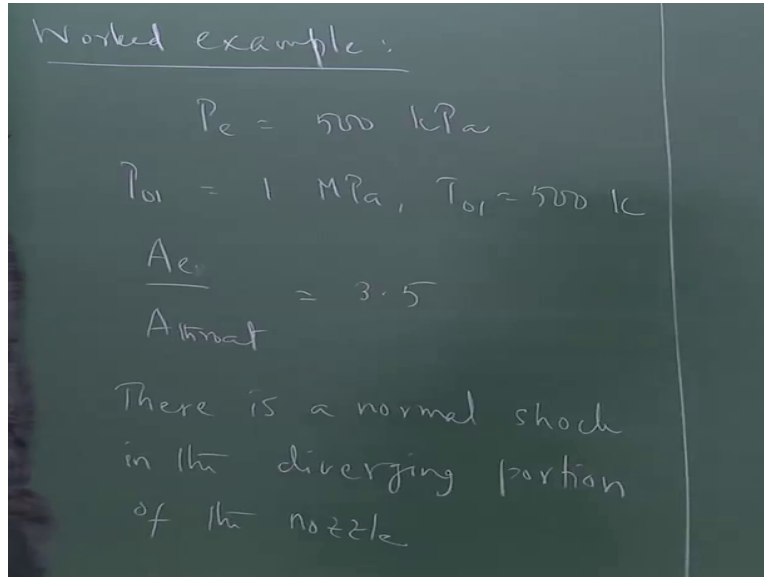


Gas Dynamics and Propulsion
Dr. Babu Viswanathan
Department of Mechanical Engineering
Indian Institute of Technology - Madras

Lecture - 17
Quasi One Dimensional Flows

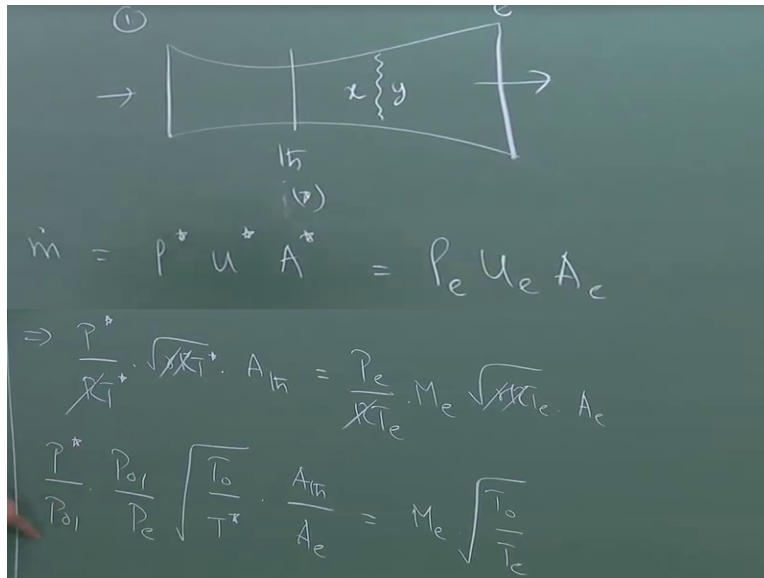
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In the previous class we were looking at a worked example, and if you remember we were looking at a case where the exit pressure P_e was given to be 500 kilopascal, and based on the other conditions if you remember the stagnation pressure P_{01} was given to be 1 megapascal, and the stagnation temperature was given to be 500 Kelvin and A_{exit}/A_{throat} was given to be 3.5. And we established that for this value of back pressure there is a normal shock in the diverging portion.

In the diverging portion of the nozzles. So what we need to do is determined the flow properties and the exit of the nozzle, that was what we were about to do.

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Now if I look at the flow situation, so this is a station 1, this is the throat section remember in this case the mass number here has become =1, so the throat section also happens to be at the sonic state, there is a normal shock here and this is the exit of the nozzle, P exit is known is 500 kilopascal. Now if I write an expression for mass flow rate through 2 sections let us say throat section and the exit section.

I can write like this $\dot{m} = \rho^* u^* A^*$ or A_{throat} remember $M=1$ here, so a throat= $\rho^* u^* A^*$. And this can be written as $\rho_e u_e A_e$, so if I rearrange this expression I can write this as $\frac{P^*}{R T^*}$ times $\sqrt{\gamma r T^*}$, and u^* can be written as square root of $\gamma r T^*$ because $M=1$ and remember $A^* = A_{throat}$, so I am explicitly indicating that here, and this can be written as $\frac{P_e}{R T_e}$ times $M_e \sqrt{\gamma r T_e}$ and this is A_e .

Now if I rearrange these expressions slightly, noticed that I can cancel out these terms here and if I write this as $\frac{P^*}{P_{01}} \times \frac{P_{01}}{P_e}$, notice that I am taking the P_e to the denominator here, the square root of γr also cancels out from these 2. And I can write this as remember there is a square root of T^* here and T_e here, so I can actually write this square root of $\frac{T_0}{T^*}$, so I multiply both sides with the square root of T_0 .

So let me write this as A_{throat}/A_{exit} so M_e is still left on the right hand side, the A_{exit} has been taken here and I have a square root of T_0 here, so I can write this is square root of T_0/T_e okay. Notice that I have multiplied both sides of the equation by a square route of T_0 to write the expression in this way okay, T_0/T_{star} is known that is nothing but $\gamma+1/2$, P_{star} or P_{01} is also known that is an isentropic flow, so I can get this value from the table.

Notice that P_{01} or P_e is also known, this is P_{01} not P_{02} , I do not know the stagnation pressure downstream of the shock wave because I do not know where the shock wave is until I know the Mach number upstream of the shockwave, I will not know the stagnation pressure downstream of the shock wave, but this is stagnation pressure upstream of the shock wave, this P_{01} I have multiplied and P_0 .

So this is known, this is known, this is known, and this is also known okay. Now T_0/T_e I can write in terms of the Mach number at exit, so that means I am left with only 1 unknown correct.

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The chalkboard shows the following equations:

$$\frac{1}{1.893} \cdot \frac{1000}{500} \cdot \sqrt{1.2} \cdot \frac{1}{3.5} = M_e \cdot \sqrt{1 + \frac{\gamma-1}{2} M_e^2}$$

Solve $M_e = 0.2735$

$$\Rightarrow T_e = \frac{T_e}{T_0} \cdot T_0 = \frac{1}{1.015} \cdot 500 = 493 \text{ K}$$

$$\frac{P_e}{P_0} \cdot P_0 = 1.056 \times 500 = 528 \text{ kPa}$$

So let us go ahead and do that and see what we get, so P_{star}/P_{01} from the tables comes out to be $1/1.893$, and P_{01}/P_{exit} can be written as 1 megapascal is nothing but 1000 kilopascal/500 and this is square root of 1.2 because it is $\gamma+1/2$ times we are also given that A_{exit}/A_{throat} is 3.5, so this is $1/3.5$, and this can be written as M_{exit} times square root of $1+\gamma-1/2$ times M_{exit} square.

So if you plug in and iterate, remember we are looking for a subsonic solution here, so I can solve for this equation for M_{exit} and get M_{exit} to be 0.2735 okay it is not very difficult to solve this equation, it is a simple quadratic equation it is not very difficult to do that. So we take this subsonic solution which is 0.273, so from this I can get my T_{exit} to be T_{exit}/T_0 times T_0 , T_0 remains the same across the shock wave also.

So I can get this to be $1/1.015$ times 500, which gives me 493 Kelvin that is static temperature at the exit, static pressure at the exit is 500 kilopascal that is already known, and stagnation pressure at the exit T_0_{exit} can be calculated as P_0/P_e times P_e and P_0/P_e for this value of Mach number we can get from the tables right, for this value of Mach number I can get P_0/P_e from the isentropic tables and this comes out to be $1.0s6$ times 500.

So the stagnation pressure at exit is 528 kilo Pascal, so at the inlet the stagnation pressure is 1000 kilopascal, and at the exit the stagnation pressure is reduced by almost 50% nearly 50%, so there is a tremendous loss of stagnation pressure. So we will still go ahead so this actually completes the requirements for the problem, but for the sake of completeness we will go ahead and try to determine where the shock is located okay, that is also very important in many applications.

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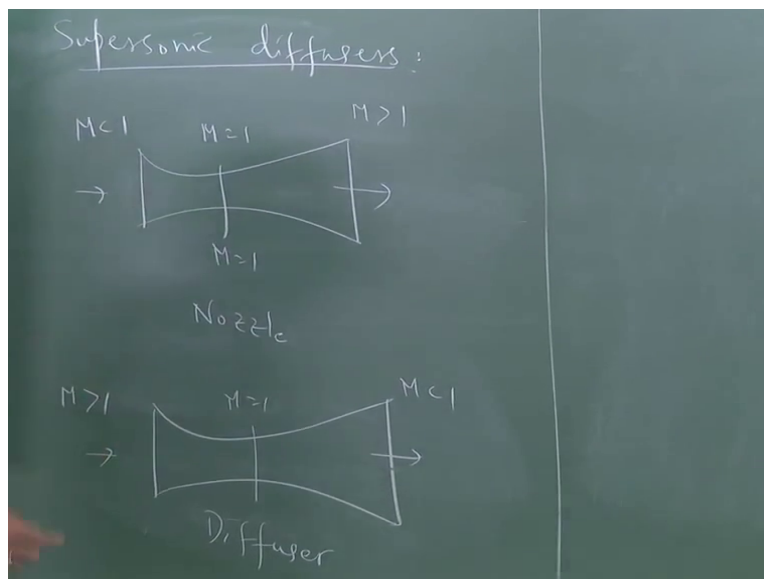
Since $\frac{P_{0,y}}{P_{0,x}} = \frac{528}{1000} = 0.528$,
from normal shock tables,
 $M_x = 2.43$
From the isentropic table,
 $\frac{A}{A^*} \Big|_{M_x=2.43} = 2.4714$.

So let me denote the state ahead of the shock by x and the state downstream of the shock by y . Now I know $P_{0,x}/P_{0,y}$ right, so $P_{0,y}/P_{0,x}$ is nothing but $528/1000$, so there is 0.528 from normal shock table I can get M_x to be 2.43 , so I know the Mach number here as $M_x=2.43$, remember this part of the flow is isentropic with $A_{star}=A_{throat}$. And so I can get, so from the isentropic tables A/A_{star} where this M occurs comes out to be 2.4714 .

So that is the A/A_{star} area ratio where the normal shock occurs, for our comparison you know that A_{exit}/A_{throat} is 3.5 , so this occurs somewhere probably in the second half of the diverging portion of the nozzle, so that completes the problem entirely okay. It is also possible to solve this problem iteratively, by assuming the shock to be located at some place, so you will do this the opposite way, you assume the shock to be at some A/A_{star} .

And then for that A/A_{star} you determine your M_x and then M_y , $P_{0,y}$ and then exit static pressure, see if this exit static pressure matches the 500 kilopascal, if it does not match then you move the shock and then do this iteratively it is possible to do it that way also okay, any questions? Okay.

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The presence of the normal shock during the start of operation of convergent divergent nozzle poses lots of problem during operation, if you remember we said that unless the $P_0/P_{ambient}$ is more than 3 , we would prefer to have a converging nozzle and still live with the loss of thrust

okay. The reason being when you add a convergent divergent nozzle the startup becomes very difficult because there is a normal shock which needs to be driven out.

So there is a lot of loss during the startup, and so convergent divergent nozzle adds a lot of complexity to this okay, so it is avoided if the stagnation pressure P_0/P_{ambient} is 3 or maybe even 4 you know it is not worth adding a CD nozzle. Now the startup difficulty that we saw with the convergent divergent nozzle is also seen in other devices, but it exhibits in a different way. One of the most important application is supersonic diffuser.

A supersonic diffuser is typically used in ramjets, which are for example either engines that used pure ramjets which would be missiles or it can also be used in supersonic transport vehicles like a Concorde or maybe fighter aircraft which use derivatives of the ramjet engine, so they have a turbojet and a ramjet option in both these cases when the flight Mach number exceeds says 2 or so, then the idea is to decelerate the air sufficiently, so that you convert the momentum of the air into pressure okay.

So then when you do that we can completely dispense with the compressor, you do not need a compressor because they slowing down the air gives you the high pressure or the pressure rise that you want, and you no longer need a compressor to increase the pressure okay, but this deceleration has to be done very carefully in something called the supersonic diffuser. The supersonic diffuser is the exact opposite of the supersonic nozzle okay.

If you remember a supersonic nozzle under design operating condition right as $M=1$ at throat, so we have a subsonic flow coming in and becomes 1 at the throat and we have a supersonic flow going out, so this is a nozzle. Now supersonic diffuser acts in the exact opposite way, we take a supersonic flow decelerated so that we get a subsonic flow at the exit, and if you remember the pressure decreases from here to here in the nozzle operation.

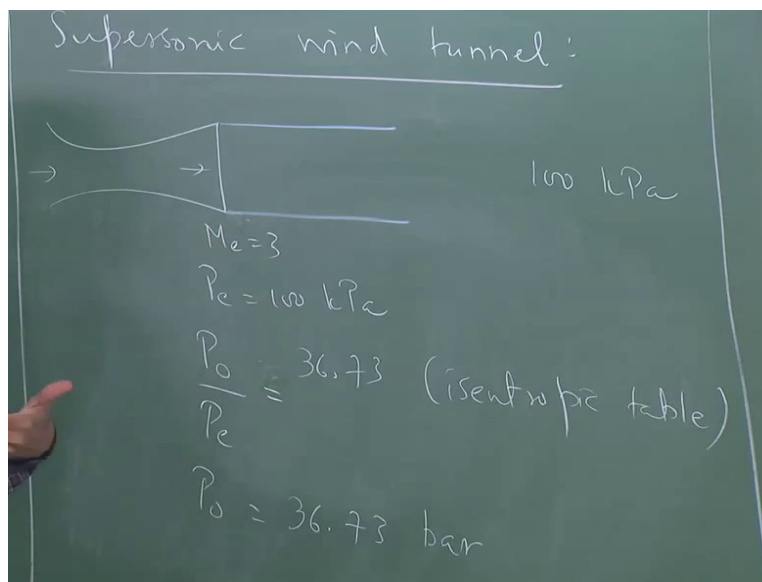
So similarly, in the diffuser operation the static pressure would increase from here to here that is the general idea, but because you have a normal shock during startup of these devices you have a lot of difficulty realizing this in practice, this sounds very good on paper but very difficult to

realize in practice and those are some of the things that we are going to look at now okay. So supersonic diffuser for example would perhaps look like this.

There are many other variations of this, this is probably the simplest one, so it takes a supersonic flow and decelerates it to a subsonic flow, this operates in the other direction and there is an increase in static pressure as we go from the free stream to this okay. So this is exposed to the free stream, so the air is coming in at a supersonic Mach number into the diffuser and we are decelerating it, this is one of the applications okay.

Now supersonic, so what we will do is we will actually work out a numerical example which illustrate this concept.

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Another application where you get into difficulty with the startup problem is the so-called supersonic wind tunnel, now many times in aerodynamics and maybe even in propulsion you need to test components and engines and other things in a supersonic stream. So let us say we want to test something in a supersonic stream, let us say that we generate a supersonic flow, so let us say that this is my supersonic flow and I have a supersonic jet coming out okay.

Let us say that this is ambient pressure at 100 kilopascal, now if I designed the nozzle then under design operating conditions just for the sake of argument let us say that the exit Mach number

here is 3, and it is operating under design condition so that the jet comes out like this, the jet is neither over expanded nor under expanded, so you get a correctly expanded jet which looks like this which is at a supersonic Mach number.

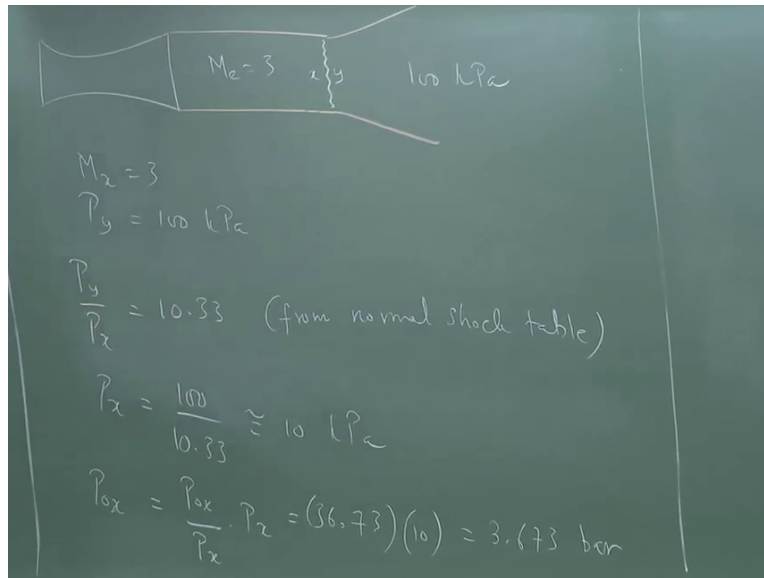
So I can insert a component into this and study the performance of the component under supersonic conditions, if I want to test let us say Mach 3 that is what I would do okay, this is correctly expanded. Now if you actually think about this case, remember when we run a compressor we talked about this also earlier, we use compressed air from storage tanks and you know where the air is stored from compressor.

So we need to spend energy to compress the air to certain pressure before we can actually run it through this to get a jet like this right. So if you look at a situation like this right, the static pressure, so if you look at this the flow when it comes out it comes out a Mach number 3 under static pressure of 100 kilopascal right, P_e in this case is 100 kilopascal because it is correctly expanded right.

So if it is correctly expanded and M_e is 3 if you go to your tables I can get my P_0 value for this case to be, what is it going to be? Let us go to the isentropic tables and if you go to the isentropic table you get the value to be for Mach number 3 36.73, so $P_0/P_e=36.73$ from isentropic tables, which means that the compressor needs to generate or produced air at a pressure of 36.73 bar or atmosphere that is going to require a lot of energy to compress air to a stagnation pressure of 36.73 bar.

So we compress air to this pressure put it in a storage tank and then connect the nozzle to the storage tank everything works fine here, this is a lot of energy. You can actually improve upon this by doing the following let us say that instead of allowing it to expand into the atmosphere this way, we want the Mach number to be 3 that is our desired test section Mach number, let us say that we do something like this.

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So same nozzle same everything but now instead of allowing the nozzle to exhaust to the ambient let us say that we put a test section, we add walls to this so this is ambient at 100 kilopascal, then we allow it to exhaust like this and we design things so that there is a normal shock which stands here. So this is a normal shock so the test section Mach number $M_e=3$, so it still satisfies the requirements for the testing.

But now I have put a test section I put walls around the test section, this is remembered this is a free jet it is a free jet which exhaust into the ambient, now we have a test section where we have supersonic flow right, so now I have a normal shock like this so let us say that this is x and this is y, this is a normal shock at Mach number 3 correct. So I can from this I can do the following, notice that $M_x=3$ right, P_y static pressure at the exit=100 kilopascal.

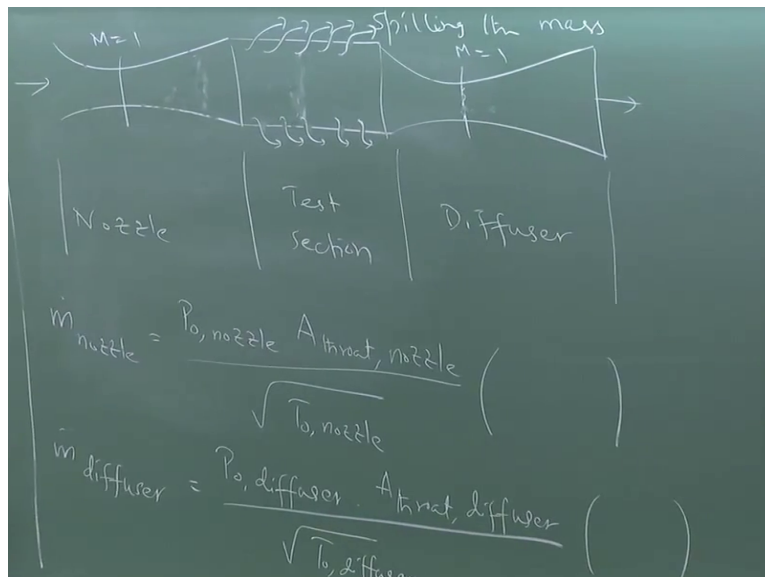
So if I go to the normal shock table I can get P_y/P_x to be, so if you go to the normal shock table for $M=3$, so from the normal shock table for shock that occurs at $M_x=3$ I get P_y/P_x to be 10.33, this means that my P_x is what is my $P_x=100 \text{ kilopascal}/10.33$, approximately 10 correct. If this is approximately 10 what is my $P_{0,x}$ Mach number is 3 so I can write $P_{0,x}$ =from the isentropic table we get $P_{0,x}/P_x$ at $M=3$ to be 36.73 that we have already seen right 36.73 times 10 do this correctly that is 10 kilopascal correct.

So this is 3.673 bar correct, now you can see what normal shock compression is doing for you, so by adding a test section in the normal shock at the end of the test section I have reduced my compressor power by how much by a factor of 10. So the compressor power now has become 1/10th of what it was if it was a free jet right, so that is what the normal shock compression does for you, because this pressure can be less.

And you are getting a substantial compression across this, this can be less so that means the stagnation pressure can also be less okay. In fact, I can make this even better if I had a diffuser to this, I can add a diffuser to this section like this, if I diffuse the flow then there is going to be another increase in pressure across this, which means this can operate at even a lower pressure, so there will be a marginal increase if I had a diffuser.

So what I am trying to tell you is, if you want to test objects and other things in a supersonic tunnel, so this would become a test section the entire setup is called supersonic wind tunnel okay. So we can see that adding a diffuser will actually benefit and we can design things properly if you design things properly we can actually reduce the amount of power that is required, but the tunnel will usually have operating problems I am sorry that tunnel will usually have starting difficulties associated with the normal shock that we normally see.

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So supersonic wind tunnel would actually look like this, so we have a convergent divergent nozzle at test section like this, and we actually put a diffuser which is what we talked about here right supersonic diffuser, so we have a supersonic nozzle at test section followed by a diffuser correct. So this is the nozzle part, this is the test section and this is the diffuser part, so flow comes in reaches the test section supersonic Mach number and then goes out like this.

So there is 1 throat here, notice that there is 1 throat here and another throat here okay. The idea is during operation the air comes in at a certain static pressure goes through becomes supersonic and then if I am able to recover the momentum of the air and convert that to static pressure, I can feed this air back into the inlet here or I am sorry if I recovered this as the momentum I can feed the air back into my storage tank.

Because I have now converted the moment into high static pressure, so I can put it into the storage tank and then use it back here, which means I can actually run this continuously with minimal energy input not 0 input but minimal energy input that is the whole idea of adding a supersonic diffuser like this. The supersonic diffuser would be very effective when it comes to converting the momentum of this high speed air into static pressure that is why we add this diffuser.

But now if you think about starting the tunnel okay, initially everything is at atmospheric pressure right let us say we start the tunnel and we go through the sequence that we described yesterday, initially we established a small flow here subsonic here, accelerate slightly again become subsonic and then just goes through okay. So we crank up the pressure, in this case we are not lowering the ambient pressure.

Yesterday, when we talked about this we were looking at keeping the inlet stagnation pressure constant and then lowering the ambient pressure. Now we are actually running the compressor, so we are actually increasing the stagnation pressure keeping the ambient pressure the same right, so I increase this stagnation pressure a little bit more, I now set up a situation where the flow accelerates M becomes $=1$ here right, and then again subsonic flow throughout, perhaps M becoming $=1$ here maybe.

Now if I crank up the stagnation pressure a little bit more what happens next? I have a normal shock right somewhere here, I set up a normal shock somewhere here, and once I set up a normal shock here the mass flow rate that comes through this right, the mass flow rate that comes through this part can be written like this right \dot{m}_{nozzle} can be written as $T_0 \text{ nozzle} \times A_{\text{throat}}$ for the nozzle/square root of T_0 nozzle.

And then we have a quantity which depends on the only on the property of the gas that will be remain the same because the same gas is going through both this, remember there is a square root of γ/R that is a big expression which is immaterial to our discussion, so I am just going to leave that as it is okay. And remember this stagnation temperature remains constant throughout there is no change in this stagnation temperature.

Now assuming that the 2 throat areas are the same, so I get $M=1$ here, so I am going to get $M=1$ here also, so that means the mass flow rate that goes to the diffuser is like this right, so the mass flow rate that goes to the diffuser can be written like this $\dot{m}_{\text{diffuser}} = P_0 \text{ diffuser} A_{\text{throat diffuser}} / \text{square root of } T_0 \text{ diffuser}$ and then same thing. Now If I compare these 2 expressions okay T_0 is the same, so I need not worry about this.

So the quantity within these parentheses is also the same so I need not worry about that. Now what can we say about P_0 nozzle and P_0 diffuser remember there is a normal shock that is sitting here P_0 diffuser there is a loss of stagnation pressure across the normal shock, so P_0 diffuser is $<$ P_0 nozzle. If the area A_{throat} areas are the same, the throat areas are the same and $M=1$ at the throat, then we can see that the diffuser is able to swallow a mass flow rate which is little bit $<$ what the nozzle is providing.

The nozzle may be providing I would say 1 kg per second or 10 kg per second, but the diffuser is able to swallow only 8 kg per second or 9 kg per second, so what happens to the remaining mass I am starting the channel right remember the sequence that we are going through we are starting the tunnel cranking up the pressure stagnation pressure trying to push the flow through the nozzle, so that we get a shock free flow with supersonic Mach number here right.

So I have come up to a point where I have a normal shock here, and this is a situation that I am faced with, so the diffuser is not able to swallow all the mass that I am pushing through the nozzle. If I do not do anything then the pressure builds up here, because of the accumulated mass, when the pressure builds up here what does it do to the shock wave? Pushes the shockwave to the front, so the shockwave can go only up to the throat.

Because once it reaches the throat $M=1$, so this shockwave becomes an acoustic wave it becomes the strength becomes infinitesimally small, so the channel becomes shock free. However, the stagnation pressure is high you artificially push the shock here, so for a moment we have shock free flow with M maybe $=1$ here and subsonic flow throughout. But the stagnation pressure we have cranked it up to a higher value.

So then that tries to push the flow again through this and we again set up a normal shock here and then the pressure builds up again it goes back. So the tunnel will never start it will just keep oscillating like this okay it will just keep going back and forth, it will never start because this shock cannot be swallowed by the diffuser, because of the mismatch in the mass flow rates correct, so this mismatch is happening because there is a loss of stagnation pressure across the shock wave.

How do we get around this? One way of getting around this is to adjust the area of the throat area of the diffuser do not make them the same, if for example I increase slightly the throat area of the diffuser then I am able to the diffuser can swallow all the mass that this is supplying. So which means when we are starting the tunnel we need to slightly open this diffuser, so the shock is here and I figure out my P_0/P_0 nozzle, and then I open up the diffuser by that much.

Then what happens the shock moves little bit down right, so then the shock goes from here to here as I keep cranking up the stagnation pressure it moves down. So what is the worst case scenario? What is the maximum opening that I have to provide for the diffuser? The maximum loss of stagnation pressure occurs when the normal shock stands in the supersonic test section

because that is where the Mach number is going to be the maximum that is where the Mach number is maximum that is where the loss of stagnation pressure is maximum.

So that also tells me the maximum area to which I need to open this, so if I open the diffuser to this area then the starting shock very easily goes through and then locates itself somewhere here during steady state operation it will locate itself. Notice that when the shockwave goes through and the initial starting shock is swallowed by the diffuser, you cannot keep the throat area of the diffuser larger than the throat area of the nozzle, you have to reduce it back to the same value as the nozzle.

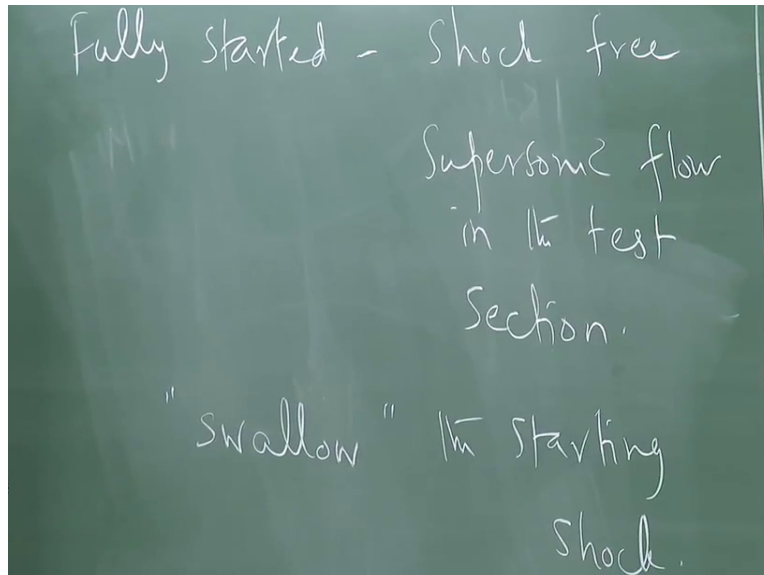
For continuous shock free operation, the 2 have to be equal, but during startup because of the normal shock and the loss of stagnation pressure across the normal shock I open it to push the flow through and then I bring it back right that is what I do. And so this starting shock with in a supersonic nozzle poses difficulties wherever it is used whether it is the nozzle or a diffuser it creates problem wherever it is used.

So we need to make sure that we understand the dynamics of what happens here, so that we can handle this properly, so opening the diffuser to accommodate the normal shock is one strategy, but that is also mechanically very complicated to implement right you need to have throat areas which are movable that is not so easy to implement in a mechanical sense in practical devices. Another way of doing this is to provide for example openings like this, so that when a normal shock stands here right.

And there is a mismatch in mass flow rate between the 2 and so we have 10 kg per second coming through only 9 kg per second going through here right, so you provide these kinds of openings so the remaining 1 kg per second can be bled off through this opening, so the flow can actually go through this. When you do this there is no problem, there is no mismatch 10 kg per second is coming through and 10 kg per second is going out, so there is no accumulation of mass right.

So the normal shock keeps moving and then eventually you close the opening, so that the shock locates itself here and the tunnel is fully started.

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When we say the tunnel is fully started what we mean by that is when we say the tunnel is fully started be mean to things shock free and supersonic flow in the test section okay. So the starting process become somewhat complicated, but once you are able to handle this you realize that the starting difficulty has to do with the mismatch of mass flow rate, which is why I said that this expression when I wrote down this expression for the first time I said this is one of the most important expressions in gas dynamics okay.

Once we realize that the mismatch is because of the different stagnation pressure we can work around this, this is actually a far more practical solution than changing the throat area, although both are used this is a far more practical solution. For example, the supersonic intake or the Concorde uses this strategy to spill the mass, so what we are doing here is spilling the mass right, so we spill the mess around the tunnel to get the shock swallowed.

So when you do this the shock keeps moving down like this, as I crank up the stagnation pressure here the shock keeps moving like this it goes out. **“Professor - student conversation starts”** excuse me (()) (37:38) no... no no this is yeah this is controlled bleeding of the flow

through the walls of the tunnel, we can actually provide that passages like this to bleed the tunnel and they are closed once the flow is established.

You cannot keep it open all the time you have to close them once the flow is completely established and the tunnel is started okay, this is a practically a viable strategy which can be used. Can we push the shock by increasing P_0 ? Push the shock where? You cannot that is what I said if you keep the throat area the same and you do not do this spilling, I come up to a certain P_0 so I have a shock here right.

Now there is a mismatch so only in 9 kg per second is going through 10 kg per second is coming in, there is 1 kg per second mass which is being accumulated, even if I increase my pressure here the shock cannot move downstream, because I am building up the mass here continuously and there is an increase in pressure. So the brute force way of using this shock down here will not work, because of the mismatch and loss of stagnation pressure across the normal shock okay.

So when you do that what will happen is this increase in pressure will tend to push the shock backwards, then you establish shock free flow, but subsonic in the test section because this stagnation pressure is already high again it will trigger a normal shock it will try to push the flow through this and again it builds up, so this will go in cycles, so you cannot push the shock through this system simply, the diffuser or the second throat has to swallow the shock okay.

So that is why if you look at this terminology swallowing the shock because you are shocking about mass flow rate, the second throat should swallow the starting shock okay. **“Professor - student conversation ends.”** But I like to use this following example in this context you know let us say that mother is trying to feed the child the food does not going easily, what is the mother say? Open wide, you cannot chow more food down the throat you know the child is not eating right that is exactly what is happening here.

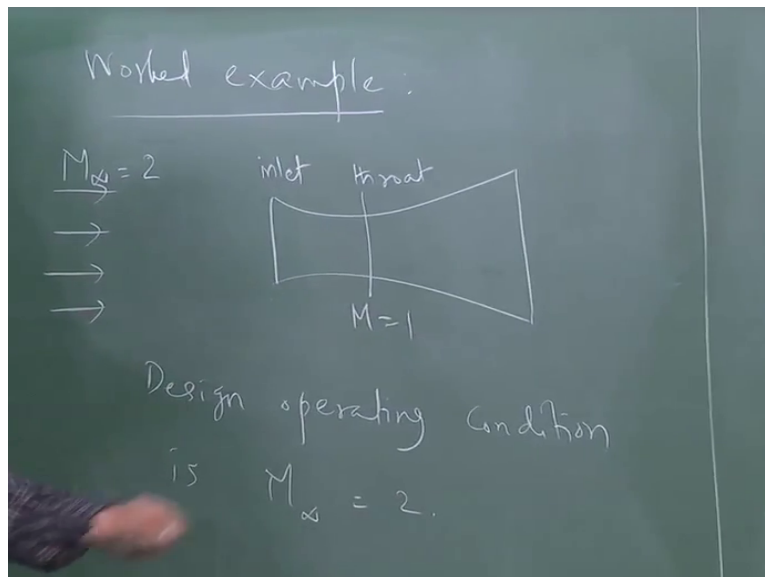
This is only capable of handling 9 kg per second if you wanted to handle more than that what you do? You open wide so that it can go through simply pushing down more mass flow rate here is not going to move the shock down here that will not happen right, you can provide these kinds

of things to handle the starting shock that is exactly what one of your classmates is doing. **“Professor - student conversation starts”** he is doing there are you not? **“Professor - student conversation ends.”**

So that is a very viable strategy and it also works okay. So the terminology also talks that way you know we remember we say that the nozzle is choked, I mean there is the maximum it can handle then we say the starting shock has to be swallowed this terminology actually there is a certain meaning behind it, this is what we be okay. You increase the area to allow the increased mass flow rate to go through.

What we will do now is a worked example which involves these types of ideas how do we change the area to get the tunnel started, and then how do we bring it back for steady state operating conditions and so on okay for a supersonic diffuser.

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So we look at the next worked example which involves a supersonic diffuser, so we are looking at the following situation. So we have a converging diverging diffuser, and we have flow so this is facing a flow let us say it is designed for a free stream Mach number, so this is the free stream which enters the nozzle, so we are supposed to decelerated it to subsonic Mach number okay. So design operating conditions is when the free stream Mach number=2.

So when the free stream Mach number=2, the flow is completely shock free and it goes from the supersonic Mach number to a subsonic Mach number if the Mach number at the throat being=1 okay. So terminology this is called the inlets, this is the inlet area and this is the throat and this is the freestream all free stream conditions will be denoted with the subscript M infinity. Now we are going to look at transient operation of this diffuser, what happens under design operating conditions?

And what happens when the operating Mach number let us say below design value which is let us say 1.5, so we will look at the operation for M infinity=2 which is the design condition and M infinity=1.5 and see how things change that is what we are going to do next.

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For $M_{inlet} = 2$, from isentropic table, we get $\frac{A_{inlet}}{A_{throat\ design}} = 1.6875$

$$\dot{m}_{design} = \frac{P_{0,\infty} A_{throat}}{\sqrt{T_{0,\infty}}} \cdot \sqrt{\frac{\gamma}{R} \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}}$$

From isentropic table, for $M_{\infty} = 2$,

$$\frac{P_{0,\infty}}{P_{\infty}} = 7.82445 \quad \text{and} \quad \frac{T_{0,\infty}}{T_{\infty}} = 1.8$$

So under design operating condition so when the inlet Mach number=2, from isentropic table we get $A_{inlet}/A_{throat\ design}=1.6875$. The mass flow rate under design operating condition through the nozzle can be written like this $\dot{m}_{design}=P_{0,\infty}$ times A_{throat} /square root of $T_{0,\infty}$ times this quantity γ/R times $2/\gamma+1$ to the power $\gamma+1/\gamma-1$ square root of that. Now I can rewrite all these quantities in terms of the static quantities under freestream condition that is how it is usually done for supersonic intakes.

Because the aircraft may be flying at a certain altitude, so we know the static temperature, we know the static pressure and we know the speed, so from which we can calculate the Mach

number, so rather than use the stagnation conditions at free stream it is conventional to use the static conditions at free stream, because they are known in this case, the aircraft or the missile flies at a certain altitude so we know these quantities right.

So from isentropic tables for $M_\infty=2$, I can retrieve the following quantities $P_0^\infty/P_\infty=7.82445$ and $T_0^\infty/T_\infty=1.8$.

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The image shows a chalkboard with the following handwritten equations:

$$\dot{m}_{\text{design}} = 5.832 \cdot \frac{P_0^\infty A_{\text{throat}}}{\sqrt{T_\infty}} \sqrt{\frac{\gamma}{R} \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}}$$

$$A_{\text{capture, design}} = A_{\text{inlet}} = 1.6875 A_{\text{throat}}$$

$$\frac{A_{\text{capture}}}{A_{\text{throat}} \bigg|_{\text{design}}} = 1.6875$$

So if I plug in these values here, I get \dot{m} for the design operating condition as 5.832 times $P_0^\infty A_{\text{throat}}/\sqrt{T_\infty}$ times $\sqrt{\frac{\gamma}{R} \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}}$. So if you think about the stream tube that enters then we can actually sketch the stream that enters the intake like this. So this much of the free stream enters the intake under design operating condition okay.

In fact, this area over here is called the capture area, so the intake may have certain geometric dimensions, but what is the cross sectional area of the stream tube that enters the intake that is called the capture area of the intake okay. So in this case the capture area is the same as the so for design operating condition A_{capture} is the same as the inlet area for design operating condition which itself is 1.6875 times the A_{throat} . So this can be written as 1.6875 times A_{throat} .

So that $A_{\text{capture}}/A_{\text{throat}}$ for design operating condition is 1.6875, so these are for the design operating condition. What we will do in the next class is see how these quantities change when we change to an operating Mach number of 1.5, so when M_{infinity} instead of being 2 when M_{infinity} becomes 1.5, how do these quantities change and how does the performance of the intake also change that is what we are going to look at in the next class.