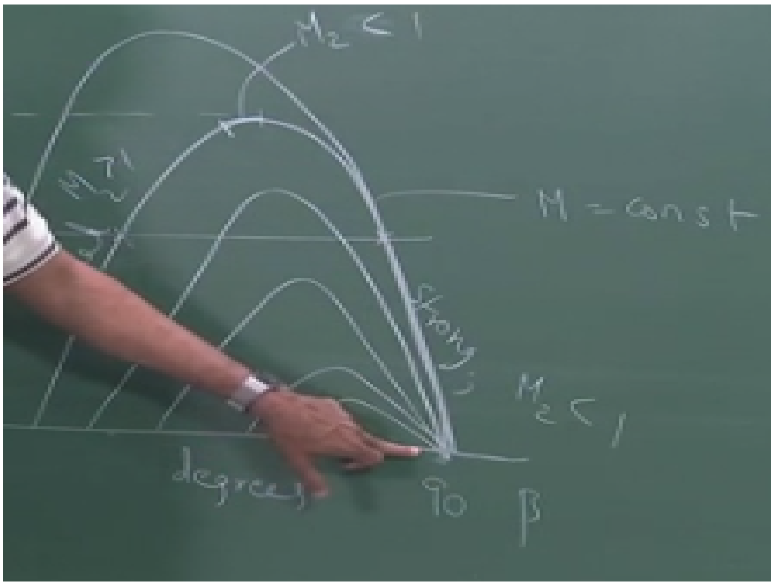

So, that is why when this collapses eventually it will collapse to a point, which is over here like this. However, for a finite value of M, which is different from 1 as we can see from here, the curve any given M = constant curve intersects the x axis at 2 locations, one here and one over here which corresponds to beta = 90, all the M = constant curves intersect here at beta = 90, but they intersect the axis here at different values of beta, all of them, right.

So, that corresponds to theta = 0. So for theta = 0, which is no flow deflection as I said, 2 solutions are possible, one when it is an isentropic process, another one is what? Normal shock wave. That is why theta = 90, so 2 solutions are possible, normal shock wave or an isentropic compression wave. We will see in the next chapter that such an isentropic compression wave is called a Mach wave.

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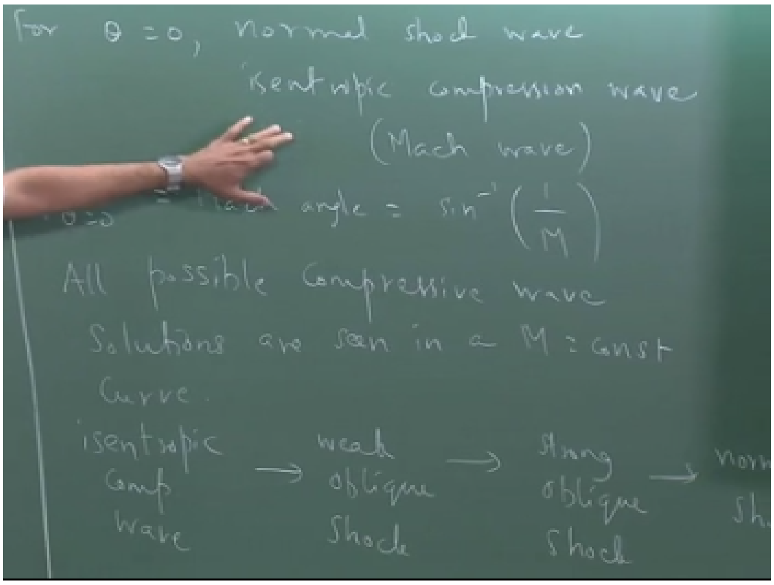

So, in the frame work of the oblique shock wave, theta may seem to be = 0 for this case. But, when you actually take a closer look, it turns out that theta is not 0 for this case, but theta is an infinitesimally small number for the Mach wave. In this frame work, we are saying theta is 0 for this solution, but in reality, it is an infinitesimally small quantity. It is a very, very weak oblique shock wave, which deflects a flow through an infinitesimally small angle.

So, seen from this perspective, it appears to be 0, but if you closer you see that the flow deflection is the small number, not 0, whereas for normal shock, it is exactly = 0, okay. In fact, the angle that you see here, each one of this m = constant curve intersects the axis at different values of beta, right. This angle is known as the Mach angle. So, beta corresponding to theta = 0 is called the Mach angle and is nothing but arc sine 1 over M.

We will show this also in the next chapter, okay. So, each M = constant curve intersects the axis at 2 locations, one corresponding to the Mach angle, another one corresponding to beta = 90 degrees. The solution corresponding to the Mach angle is an isentropic compression wave. No change in entropy in this case and the other one is a normal shock wave, which is the strongest compression wave possible with the highest loss of stagnation pressure.

So, seen from this perspective, you can see that this each one of this curve if I take a single curve here, let us say the curve corresponding to M = 6 here, the solution here is an isentropic compression wave and then, if I travel along this curve, I get weak oblique shock with increasing loss of stagnation pressure. Then, I start to get the strong solution and the loss of stagnation pressure keeps increasing until I reach the normal shock wave for which the loss of stagnation pressure is the highest possible.

So, a single curve represents all possible compressive wave solutions, okay. So, all possible compressive wave solutions in the M = constant curve. So, what are these solutions? We are saying an isentropic compression wave which then becomes a weak oblique shock, which then becomes a strong oblique shock, which then becomes a normal shock. So, these are the possible compressive wave solutions in gas dynamics.

And, a single m = constant curve exhibits all these solutions. Remember, the most important thing is this is an attached shock wave solution. What we have exhibited in this diagram are solutions for the theta, beta, M curve which assumes the shock wave to be attached at the corner. Any questions? **"Professor - student conversation starts"** well, three dimensional shock waves, we are talking about plane shock waves here.

Three dimensional would mean spherical shock wave front and it can be applied. It is like theta, beta, three dimensional. No, the theory can be applied. If you remember, I said earlier that you know an acoustic wave travels like this. So, if you look at a small section of a spherical wave front, the flow can be essential assumed to be one dimensional and whatever, we are doing is applicable in that sense. It is applicable provided curvature effects are small.

If curvature effects are very large, then it is not applicable. Because, then it is not one dimensional, right. If the radius of the sphere is much large compared to the other dimensions that we are looking at, then we can assume one dimensional flow. Sir, 'ya,' if the values at theta tends to 0 or interpolated or is it practically possibly to have a shock on a flat plate. No, we are not saying that it is possible to have a shock on a flat plate.

That is why I said that this theta = 0 for this case is only when you draw a diagram like this, where the scale runs from 0 to 45 on the y axis. If I draw a diagram where the scale runs from 0 to let us say 0.01, then this curve will terminate at 0.01, it will go to 0, okay. So, the isentropic shock wave turns a flow through an infinitesimally small angle which we will call as d theta in the next chapter and derive an equation for that.

Whereas, this even if I draw a scale 0 to 0.1, normal shock wave will terminate a theta = 0, whereas theta is not identically = 0 for an isentropic shock wave. It is very small, but not identically = 0, in fact, if you wish I can write it like this, so theta is not exactly = 0, but theta is a number of yes much, much smaller than 1 for this case, okay. So, we cannot have a compression wave on a flat plate.

Sir, very strong oblique shock possible? Very strong oblique shock is a normal shock; normally you will not see a very strong oblique shock. You must keep in mind that this portion of the curve is very steep, the strong shock portion of the curve is very steep as you can see from this diagram also, the strong shock portion is very, very steep.

So, when you try to provoke strong shock what normally happens in reality, is it you will get a normal shock, but not a strong oblique shock, because the portion of the curve is very steep. It usually becomes a normal shock rather than a strong oblique shock. Strong oblique shocks are seen only when the shock wave detaches from the attachment point that we will discuss in the next module. "**Professor - student conversation ends**".

So, we said that the single M = constant curve represents all possible solutions, so as you can see here there is no loss of stagnation pressure because it is an isentropic process. There is a loss of stagnation pressure here; it is irreversible turning through a finite angle. The loss of stagnation pressure is more and loss of stagnation pressure is the most for normal shock and we can calculate P02 or P01 this way.

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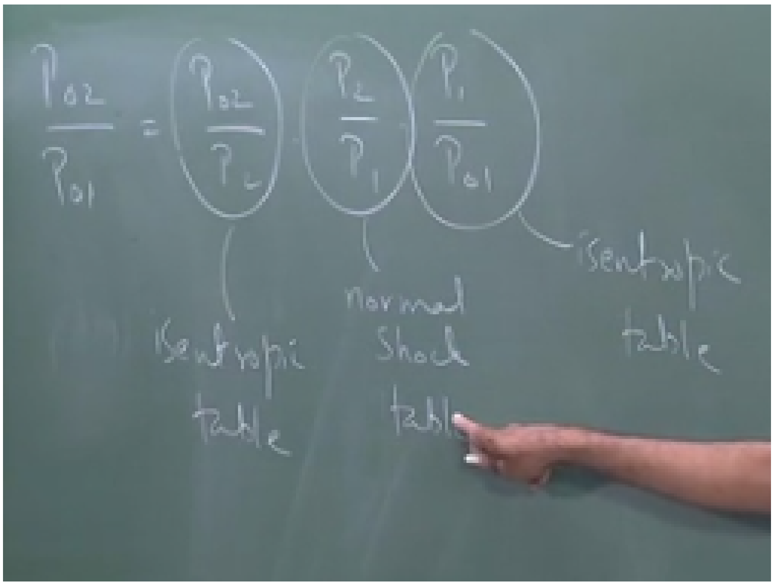

So for an oblique shock, so when you remember we said that for the normal component of the oblique shock wave we can use the normal shock tables. So, you can use the normal shock tables only to retrieve static quantities, not stagnation quantity, so you can use the normal shock table to get P2, T2, may be rho2, but not P02, because P02 is frame dependent and in a normal shock wave, we are moving along with the shock wave.

Whereas, in the case of an oblique shock wave, we are not only moving along with the shock wave, but we are also moving along the shock wave, which means stagnation quantities cannot be retrieved from the normal shock table for this problems. So, you must write it this way and for the given M2 and given M1, I can calculate this, I know P2 over P1 from normal shock table. So, this I can get from normal shock table.

So, this I can get from isentropic table and this also I get from isentropic table. So, that is how we calculate loss of stagnation pressure across an oblique shock wave. 'Ya' **"Professor - student conversation starts"** P02/P01 will be = P0n2/P0n1. No, that is what I said this is stagnation pressure. There is no component for a stagnation pressure. What you mean by p0n2?

Remember, stagnation pressure by definition is when the velocity you take the flow, you decelerate the flow isentropically to velocity 0 that means all components are becoming 0, so there is no p0n2. The frame of reference is different that is why we are not able to use the other one. Loss of stagnation pressure takes place because of the n2 only, normal component only that law should be (()) (23:16).

No, that is what I said; stagnation pressure is a frame dependent quantity. We are using different frames. When we use the normal shock table, you are using a different frame of reference. When you are looking at an oblique shock, you are looking in a different frame of reference, which is why that you cannot use the normal shock table to calculate this. It is true that the loss of stagnation pressure comes because of the normal shock component.

But, the value happens to be frame dependent. Remember, we are able to calculate this without any difficulty, so we are accounting further, right. But, the value depends upon the frame. So, what you will do next is workout several examples illustrating this idea. You go ahead, if it is attached form shock waves and never seen its wavelength.

Is that something to do the entropy change between the strong shock wave and the weak shock wave? Not exactly, actually both are allowed, the entropy change is positive for the weak as well as the strong shock. I mean the entropy is smaller for, 'ya,' the entropy is smaller for the weak for the shock wave and the higher for the strong wave. But, I would be hesitant to use your argument that because this is smaller wave.

See this because we also see normal shock wave, which is the highest entropy change. So, my understanding on this is that because the strong shock portion of this curve is so steep that in reality if you try to go beyond this theta max point or if you try to get the other solution, it is never seen because it will call us into a normal shock rather than show a strong oblique shock, okay. "**Professor - student conversation ends.**"

Okay, so we are going to look at different worked examples. Let us start with worked example 1. The problem is very simple, this will basically demonstrate the use of oblique shock table, supersonic flow at M = 3, static pressure 100 kPa and static temperature 300 K; it is deflected through 20 degrees at a compression corner. Determine the shock wave angle and the flow properties downstream of the shock wave.

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So, we have a compression corner of angle 20 degrees, so flow at M1 = 3, approaches the corner and we have an oblique shock wave and the flow is deflected through 20 degrees, P1 is given to be 100 kPa and T1 is given to be 300 Kelvin. We are asked to calculate the properties downstream of the shock wave. So, for M1= 3 from the isentropic table, we have P01 over P1 = 36.73 and T01 over T1 = 2.8.

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From which, I get P01 = 3.673 megapascal and T01 to be 840 Kelvin. Now, we need the wave angle here, we know M, we know theta, we want the value for beta, right. So, solving the equation is one way of doing this, but actually tabulated the solutions to the equation and that is what we are going to see now. So, we can see this is the table corresponding to theta, beta, M. So, here we can see for M = 3 and theta = is goes up to 18.

So, we will go to the next page, M = 3, theta = 20, we have beta to be 37.76, right, beta has 37.76 from the oblique shock table. Therefore, Mn1 = M1sinebeta. So, we can see now why we needed beta, only when we have beta, we can calculate Mn1, so Mn1 for this case then comes out to be 1.837, so with this value of Mn1, we go to the normal shock table. For mn1 = 1.837, remember when we go to the normal shock table, we treat this as M1, okay.

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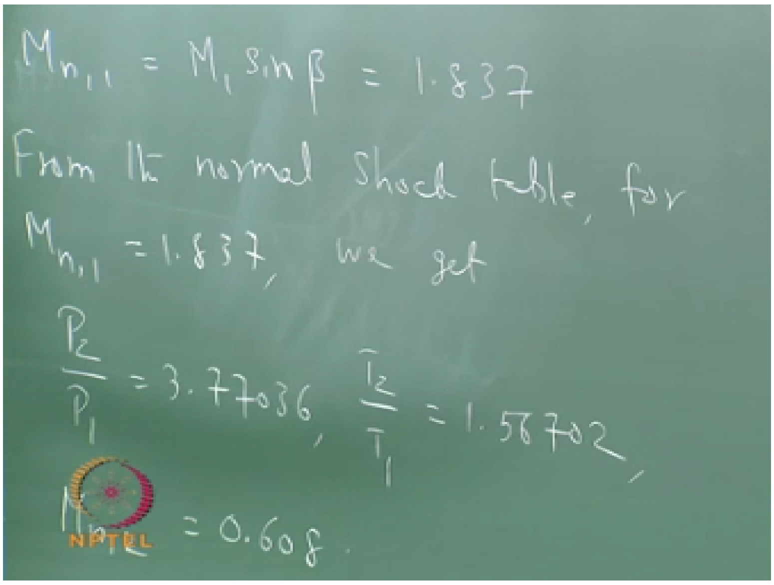

In the context of the oblique shock wave, this is Mn1 for the normal shock table; however, this will be M1. So, for this value of M1, we interpolate and retrieve the following quantities. Remember, what kind of quantities can be retrieved, Mn2, P2 and T2, none of the other quantity. So, we get P2 over P1 to be 3.77036, we get T2 over T1 to be 1.56702 and Mn2, the table gives us M2, but we take it as Mn2, so Mn2 = 0.608.

So, these are the quantities that we are allowed to retrieve from the normal shock tables. Remember, there is no problem in getting Mn2, because this is in the frame of reference compatible with the normal shock wave. So, we are allowed to do this and these are static properties which are frame independent. So, these are the only things we are allowed to retrieve from the normal shock table.

Hence, we can calculate M2, which is = Mn2 divided by sine of beta- theta, we derived this relationship yesterday, so we get M2 to be = 2 and P2 can be calculated P2 over P1 = this, so P2 is 377 kilopascal and T2 = 470 Kelvin. Now, P02 = P02 over P2 times P2 and we get P02 over P2 from the isentropic table. So, for M2 = 2, we get this to be 7.82445 times 377, so P02 comes out to be 2.95 MPa.

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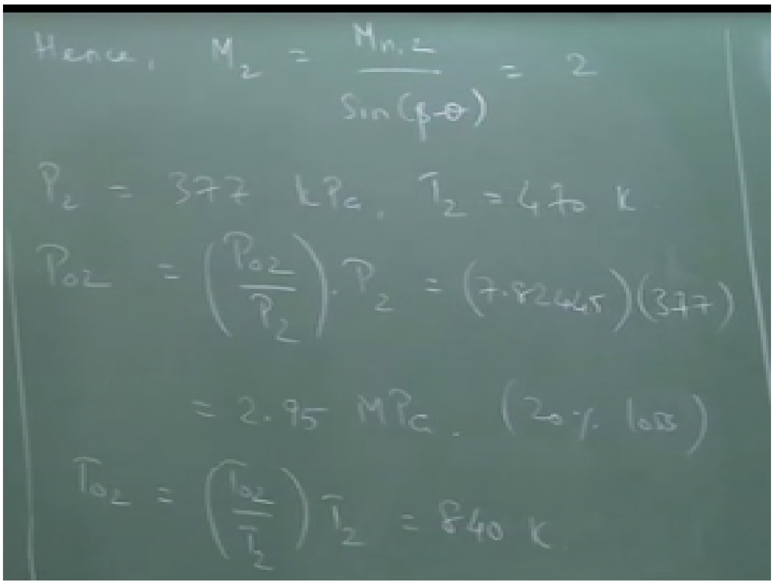

T02 = T02 over T2 times T2 and if you plug in the numbers, we get T02 to be 840 Kelvin. Notice that, P01 was 3.673 MPa and P02 is 2.95 MPa that is about a 20% loss of stagnation pressure, right. So, this is a 20% loss. Had this been a normal shock at M = 3, right. This was M = 3, had this been a normal shock at M = 3, the loss of stagnation pressure would have been around 67%, okay. In comparison, this is 20%.

Notice that, the stagnation temperature, what is happening with this stagnation temperature? Stagnation temperature is remaining the same even in this case also, okay. If we do that correctly, perhaps notes, let me just check the solution, yes we are okay. 'Ya,' okay. Let us go to the next worked example. The next worked example is a very involved example, it goes like this.

A converging, diverging nozzle with an exit to throat area ratio of 2.637, operates in an over-expanded mode and exhaust into an ambient pressure of 100 kilopascal. So, the figure is given, so P ambient 100 kilopascal. So, this is 1, I am calling this, this is the exit. So, A exit over A throat = 2.637. The inlets stagnation conditions are 300 Kelvin and 854.5 kilopascal, so P0 is 854.5 kPa, T01 = 300 Kelvin.

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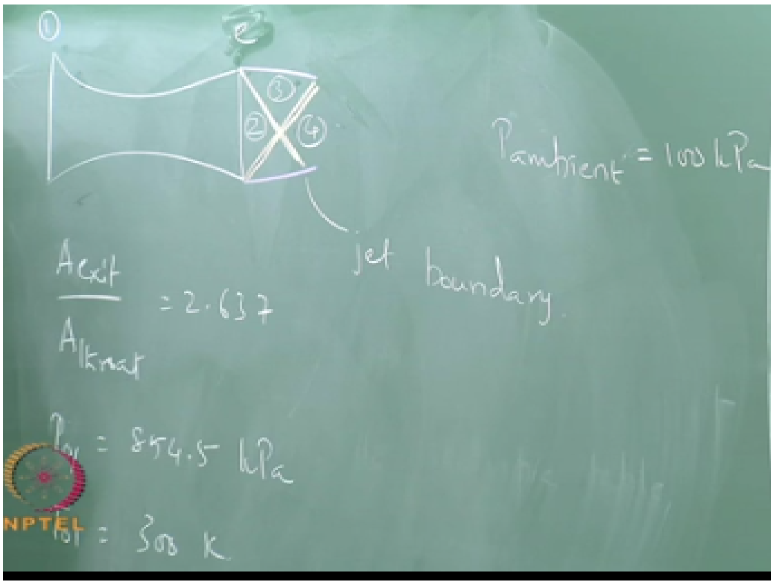

We are asked to determine the flow properties at the exit and also the angle made by the edge of the jet with the horizontal. So, let me sketch the situation that is given in the problem. You know that when it is over-expanded, it is given that over-expanded, so that means that the flow has to be compressed after it comes out. So, that means we trigger oblique shock waves from the lip of the nozzle, right.

We trigger oblique shock waves from the lip of the nozzle and the jet is then deflected like this. So, we are asked to calculate the angle that the jet boundary makes with the horizontal and we were asked to calculate the flow properties at 2, 3 and 4. Some approximations are required here, for example, 2 shock waves like this intersect, the flow is little bit more complicated than what we have studied so far.

But, we will assume that ours is an extremely good approximation and it is and we will go ahead with the calculation. So, we will go up to the point when the shock wave intersects the jet boundary. So, we need to look at more theory to continue the problem further down. So, we will go up to 2, 3 and 4. So that is what we have been asked to calculate. So, for A exit over A throat.

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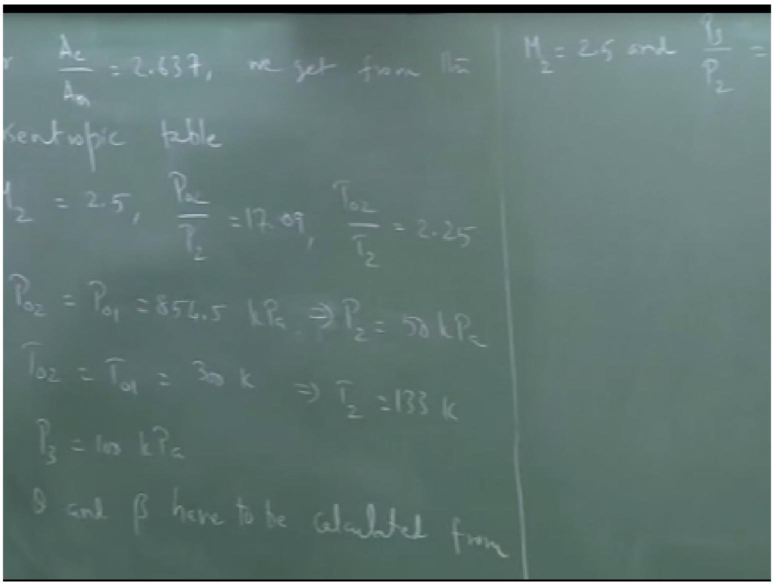

Remember, the flow is choked at the throat, so that means A over A star of 2.637, I get from the isentropic table, M2, the Mach number with which the flow comes out is 2.5 and I also get P02 over P01, let us take the look at that also. P02 over P2 = 17.09 and T02 over T2 = 2.14, P02 = P01 because the flow is isentropic up to that point, right. So, P02 = P01 = 854.5 kPa and T02 = T01 = 300 Kelvin.

Because the flow is isentropic until state 2, which means I can get from this P2 can be calculated as 50 kilopascal and from this T2 can be calculated as 133 Kelvin, okay. Now, we have to go across the first oblique shock wave from 2 to 3. Now, remember the jet boundary looks like this, okay. The ambient pressure is at 100 kPa, so what is the static pressure in 3? This is directly exposed to the ambient, so that means P3 is 100 kilopascal.

So, notice that this is the very interesting oblique shock wave calculation. I know M2, but I do not know theta, I also do not know beta, but I know the pressure ratio across the oblique shock wave, okay. We said theta, beta, M, but now we do not have theta, we also do not have beta, however, I have static pressure ratio across the shock wave. So, I have to work backwards from there to get my theta and beta, okay.

So, let us write it down, theta and beta. I have to be calculated from the fact that M2 = 2.5 and P3 over P2 = 2, okay. So, this is where the calculation procedure gets tricky from the normal shock, how do we get this pressure ration for an oblique shock wave? Do we get it from? If you remember the previous example, we retrieve the static quantities from the normal shock table.

So, from the normal shock table, for P3 over P2 = 2, we get Mn2 = 1.36. So, we go to the normal shock table, we go down the column, P2 over P1, see where it is becoming 2. So, the Mach number corresponding to that is 1.36. Remember, this Mach number would be labeled M1 in the normal shock table, okay. So, we have to be very careful with the notation and the numbering, okay. So, this would be M1 from the normal shock table.

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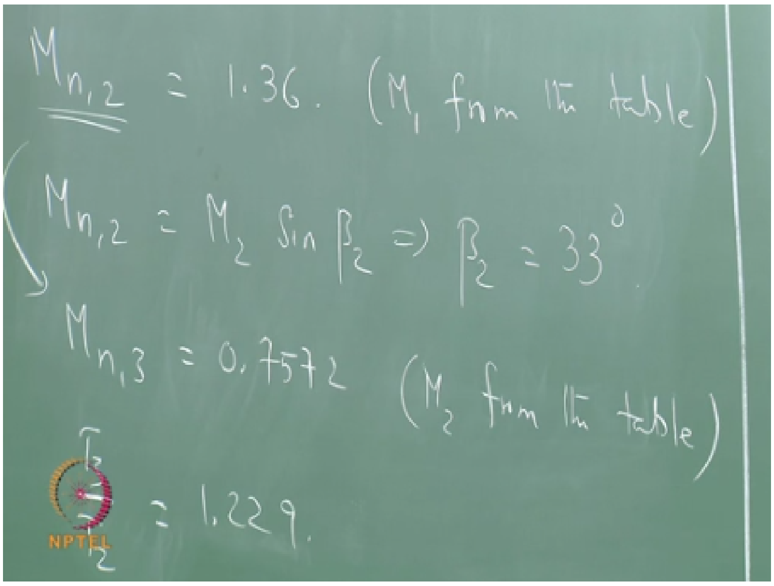

But in our notation, we have to label this as Mn2, okay. Just to make things clear, let me highlight like this, M1 from the normal shock table, but in the oblique shock, where you to context, we have to label this as Mn2 and you know that Mn2 = M2sinebeta implies that let us called this M2 sinebeta2, just to be consistent, so this implies that beta2 = 33 degrees. So, we have obtained beta2 now, okay.

In addition, we are also allowed to retrieve from the normal shock table, other quantities, which are Mn3 = 0.7572, so this will be labeled M2 from the table and we are allowed to retrieve the ratio of static temperature T3 over T2 = 1.229, okay. So, these are the quantities that we are allowed to retrieve from the tables. So, T2 is known, I can calculate there are some quantities that I can calculate from this thing.

Remember, T2 is know, I know T2 from here 133 Kelvin, so T3 = T3 over T2 times T2 and that gives me 1.229 times 133 which is nothing but 164 approximately we say that this is 164 Kelvin. Now, we still need to calculate M3, how are we going to calculate M3 here? If you are thinking of using this relationship, in fact, we are going to use this relationship, Mn3, if you remember from our previous class is M3 times sinebeta2 – let us say theta2.

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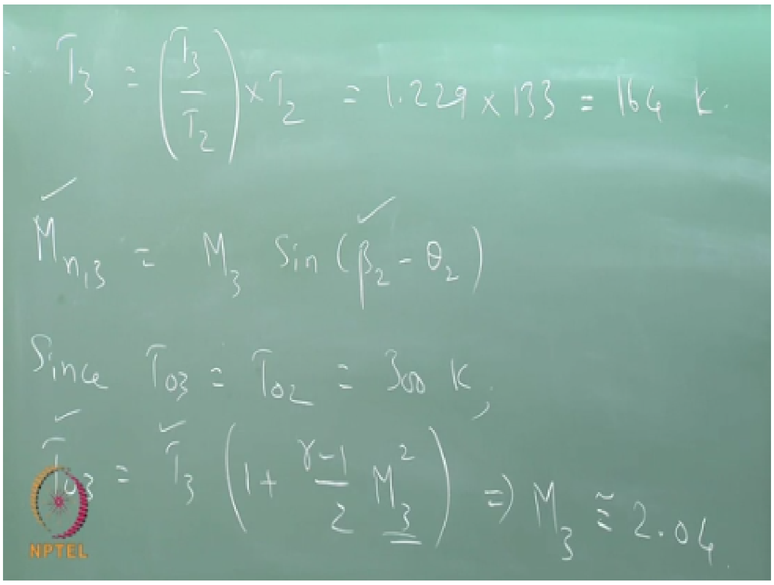

I know Mn3 from a normal shock table, so this is known, this is also known, but I do not M3 or theta2. So, how are we going to calculate? Either this or this, 'ah' that is what we are going to make use of. Next, since this stagnation temperature remains constant and I know the static temperature, right. From the definition of stagnation temperature, I can write this, T03 is known, T3 is known, I can calculate M3 from this.

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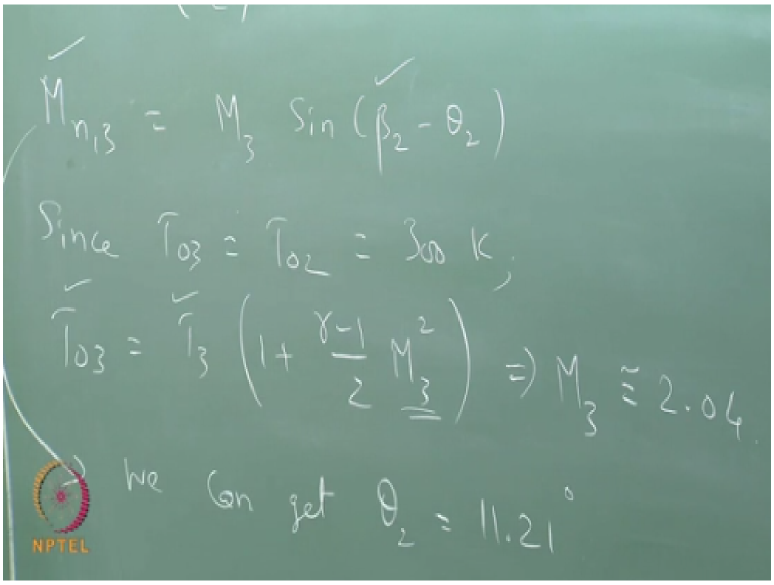

So, this gives me M3 = approximately 2.04. So, by using this relationship, we can get theta, which is the flow deflection angle as 11.21 degrees. So now, I know Mn3, I know M3, I know beta, so I can calculate this angle theta as 11.21 degrees. So, this is the flow deflection that the fluid has undergone. Remember, what we will do in the next class, continue this, look at this solution in little bit more detail and then continue further.