Gas Dynamics and Propulsion
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Lecture - 19
Quasi One Dimensional Flows

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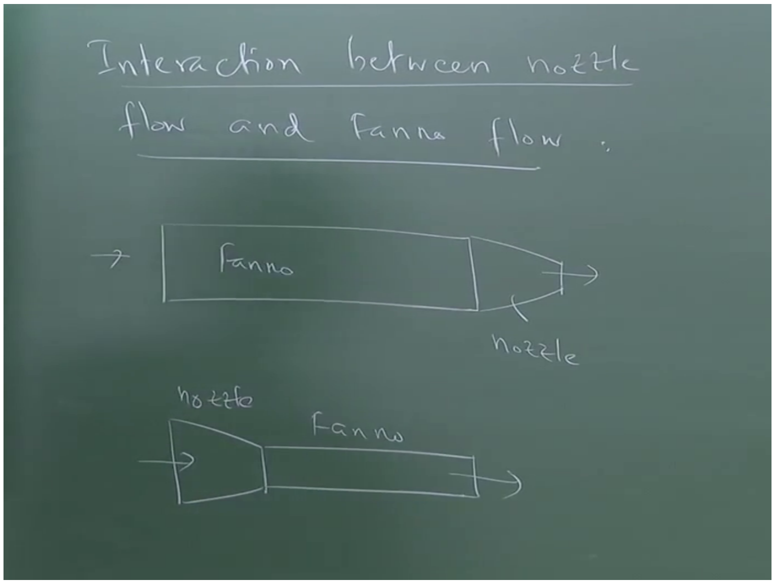

In the previous class, we look at the interaction between nozzle flow and Fanno flow, under this category we looked at 2 situations. One when convergent nozzle is located downstream of a pipe or duct so this was the situation that we looked at, so we had a converging nozzle which was located downstream of a pipe, so we have a flow coming in like this and going out like this, so we had Fanno flow in this section and this is the nozzle, so we had isentropic flow in the nozzle section.

And then we also considered a situation when we had a nozzle which proceeded pipe like this pipe or a duct like this, so we had flow which was going through, so we had Fanno flow here and this was the nozzle so we had isentropic flow in the nozzle, we look that both these cases in the previous lecture. And today we are going to look at the interaction between flow in the convergent divergent nozzle and duct.

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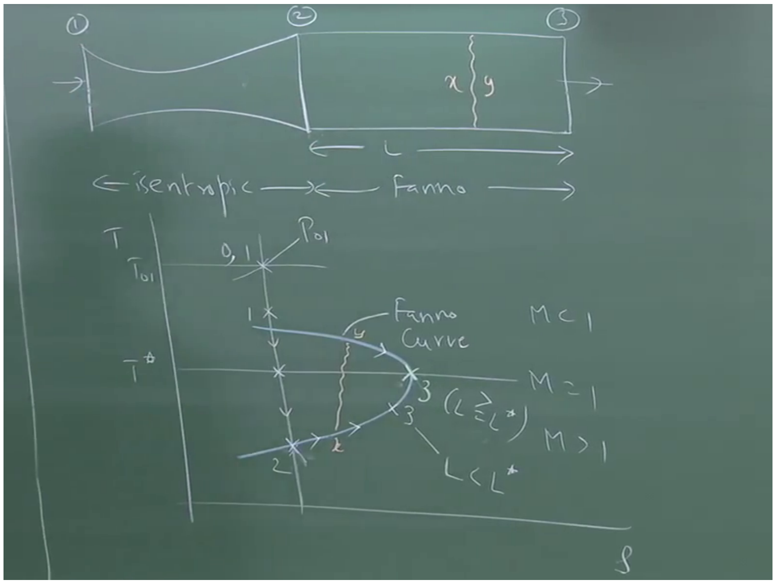

So the situation that we are looking at is something like this, so we have a convergent divergent nozzle which precedes duct or a pipe, so we have a duct or a pipe which looks like this, so you have flow entering this way and exiting this way. So let us label this as section 1, this is section 2 and this is section 3, and the flow is isentropic in this part in the nozzle part, and we have Fanno flow in this part of the domain.

So if I sketch the process on a TS diagram, the diagram would look something like this, so let us take a TS diagram and let us say that this is my initial stagnation state, so let us say this is T01 and let us say this is P01, so that this is stagnation state 0, 1. And we started let us say subsonic state which is 1 as given here like this, so that is this the subsonic state where the Mach number is <1, and let us say that this is T star corresponding to this value, so this is M=1 and this is M<1 and this is M>1.

So the flow enters at a subsonic Mach number, the subsonic Mach number and then the flow reaches a supersonic Mach number at state 2, so it undergoes an isentropic expansion process from state 1 to state 2 with throat being at the sonicstate and after doing this we have a Fanno process, so the Fanno curve for the given mass flow rate may look something like this. So the Fanno car looks something like this, so this is my state 2.

And the Fanno curve which passes through state 2 may look something like this, and if L is less than, if the length of the duct L<L star corresponding to Mach number M2, then state point 2 or state point 3 may lie somewhere here, so the flow proceeds this way from 2 to 3 we have Fanno flow here to here, so this is the Fanno curve. So this is what we get if L<L star, so this is the end state for L<L star corresponding to M2.

If L=L star corresponding to M2 then the exit state would lie here, so this is when L=L star corresponding to M2. Now if L is actually >L star corresponding to M2, then there is going to be a normal shock somewhere in the diverging part of the I am sorry there is going to be a normal shock in the duct somewhere and then the flow becomes subsonic across the normal shock and then it goes from there.

So the same situation if I try to illustrate on this diagram, then the flow may something like this, so there is a normal shock and the corresponding position maybe something like this, so let us call this state x and this state as y, so this is x and this state is y, so the flow goes from 1 to 2 to x and then it jumps to y, and then depending upon the exit condition it may reach state 3 or the sonicstate even if L>L star.

So it will definitely have reached state point 3 if L=L star, if L>L star then we have a normal shock and it may reach the sonicstate even in this case depending upon the ambient or the backpressure condition okay, so these are the three possible solutions. That note that in this case the mass flow rate once we have this because this flow is isentropic, the mass flow rate in this case is not change as a result of the addition of the duct.

The only thing that can change probably are the exit pressure, and the presence of a shock you may or may not have a shock depending upon the length of the duct. Unlike the previous case the mass flow rate is not change in this case unless you make this length so long that you go all the way back and change the flow in the nozzle which is very unlikely okay. So this covers all aspects of interaction between a nozzle flow and Fanno flow.

We can do similar thing or almost identical thing for interaction between nozzle flow and a Rayleigh flow okay, it is not very difficult to do we will try to do that through a numerical example. So the next thing that we are going to do is a numerical example first one involving a nozzle flow and the Fanno flow, and then another one involving nozzle flow and Rayleigh flow, we will do 2 examples.

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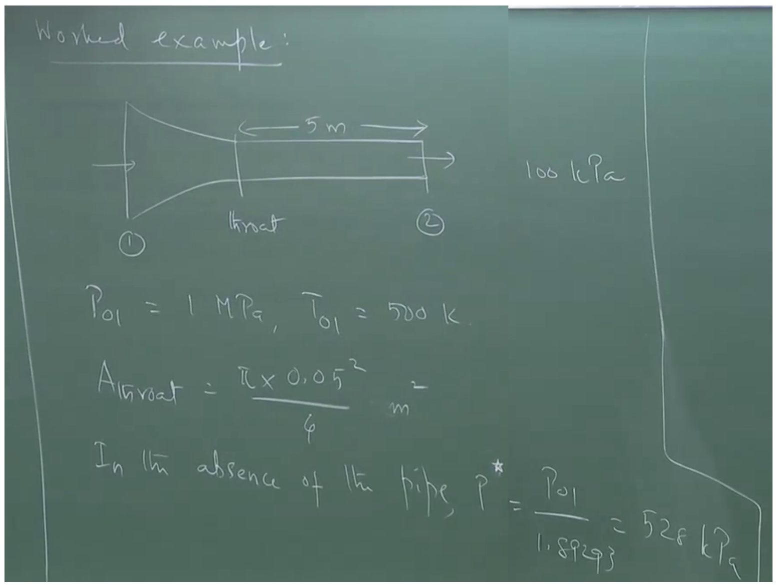

Let us do the first one, so the example reads like this. Air flows through nozzle pipe combination which looks like this, so air flows through the nozzle pipe combination shown in the figure, so this is 1, this is throat and this is labelled as 2, this stagnation conditions at the nozzle inlet are 1 megapascal and 500 Kelvin, the pipe diameter is 0.05 meters and it is 5-meter long. Determine the reduction in mass flow rate due to the presence of the pipe take f to be 0.024 and the back pressure to be 100 kilopascal okay.

So let us write down the information that is given in the problem, the length of the pipe is given to be 5 meters and P01 is given to be 1 megapascal, T01 is given to be 500 Kelvin, the diameter of the pipe is also given, so area of throat is pi times the diameter of the pipe which is 0.05 meters, so that is also known okay so let us leave it like this, so that is known, so this is the area of the throat in meter square.

If I do not have the pipe then you can see that the flow through the nozzle is definitely going to be choked because P star corresponding to this P01 is quite high, the ambient pressure is given to be 100 kilopascal so that is given in the problem, so P star corresponding to P01 is going to be much higher than 100 kilopascal, so in the absence of the pipe the nozzle is definitely choked. Let us write like this P star can be obtained from the because of flow is isentropic we can obtain the value from the gas tables.

Let us go ahead and do that, P0 is given, we can calculate P star like this so P0/P star for this case is 1.89293, so this is P01/1.89293 and this value comes out to be. **"Professor - student conversation starts"** Can someone calculate this value and tell me 528 kilopascal. **"Professor - student conversation ends."** It is important to actually calculate the P star in these cases where the ambient pressure is given, you cannot simply assume M to be 1.

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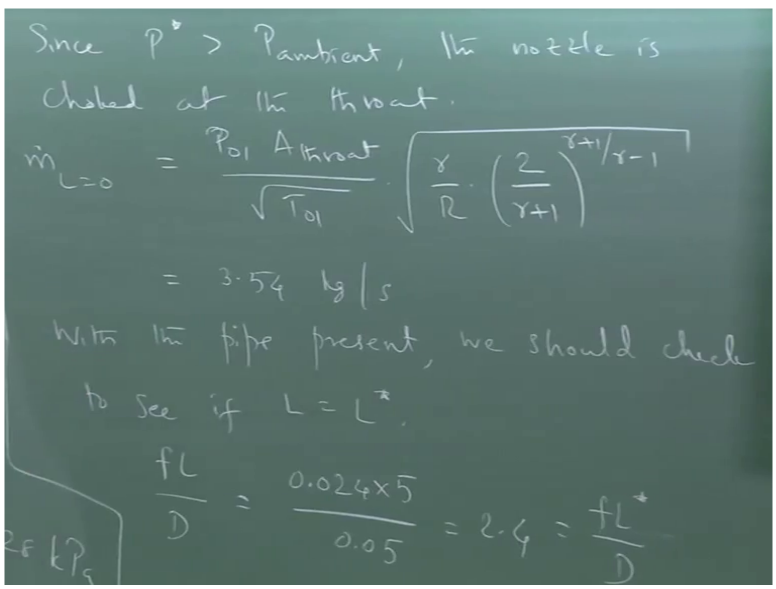

You need to check to see whether M is indeed 1 or not, so we calculate P star for this case, since P star>P ambient which is 100 kilopascal, the nozzle is choked at the exit, in this case at the throat section, which means I can calculate my mass flow rate corresponding to no pipe being present, so that corresponds to length of pipe being 0, this is nothing but P01 A throat/square root of P01 times gamma/R times 2/gamma+1 raise to the power gamma+1/gamma-1.

And if you plug in the values and take gamma to be 1.4 for this case you get this to be 3.54 kilogram per second for the case when the pipe is absent. When the pipe is present the stagnation pressure at the throat is known right still that is=P01 okay, so first we need to see whether the flow is choked at the exit or not okay. So what we do is the following, if so with the pipe present check to see if L=L star, once we do that other things can be easily done that is not a problem.

So we go to the Fanno table f is given to be 0.02, so if times L/D=0.024 times 5/D is 0.05, and if you evaluate this, this comes out to be 2.4 and we assume this to be L star right we take this to be L star, so we say that this is L star and we want to see whether the exit pressure P star>P ambient or not.

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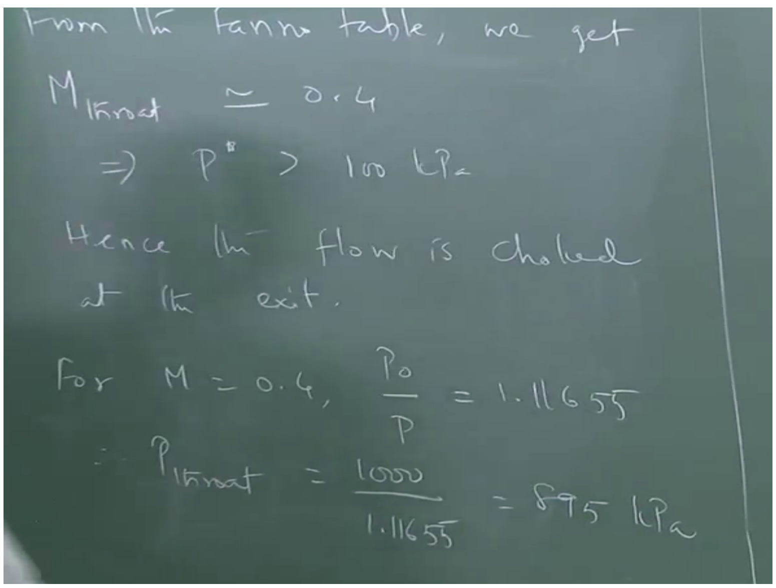

So from the Fanno table we get for this value of f L star/D we get M throat to be approximately 0.4, and we also get P0/P star from the table, from the Fanno table we get P/P star and also P0/P star. So from this we show that P star>100 kilopascal which is the ambient pressure, hence the nozzle is choked okay, is this clear? “Professor - student conversation starts” go ahead, P star is at exit of the pipe. Yeah P star, remember we have assumed L to be=L star, so P star is at the exit of the pipe okay.

In fact we can if you want we can go through this that is not a problem let us go ahead and do this calculation okay. “Professor - student conversation ends.” So let us go to the table and get for M=0.4 P0/P can be evaluated as 1.11655 from the isentropic table, P0 is known therefore P throat=P0 is given to be 1 megapascal, so that is 1000 kilopascal/1.11655. **"Professor - student conversation starts"** can someone tell me what this is, so this is 895 kilopascal. **"Professor - student conversation ends."**

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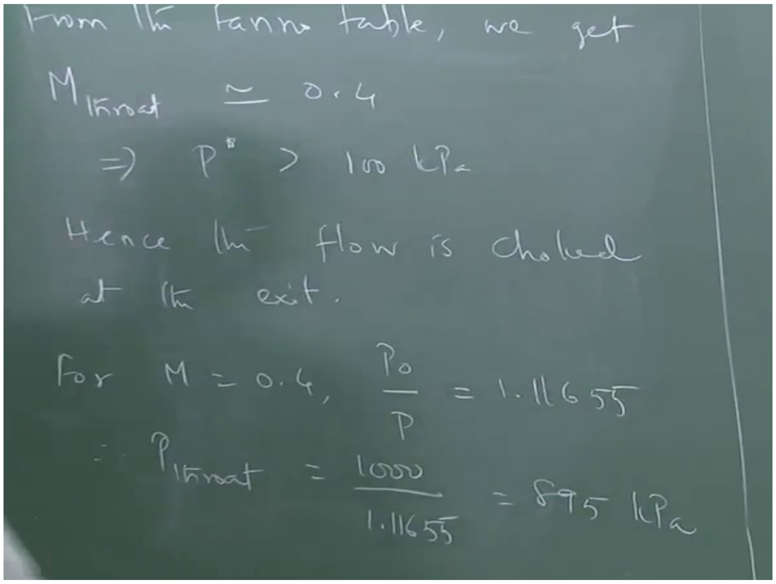

Now from Fanno table for M=0.4 P/P star is 2.69582, so P throat is known, so from this I can calculate P star to be 895 kilopascal/2.69582 which is 332 kilopascal. So if I assumed L to be=L star and then go through with this calculation I get my P star to be 332 which is more than 100 kilopascal which is the ambient pressure, hence I am safe in saying that the flow is indeed choked at the exit.

If the ambient pressure is given you must always check this to see that the process more, because it cannot be less than the ambient pressure, if it is going to be subsonic at the exit, then the exit pressure has to be equal to the ambient pressure and there is no possibility of over expansion in this case okay. So you must establish if the ambient pressure is given in the problem, so this is safe we are okay.

So now the mass flow rate for a duct with length 5 meter being present here can be calculated like this, rho throat times u throat times A throat, so rho u A at the throat section. And I can rewrite this as follows, rho is nothing but P/RT, and u can be written as M times square root of gamma RT, and A we leave as it is, so this evaluated at the throat section. And if I rearrange this in terms of the stagnation quantities.

I can write this as P/P01 times P01 times so the square root of gamma/R comes out like this, and there is a square root of T in the denominator, so I can write this as, all evaluated at the throat section okay. So notice that I have multiplied and divided by a P01, and I have multiplied and divided by a square root of T01, so I know M throat, I know A throat, I know T01, so this can be evaluated and I know this also. All the quantities here can be evaluated and this mass flow rate comes out to be 2.235 kg per second.

So you can see that the mass flow rate because of the duct has reduced from 3.54 kilogram per second to a value of 2.235 kilogram per second, so if I add a 5-meter pipe to the end of the nozzle, the mass flow rate reduces by about 37% approximately, any questions or doubts? Okay. Let us do another example, this one involving interaction between a nozzle and Rayleigh flow not exactly Rayleigh flow but with heat addition.

This problem reads like this, in an aircraft jet engine fitted with a constant area afterburner and a converging nozzle, air enters the nozzle with stagnation temperature and pressure of 900 Kelvin and 0.5 megapascal when the afterburner is not lit. If the afterburners lit the stagnation temperature increases to 1900 Kelvin with a 15% loss in stagnation pressure at the nozzle inlet, if the mass flow rate has to be maintained at 90 kilogram per second.

Determine the required nozzle area in both cases, also determine the thrust augmentation with afterburner operation assuming that the engine is on a static test and at sea level within bracket ambient pressure 0.1 megapixel assume isentropic process for the nozzle, for the fluid that passes through the afterburner take gamma to be 4/3 and CP to be 1.148 kilo joule per kg per Kelvin. So this is the second worked example.

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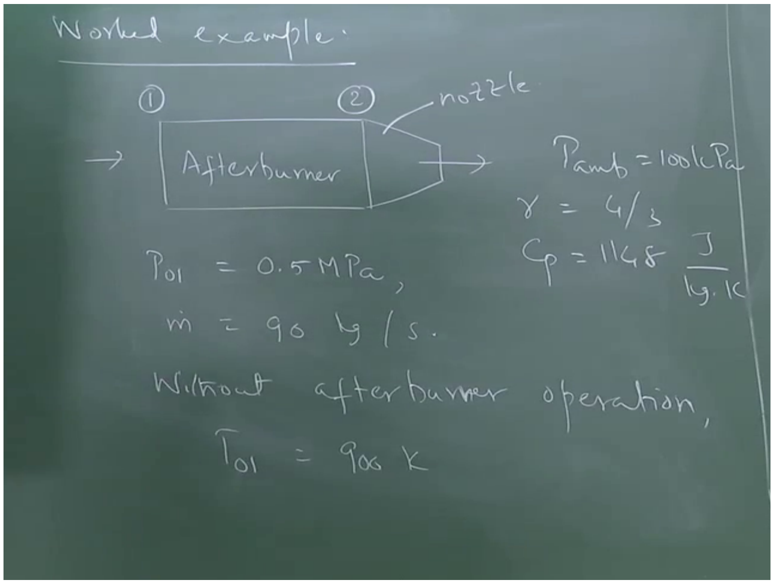

Let us sketch the problem or the situation given in the problem, so we have a constant area afterburner followed by a convergent nozzle, so heat is when the afterburner is so this is the afterburner constant area afterburner, and this is the nozzle 100 kilopascal, and P0 so let us call this 1, let us call this 2, and this is the exit section so there is no problem. So P01 is given to be 0.5 megapascal, and T01 is given to be 900 Kelvin this is without the afterburner.

So that is the first case that we are going to look at without afterburner what is the mass flow rate the throat area and the thrust that is required, m dot is given to be 90 kg per second. So let us write this problem statement slightly differently, for both cases gamma is given to be 4/3 and CP is given to be 1148 joule per kg Kelvin, so without afterburner operation the T01 is given to be 900 Kelvin, so since the ambient pressure is given you first need to check whether the nozzle is checked or not right.

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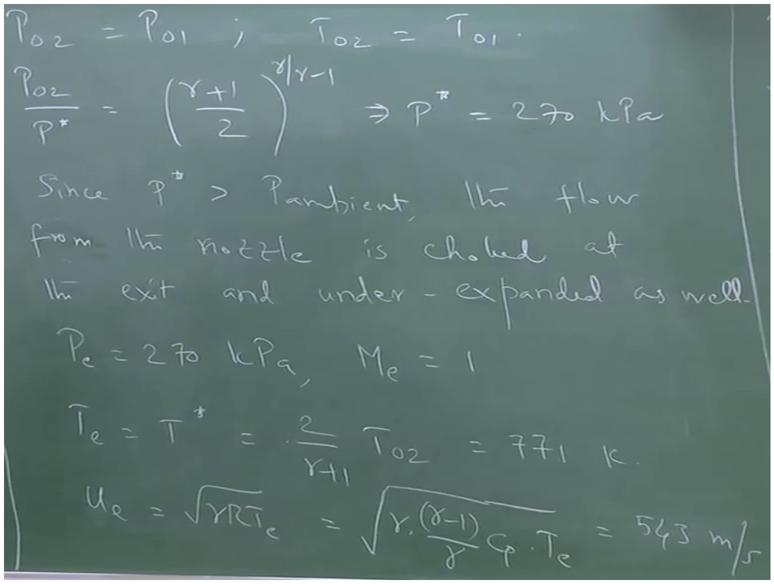

So when the afterburner is not operating there is no heat addition here, so stagnation quantities everything remains the same, it is as if the duct is not present at all, so P02=P01, so in this case P02=P01 and T02=T01 when the afterburner is not operating. And P02/P star as you know=2/gamma+1 correct I am sorry gamma+1/2 raise to the power gamma/gamma-1, and if I calculate this.

So from this if I calculate P star with the value of P02 that I have I get P star to be 270 kilopascal which >the ambient pressure that means the nozzle, the flow in the nozzle is choked and under expanded right, so this is the flow that we say from the nozzle is choked at the exit and under expanded also. So P exit=270 kilopascal, M exit=1, and T02 is known, so I can calculate T exit=T star, and that is nothing but 2/gamma+1 times T02, and this comes out to be 771 Kelvin.

And the exit velocity ue is square root of gamma times R times Te right, because the Mach number at exit is 1, the ue square root of gamma R Te, and if you remember this is gamma, CP is nothing but that gamma times R/gamma-1. So we can we write this in terms of CP is given in the problem, so I can write this as the gamma-1/gamma times CP times Te, so this gives me the velocity at exit to be 543 meter per second.

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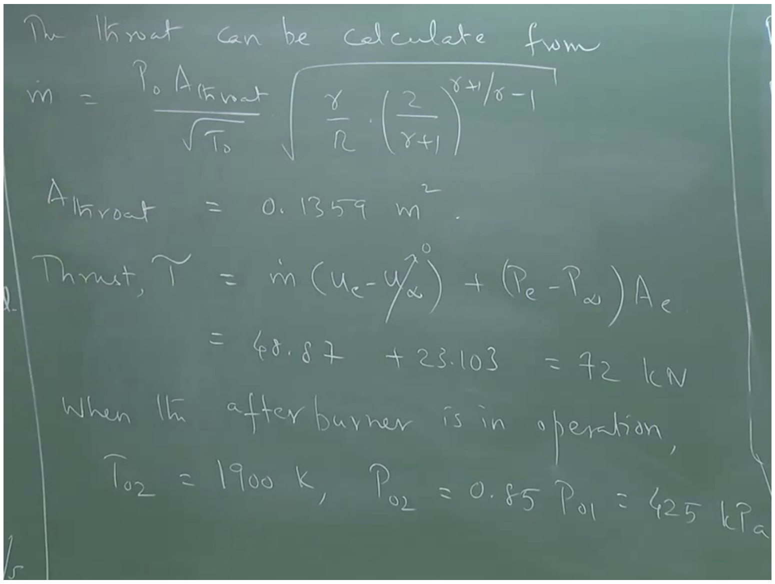

The nozzle is choked and the mass flow rate is given to be 90 kg per second, so I can use the expression for choked mass flow rate in an nozzle, and the throat area can be calculated from our usual formula m dot=P0 A throat/square root of T0. So therefore, A throat when the afterburner is not in operation, if you substitute the numbers this comes out to be 0.1359 meter square okay, and by using the impulse function we can calculate the thrust.

Although we have not shown this formula we will show this later on, but first it is easy to use it now, so the thrust is given by the expression m dot times ue-u infinity+, and since it is mounted on test and u infinity is 0, so this is expression for the thrust produced by the afterburner. And if you substitute the numbers, notice that since the flow is under expanded P-P infinity is a positive number, so we are going to get some thrust from both the terms.

So this comes out to be 48.87 from the first term, and from the second term we get 23.103, so it sums up to 70 approximately 72 kilonewtons of thrust, this is when the afterburner is not lit that means we are not changing the stagnation temperature here. So when the afterburner is in operation, we need to do the same calculation, so let us see what happens when the afterburner T02 is given to be mass flow rate has to be maintained the same, so that does not change.

T02 is given to be 1900 Kelvin, and P02 is given to be 0.85 times P01 it said that there is a 15% loss of stagnation pressure, so this comes out to be 0.85 times P01 and that is nothing but 425 kilopascal.

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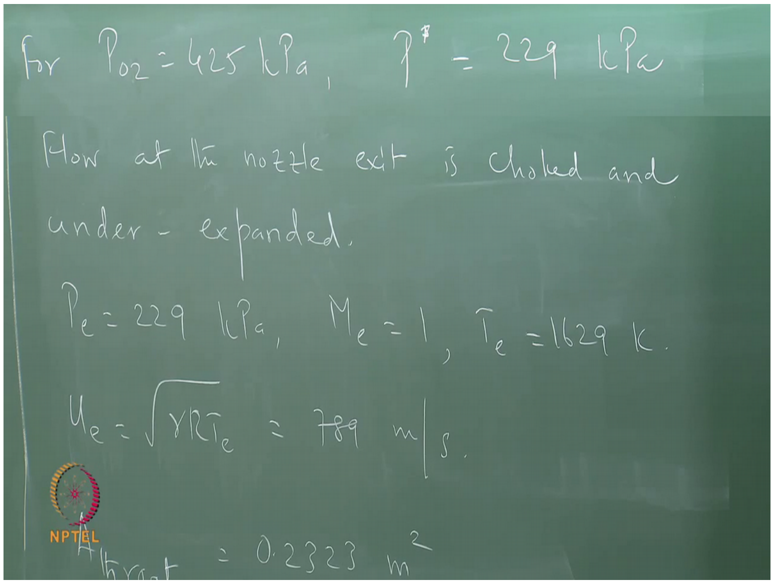

And we calculate P star once again, for P01 I am sorry for P02=425 kilopascal, P star works out to be 229 Kelvin kilopascal which is still >the ambient pressure, so flow at the nozzle exit is choked and under expanded. Therefore, P exit=229 kilopascal, M exit=1, and T exit can also be calculated from the given value of the stagnation temperature, so T exit comes out to be 1629 Kelvin that=T star right.

And velocity at the exit can be calculated as the same manner as before, so square root of gamma R times T exit, and if you substitute the numbers you get this to be 789 meter per second, so the thrust now with the afterburner operating if I substitute into this expression, so the thrust the afterburner operating comes out to be 71 for the first term+30 for the second term that is 101 kilonewtons.

"Professor - student conversation starts" The exit area is changed. I am sorry I am going to calculate that okay, so shall we calculate that before doing this okay. **"Professor - student conversation ends."** Mass flow rate remains the same, now I have my quantities, so I can use this formula to calculate the new exit area P0 is changed, T0 is changed, so remember by looking at the numbers you see that P0 has decreased from before, T0 has increased from before, M has remained the same.

So that means A throat has to increase to accommodate the same mass flow rate, so if you calculate for these values A throat with the afterburner in operation has to be more and it comes out to be 0.2323-meter square. So you can see that the area has to be throat area has to be increased to accommodate the same mass flow rate now. If I substitute this value of the area exit area into my expression for thrust which is over here.

So if I substitute mass flow rate is the same, I have a new exit velocity, new exert pressure, ambient pressure is the same, I have new exit area. If I substitute these values into this, then I get thrust with the afterburner in operation to be 71+30 that is 101 kilonewton, which is roughly 40% increase in the thrust. So this shows that the afterburner is an extremely useful and convenient device for short term thrust augmentation, it is very simple in construction.

It is not very efficient, but if you want short term thrust augmentation then it is very effective way of achieving that, you get a 40% increase by injecting more fuel into the stream here burning in, increasing the stagnation temperature and then expanding it in the nozzle with increased area. The only additional complexity is that the area of the nozzle has to be increased, so which means you need a variable throat area nozzle when an afterburner is connected to the engine.

And we will see how these nozzles are fabricated later on when we talk about after burning engines okay, but this demonstrates the augmentation in thrust that is possible okay. Now one point that we discussed earlier has to do with P0/P ambient, now if you take the case without afterburner operation you can see that P0 is 500 megapascal, so P0/P ambient is 5 correct, the question is since P0/P ambient is 5, and I am using only a convergent nozzle.

If I had used a convergent divergent nozzle how much would the thrust have increased, would I have gotten a lot more extra thrust or is it about the same not worth doing a convergent divergent nozzle, we will examine that next just to complete the problem okay. So we are looking at a situation when the afterburner is not operating, so that is a simple thing to look at and we are going to use a convergent divergent nozzle right.

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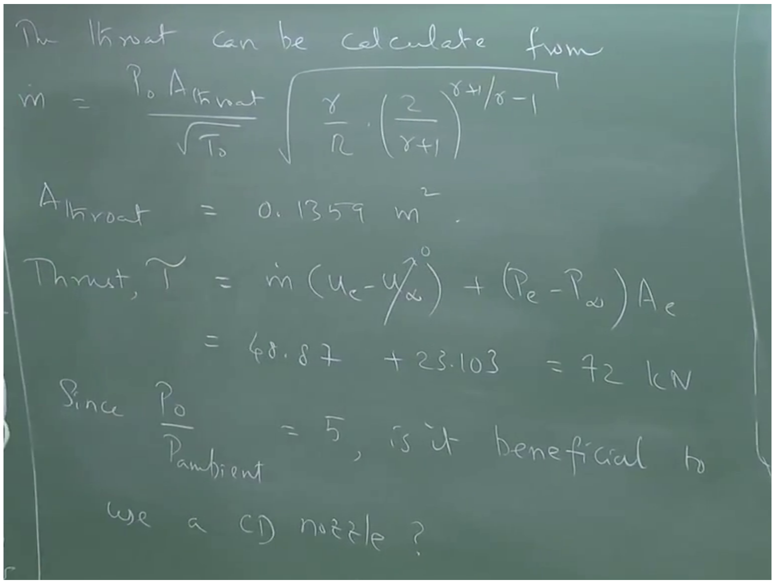

So since P0/P ambient=5 in this case 500 kilopascal/100 kilopascal it is 5 is it better or beneficial to use, earlier we stated that if P0/P ambient is more than 3 or 4 approximately it may be better to use a convergent divergent nozzle, let us see whether that is true or not. So if we are going to use a CD nozzle and we expand the flow correctly to the ambient pressure right.

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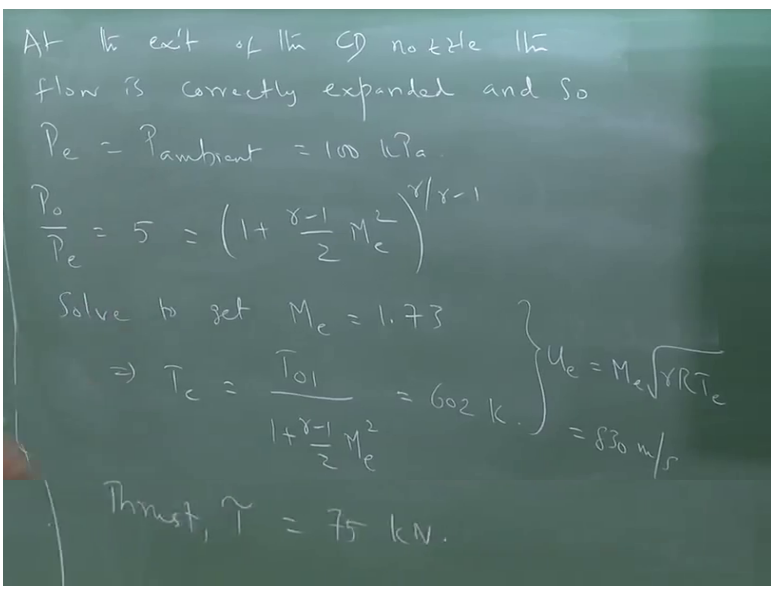

So at the exit of the CD nozzle, so the flow is correctly expanded and so Pe=P ambient=100 kilopascal, so this means P0/Pe=5, and we also know that since the flow is isentropic, this is nothing but 1+gamma-1/2 times Me square raise to the power gamma/gamma-1. So I substitute gamma=4/3 and I can solve this equation to obtain Me right, to solve to get Me to be 1.73, in fact if I expand it the flow all the way to atmospheric pressure my exit Mach number would be supersonic and correctly expanded.

So once I have Me I can get my exit temperature static temperature I know the stagnation temperature, so I can get my exit static temperature T01/1+gamma-1/2 times Me square and this gives me 602 Kelvin, and using these 2 values exit member and static temperature I can get my u exit as Me times square root of the gamma R Te, and the exit velocity comes out to be 830 meter per second, so the exit velocity with the converging nozzle came out to be 543 meter per second.

Now the exit velocity with convergent divergent nozzle comes out to be 830 meter per second, but Pe=P ambient which is also P infinity, so if you now evaluate thrust that is that the nozzle is going to develop, notice that if you look at these 2 terms, now with the convergent divergent nozzle this is going to be 0, because Pe=P ambient only this contributes the thrust, so thrust in this case is going to be, so if you substitute the numbers you get the thrust to be about 75 kilonewtons.

Earlier it was 72 kilo newton now it is about 75 kilo newton an improvement of 3 kilonewtons, so definitely there is an improvement and the improvement will become better as your P0/P ambient keeps increasing it is 5 in this case as it keeps getting better it will become more and more. So this shows that for these kinds of ratios P0/P ambient it may not be worth having a convergent divergent nozzle because of the normal shock during startup and the complexity that brings in it may be better off just using a converging nozzle in this case okay any questions okay.

So that concludes our chapter on Quasi One Dimensional Flows, we will start the next chapter on Oblique Shock Waves.