Gas Dynamics and Propulsion
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Lecture - 16
Quasi One Dimensional Flows

In the previous class, we looked at flow through a convergent nozzle.

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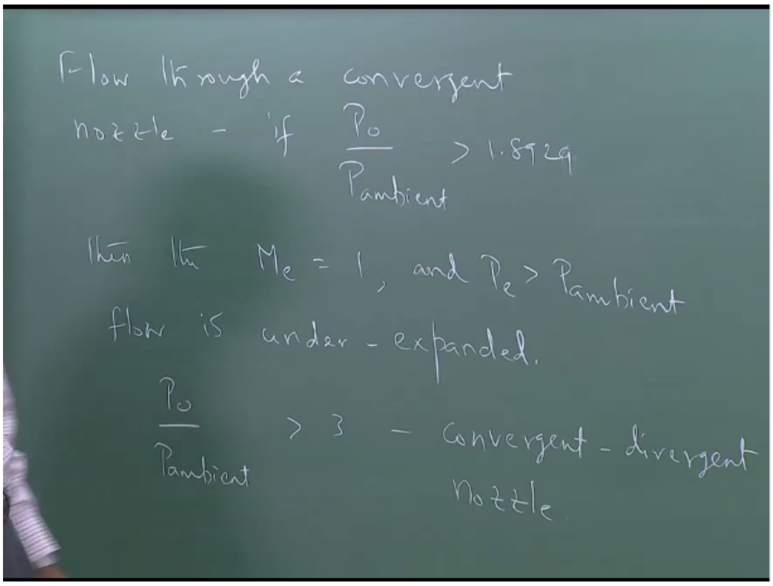

Flow through a convergent nozzle - if $\frac{P_0}{P_{ambient}} > 1.8929$

then the $M_e = 1$, and $P_e > P_{ambient}$

flow is under-expanded.

$\frac{P_0}{P_{ambient}} > 3$ - Convergent - divergent nozzle.

And we showed that irrespective of whether the flow is pushed through the nozzle which means changing the stagnation pressure at the inlet and keeping the ambient pressure constant or whether you pull the flow through the nozzle where you keep the stagnation pressure at the inlet constant and change the ambient pressure.

Irrespective of that we showed that if the ratio P0/P ambient, if it was greater than 1.8929 then we said that the flow at the exit is under-expanded. Then M exit=1 and P exit>P ambient and we also said that the flow is under-expanded and one of the consequences of the under expansion was that the subsequent expansion of the fluid takes place in the air and unless the expansion takes place against a solid surface, we cannot really effectively convert that into thrust right.

So this is undesirable for that reason and must be avoided. So in general if P0/P ambient>3, if P0/P ambient becomes greater than 3 then it is usually preferable to have convergent-divergent nozzle rather than a convergent nozzle alone. As we are going to see next the convergent-divergent nozzle although it may be better at recovering the thrust it comes with its own problems okay.

And in view of the problems that a convergent-divergent nozzle poses normally this decision whether to use a convergent nozzle or a convergent-divergent nozzle is taken based on this ratio. If this ratio becomes too high maybe more than 3 then we are really wasting a lot of thrust and we should recover that as thrust so we decide to have convergent-divergent nozzle and we have to deal with the problems that this brings.

But it would be well worth having this kind of a thing. So for example if you take a rocket engine right, the ratio P0/P ambient maybe as high as 10, 20, 50 or even 100 whereas in aircraft engine, this ratio remember this is stagnation pressure just upstream of the nozzle. So by the time the air if you remember the numbers that we talked about earlier the pressure ratio is about 30 or so in an aircraft engine.

That means the stagnation pressure generally increases by a factor of 30, there is no change of stagnation pressure in the combustor and then we extract work in the turbine. So the stagnation pressure just upstream of the nozzle because of the work extraction of the turbine will probably be reasonably close to this value, which is why we use a convergent nozzle in an aircraft engine whereas we use a convergent-divergent nozzle in a rocket okay.

So that decision is always based on this ratio. This 3 is just a rule of thumb, depending upon the complexity and the other issues we may even say up to 5, I can afford to lose the thrust because the convergent nozzle is less complex than the convergent-divergent nozzle okay and we are going to see the additional complexities that a CD nozzle brings in next that is what we are going to see next.

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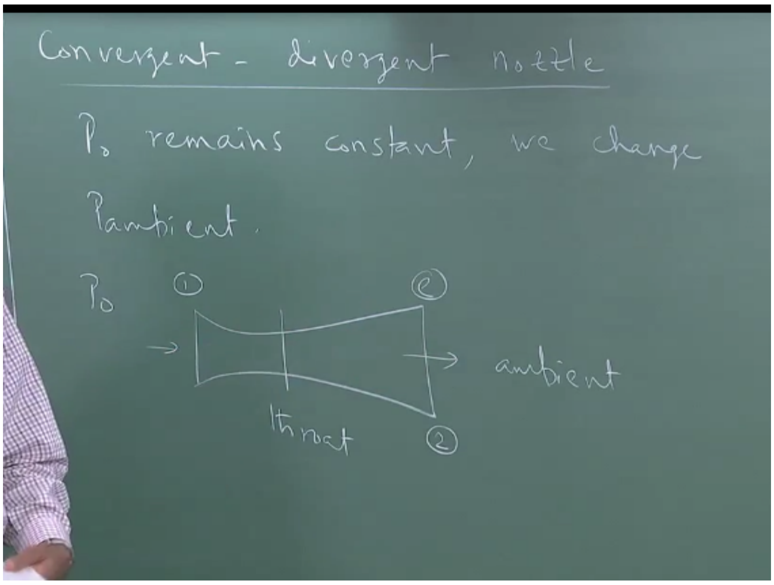

Convergent - divergent nozzle

P_0 remains constant, we change $P_{ambient}$.

So once we have this type of a situation we wish to use a convergent-divergent nozzle and let us see how a flow through a convergent-divergent nozzle is established. So here also it is possible to push the flow through the nozzle or pull the flow through the nozzle and for the sake of simplicity, we are going to look at a situation where the P0 remains constant in such changing the ambient pressure.

So we start with the high value of ambient pressure and then we keep lowering the ambient pressure to establish a flow through the nozzle and we look at the sequence of events that take place in the nozzle. Many things take place in the nozzle and we will try to discuss all aspects using separate plots for each one of this. The most important plot that we will use is a TS diagram.

In addition, we will also look at changes in pressure along the length of the nozzle and so on okay. So let us assume that we have a convergent-divergent nozzle that looks like this. Let say that so this is state 1, this is the throat and this is actually the exit and this is the ambient and let say that the inlet stagnation pressure is P0 and that remains fixed. So this we use either e to denote the exit state.

Or keeping it to be consistent with what we did earlier I can also use number 2 to denote the exit state. Both these things can be used interchangeably. There are no issues with this okay. So let us say that when we start the pressure everywhere is same. We have a certain P0. Let say that we lower the ambient pressure to a value below let say stagnation pressure, so we establish a small mass flow rate through the nozzle.

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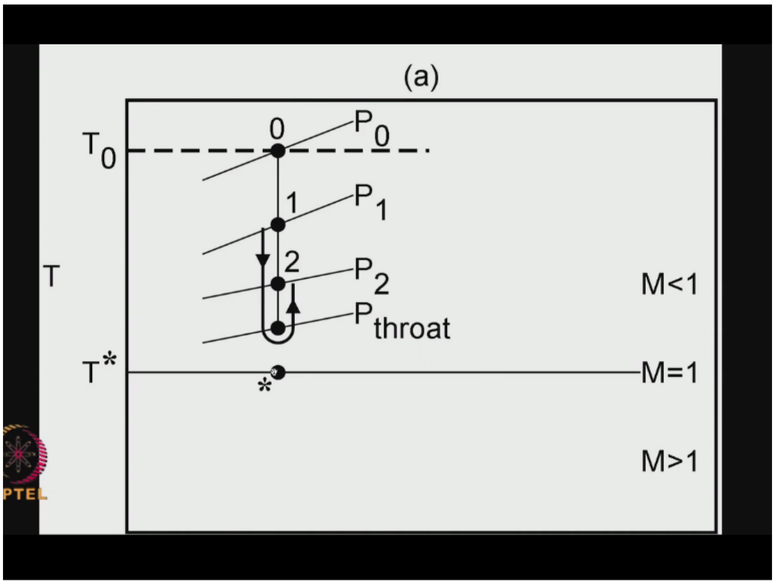

So if you look at the TS diagram corresponding to the state you can see it here, so state 1 has a certain static pressure P1 and the corresponding stagnation state is shown here. So this is T0 and this P0 and you can see that when we lower the ambient pressure a little bit, the flow comes in at a subsonic Mach number.

And because it is a convergent portion it accelerates in the convergent portion which is a lower value of pressure at the throat which is not equal to the sonic pressure, which is not = P star so it accelerates up to a certain value here of Mach number which is also not equal to the sonic Mach number and then once it enters the divergent portion because the flow is still subsonic right, the flow decelerates and reaches the exit pressure P2 okay.

Notice that for this case the exit pressure P2=to the ambient pressure because the Mach number at the exit is subsonic okay. So that is what we are seeing for this value of the ambient pressure. So I have lowered the ambient pressure just a little bit to establish a small mass flow rate through the nozzle okay. What I do next is lower the ambient pressure little bit more right.

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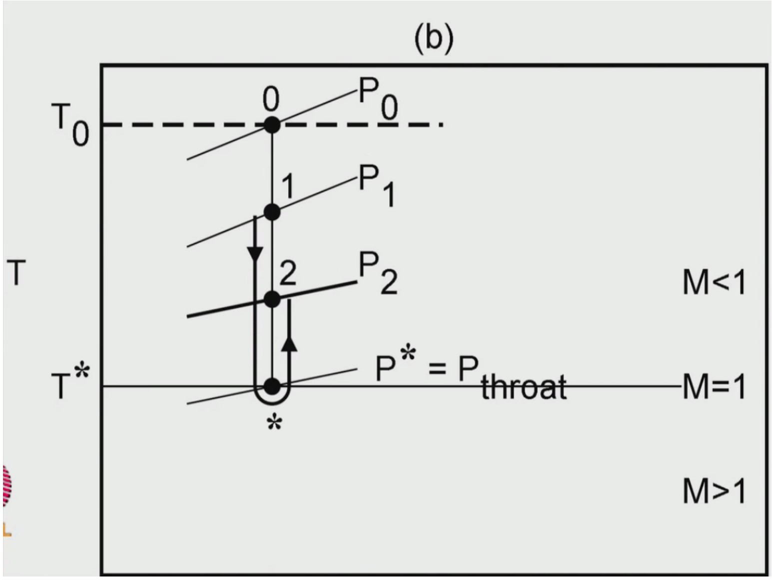

So I lower the ambient pressure a little bit more and very quickly I hit this condition where I start from P1 which is the inlet pressure, stagnation pressure remains the same, the subsonic flow accelerates in the converging passage, but just attains M=1 at the throat and then the exit pressure is high enough that the flow does not continue to accelerate, but it decelerates and reaches an exit pressure P2, which is = P ambient in this case also okay.

The flow in this case is isentropic and the Mach number at the throat has just become = 1 and if you remember we said that for a given value of A/A star, notice that once M reaches 1 at the throat A throat=A star right. So we said that corresponding to a given value of A/A star, there are 2 possible solutions, 1 the subsonic, other 1 is the supersonic solution. This is fully isentropic with the throat Mach number = 1.

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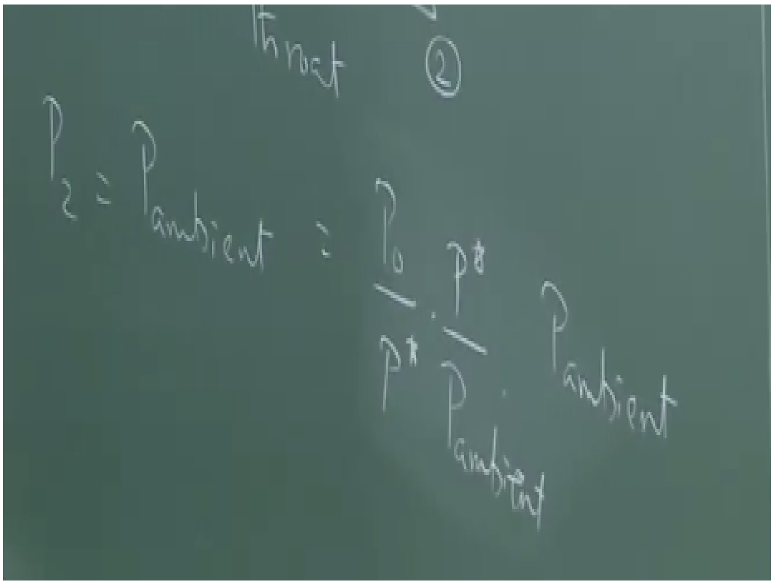

So that means that for this particular condition the P2 is the same as P ambient.

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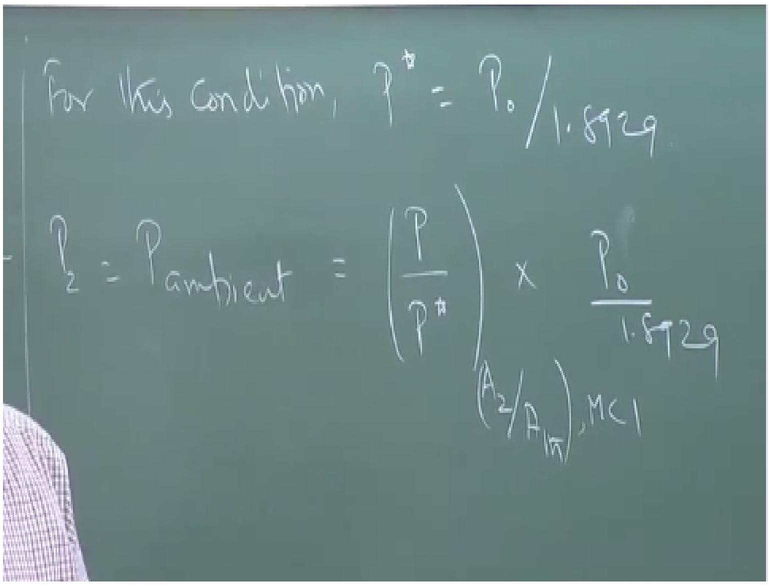

For this condition, $P^* = P_0/1.8929$

$$P_2 = P_{ambient} = \left(\frac{P}{P^*}\right)_{(A_2/A_{th}),\, M<1} \times \frac{P_0}{1.8929}$$

So for this condition P star=P0/1.8929 and so P2=P ambient=P/P star corresponding to A2/A throat for the given value of A2/A throat right. For a given value of A2/A throat I can go to my isentropic tables and retrieve this value of P/P star. Remember there are 2 values in this case we retrieve the subsonic value right. So once I retrieve this value, I multiply this by P star which I already know from here.

So that is P0/1.8929 that gives me the ambient pressure so this ambient pressure corresponds to this value and this we are able to do for 2 reasons, 1 flow is isentropic and number 2 the Mach number at the throat is = 1 okay. Please remember that the previous case also the flow is isentropic, but we are not able to use this expression in the previous case to calculate the ambient pressure.

Because the Mach number of the throat is not = 1 okay. So in this case, the Mach number at the throat is = 1 so we are able to calculate the P this way. So this value of back pressure is such that the flow attains so this value of pressure is high enough that the flow reaches M=1 and then once it reaches M=1 at the throat whether it becomes supersonic or continues to remain subsonic depends upon the exit condition.

So this was something that we said earlier so the exit condition is such that the flow still remains subsonic, it decelerates in the divergent portion.

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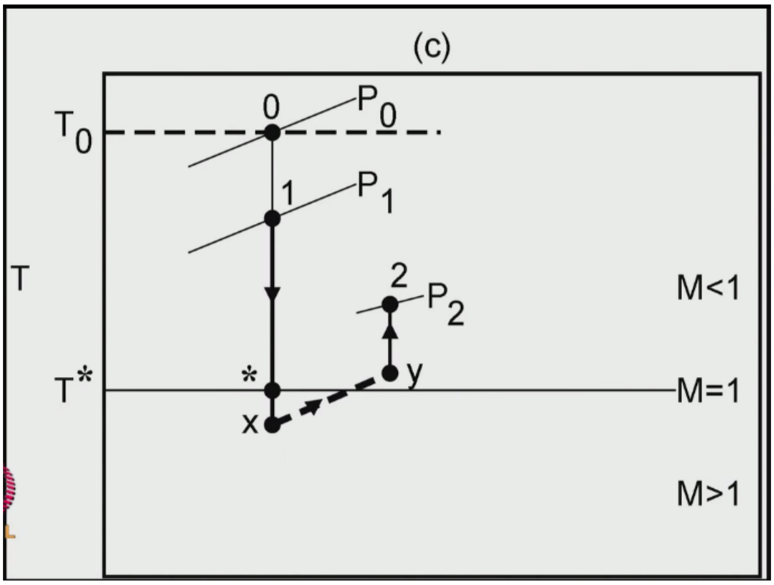

Now if I lower my ambient pressure little bit more let say I lower the ambient pressure little bit more then this is the scenario I have okay. So now as you can see because I have lowered the ambient pressure from the previous value, the flow after reaching M=1 at the throat as you can see M=1 at the throat, the flow continues to accelerate because it sees a favorable pressure at the exit.

However, the pressure is still high enough at the exit that it cannot accelerate throughout and so you have a normal shock, which stands in the divergent portion of the nozzle okay. So now you have a normal shock, which stands somewhere near here okay and then the flow become subsonic and decelerate in the diverging passage as you can see here.

Notice that the Mach number I had of the shock wave denoted by state x is just a little bit more than 1 and the Mach number downstream is also just a little bit less than 1. The shock is not very strong because it is happening at Mach numbers close to 1, it is not a very strong shock but there is a loss of stagnation pressure because of the irreversibility. So the flow is isentropic from 1 to x and then isentropic from y to 2.

But across x and y, there is an increase in entropy okay. Now also notice that the process curve from 1 up to throat remains the same as before, it cannot change anymore because M is already = 1 at the throat even in the previous case. So that means any change in the downstream pressure will not be able to propagate upstream of the M=1 location in the sonic state okay, which means that the mass flow that comes through will also remain the same right.

This is not going to change anymore. The same mass flow rate is going to go through now okay. Now what happens if I lower the pressure some more right.

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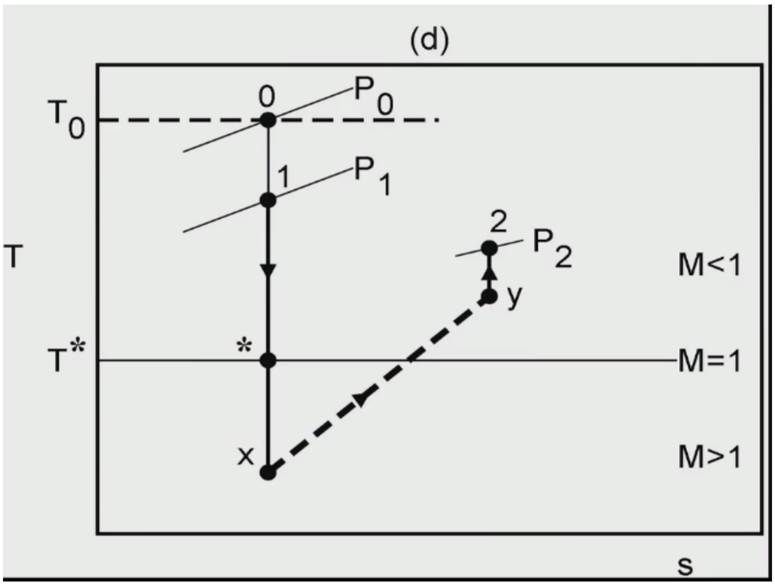

if I lower the pressure some more then I get a situation like this where the flow accelerates even more in this position, even more in the divergent part of the nozzle because it sees a further lowering of the pressure, it accelerates little bit more but then when it accelerates a little bit more, the shock also become stronger because as you can see the further away move from this M=1.

This is the M=1 line, so the further away move from the M=1 line, the higher the Mach number and the stronger the shock and the more the loss of the stagnation pressure right and then you can see that this Mach number is also much less than before because the shock is now stronger and you finally reach this state. So now the loss of stagnation pressure even more.

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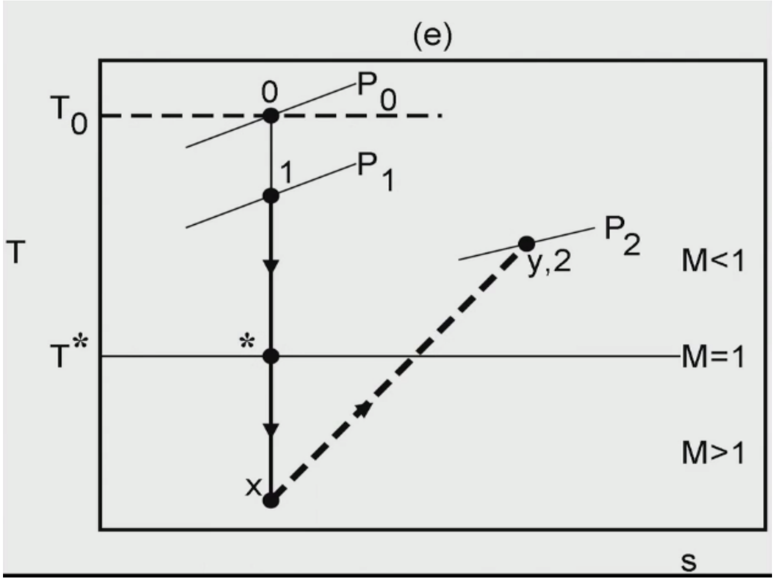

If I lower the pressure a little bit more okay, now I get into a situation where so I keep lowering this ambient pressure initially I had a normal shock here, when I lower it a little bit the normal shock moves from here and positions itself over here. I will show this figure in a minute okay, but let me just go through this. So it positions itself here. If I lower it a little bit more, then the normal shock moves further downstream okay.

And then we lower it enough that it finally positions itself just at the exit so that is when the normal shock is the strongest and the loss of stagnation pressure is also the highest possible okay and that is what is shown in this figure. Notice that the y state and 2 are both at the same location so that means the normal shock stands just at the exit and this Mach number is also the highest possible.

Because the flow is now accelerating fully from the inlet all the way to the exit, still not isentropic because there is a normal shock just at the exit right. So the loss of stagnation pressure is quite high here and we get into the situation. So this is the worst possible operating condition for the nozzle because the loss of stagnation pressure is the highest possible in this case okay.

Notice that this part of the curve from 1 to x will continue to remain the same as before because all states which are upstream of equal to this is M > 1 so all states which are upstream of M>1 will not be affected by changes downstream that we are making okay.

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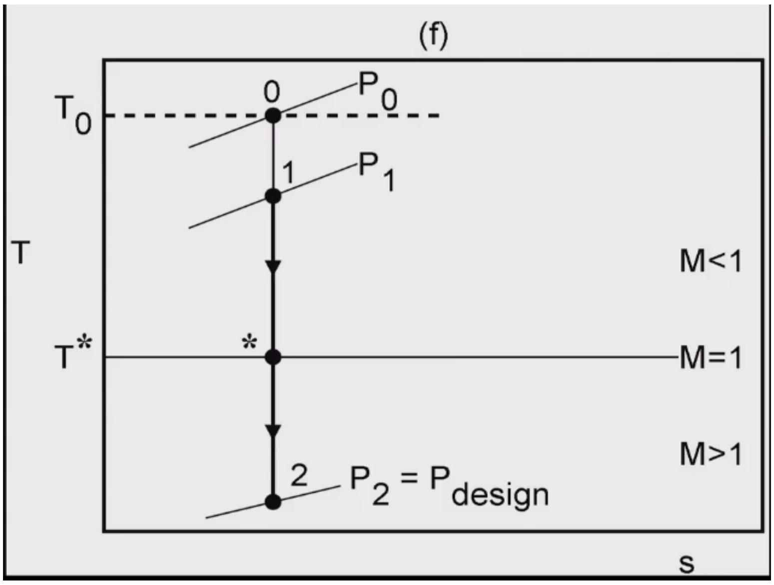

So now what happens if I lower the ambient pressure a little bit more then the shock jumps out of the nozzle and we get a fully isentropic flow okay, shock free isentropic flow and the exit pressure in this case is usually called the design pressure okay. **"Professor - student conversation starts."** What so if ambient pressure is between we design and socket exit that pressure in that range then?

No, we will discuss this when we go to the next figure okay. I have a discussion relating to this in the next figure we will discuss it there. **"Professor - student conversation ends."** So this solution again notice that this solution is completely isentropic and the Mach number of the throat is also = 1.

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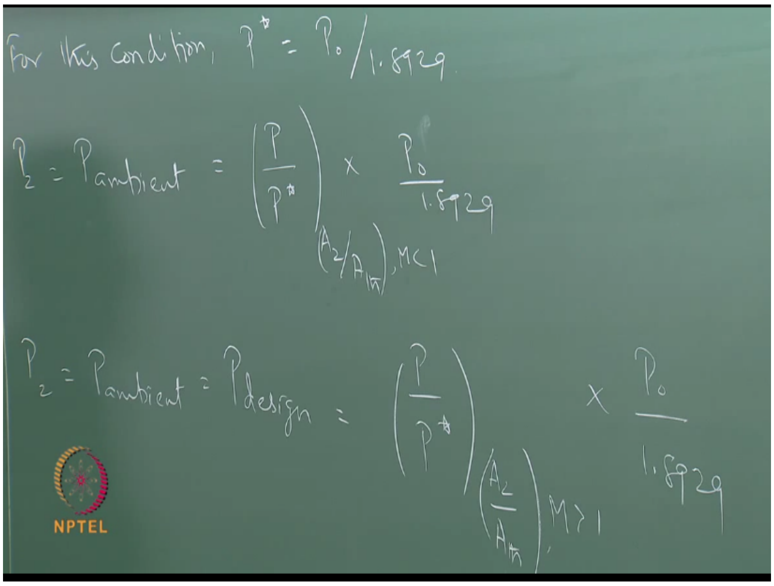

For this condition, $P^* = P_0/1.8929$

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So this corresponds a situation when the ambient pressure P2=P ambient and this pressure is also called the design pressure is the same as this except that we have pick up the value corresponding to this A/A star, we pick up the supersonic value from the table. Here we picked up the subsonic value, here we pick up the supersonic value from the table times P0/1.8929.

What you need to understand and appreciate is the change in the exit state from here to here with just a slight reduction in the ambient pressure okay. That is an extremely important point that you need to appreciate okay. Let us go the next one and then maybe we can discuss this a little bit more. So now we will discuss whatever you are talking about.

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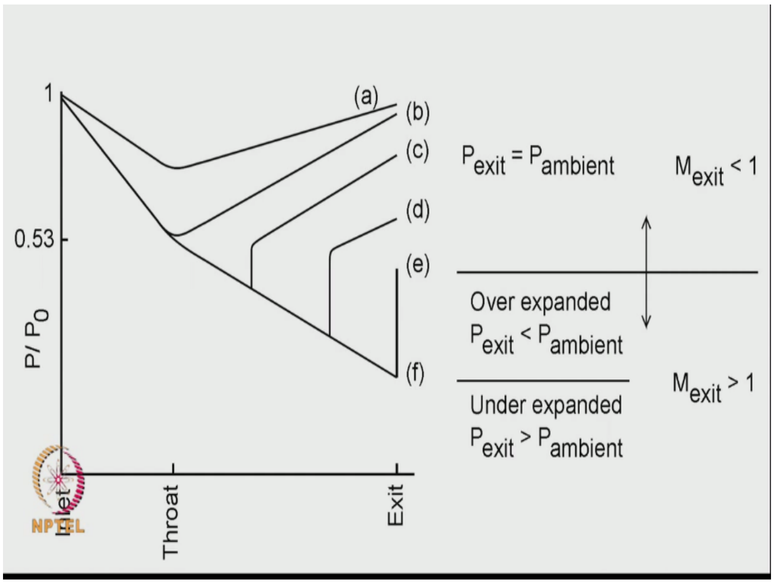

Now we talked about a sequence of events starting from a little bit of flow to these situations. So if you remember let me just go back a little bit and show you that so we are going to for each one of this case that we are talking about we are going to show the variation of pressure along the length of the nozzle for each one of the case that we discussed now okay.

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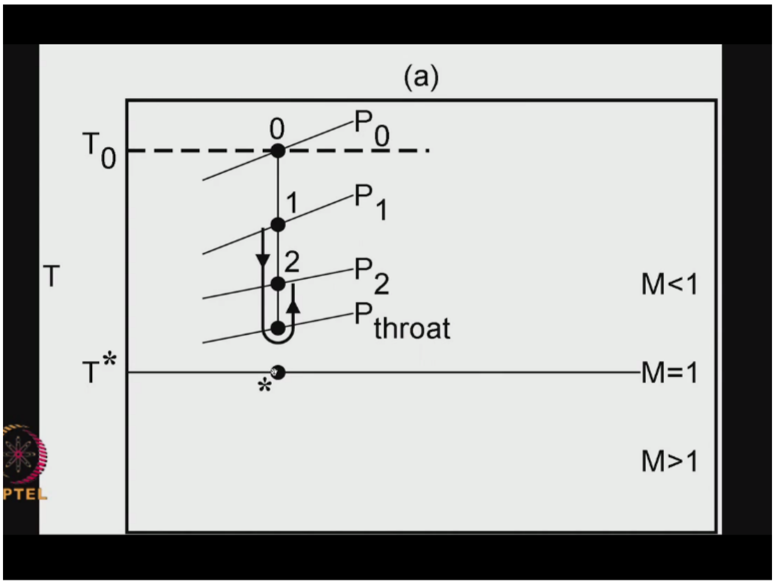

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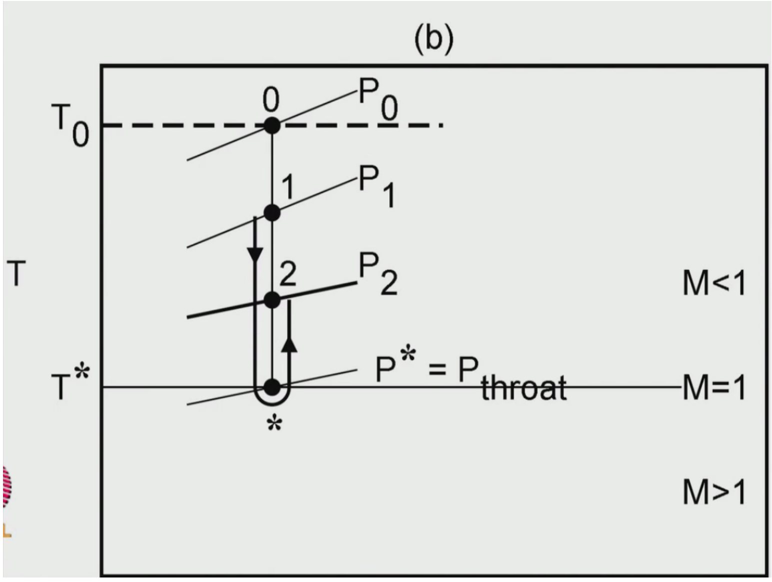

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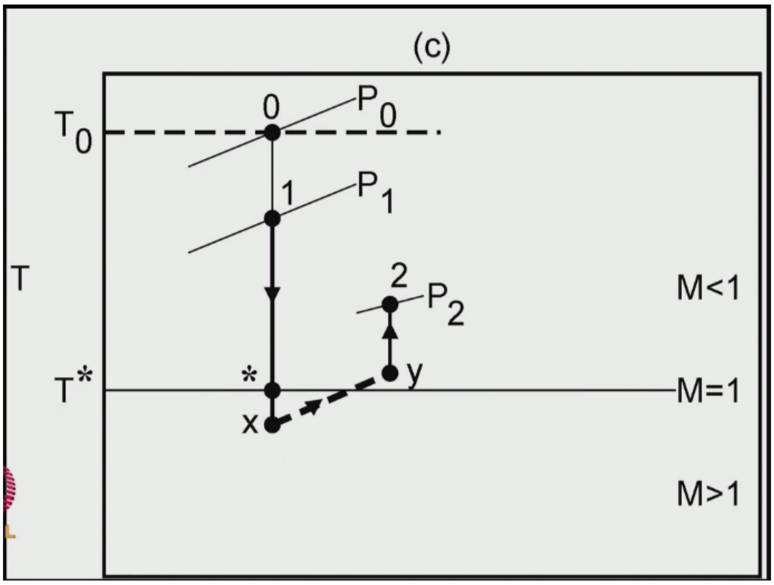

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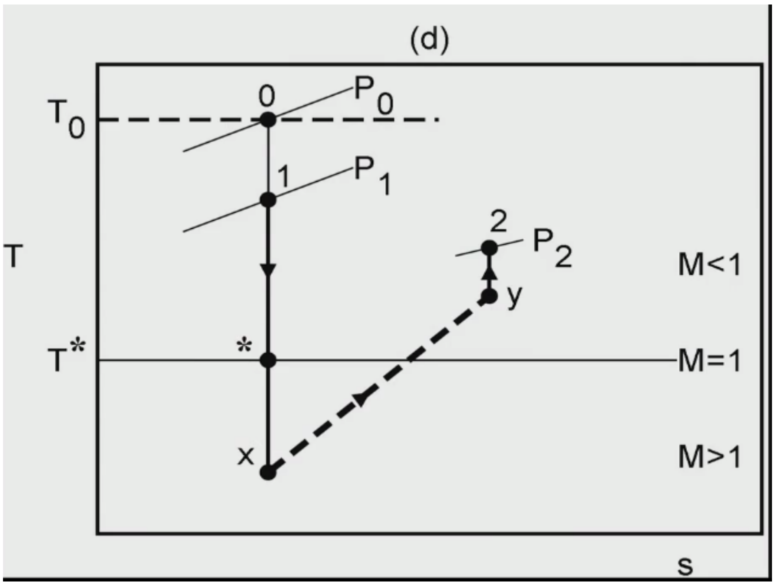

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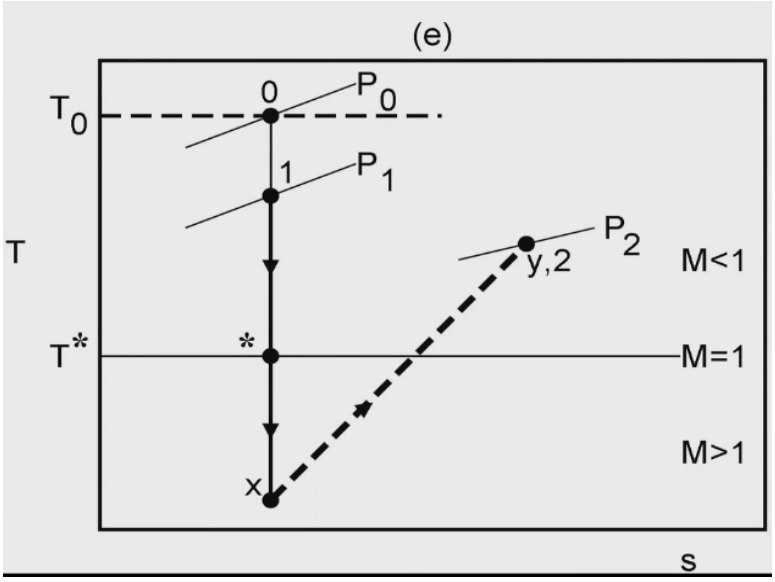

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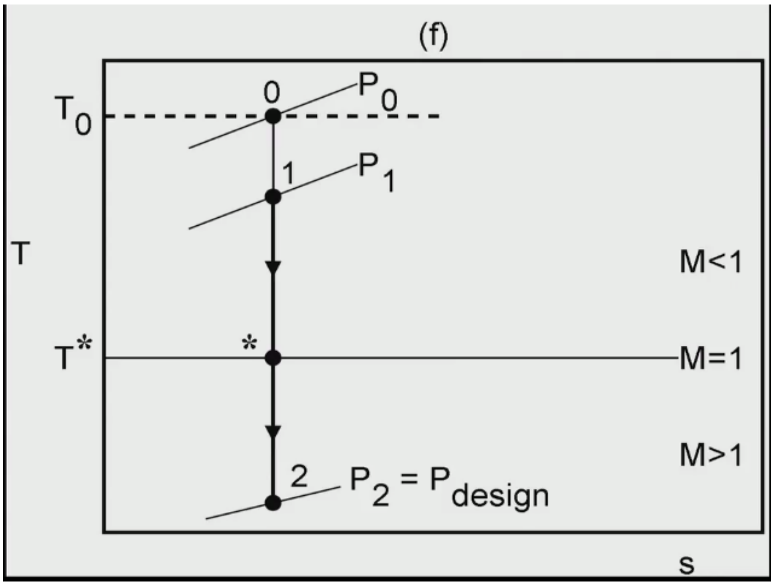

First one was this, second one when the throat Mach number became 1, third one when there was a normal shock just downstream of the throat fourth one further downstream, fifth one when the normal shock was at the exit and then shock free okay. So we are look at variation of pressure along the length of the nozzle in this case. So this is the inlet, this is the throat section, this is the exit section okay.

So notice that first case, the pressure of the throat is not = P star so we just establish a flow so it is subsonic fully, accelerates a little bit in the convergent portion then decelerates, fully subsonic, isentropic but throat pressure is not equal to critical pressure. B, same situation, isentropic, accelerating in the convergent portion, decelerating in the divergent portion, but this P=P star as you can see from here.

So here we have written P/P0 okay so this 0.53 is 1/1.8929 okay. So we have written that here so this P=P star. Now we lower the pressure a little bit more so you see a shock just downstream of the throat. Notice that the pressure remains same, the pressure variation remains the same up to the throat right because M is already = 1 at the throat then it changes further down here.

And then you get a shock wave, which stands here deceleration here. I lower the ambient pressure a little bit more, the shock moves as you can see further downstream and again you get a normal shock like this. Notice that because this part of the flow remains the same, the mass flow rate also remains the same. So the mass flow rate increases from here, initially it was 0, it keeps increasing until P throat becomes = P star.

Once P throat becomes = P star, mass flow rate then remains the same right. So now we are at a situation that the normal shock is here. If I lower the ambient pressure little bit more, the normal shock stands at the exit as you can see. What is special about situations a through e is that the Mach number at the exit is subsonic and consequently the pressure at the exit is also equal to the ambient pressure.

Now if I lower the ambient pressure little bit more, then the normal shock jumps out of the nozzle and the flow becomes fully isentropic, so the pressure variation in this case is along this line without this it is along this line, shock free but notice that in this case the ambient pressure is probably let say as a value somewhere over there, but the exit pressure is somewhere over here.

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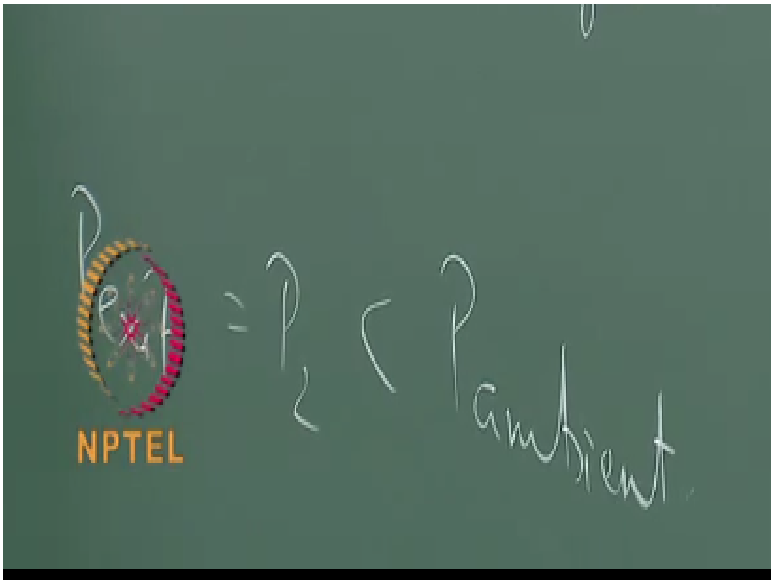

So when this happens you notice that we now have a situation where corresponding to this case the P exit or which is P2 is actually less than the ambient pressure right. Do you see this point? This is an extremely important point which does not happen in the case of a convergent nozzle okay.

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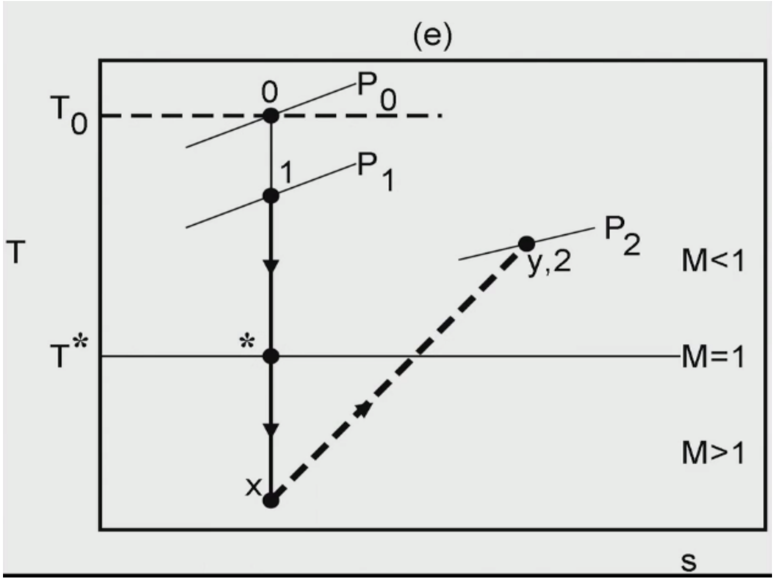

Let us go through this one more time. So if you go to this situation notice that when the shock stands at the exit, this is my exit pressure which is also equal to the ambient pressure because the flow is subsonic. So I lower my ambient pressure just a little bit from here when I do that the flow becomes something like this. So the P ambient isobar is still over there whereas the exit pressure now is over here.

So the exit pressure is actually much less than the ambient pressure in contrast to what we saw in the case of a convergent nozzle. Now this means the flow is actually over expanded because it is coming out at a pressure which is less than the ambient pressure.

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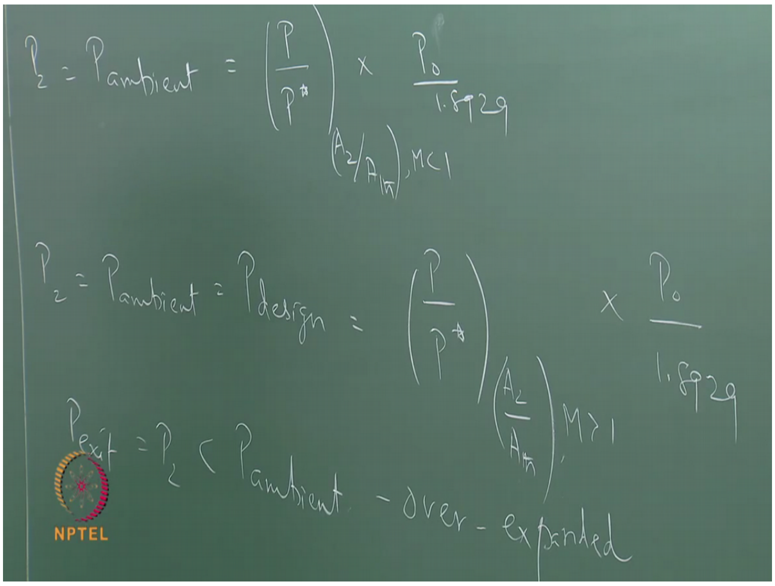

$$P_2 = P_{ambient} = \left(\frac{P}{P^*}\right)_{(A_2/A_{th}),\,M<1} \times \frac{P_0}{1.8929}$$

$$P_2 = P_{ambient} = P_{design} = \left(\frac{P}{P^*}\right)_{\left(\frac{A_2}{A_{th}}\right),\,M>1} \times \frac{P_0}{1.8929}$$

$P_{exit} = P_2 < P_{ambient}$ - over-expanded

So the flow in this case is said to be over expanded. We can never have over expanded flow in a convergent nozzle because the Mach number can never become supersonic, can always be only 1, it can never become supersonic, which means that in this case after the flow comes out right, it needs to equilibrate with the atmosphere which means it needs to be compressed okay.

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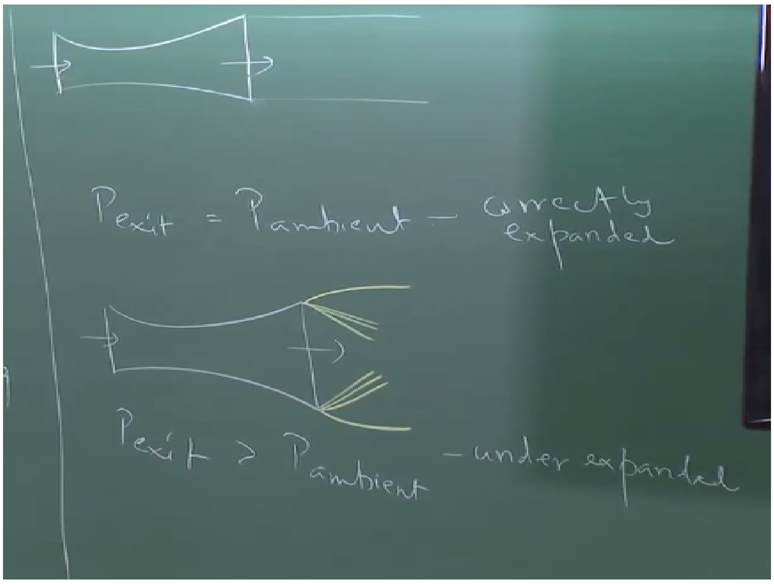

$P_{exit} = P_{ambient}$ – correctly expanded

$P_{exit} > P_{ambient}$ – under expanded

So if you look at the jet as it comes out, the jet looks something like this. We will discuss the structure of the jet in much greater detail in the subsequent chapters, but for now what we are going to say is if P exit=P ambient as we said earlier, the jet comes out like this. This is the jet, the diameter of the jet=the exit diameter and if P exit is more than P ambient then that means the flow has to be expanded outside.

So the jet actually swells right expansion means swelling. So this corresponds to the case when P exit=P ambient. Now if P exit>P ambient that means the flow has to be this is under expanded and this is correctly expanded. So if this happens then as we said earlier, the jet swells when it comes out so it initially swells like this and then it will equilibrate and then keep going like that okay.

So normally what happens is you get a sequence of expansion fans here which expands the flow as it comes out and then tries to equilibrate it with the ambient. So that is why the jet actually the diameter becomes larger than the exit diameter. Now in the case that we just discussed the flow is actually over expanded, which means it needs to be compressed and let us see what that looks like.

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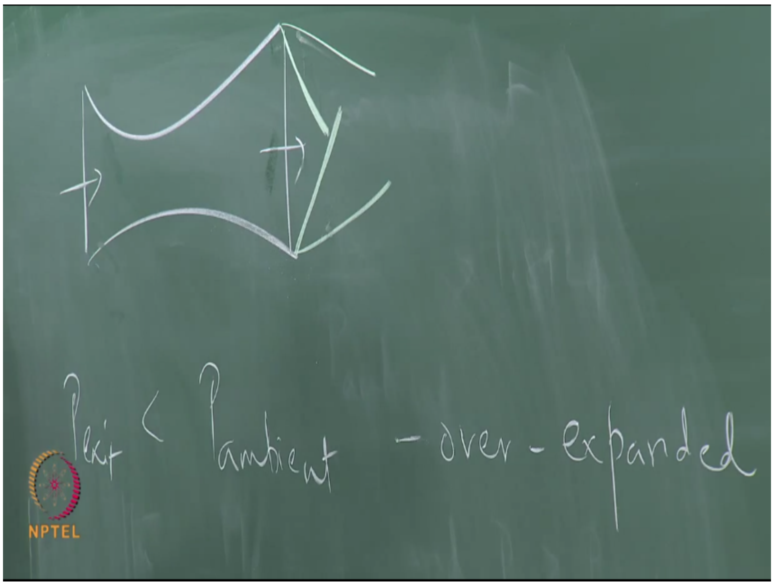

So here we have a situation where the exit pressure P exit is less than P ambient and this is over expanded situation so that means the jet has to be compressed outside the nozzle so which means if it is compressed then if it has to be expanded then the jet swells if it has to be compressed then it shrinks okay. So the jet becomes something like this and we usually get an oblique shock, which is generated from the corner.

So what happens is when the normal shock sets just at the exit, it is a normal shock, remember what did we say about normal shock, it is called a normal shock because there is no change in flow direction after the flow goes through the nozzle right. So when you had a normal shock here, the pressure was very high when you lower the ambient pressure, the normal shock jumps out, but it is no longer a normal shock, it becomes an oblique shock okay.

It is no longer a normal shock, it becomes an oblique shock and there is a change in flow direction in across an oblique shock okay. So this is something that we will discuss further down. So this is what happens so the jet is compressed when it comes out in this case okay and that is what is illustrated in this figure also let us take a closer look at this figure.

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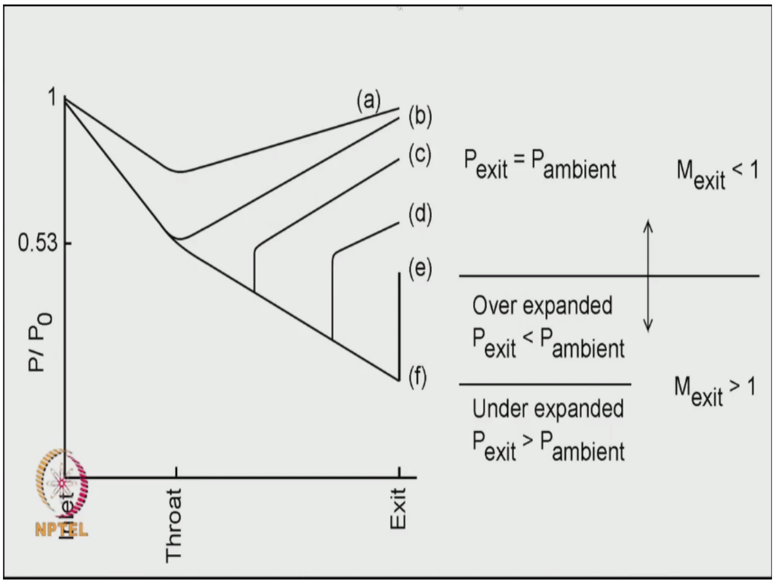

So you can see that when we just lower the pressure here from this location up to the situation when the ambient pressure becomes equal to the exit pressure the flow is going to be over expanded. So it was subsonic up to here and then it becomes supersonic beyond this point okay. So now we can see that from here up to here, the exit pressure is less than the ambient pressure correct.

At this location, the exit pressure becomes equal to the ambient pressure. Now I can continue to lower the pressure even more and if I do that then we can see that the flow becomes under expanded. Now the exit pressure is actually more than the ambient pressure and the jet is under expanded, which is the same as what we saw with the convergent nozzle. This kind of situation arises with the rocket nozzle.

For example, you may design the rocket nozzle for operation at a certain altitude right. Let say you design it for operation at 10 kilometers where the ambient pressure is some value less than the sea level static pressure. So when the nozzle takes off at sea level, the ambient pressure is actually more than the exit pressure or exit pressure is less than the ambient pressure.

So the flow is going to be over expanded until the rocket reaches the design altitude where it becomes correctly expanded and then when it continues to go above, the exit pressure is actually more than the design pressure, which means it is under expanded now. So it starts out as over expanded, correctly expanded, then under expanded. So this kind of situation is seen in real life with these types of nozzles with the convergent-divergent nozzle okay.

Now let us take a look at the importance of this in a propulsion context. Why is this important in a propulsion context because remember we use the convergent-divergent nozzle so that we could recover the thrust, some of the thrust which was being lost you know we wanted to recover. So what is the impact of this type of a situation on the thrust? So as you already said if the flow is under expanded, then the expansion takes place outside.

As you saw here if the flow is under expanded, then the expansion takes place outside even in this case the expansion takes place outside and there is going to be a loss of thrust because the expansion is not against a solid surface so there is a loss of thrust. What about over expanded? That is what we have to see next. We can look at this with a similar kind of diagram that we drew earlier okay.

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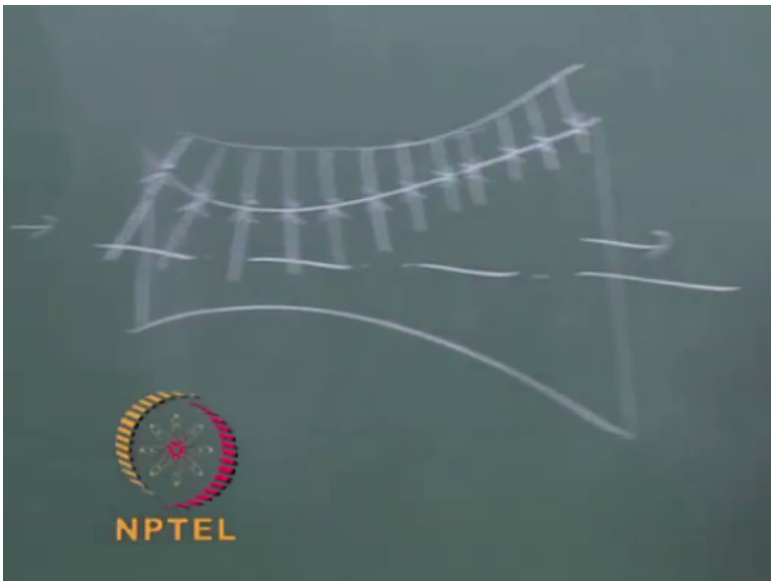

So let us go to over expanded situation same situation I am going to draw. I am going to draw the pressure arrows. What I am going to do now is I am going to draw arrows just like we did before corresponding to the local pressure. Ambient pressure is a certain value right. So let say that the ambient pressure looks like this. The ambient pressure is constant. This is my ambient pressure the vector corresponding to ambient pressure.

And if you sketch the pressure inside the nozzle right, so this is going to look like this and the pressure continues to change so initially the vector maybe very long. Notice that the pressure is very high initially and then the pressure decreases as we showed. Notice that at some point here if the flow is over expanded, at some point in the divergent portion the pressure inside the nozzle becomes less than the ambient pressure correct.

If you think about it if you look at this figure here notice that if the ambient pressure is at this value for example the pressure at this part of the nozzle becomes equal to the ambient pressure so from that point onwards the actual pressure inside the nozzle is less than the ambient pressure and that is what you are also seeing here. So at some point inside the nozzle the pressure becomes less than the ambient pressure.

In this case if you remember the flow through the nozzle is in this direction. So we are trying to develop a thrust force in which direction? Going this way right so if this pressure becomes less than the ambient pressure then this part of the nozzle is actually producing drag and not thrust. In fact, I can actually remove this part of the nozzle. It is as if I actually have a nozzle which is only shorter than the entire nozzle.

Because this length of the nozzle is working against me so that is the problem with over expanded flow. Over expanded flow is highly undesirable in a propulsion application because of the loss of thrust. Under expanded is somewhat better because the pressure is higher here so you get thrust throughout plus you also get some thrust from here, but the expansion against the air is not good so that we can try to avoid.

But over expanded flow is very bad because you start losing thrust in part of the nozzle itself, so this part of the nozzle is actually not doing what it is supposed to be doing. In fact, I will be better off if I remove this part of the nozzle correct. So as I said if you design a rocket engine for operation at a certain altitude and you operate it at altitudes below that then the flow is over expanded.

And when the over expanded so the thrust that the engine produces will also be less than what you want which is why actual rockets will use multiple stages. So each stage is designed for a certain altitude and the nozzle will only operate around that altitude not too far away from that altitude. So you cannot have a single rocket nozzle from sea level all the way to outer space.

Because of this problem you try to have multiple stages, each stage being optimized for a certain altitude or operation around a certain altitude okay that is generally what is done to avoid this problem. Another way to avoid this problem is to use something called an altitude compensating nozzle. There are nozzles which actually make the adjustment as the altitude changes.

They will automatically adjust themselves to compensate further loss of thrust so those are actually much better. Although, they have not been realized in practice, they are very good on principle okay. We will try to discuss this later on when we have time okay if we have time okay. Any questions so far? What we are going to next is work out an example illustrating some of these concepts.

So the problem statement reads like this. A converging-diverging nozzle with an exit to throat area ratio of 3.5 operates with inlet stagnation conditions 1 MPa and 500 kelvin. Determine the exit conditions when the back pressure is a 20 kilopascal and b 500 kilopascal?

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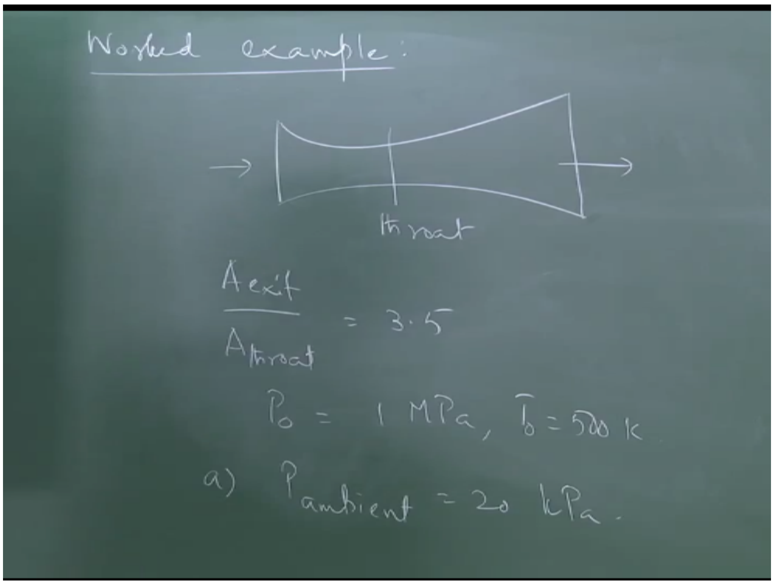

Worked example:

$$\frac{A_{exit}}{A_{throat}} = 3.5$$

$P_0 = 1$ MPa, $T_0 = 500$ K

a) $P_{ambient} = 20$ kPa.

So we have a convergent-divergent nozzle. So this is the throat, so it is given that the exit area A exit to A throat=3.5. It is also given that P0=1 megapascal and T0=500 kelvin. So we are asked to determine the exit state for 2 different back pressures, the first back pressure being 20 kilopascal, So here we have P ambient=20 kilopascal.

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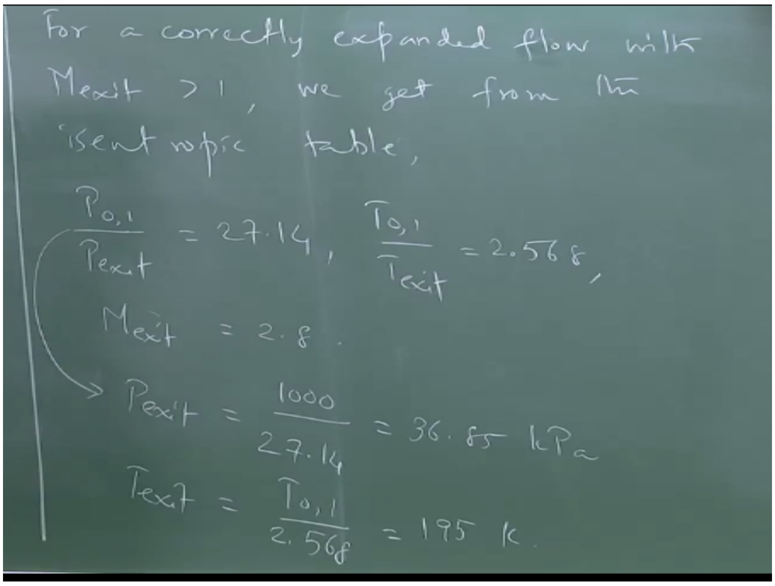

For a correctly expanded flow with $M_{exit} > 1$, we get from the isentropic table,

$$\frac{P_{0,1}}{P_{exit}} = 27.14, \quad \frac{T_{0,1}}{T_{exit}} = 2.568,$$

$M_{exit} = 2.8$.

$$P_{exit} = \frac{1000}{27.14} = 36.85 \text{ kPa}$$

$$T_{exit} = \frac{T_{0,1}}{2.568} = 195 \text{ K}.$$

So we go to the tables and for a correctly expanded flow with Mach number at the exit being supersonic we get from the isentropic table let say P0,1 just to be on the safe side over P exit is equal to so here 1 is the inlet state so let me just mark this as 1 to be the inlet state so P0,1/P exit is 27.14 corresponding to supersonic exit Mach number and T0,1/T exit to be 2.568 and M exit=2.8.

Since P0,1 is known so from this I can calculate P exit=P0,1 which is 1000 kilopascal/27.14 so that gives me 36.85 kilopascal. So the given value of exit pressure ambient pressure is 20 kilopascal so there are no issues so the flow is actually under expanded right so flow is under expanded so T exit can also be calculated from the above expression T exit=T0,1/2.568 and I get this to be 195 kelvin right.

So the flow is under expanded with exit pressure equal to this, exit static temperature equal to this and exit Mach number equal to this. These are the exit conditions when the ambient pressure is equal to this value. The next part of the problem asks us to determine the exit conditions when the ambient pressure is 500 kilopascal. So we know that this pressure is quite high okay, but what we do not know is whether there is a normal shock in the divergent portion of the nozzle or not.

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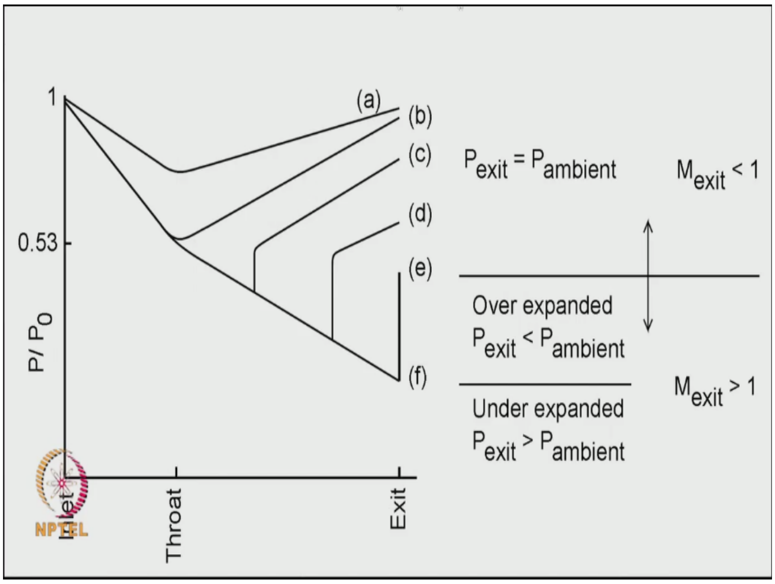

So if you revert back to this figure we can see that the design pressure P design here is equal to as we just calculated P design=36.85 kilopascal so this is 36.85 kilopascal. We are asked to determine the exit condition when the exit pressure is 500 kilopascal? So what we want to find out is so this is the other isentropic solution. So I will try to determine what this pressure value is.

Once I do that then I will know whether the flow is shock free or not. So I will determine the exit pressure corresponding to this situation b that is sketched here and then see whether 500 kilopascal falls between these 2. If it falls between these 2 that means, there is a normal shock in the divergent portion of the nozzle otherwise the flow is shock free okay. That is what we are going to do next. So we do the same thing we go back to the tables and let us take a look at the tables here.

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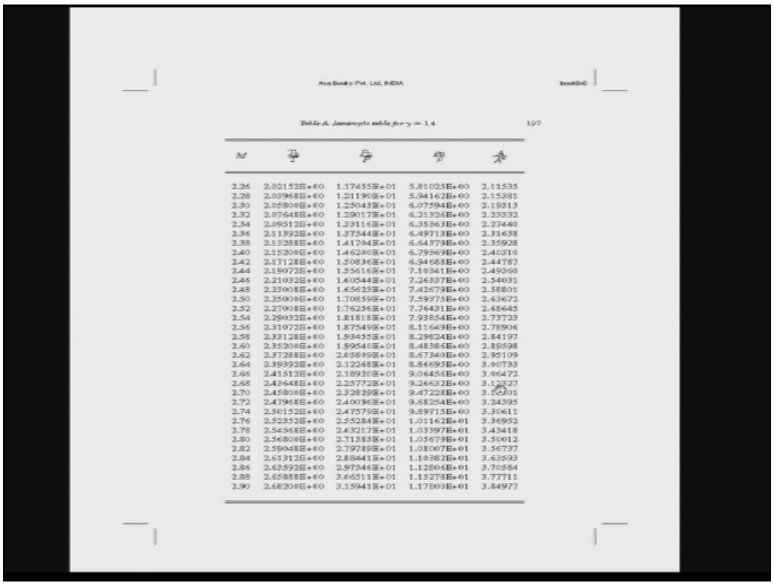

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Table A. Isentropic table for $\gamma = 1.4$ 197

M	$\frac{T_0}{T}$	$\frac{P_0}{P}$	$\frac{\rho_0}{\rho}$	$\frac{A}{A^*}$
2.26	2.02152E+00	1.17455E+01	5.81025E+00	2.11535
2.28	2.03968E+00	1.21190E+01	5.94162E+00	2.15381
2.30	2.05800E+00	1.25043E+01	6.07594E+00	2.19313
2.32	2.07648E+00	1.29017E+01	6.21326E+00	2.23332
2.34	2.09512E+00	1.33116E+01	6.35363E+00	2.27440
2.36	2.11392E+00	1.37344E+01	6.49713E+00	2.31638
2.38	2.13288E+00	1.41704E+01	6.64379E+00	2.35928
2.40	2.15200E+00	1.46200E+01	6.79369E+00	2.40310
2.42	2.17128E+00	1.50836E+01	6.94688E+00	2.44787
2.44	2.19072E+00	1.55616E+01	7.10341E+00	2.49360
2.46	2.21032E+00	1.60544E+01	7.26337E+00	2.54031
2.48	2.23008E+00	1.65623E+01	7.42679E+00	2.58801
2.50	2.25000E+00	1.70859E+01	7.59375E+00	2.63672
2.52	2.27008E+00	1.76256E+01	7.76431E+00	2.68645
2.54	2.29032E+00	1.81818E+01	7.93854E+00	2.73723
2.56	2.31072E+00	1.87549E+01	8.11649E+00	2.78906
2.58	2.33128E+00	1.93455E+01	8.29824E+00	2.84197
2.60	2.35200E+00	1.99540E+01	8.48386E+00	2.89598
2.62	2.37288E+00	2.05809E+01	8.67340E+00	2.95109
2.64	2.39392E+00	2.12268E+01	8.86695E+00	3.00733
2.66	2.41512E+00	2.18920E+01	9.06456E+00	3.06472
2.68	2.43648E+00	2.25772E+01	9.26632E+00	3.12327
2.70	2.45800E+00	2.32829E+01	9.47228E+00	3.18301
2.72	2.47968E+00	2.40096E+01	9.68254E+00	3.24395
2.74	2.50152E+00	2.47579E+01	9.89715E+00	3.30611
2.76	2.52352E+00	2.55284E+01	1.01162E+01	3.36952
2.78	2.54568E+00	2.63217E+01	1.03397E+01	3.43418
2.80	2.56800E+00	2.71383E+01	1.05679E+01	3.50012
2.82	2.59048E+00	2.79789E+01	1.08007E+01	3.56737
2.84	2.61312E+00	2.88441E+01	1.10382E+01	3.63593
2.86	2.63592E+00	2.97346E+01	1.12806E+01	3.70584
2.88	2.65888E+00	3.06511E+01	1.15278E+01	3.77711
2.90	2.68200E+00	3.15941E+01	1.17800E+01	3.84977

So if you remember these are the tables that we looked up earlier you can see from here. Isentropic table and we said that A/A star is given to be 3.5 and we wanted a supersonic solution so we can see that A/A star 3.5 comes, you can say that it comes somewhere here and this was a supersonic Mach number that we picked up earlier 2.8 and A/A star 3.5.

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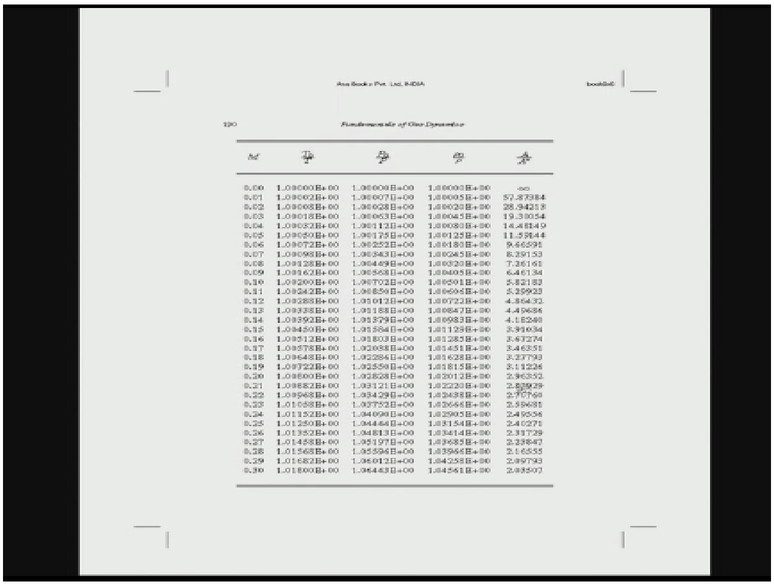

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M	$\frac{T_0}{T}$	$\frac{P_0}{P}$	$\frac{\rho_0}{\rho}$	$\frac{A}{A^*}$
0.00	1.00000E+00	1.00000E+00	1.00000E+00	∞
0.01	1.00002E+00	1.00007E+00	1.00005E+00	57.87384
0.02	1.00008E+00	1.00028E+00	1.00020E+00	28.94213
0.03	1.00018E+00	1.00063E+00	1.00045E+00	19.30054
0.04	1.00032E+00	1.00112E+00	1.00080E+00	14.48149
0.05	1.00050E+00	1.00175E+00	1.00125E+00	11.59144
0.06	1.00072E+00	1.00252E+00	1.00180E+00	9.66591
0.07	1.00098E+00	1.00343E+00	1.00245E+00	8.29153
0.08	1.00128E+00	1.00449E+00	1.00320E+00	7.26161
0.09	1.00162E+00	1.00568E+00	1.00405E+00	6.46134
0.10	1.00200E+00	1.00702E+00	1.00501E+00	5.82183
0.11	1.00242E+00	1.00850E+00	1.00606E+00	5.29923
0.12	1.00288E+00	1.01012E+00	1.00722E+00	4.86432
0.13	1.00338E+00	1.01188E+00	1.00847E+00	4.49686
0.14	1.00392E+00	1.01379E+00	1.00983E+00	4.18240
0.15	1.00450E+00	1.01584E+00	1.01129E+00	3.91034
0.16	1.00512E+00	1.01803E+00	1.01285E+00	3.67274
0.17	1.00578E+00	1.02038E+00	1.01451E+00	3.46351
0.18	1.00648E+00	1.02286E+00	1.01628E+00	3.27793
0.19	1.00722E+00	1.02550E+00	1.01815E+00	3.11226
0.20	1.00800E+00	1.02828E+00	1.02012E+00	2.96352
0.21	1.00882E+00	1.03121E+00	1.02220E+00	2.82929
0.22	1.00968E+00	1.03429E+00	1.02438E+00	2.70760
0.23	1.01058E+00	1.03752E+00	1.02666E+00	2.59681
0.24	1.01152E+00	1.04090E+00	1.02905E+00	2.49556
0.25	1.01250E+00	1.04444E+00	1.03154E+00	2.40271
0.26	1.01352E+00	1.04813E+00	1.03414E+00	2.31729
0.27	1.01458E+00	1.05197E+00	1.03685E+00	2.23847
0.28	1.01568E+00	1.05596E+00	1.03966E+00	2.16555
0.29	1.01682E+00	1.06012E+00	1.04258E+00	2.09793
0.30	1.01800E+00	1.06443E+00	1.04561E+00	2.03507

Now for the same A/A star we want to pick up the subsonic value right which means I do this so you can see that for A/A star of 3.5 corresponding to the subsonic portion, the Mach number comes somewhere here between 0.16 and 0.17 so what is the exit pressure corresponding to this. So we pick up the P0/P from here and then calculate the exit pressure corresponding to that okay.

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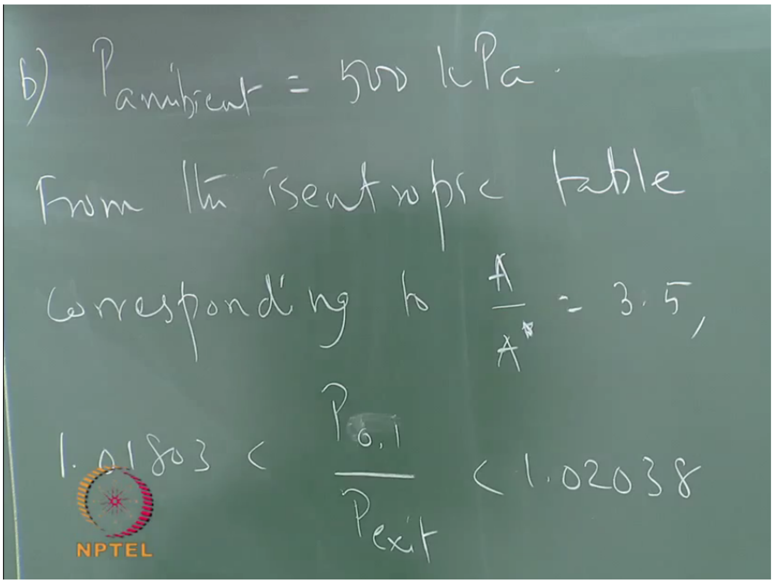

b) $P_{ambient} = 500$ kPa.

From the isentropic table

Corresponding to $\frac{A}{A^*} = 3.5$,

$$1.01803 < \frac{P_{0,1}}{P_{exit}} < 1.02038$$

When it falls in between 2 values so you can see that from the isentropic tables corresponding to A/A star=3.5 we see that the exit pressure lies in between 2 entries so basically it lies between these 2 things 1.01803<P0,1/P exit<1.02038. So where do these numbers come from?

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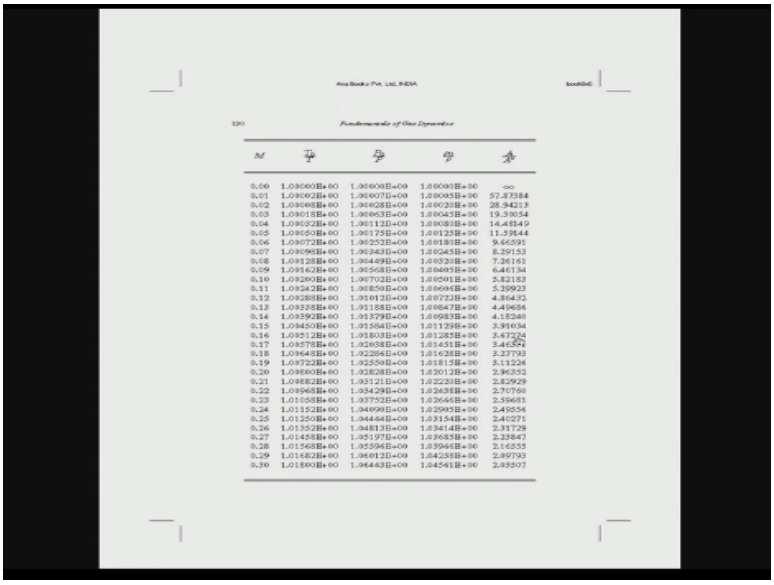

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M	$\frac{T_0}{T}$	$\frac{P_0}{P}$	$\frac{\rho_0}{\rho}$	$\frac{A}{A^*}$
0.00	1.00000E+00	1.00000E+00	1.00000E+00	∞
0.01	1.00002E+00	1.00007E+00	1.00005E+00	57.87384
0.02	1.00008E+00	1.00028E+00	1.00020E+00	28.94213
0.03	1.00018E+00	1.00063E+00	1.00045E+00	19.30054
0.04	1.00032E+00	1.00112E+00	1.00080E+00	14.48149
0.05	1.00050E+00	1.00175E+00	1.00125E+00	11.59144
0.06	1.00072E+00	1.00252E+00	1.00180E+00	9.66591
0.07	1.00098E+00	1.00343E+00	1.00245E+00	8.29153
0.08	1.00128E+00	1.00449E+00	1.00320E+00	7.26161
0.09	1.00162E+00	1.00568E+00	1.00405E+00	6.46134
0.10	1.00200E+00	1.00702E+00	1.00501E+00	5.82183
0.11	1.00242E+00	1.00850E+00	1.00606E+00	5.29923
0.12	1.00288E+00	1.01012E+00	1.00722E+00	4.86432
0.13	1.00338E+00	1.01188E+00	1.00847E+00	4.49686
0.14	1.00392E+00	1.01379E+00	1.00983E+00	4.18240
0.15	1.00450E+00	1.01584E+00	1.01129E+00	3.91034
0.16	1.00512E+00	1.01803E+00	1.01285E+00	3.67274
0.17	1.00578E+00	1.02038E+00	1.01451E+00	3.465[illegible]
0.18	1.00648E+00	1.02286E+00	1.01628E+00	3.27793
0.19	1.00722E+00	1.02550E+00	1.01815E+00	3.11226
0.20	1.00800E+00	1.02828E+00	1.02012E+00	2.96352
0.21	1.00882E+00	1.03121E+00	1.02220E+00	2.82929
0.22	1.00968E+00	1.03429E+00	1.02438E+00	2.70760
0.23	1.01058E+00	1.03752E+00	1.02666E+00	2.59681
0.24	1.01152E+00	1.04090E+00	1.02905E+00	2.49556
0.25	1.01250E+00	1.04444E+00	1.03154E+00	2.40271
0.26	1.01352E+00	1.04813E+00	1.03414E+00	2.31729
0.27	1.01458E+00	1.05197E+00	1.03685E+00	2.23847
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0.29	1.01682E+00	1.06012E+00	1.04258E+00	2.09793
0.30	1.01800E+00	1.06443E+00	1.04561E+00	2.03507

Again if you go to the table you can see that so for 3.5 falls between these 2 right so you can see 1.01803 and 1.0238 so these are the 2 values that we are talking about, so it falls between these 2 values and if you convert this to pressure then this actually says that the exit pressure lies somewhere between 980 kilopascal and 982.29 kilopascal. So the given value is 500 kilopascal correct.

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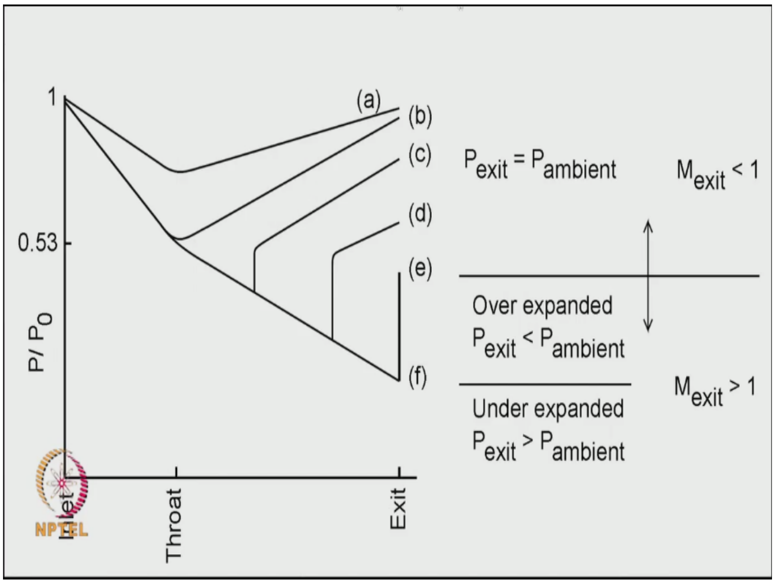

So if you revert back to the other figure you can see that so you can see that this pressure the pressure corresponding to b is now about 982 kilopascal and this was about 36 kilopascal corresponding to f, the given value of 500 kilopascal lies between b and f so that means there is a normal shock somewhere at the divergent portion of the nozzle okay and so that is what we have to calculate next.

So normal shock stands somewhere in the divergent part of the nozzle. So what we need to do now is figure out where the normal shock is going to sit okay. So as you can see from here, the normal shock can be anywhere in the divergent part of the nozzle. So for the given value we need to figure out where the normal shock sits in the divergent portion. This can be done in 2 different ways, one is an iterative method, which maybe slightly quicker.

Other one is slightly more involved, but it will give you an answer in 1 shot okay. We will do this in the next class.