Gas Dynamics and Propulsion
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Lecture – 20
Oblique shock waves

So, we begin a new chapter today on oblique shock waves. So, we will start this chapter with recap of what we learnt about normal shock waves and we will see how an oblique shock wave differs from a normal shock wave. The application and other uses of oblique shock waves and where they occur and so on, we will discuss afterwards. So, if you recall we started over discussion on normal shock waves.

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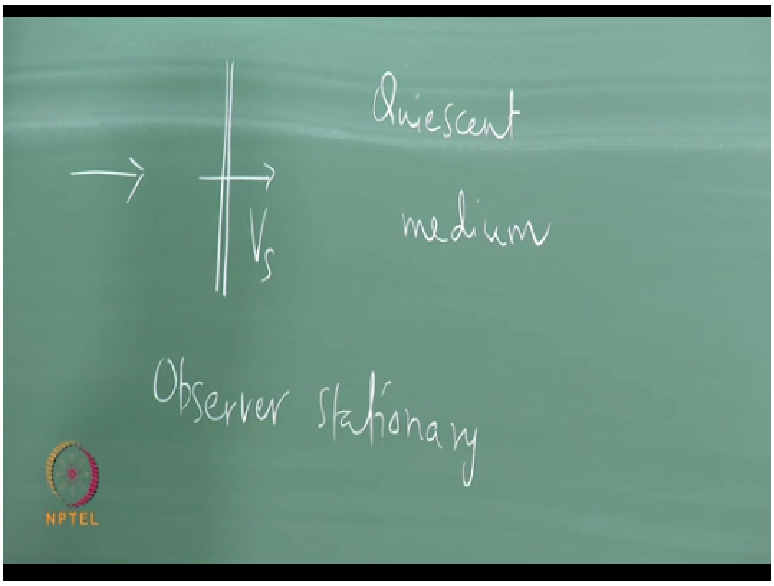

I considering the situation like this where a normal shock propagated with a velocity v is into quiescent medium and this was in a frame of reference that they observer is stationary and as a result of passage of the shock wave, so the shock wave is propagating like this and as a result of passage of the shock wave, the fluid in this section attains a certain velocity, there is increase in pressure, increase in temperature and so on.

But, the fluid ahead of this is quiescent without any velocity. So, this was the scenario that we were looking it and then, we changed the frame of reference. We said that you know, we will get into a frame of reference where the observer moves along with the shock wave, so it moves along with the shock wave, then we drew this kind of a sketch.

Now, the shock wave appears stationary as the observer is moving along the shock wave, the force seems to approach the certain velocity and received with the certain other velocity. So, the velocity with reach the flow approach is shock wave is we labeled that as u1 which is actually equal to the shock speed. So, this was labeled as 1 and this was labeled as 2 and so, state 2 here the flow receives with the certain velocity u2.

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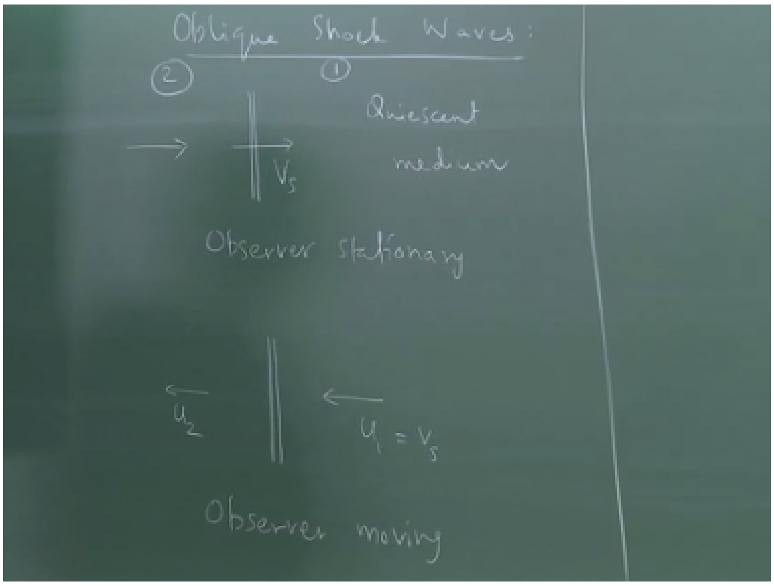

And we saw that or we showed that u2 is < u1 that is the permitted solution and the process is the compression process. An expansion shock wave was forbidden by second law of thermodynamics. So, these are the highlights of our discussion on normal shock wave. What is that the velocity vector both before and after in this frame of reference where the observer moves.

In this frame of reference, notice that the velocity both before and after is normal to the shock wave and hence the name normal shock wave and there is no change in the direction of the fluid after it passes through the shock wave, right. These are the highlights of the normal shock wave of compression process. Oblique shock wave, we start with the same scenario, okay.

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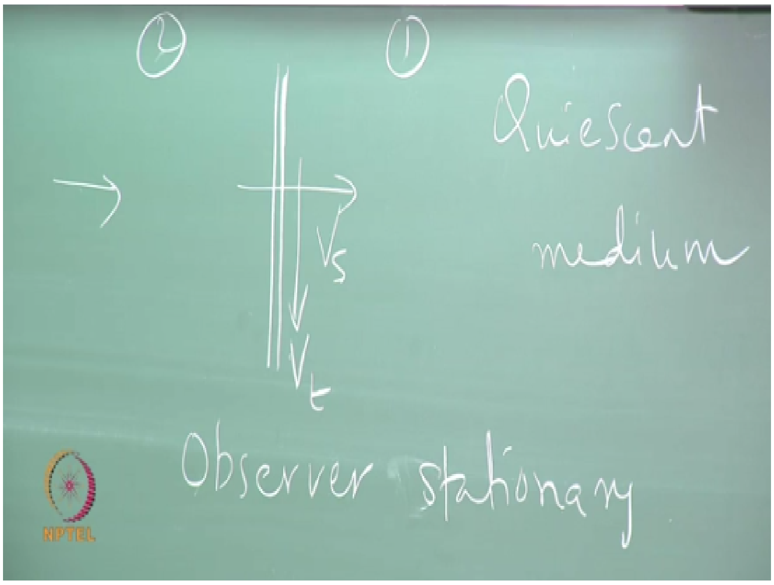

We start with the same scenario, the only difference is, so let it be draw this here the oblique shock wave and let us label this as 1 just likes before label this as 2, so this is in a frame of reference where the observer is stationary. Now, let us say that the observer now gets on to a frame where not only does the observer moves along with the shock wave, the observer also moves along the shock wave.

Please notice the difference, here the observer moves along with the shock wave, here the observer moves along with the shock wave and along the shock wave. Meaning, the observer also let us say travels this wave with the certain speed, which I am going to label as V sub t, the observer can go this way or that way, both are equivalent that is not an issue. So, the observer now moves along the shock wave, in addition to moving along with the shock wave, okay.

Now, when we go to this frame of reference where the observer is moving in this manner. Now, I see that the velocity of the fluid that is approaching me as 2 components, one component in one direction and another one which is normal to this. So, let me draw the shock wave. So, because the observer is moving this way, the flow seems to approach the observer in this direction with the velocity which is equal to this velocity, correct.

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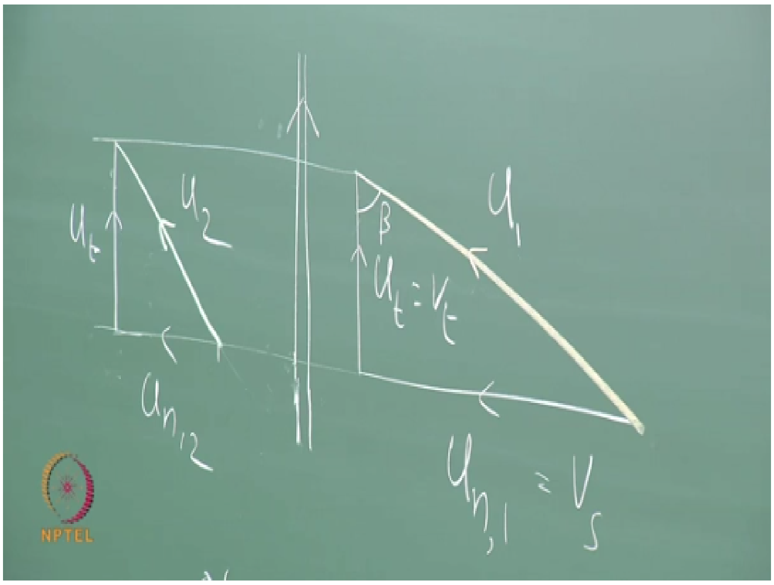

So, this is the tangential component of velocity which I can label like this. Now, because the observer is also moving this way, the flow seems to approach with the velocity in the normal direction with the value = vs just like what we saw here, correct that is the normal component. So, I am going to show the normal component like this. So, I am going to use the subscript n, here I use the subscript t to denote tangential component.

Here, I am going to use subscript n to denote normal component, this normal component is = vs, right. The observer has 2 component of motion, one which is along the shock wave. So, the fluid acquires that in the opposite direction, one along with the shock wave or normal to the shock wave. So, the fluid acquires that velocity in the opposite direction. This is quiescent medium, so it looks like this.

So, the resultant velocity vector in this case is going to be like this. So, this is u1, this is vector u1. What is that u1 is not normal to the shock wave, here u1 is normal to the shock wave, here u1 is not normal to the shock wave. Now, what happens to the flow after the passage of the shock wave that is state 2. Now, there is no shear force or any other force acting in the tangential direction to the shock wave.

So, the tangential component of the velocity will remain the same across the shock wave, there is no change, right. Only the normal component of the velocity is reduced as a result of compression across the shock wave. So, normal component reduces, tangential component remains the same, okay. So that means if this is my tangential component, which remains the same, my normal component is < what it was before. Let us say something like this.

So, let us called this un2 this is normal to the shock wave. So, now the resultant vector, this is u2 is that clear. Tangential velocity vector or tangential component remains the same. Normal component alone decreases just like it decrease in going from here to here. In fact, the reduction is actually the same. If you consider only the normal component, it is as if it is going through a normal shock wave.

Whatever, we have developed here the theory is applicable here also. Only thing is we have to now worry about directions, okay. This direction, so now, another thing that we must keep in mind is when the observer starts moving along the shock wave this way. The shock way also appears to slide upwards, right. So, the shock wave now, you know that acquires a direction.

So, the shock wave goes like this because the observer is moving along the shock wave like this. The shock wave seems to slide above the observer. So that it means the shock wave has the direction now in contrast to what we had before. There was no directionality to the shock wave before now. There is the directionality to the shock wave in this frame of reference (()) (09:08) this is the observer moving frame of reference, okay.

This angle is usually denoted as beta or the wave angle that is the angle that the shock wave, this vector makes with the vector u1, the acute angle between that shock wave vector and this vector u1. I will show this in a slightly different orientation, so that you become more comfortable, but this is the picture that you will see if you go from this frame of reference to this frame of reference.

Now, you want to find out one more thing is the flow deflected or not is the other thing that we want to see, here there was no change in the flow direction. Flow was not deflected at all. Here, we wish to see if it is deflected or not and by how much. We can easily see that the velocity vectors are not parallel to each other obviously because un2 is smaller, so which way is the flow deflected towards the shock wave or away from the shock wave?

If you take this length and put it here, right, this will come up to let us say somewhere up to there, this would be un1. If I superimpose this on this, this would be approximately un1. Then, the resultant velocity would look like this. So that means the flow upon passing through the shock wave is deflected towards the shock wave via certain angle. That angle is called the flow deflection angle.

This angle is called the wave angle. I am going to make this more clear in a minute. Now, the important point here is this angle is measured with respective to u1 and the direction of the shock wave that is why it is called the wave angle or the shock angle. Now, lies become easy if I rotate this figure. If I rotate this figure, so that u1 becomes horizontal, then things are little bit clearer to see, okay.

So, what is that this is the true picture, for our convenience and ease of use, we are going to rotate this through an angle beta. So, that this becomes horizontal. So, the flow deflection becomes easy to see. Flow reflection is not so easy to see in this diagram. So, I am going to rotate this diagram this way counter clockwise by an angle beta, right. So, if I rotate this in the counter clockwise direction, when I do that my shock wave rotates through an angle like this, right.

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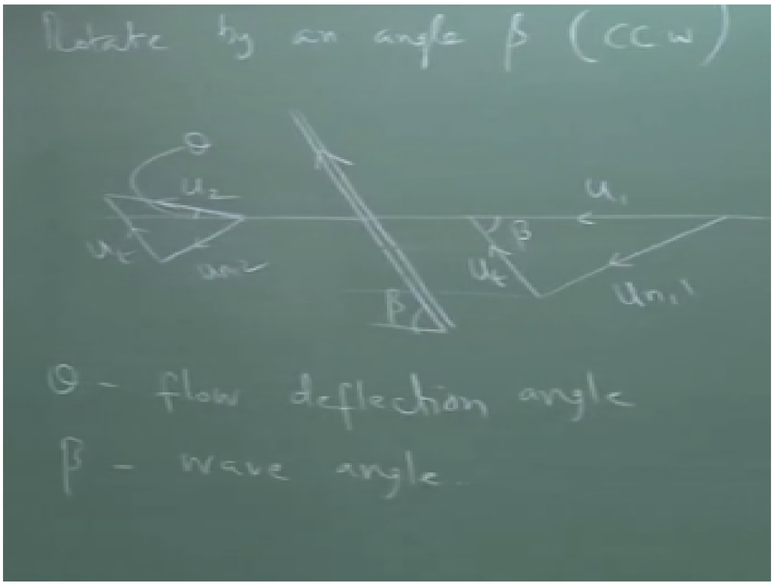

This is the direction and this angle is, what is this angle? Beta. Thus, I have rotate through beta, so this angle is beta and so, this is my u1, which has become horizontal now, this is my ut and this is my un1, okay. So, I have rotated this. This is not a new reference frame. This is the same reference frame, but instead of looking at this figure like this, if you will, I am looking at the figure like this, so that u1 is horizontal, you must understand that.

This is not a new reference frame, this is the same reference frame, okay. I am tilting my head, so that u1 becomes horizontal, so that is what has happened here. So this is u1, this angle is beta and again notice that this angle is also beta. Now, when I draw the diagram on this side, again this is ut. I am sorry, un2 is slightly, this is ut, this is u2, okay. Notice that, ut remains the same, un2 is < un1 and now, you can see that this was the original velocity vector direction.

What is this angle? That is the flow deflection angle, usually denoted by theta. So, this is the flow deflection angle theta. So, let us write it down explicitly theta is the flow deflection angle and beta is the wave angle. What is that, both these angles are measured with respect to u1. Please bear this in mind, just because we have drawn this horizontally does not mean that these angles are measured from the horizontal;

Both the angles are measured with respect to u1. So, the flow deflection angle is the angle through which the flow is turn with the respect to vector u1 and wave angle is the angle that the vector long away makes with u1, not the horizontal. This is only for ease of illustration, nothing more than that, okay. So, let us write done few ideas from these 2 of them, we will go from here, okay.

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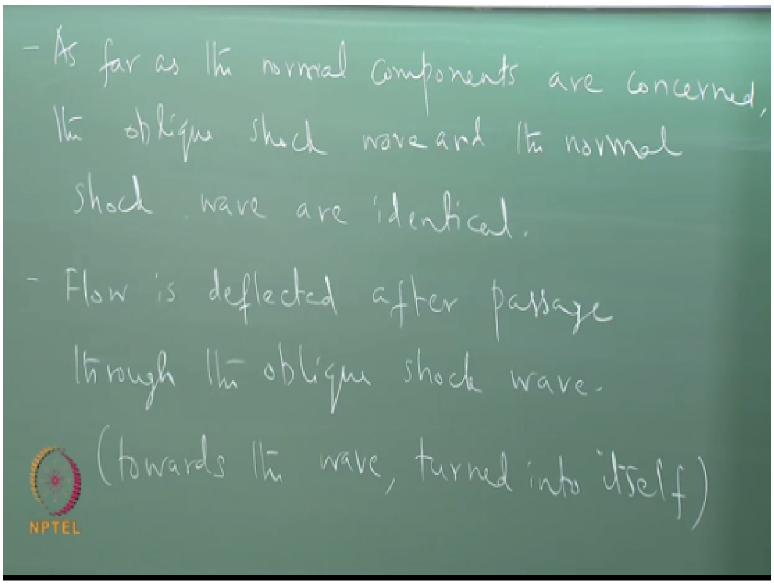

Notice that in so far as the normal component is concern, the oblique shock wave and the normal shock wave are the same, okay. So, as far as, the normal components are concern, they are both the same, the oblique shock wave and the normal shock wave are identical. Why is this important?

Because if I have a flow that is approaching a shock wave like this with let us say static pressure p1 and static temperature t1 and velocity u1 with the normal component equal to this. How do I get the static pressure? and static temperature after the passage of the shock. I used the normal shock table, but instead of using m1, when I go to the normal shock table, I used this Mach number, mn1 is what I used when I go there.

Remember, static quantities are frame independent, right. So, the static pressure and static temperature that I calculate using the normal shock relationship for this will be the same as for this frame of reference also. Stagnation pressure will not be the same. Static pressures will be the same, so that is why this is very important. So, static quantities across the oblique shock waves can be calculated using the normal velocity component.

Just like you will do for normal shock waves, okay. Next, flow is deflected after passage through the shock wave. In fact, which way is the flow deflected, the flow is deflected towards the wave, right or equivalently, there is another terminology that is usually use, the flow is turned into itself is what people use, although that is very confusing. I preferred this, because this has the direction.

So, we can easily see whether the vector is shifting towards this direction or moving away from this direction. So, I do not like this other terminology, but that is also used. Flow is turned into itself. This is little bit confusing because when you look at some complicated situations, you really do not know which is turning into itself and which is turning away from itself. Whereas, this one irrespective of the orientation.

Once you draw this, sketch the direction is always clear whether what you are drawing is a shock wave or not. Because later on, we are going to look at expansion waves also, not the counter parts of this, but infinitesimally strong expansion waves, which will actually deflect the flow away from itself. So, it can be confusing, so towards the wave is a much better way to describe the flow deflection, okay.

Next, probably most important point, let us say that we consider a situation where we have something like this and something like this and let us further state that u1 is same in both cases, p1 is a same, t1 is a same that means m1 there and m1 here also same. However, mn1 is always going to be < m1, right. So, the loss of stagnation pressure across this shock wave is going to be < the loss of stagnation pressure across this shock wave for the same Mach number.

Because the normal component is always less, okay. Since the normal component of the velocity < the magnitude of the velocity itself, loss of stagnation pressure is less in an oblique shock wave. Why is this important? This is important because in practical devices, we saw that beyond the Mach number of 2, the loss of stagnation pressure in the normal shock wave was 70% as much as 70% or so.

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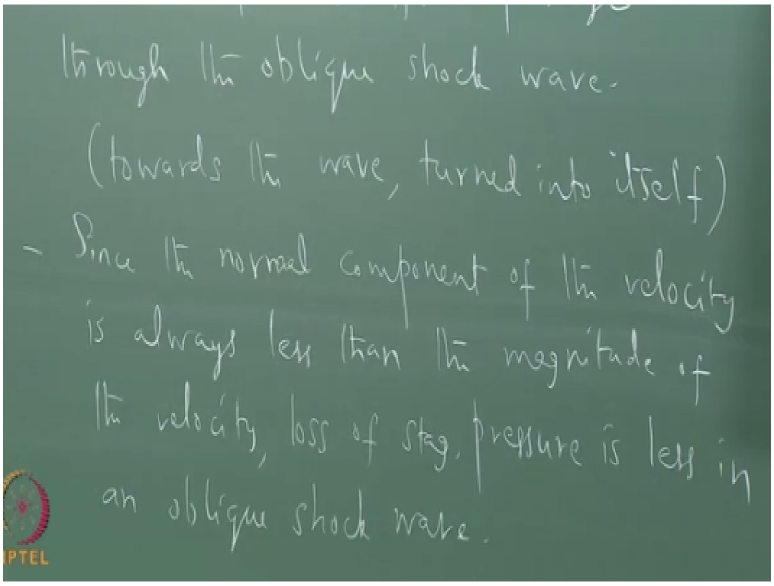

But, it is actually not possible or very impractical to compress flows at high Mach number using a normal shock wave. So, the strategy in practical devices, is to exploit this fact, compress them through a series of oblique shock waves and once a Mach number gets below 2, terminate with the normal shock wave that is very effective, very efficient, not efficient, but very effective (()) (21:43) normal shock is effective, but not efficient, so utilize the best aspect.

Decelerate the flow from a Mach number of let us say 4 or 5, may be 2 using a series of oblique shock waves which has a lesser loss of stagnation pressure and then terminate with the normal shock wave. So, oblique shock waves find a lot of use mainly because of this reason, otherwise everything is similar to this, pressure rises across the oblique shock wave, temperature increases and the normal component of velocity decreases.

The tangential component of velocity remains the same, okay. Now because the normal component of velocity alone decreases and not the tangential component across an oblique shock wave, the flow becomes subsonic only with the respective to a normal component, not the total velocity component, mn1 is supersonic, mn2 is always subsonic because that is the normal shock wave. However, m1 is supersonic and m2 can also be supersonic.

Most of the situations m2 is also supersonic, okay. That is the next important point, which we will write down here. Mn1>1, mn2<1 always, m1>1, m2>1 most of the time, we will see when this is not satisfy. So, where are normal shocks used, as I said normal shocks, I am sorry. Where are oblique shocks used? Oblique shocks are very effective for decelerating and compressing a supersonic flow.

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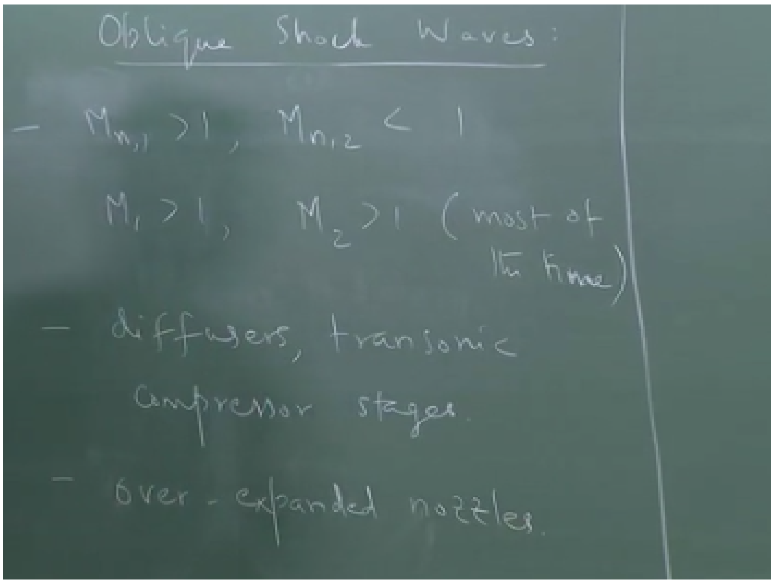

So, they are used in diffuses and also turbomachinery blade passages, if you have a transonic stage, where the stage handles mixed flows, then it is used in transonic stages also; for example, transonic compressive stages. So in this application, you design the geometry, so that you generate shock waves of a certain angle and strength. So, this is controlled. You controlled the shock wave angle and the flow deflection and the strength of the shock waves in this application.

There are other applications, where you do not really control this and that is what we saw when we discuss nozzle flows, over expanded nozzles, when it comes outside there is a shock wave that is oblique shock waves that is generated which compresses the flow, right. So there, the strength is determined by the back pressure and the nozzle geometry, so this appears also in over expanded nozzles. So, here we do not really control the angle.

It is determined by other fluid dynamic parameters. Here, the geometry is designed to generate or trigger an oblique shock at a certain angle to accomplish a certain amount of fluid deflection, okay. We are going to see the dynamics of this. Basically, what we want to do is, just like what we did for the normal shock wave. Given m1, we were able to find everything else, the pressure wise across the shock wave, m2, temperature wise, p02 or p01.

We are going to do the same thing here, but here there is one additional quantity, what is that? There are two additional quantities that is wave angle and the flow deflection angle. So, given m1, theta and beta, how are we going to determine, how do we determine the downstream Mach number and all other flow properties that is our next task. So, how do you actually trigger an oblique shock in a practical application?

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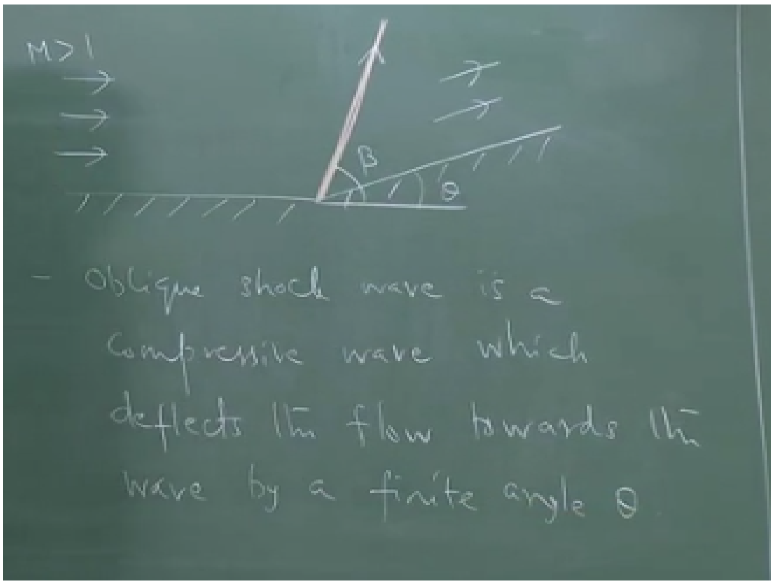

And, we said that in some cases, we design; so that we trigger an oblique shock at the certain angle. Let us consider the following situation, let us say that we have supersonic flow, we just flowing along the surface like this and it encounters a corner like this. So, let us say that in at the corner, it is inclined at an angle theta to the horizontal. So, when a supersonic flow encounters a corner like this, the flow has to be deflected through an angle theta or it has to be turned into itself.

So, what happens is, so something is generated from this corner and in this case, you know that after passage through the wave. Let us say that this is some wave, we do not know whether it is an expansion process or a compression process, but we do know that after passing through the wave, the flow is deflected towards the wave, right. We can see that it is deflected the wave through an angle theta.

Remember the wave direction itself is like this, so a wave is generated from the corner goes like this, right. So, the flow is deflected towards the wave, so that means that this is an oblique shock. So, this is how we trigger an oblique shock wave of a certain strength and angle. So, this is the deflection angle. This angle is the wave angle beta. So, if I design this theta then for a given Mach number, I fix the theta, I get a wave of certain strength and the properties are also for certain value.

So, I trigger a series of shocks like this, I can successively decelerate the flow and bring it to a value that I want, that is how we trigger oblique shock waves in practical applications. So, the corner is design to generate an oblique shock wave which deflects the flow through this angle and a wave angle beta. What is that the opposite version, where we have an expansion fan, which deflects the flow through an angle theta away from itself is forbidden, okay.

So, the oblique shock is a compressive wave which deflects the flow towards the wave by a finite angle theta. Now, the counterpart to this where a wave deflects the flow away from itself through a finite angle theta, which should be an expansion process is not allowed by second law of thermodynamics. Let us write that down also. So, an expansion wave which turns the flow away from the wave through a finite angle is forbidden by the second law.

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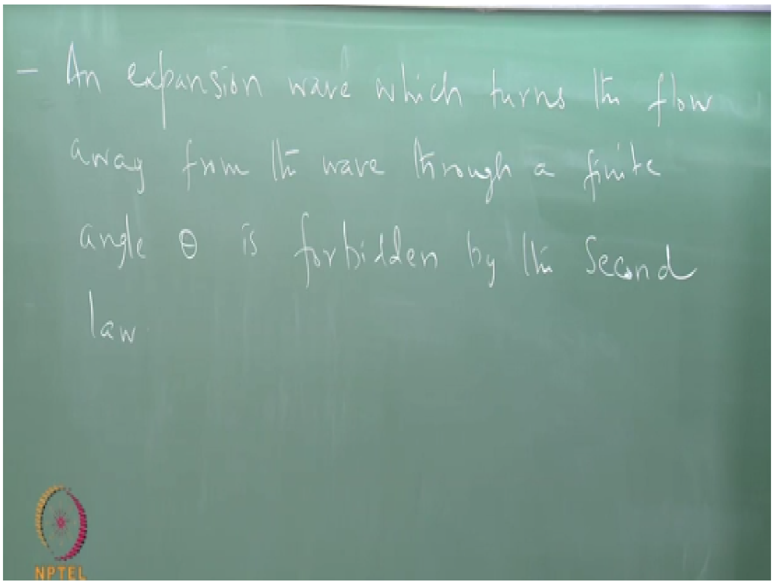

The important point here is that turning through a finite angle. You will see in the next chapter that if the flow turning is through an infinitesimally small angle, then expansion waves are allowed that is an isentropic process, entropic and remain the same. This will actually require the entropy to decrease that is not allowed in an adiabatic flow. If the flow turning is through an infinitesimally small angle, then such expansion waves are allowed, okay.

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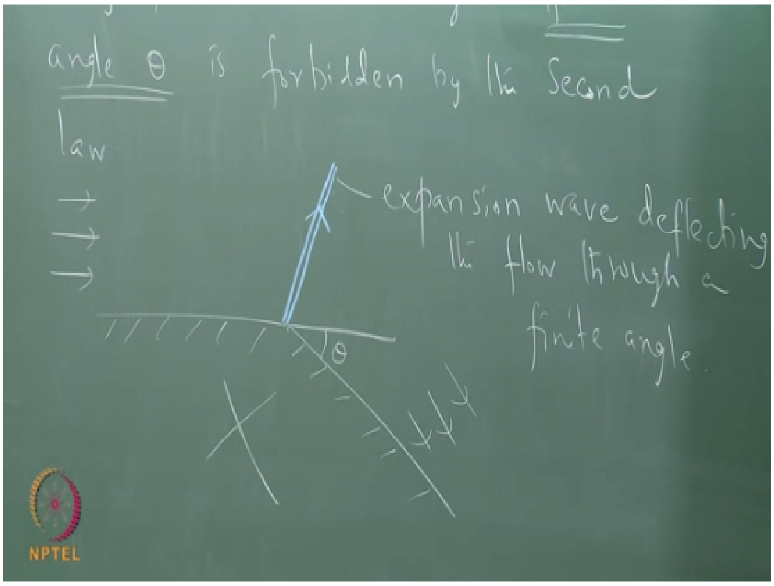

So, most important point here is that what is not allowed is turning through a finite angle in an expansion corner is not allowed, okay. So, the situation that we are talking about is the exact opposite of this. So, if we have and instead of corner which goes like this, let us say we have a corner goes like this. So, this is deflected, trying to deflect the flow away from itself through an angle theta, right.

So, an expansion wave which can accomplish this just like this of a finite strength is not allowed. So, this is an expansion wave which deflects the flow through a finite angle. This is not allowed by second law of thermodynamics. **"Professor - student conversation starts"** yes, sir the value of un1 will it be always >1, un1, yes because as far as the normal component is concern, we said that this is the normal shock wave.

So, normal shock wave, the Mach number approaching a flow is always >1. If u1 is letting near to 1 and it is a component, then it will become an acoustic wave, it will become a shock wave of infinitesimally small strength, right. It cannot be subsonic. Because if it subsonic, then we have other problem, right, that is not allowed by second law. It has to be either an expansion process, we cannot have a compression process where the flow is subsonic and it is compressed further.

The shock wave always moves with supersonic speed in a quiescent medium, which is why the flow always approaches the shock wave with the supersonic Mach number. The smallest possible is an acoustic wave which moves with the speed of sound, so that cannot be < that. A wave solution as we discussed in our earlier chapter, a wave solution is permitted only if the speed is sonic or supersonic.

The nature of the governing equations is such that the equations are hyperbolic when the velocities are supersonic. So, it is only when the flow the equations behave with the hyperbolic character that wave solutions are permitted. The flow velocity is subsonic, then the equations behave in a manner called elliptic and there is no wave solution for an elliptic equation. Wave solutions are permissible or permitted only for hyperbolic equations.

"**Professor - student conversation ends"**. Okay, so what we are going to do know is relate m1 beta and theta to m2 and other downstream properties. So, let us write down the governing equations. They are almost as same as what we wrote down for normal shock. So, we are going to write it in terms of the normal components. Rho1un1 = rho2un2, p1+ rho1un1 square = p2+ rho2un2 square.

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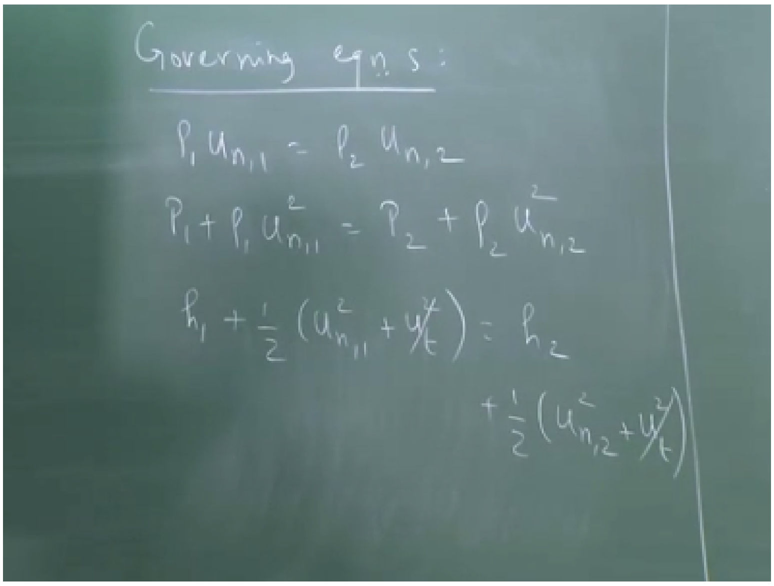

Energy equation remains the same, h1+ one half un1 square+ ut square = h2+ one half un2 square+ ut square and ut remains the same across we know that, so ut square cancels out. So, we are left with the almost the same thing as what we had before. In addition, so this implies that t02, in the eval, we can leave this, we will pick it up later this is fine. So, this is the governing equation.

And, what we want to do is, obtain a solution which relates m1 theta and beta to the downstream properties, okay. Now, if you look at the velocity triangles here, you can see that this angle, we have to calculate this angle, this is 90 degrees, right, u1 and ut are perpendicular to each other, so this is 90 degrees and this angle is beta, so that this angle is, what is this angle? Beta- theta, right.

So, I am going to write down the following equation from this velocity triangle. Un1 = u1 cosine beta and un2 = u2sine beta- theta. **"Professor - student conversation starts"** yet I do this right, oh, sine beta, thank you. "**Professor - student conversation ends"**. And, we also have ut = u1 cosine beta which is also equal to u2 cosine beta- theta. So, from these 2, I can write this as un1 over un2 = tangent beta divided by tangent of beta- theta, okay.

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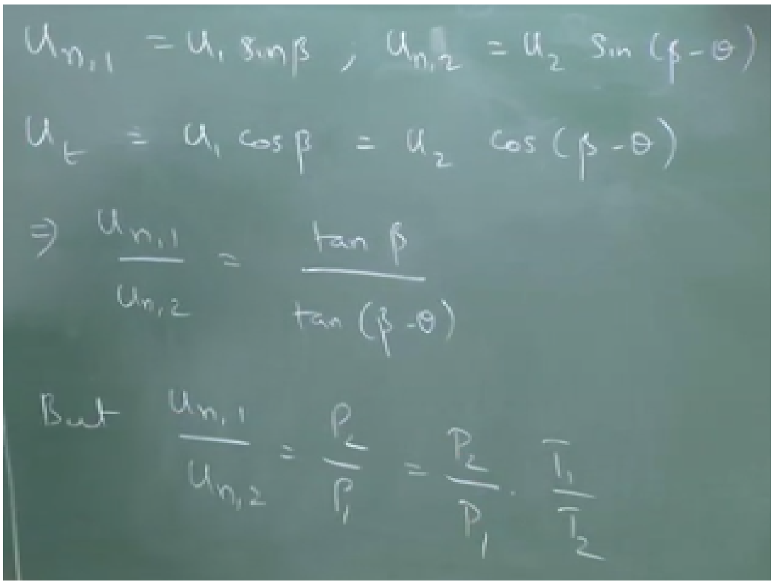

That is u1 over u2, so if I divide these 2 expressions, I get u1 over u2 and I can get u1 over u2 as cosine beta- theta divided by cosine beta. So, I have eliminated that, so I end up with something like this. This is one relationship for un1 over un2, but I can also see from here that un1 over un2 = rho2 over rho1, so I can do that also. So, from the continuity equation, I can write the following and if I use the equation of state.

I can write this as p2 over p1 times t1 over t2. Now in the right hand side, remember these are static quantities and we already wrote down relationships when we discuss normal shock waves. We already wrote down relationships for these 2 in terms of m1 and m2 at the time, so the m1 and m2 that we used there, now become mn1 and mn2, fine that is what we are going to do.

So, if you rewrite this, this can be written as if you use those relationships from normal shock wave, I can actually write this as gamma+ 1 times mn1 square divided by 2 gamma- 1 times mn1 square. So, this we obtain in the right hand side, we have obtained from normal shock relations after replacing the m1 and m2 there with mn1 and mn2, okay. Remember, we knew mn2 also; we had eventually solve the equation.

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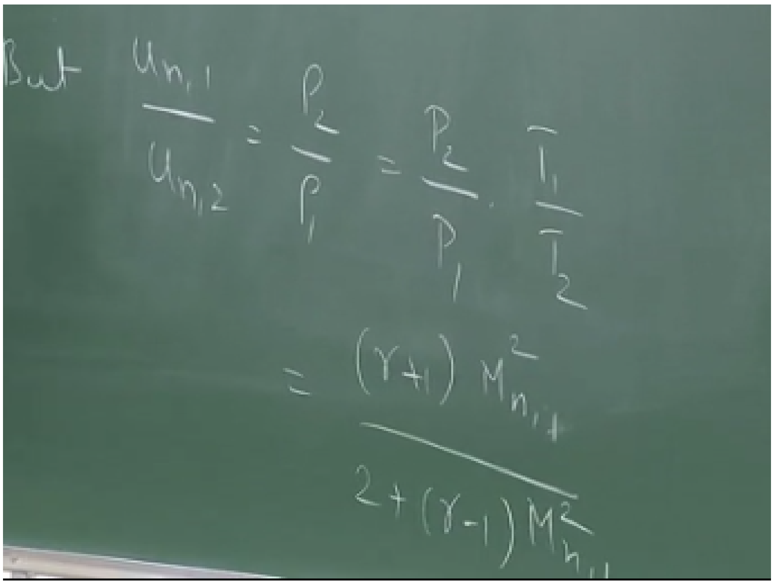

So, we know mn2, so I have written everything in terms of mn1. But, we also know that mn1 from these velocity triangles or if you look at this relationship divided both sides by square root of gamma rt1, what do I get, I get mn1 here, right. I get m1 here, mn1 = m1 sine beta. So, this is the same relationship, I am not doing anything different, so mn1 is m1 sine beta and mn2 = m2 sine beta- theta.

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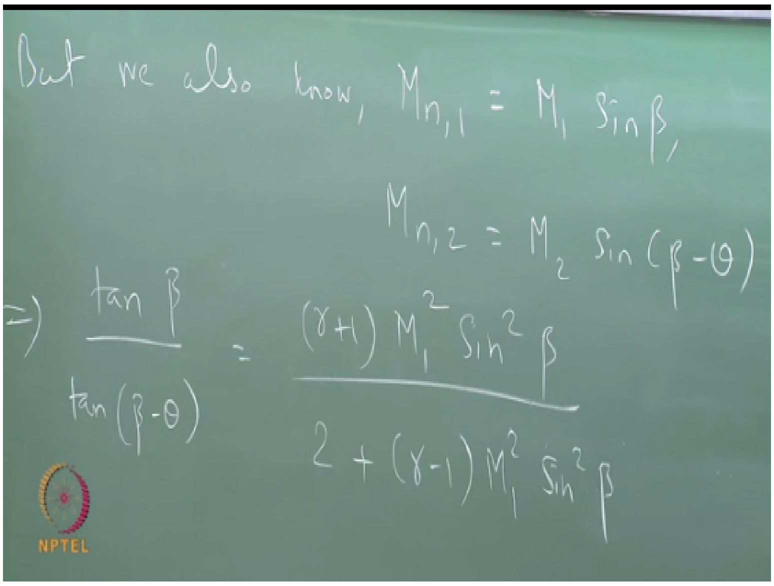

Speed of sound is the same, irrespective of the velocity component, right, which is why I can divide both sides of this also by square root of gamma rt2 and we will get mn2 = m2sinebeta-theta, so I have all things, so I take this, I substitute that here and equate that to here. We finally get if you equate the 2 relationship for un1 over un2, I get tan beta divided by tan beta-theta = gamma+ 1 times m1 square sin square beta divided by 2+ gamma- 1 times m1 square sine square beta and this was pretty much what I wanted, correct.

I wanted a relationship which connected m1 with beta and theta, m1beta and theta with m2 and other downstream properties, so that is what I have now. So, once you give me one quantity or two quantities in this, I can evaluate all the other things. It is easy to rewrite this slightly. If it is better to rewrite this slightly like this just simple rearrangement gives you tan theta = cotangent beta times m1 square sine square beta- 1 divided by m1 square times gamma+ cosine2 beta+ 2.

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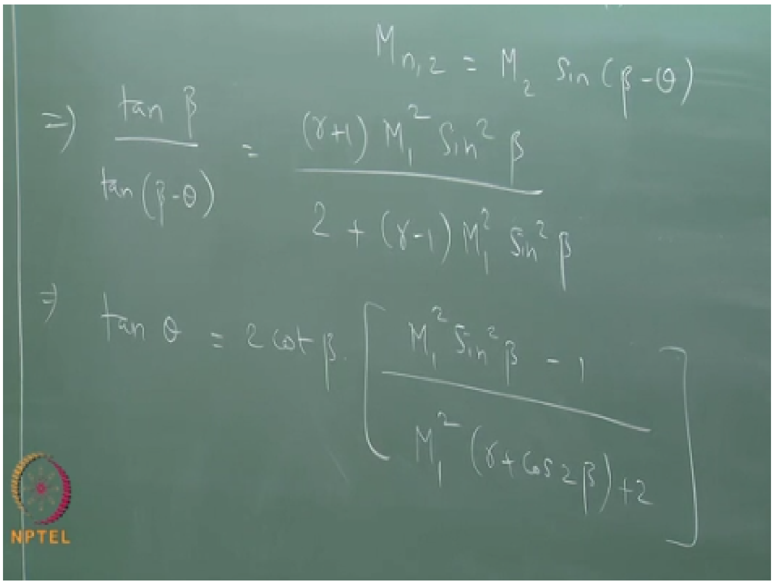

So, previously in the normal shock wave, there was only one quantity given m1, we wanted to calculate m2 and all the other properties. Here, there is one more, given m1 and either beta or theta, I can calculate all the other quantities. That is what this is telling you. So, given m1 or given any two quantities here from the list theta, beta, m1, the other quantity can be calculated from this relationship and this relationship is known famously as the theta, beta, m relationship.

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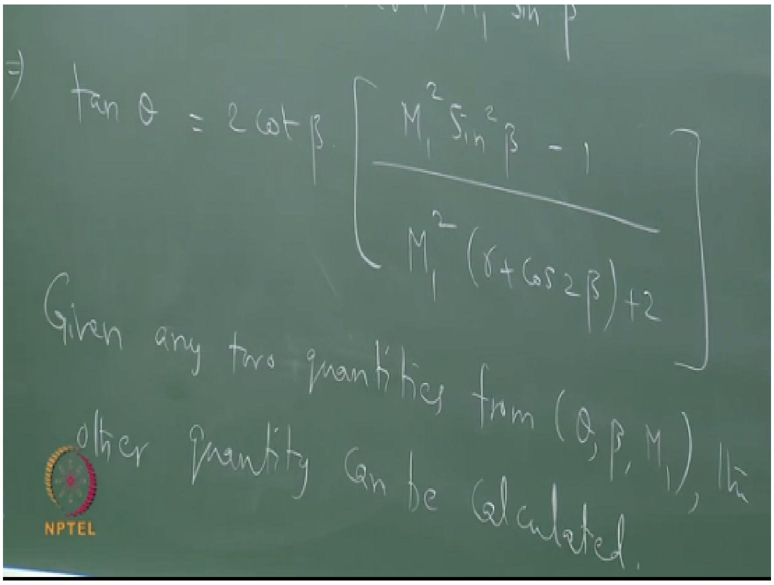

So, this is known as the, what we need to do next is look at this solution and then see what the constraints are for practical application. For example, things like for any Mach number m1, can I deflect the flow through any angle that I want, number 1 or for a wave angle, given wave angle and m1 what kind of flow deflection angles are possible. Can I deflect the flow through any angle I want?

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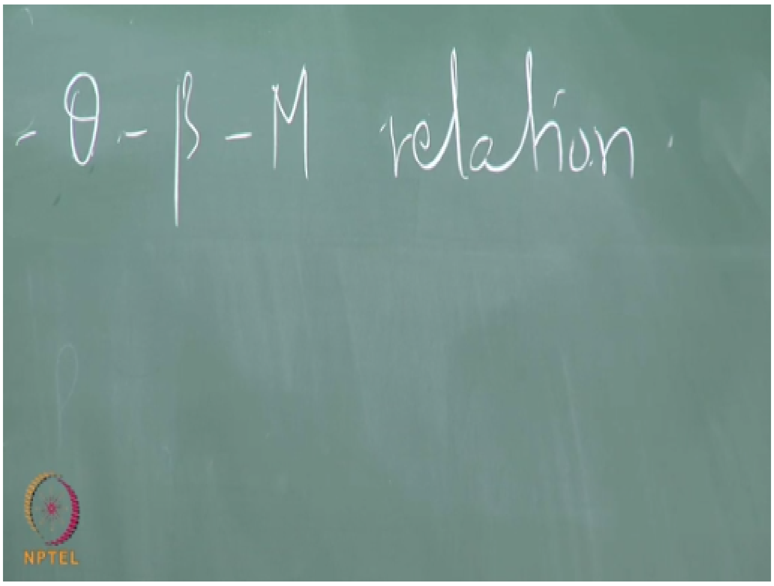

So, there are many such inferences and questions that we need to look at. Another important question is this appears to be a nonlinear equation. The trouble with nonlinear equations is they usually have more than one solution that is the case which are the solutions which are meaningful and do not violate second law and so those are the one that are going to be seen in real life application.

So, there is more than one solution which does not violate second law, then which one will be see in an application that is also important. So, those are the kinds of issues that we need to look at next when we solve these equations. And, try to obtain solutions and then infer the behavior of this equation. It is easier not to solve this equation, but though actually look at this equation in the parametric way.

So, I keep m1 fixed and I keep let us say theta fixed and I vary beta, then I look at the values, the range of values that (()) (46:58). So, I can do things like that then construct curves m = constant curves as the value vary through the other 2 extremes. I can look at m = constant curves and then infer the behavior of the solution from that other than solving for them directly. Remember that has been our strategy throughout.

We do not want avoid solving equations even when we looked at really flow or fan of flow or normal shock wave, we prefer to tabulate rather than solve that is the much more practical strategy. Here also we will do the same thing. You will not attempt to solve this equation. But, we will try to tabulate the solutions or draw curves and then draw inferences from that. So inferences from the theta, beta, m curve is something that we will take up in the next class.